Pion photoproduction in the $\Delta(1232)$ region and chiral bag models

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(Received 16 June 1982; revised manuscript received 16 December 1982)

We calculate pion photoproduction near the $\Delta(1232)$ resonance using a version of the chiral bag model based on an effective Lagrangian that reproduces correct pion-nucleon phenomenology through third order in the pion field. The pionic corrections to the $M1$ (or $M1+$) amplitude are common to a broad class of chiral bag models and substantially improve agreement with experiment. We treat also the $E2$ (or $E1+$) multipole which is accessible in our approach since the effective Lagrangian induces quark-quark tensor coupling through pion exchange; this in turn permits the admixture of baryon components involving quark excitation to d states. The resulting $E2$ admixture is consistent with the rather loose bounds set by the present experimental information on this quantity. The present work neglects the tensor force and d state arising from gluon exchange between quarks.

I. INTRODUCTION

The MIT bag model[1,2] has been quite successful in reproducing the static properties of hadrons. As is well known, however, it breaks chiral symmetry and generally gives a poor accounting of pionic properties. This has motivated the consideration of chiral bag models,[3–7] in which the pion is incorporated as a separate, local field. In particular, the cloudy bag model (CBM)[5] has yielded a good description of many features of pion-baryon interactions, and, especially, of the 3,3 πN resonance.

In a previous study[8] we found that a modified version of the CBM allows for straightforward inclusion of the phenomenology of third-order pion-field effects, as are required, for instance, in the $\pi N \rightarrow \pi\pi N$ reaction. This phenomenology does not emerge simply in the conventional CBM, and the effective Lagrangian that we construct[8] in order to produce it then implies somewhat different quark-pion dynamics. As in the conventional CBM, the pion field penetrates the bag interior in our approach, but, unlike the usual version, our Lagrangian induces pion-quark interactions in the interior and not exclusively on the bag surface. (It has on occasion been suggested[6] that the penetration of the bag by the pion field in the CBM may be viewed as a crude representation of quark-antiquark excitations in the bag.) Our Lagrangian derives from other CBM Lagrangians through a chiral transformation in a fashion similar to the generation of the chiral Lagrangian of Weinberg[9] from the σ model of Gell-Mann and Levy[10]; similar approaches, valid through second order in the pion field, have been considered by Thomas.[5] (For the sake of completeness the essential features are presented briefly in Sec. II.)

As a consequence of the interaction of pions and quarks inside the bag in the present model, we find as one of its striking features a pion-mediated quark-quark force—the direct analog of the NN tensor force as generated by one-pion exchange which admixes the deuteron d state. This force produces d-state components in baryon states. Such admixtures have been discussed by several authors[11,12] in connection with corrections to the static properties of the hadron. Vento *et al.*[11] found that a non-negligible d-state admixture for the nucleon was desirable in order to improve the value of the axial-vector weak-coupling constant g_A. Hulthage *et al.*[12] used this d state to correct the values of the hadron masses and of g_A. (It has been suggested[13] that the inclusion of quark states with $l>1$ may lead to the divergence of the quark self-energy because of the sharp bag boundary. Since we are here restricting ourselves to a phenomenological treatment of the lowest excitations only we do not address this problem.)

Our main purpose in the present work is to study within the framework of our model the influence of quark d-state admixtures in baryons on photopion production in the 3,3 resonance region. We shall confront our results with present information on $E2$ contributions in that region which are expected to reflect such admixtures directly. At the same time we shall require information on the dominant $M1$ multipole in the $\Delta(1232)$ range, where it emerges (Sec. III) that the inclusion of pionic effects leads to an improvement for the agreement of the amplitude with experiment; this $M1$ result is not specific to the present version of the CBM, but has not been reported previously. In the present calculations we encounter the well-known problem of the proper treatment of the center-of-mass (c.m.) motion of the bag, reviewed by Jaffe[4] and by Thomas.[5] This difficulty arises because there as yet exists no complete formulation of the chiral bag model possessing translational invariance. Nevertheless, it is to be expected that the spin flip involved in the $M1$ $N\rightarrow\Delta$ transition and the $l=2$ orbital excitation entering in the $E2$ multipole are not heavily influenced by spurious c.m. motion. This is because these excitations have small overlap with states generated by boosting the baryon as a whole. These expectations appear to be borne out by the success of the static treatment of the Δ in πN scattering.[5]

The experimental situation[14–18] on $E2$ components in the $\Delta(1232)$ region is, unfortunately, not very clear. The analyses of the experimental data tend to use either a formalism based on fixed-t dispersion relations[19] or isobar-model fitting.[16,17] In the former the πN scattering phase shifts are used in order to calculate the resonant and nonresonant amplitudes, while in the latter the resonance parameters are exploited together with Born terms and background fits. From these analyses it is difficult to extract unambiguously a value for the $E2$ amplitude at resonance. We shall compare our results with the relatively recent study of Metcalf and Walker[17] which seems to provide a very good fit to the data.

On the theoretical side, Chew, Goldberger, Low, and Nambu (CGLN)[20] pioneered the approach to this problem based on dispersion relations. Their evaluation indicated a vanishing of the $E2$ amplitude at resonance. Other subsequent applications of this technique[21–24] yielded differing results. We shall here calculate the $E2$ amplitude by coupling quark s states to quark d states through the tensor force arising from the pion-quark interaction [Figs. 1(a)—1(c)], and adding the contribution of the seagull terms Fig. 1(d), as well as through the terms in Figs. 2(a)—2(c) treated through dispersion relations by CGLN; for these latter our results are the same as theirs except for the appearance of the particular form factors dictated by the CBM.

II. GENERAL FEATURES OF THE MODEL

We start from the chirally invariant Lagrangian[3]

$$\mathscr{L}=\sum_a\left[\frac{i}{2}\bar{q}_a\overleftrightarrow{\partial\!\!\!/}q_a-B\right]\theta_V-\frac{1}{2f}\sum_a\bar{q}_a\left(\sigma+\frac{i}{f}\underline{\tau}\cdot\underline{\pi}\gamma_5\right)q_a\Delta_S+\tfrac{1}{2}(\partial_\mu\underline{\pi}\cdot\partial^\mu\underline{\pi}+\partial^\mu\sigma\partial_\mu\sigma)-\tfrac{1}{2}m^2\underline{\pi}^2\,, \tag{1}$$

where q_a is the quark field of color a, $\underline{\pi}$ is the pionic field, Δ_S is a surface δ function for bag surface, S, θ_V a step function for the bag interior with volume V, and m is the pion mass. The σ field is chosen as in the nonlinear model[10]

$$\sigma=(f^2-\underline{\pi}^2)^{1/2}\,. \tag{2}$$

The Lagrangian of Eq. (1) fulfills PCAC (partial conservation of axial-vector current) in the limit of vanishing pion mass.

We now transform the Lagrangian by the chiral transformations

$$q_a\rightarrow Sq_a=\frac{1}{[2f(f+\sigma)]^{1/2}}(f+\sigma+i\underline{\tau}\cdot\underline{\pi}\gamma_5)q_a\,, \tag{3a}$$

$$\underline{\pi}\rightarrow V(\underline{\pi})=\frac{2f}{f+\sigma}\underline{\pi}\,, \tag{3b}$$

obtaining the effective Lagrangian of the present model,

$$\mathscr{L}=\sum_a\left[\frac{i}{2}\bar{q}_a\overleftrightarrow{\partial\!\!\!/}q_a-B\right]\theta_V-\tfrac{1}{2}\sum_a\bar{q}_aq_a\Delta_S+\sum_a\bar{q}_a\gamma^\mu\frac{1}{1+g^2\underline{\pi}^2}[g\gamma_5\underline{\tau}\cdot\partial_\mu\underline{\pi}-g^2\underline{\tau}\cdot(\underline{\pi}\times\partial_\mu\underline{\pi})]q_a\theta_V$$
$$+\frac{1}{2}\frac{1}{(1+g^2\underline{\pi}^2)^2}\partial^\mu\underline{\pi}\cdot\partial_\mu\underline{\pi}-\frac{1}{2}\frac{1}{1+g^2\underline{\pi}^2}m^2\underline{\pi}^2\,, \tag{4}$$

where $g=1/2f$. The fields obey

$$i\not{n}q_a(x)=q_a(x)\quad\text{for } x\in S\,, \tag{5a}$$

$$i\partial\!\!\!/ q_a(x) = -\gamma^\mu(1+g^2\underline{\pi}^2)^{-1}[g\gamma_5\underline{\tau}\cdot\partial_\mu\underline{\pi} - g^2\underline{\tau}\cdot\underline{\pi}\times\partial_\mu\underline{\pi}]q_a(x) \text{ for } x\in V\,, \tag{5b}$$

$$(\partial^2+m^2)\underline{\pi} = -ig\sum_a \bar{q}_a\gamma_5\underline{\tau}q_a\Delta_S + O(\underline{\pi}^3)\,, \tag{5c}$$

where n_μ is the outward normal to the bag surface. The Lagrangian of Eq. (4) possesses chiral symmetry under the transformations

$$q_a \to (1-ig\underline{\tau}\cdot\underline{\pi}\times\underline{\epsilon})q_a\,, \tag{6a}$$

$$\underline{\pi}\to\underline{\pi}-2\underline{\pi}\cdot\underline{\epsilon}\underline{\pi}-\frac{\underline{\epsilon}}{g}(1-g^2\underline{\pi}^2)\,, \tag{6b}$$

with an axial-vector current

$$\begin{aligned}\underline{J}^A_\mu &= (1+g^2\underline{\pi}^2)^{-1}\sum_a \bar{q}_a[2g\underline{\tau}\times\underline{\pi}\gamma_5+2g^2\underline{\tau}\cdot\underline{\pi}\underline{\pi}+\underline{\tau}(1-g^2\underline{\pi}^2)]\gamma_5\gamma_\mu q_a\theta_V\\ &\quad -(1+g^2\underline{\pi}^2)^{-2}\left[2g\underline{\pi}\cdot(\partial_\mu\underline{\pi})\underline{\pi}+\frac{1}{g}(1-g^2\underline{\pi}^2)\partial_\mu\underline{\pi}\right]\,.\end{aligned} \tag{7}$$

Earlier we found[8] that the terms in Eq. (4) to third and fourth order in the pionic field agree with the phenomenology of the $\pi N\to\pi\pi N$ reaction, while the same terms in the conventional CBM misrepresent the experimental situation rather badly. The terms linear in $\underline{\pi}$ will be used in the calculation of the photoproduction amplitudes (Sec. III).

In order to generate the necessary photon couplings we proceed by introducing the minimal substitution

$$\partial_\mu\to\partial_\mu - ieA_\mu\,, \tag{8}$$

where A_μ is the photon field. We obtain

$$\mathscr{L}_{q\gamma} = \sum e_q\bar{q}_a\vec{\gamma}\cdot\vec{A}q_a\,, \tag{9}$$

$$\mathscr{L}_{\pi\gamma} = -ie(\pi^*\vec{\nabla}\pi - \pi\vec{\nabla}\pi^*)\cdot\vec{A}\,, \tag{10}$$

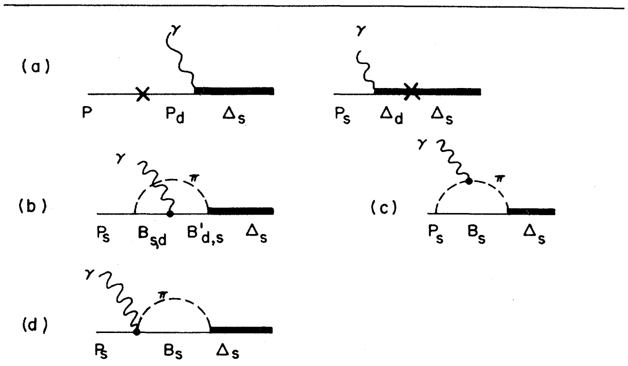

FIG. 1. Diagrams used in the calculation of the $E2$ amplitude. B_S represents a nucleon or Δ isobar with all quarks in $l=0$ states, and B_d represents such a baryon with a quark in a d orbital. The × insertion represents the effects of the (nonstatic) quark-quark tensor force as mediated by pion exchange.

$$\mathscr{L}_{\pi q\gamma} = ieg\sum \bar{q}_a\vec{\gamma}\cdot\vec{A}\gamma_5(\tau^-\pi-\tau^+\pi^*)q_a\,, \tag{11}$$

where $\pi=2^{-1/2}(\pi_x+i\pi_y)$ is the field creating a π^+ particle or destroying a π^-. Using the solutions of the static cavity[1,2] we obtain a form factor for the "catastrophic term," Eq. (11), of the form (between s states)

$$f(r) = \left[j_0{}^2\left[\frac{\omega_0 r}{R}\right]-\tfrac{1}{5}j_1{}^2\left[\frac{\omega_0 r}{R}\right]\right]\,, \tag{12}$$

where[1,2] $\omega_0=2.04$ is the lowest eigenmode in the MIT bag of radius R.

III. THE PHOTOPRODUCTION AMPLITUDES

A. The $M1$ amplitude

Photoproduction in the Δ(1232) resonance region proceeds mainly through the $M1$ transition.[14–24] The decomposition of the transition amplitude in terms of helicity amplitudes is well known[25] and yields

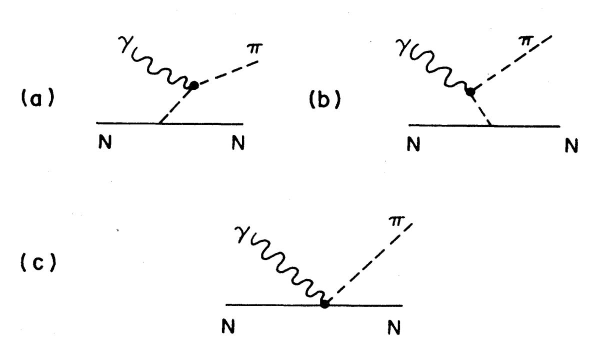

FIG. 2. Born terms for photoproduction of $M1$ and $E2$ amplitudes (notation as in Fig. 1).

$$A_1=\frac{1}{\sqrt{2q}}\langle\Delta(0,\tfrac{1}{2})|\vec{\epsilon}\cdot\vec{j}\,|N(\vec{q}\,',-\tfrac{1}{2})\rangle , \quad (13a)$$

$$A_3=\frac{1}{\sqrt{2q}}\langle\Delta(0,\tfrac{3}{2})|\vec{\epsilon}\cdot\vec{j}\,|N(\vec{q}\,',\tfrac{1}{2})\rangle , \quad (13b)$$

where $\vec{q}$ is the photon momentum, $\Delta(0,\mu)$ represents a Δ isobar of zero momentum, spin projection μ, and $N(\vec{q}\,',\nu)$ denotes a nucleon of momentum $\vec{q}\,'$ and spin projection ν. The calculation of these amplitudes in the MIT bag model[25] yields values for A_1 and A_3 that are 30% less than the experimental values.[26]

These amplitudes are here calculated using the diagrams of Fig. 3. In all cases we construct baryon (N,Δ) states within the nonrelativistic quark model as was done by the MIT group[16] and for the CBM,[5] and the calculation proceeds in the nonrelativistic approximation. From the diagrams of Fig. 3, with the neglect of recoil effects and the use of the standard multipole expansion of the electromagnetic field, we arrive at an expression for the $M1$ amplitudes of Eq. (13),

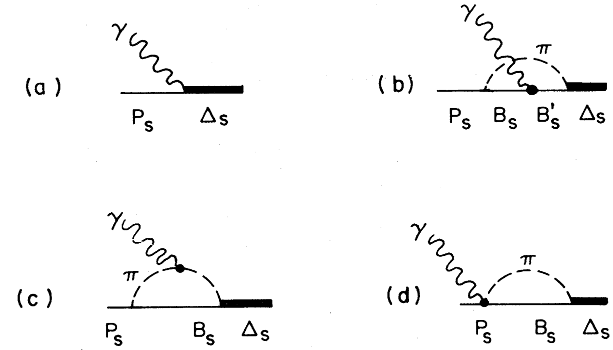

FIG. 3. Diagrams entering in the calculation of the $M1$ amplitude (notation in Fig. 1).

$$A_1(M1)\cong\frac{1}{\sqrt{3}}A_3(M1)=-\left[\frac{\sqrt{2}}{3}+\frac{\sqrt{2}g^2N_0^{\,4}}{324\pi^2}(125J_1+76J_2+8J_3+100J_4)\right]\langle M1\rangle-\frac{5}{9}\frac{g^2N_0^{\,4}}{\pi^{7/2}\sqrt{\omega_\gamma}}J_5 , \quad (14)$$

where the first, leading term arises from Fig. 3(a) and the square-bracket term stems from Fig. 3(b). In the latter,

$$J_i=\int G_i^{-1}(k)a^2(k)\omega_k^{-1}k^4dk,\quad i=1,2,3,4 \quad (15a)$$

and

$$G_1=\omega_k(\omega_k+\delta-\omega_\gamma) , \quad G_2=(\omega_k+\delta)(\omega_k+\delta-\omega_\gamma) ,$$
$$G_3=(\omega_k+\delta)(\omega_k-\omega_\gamma),\quad G_4=\omega_k(\omega_k-\omega_\gamma), \quad (15b)$$

while

$$a(k)=\int_0^R r^2dr\left\{j_0(kr)\left[j_0^{\,2}\left[\frac{\omega_0 r}{R}\right]-\tfrac{1}{3}j_1^{\,2}\left[\frac{\omega_0 r}{R}\right]\right]-\tfrac{4}{3}j_2(kr)j_1^{\,2}\left[\frac{\omega_0 r}{R}\right]\right\} , \quad (16)$$

where $\omega_k=(m^2+k^2)^{1/2}$, ω_γ is the photon laboratory energy, and δ is the $\Delta(1232)$-proton mass difference. In Eq. (14) the normalization factor is

$$N_0=\left[\frac{\omega_0}{2(\omega_0-1)R^3}\right]^{1/2}\frac{1}{j_0(\omega_0)} \quad (17)$$

and

$$\langle M1\rangle=\frac{2N_0^{\,2}}{\sqrt{2\pi\omega_\gamma}}\int_0^R j_1\left[\omega_\gamma r\right]j_1\left[\frac{\omega_0 r}{R}\right]j_0\left[\frac{\omega_0 r}{R}\right]r^2dr . \quad (18)$$

The photon-pion interaction, Fig. 3(c), yields the last term in Eq. (14), in which

$$J_5=\int_0^\infty\frac{a(k)a(k')k^3dk\,k'^3dk'}{\omega_k\omega_{k'}[(\omega_k+\omega_{k'})^2-\delta^2}\left[\frac{\delta}{\omega_k}-5\frac{\delta}{\omega_{k'}}\right]\int_0^\infty r\,dr\,j_1(\omega_\gamma r)j_1(k'r)j_1(kr) . \quad (19)$$

TABLE I. $M1$ transition amplitudes.

Helicity amplitude	$A_1(10^{-3}\text{GeV}^{-1/2})$	$A_3(10^{-3}\text{GeV}^{-1/2})$
MIT bag model (Ref. 25)	-102	-176
Chiral bag model (unmodified ω_0)		
$R=1$ fm	-139	-263
$R=0.9$ fm	-145	-275
$R=0.8$ fm	-155	-294
Chiral bag model (modified ω_0)		
$R=1$ fm	-130	-250
$R=0.9$ fm	-133	-252
$R=0.8$ fm	-141	-254
Experiment (Ref. 26)	-141 ± 7	-259 ± 10

From Eq. (14) we then obtain the $M1$ amplitudes at resonance through numerical integration.

Table I displays the $M1$ amplitudes compared to the experimental values[26] and the MIT-bag predictions.[25] It is seen that the pion corrections of the CBM nearly fill the gap between experiment and the MIT bag values. Changing the bag radius from $R=1$ fm to $R=0.9$ fm yields an overestimate of the amplitude since the pionic effects increase with diminishing bag radius. Nevertheless when the bag radius is less than 1 fm one has to correct the bag eigenfrequencies[8] by a non-negligible amount which compensates in part for the pionic effect. Using the appropriately modified values[8] of ω_0 we arrive at a result that agrees very well with the experimental results when a bag radius of 0.9 fm is used, as can be seen in row 6 of Table I. A large hadron radius $R>1.2$ fm will cause the suppression of all pionic effects while, on the other hand, if small values $R<0.7$ are preferred, higher-order pionic terms must be taken into account. The results of Table I show about 20% contribution of the pionic effects to the $M1$ amplitude in the $\gamma N\to\pi N$ process (in rough agreement with the pionic effects in the results of the CBM (Ref. 5) for the $\pi N\to\pi N$ process at resonance).

In sum, we see that the $M1$ results are consistent with a bag radius in the often-used[1-6] range $0.8\le R\le 1.2$ fm.

B. The $E2/M1$ ratio

In order to study the small component of the $E2$ multipole in the $\Delta(1232)$ region, we first consider the possible quark d orbital excitations that can admix with a nucleon or a Δ-isobar state. Of the two available d states, $d_{3/2}$ and $d_{5/2}$, we consider mainly the $d_{3/2}$ state which is higher in energy by some 150 MeV but was here still found to contribute an amplitude larger by nearly an order of magnitude than that for the $d_{5/2}$ state. From Eqs. (5a) and (5b) we obtain

$$q_{3/2,m}(\vec{r},t)=N_1\begin{bmatrix} j_2\left[\dfrac{\omega_1 r}{R}\right] \\ -i\vec{\sigma}\cdot\hat{\vec{r}}\,j_1\left[\dfrac{\omega_1 r}{R}\right]\end{bmatrix}\times u_m\chi e^{-i\omega_1 t}\,, \tag{20}$$

where N_1 is the normalization factor

$$N_1=\left[\frac{\omega_1}{2(\omega_1-2)j_1{}^2(\omega_1)}\right]^{1/2} \tag{21}$$

and

$$u_m=\frac{1}{\sqrt{5}}\begin{bmatrix}(\frac{5}{2}-m)^{1/2}Y_2^{m-1/2}(\hat{\vec{r}}) \\ -(\frac{5}{2}+m)^{1/2}Y_2^{m+1/2}(\hat{\vec{r}})\end{bmatrix} \tag{22}$$

and χ is an isospinor. The eigenfrequency ω_1 is determined by Eq. (5b) through the boundary condition $j_2(\omega_1)=-j_1(\omega_1)$, which yields for the lowest such mode the value $\omega=5.12$ The Appendix displays all the N and Δ states containing a quark in a $d_{3/2}$ orbital. Configurations such as $(p_{1/2})^2$, $p_{1/2}p_{3/2}$, $(p_{3/2})^2$ do not contribute directly to the lowest-order photoabsorption and were thus omitted. We also neglected the mass shifts due to the different color-spin interactions between quarks when a d-state quark is present so that $\Delta E\equiv(\omega_1-\omega_0)/R=648$ MeV for $R=1$ fm.

The $E2$ amplitudes for the transitions of Eqs. (13) are obtained from the diagrams of Fig. 1 as

$$\begin{aligned}A_1(E2)\cong\frac{1}{\sqrt{3}}A_3(E2)=&-\frac{N_0{}^3N_1g^2}{18\sqrt{5}\pi^2}\frac{\langle E2\rangle}{\sqrt{2\omega_\gamma}}\frac{1}{\omega_1-\omega_0}[25I_1-10I_2-\sqrt{5}(8I_3-5I_4)+\tfrac{25}{3}I_5+\tfrac{80}{3}I_6\\&+\tfrac{10}{3}I_7-\tfrac{25}{3}I_8+\tfrac{20}{3}I_9+\tfrac{10}{3}I_{10}]\\&-\frac{g^2N_0{}^4}{36\pi^{5/2}\sqrt{\omega_\gamma}}(I_{11}+I_{12}-I_{13})\,,\end{aligned} \tag{23}$$

where the integrals are

$$I_i=\int_0^{\infty} F_i^{-1}(k)a(k)b(k)k^3dk,\quad i=1,\ldots,13 \tag{24}$$

and the functions F are shown in Table II in terms of the auxiliary integrals

$$b(k)=\int_0^R r^2dr\left\{j_2(kr)\left[j_0\left[\frac{\omega_0 r}{R}\right]j_2\left[\frac{\omega_1 r}{R}\right]-\tfrac{1}{3}j_1\left[\frac{\omega_0 r}{R}\right]j_1\left[\frac{\omega_1 r}{R}\right]+\tfrac{2}{3}j_0(kr)j_1\left[\frac{\omega_0 r}{R}\right]j_1\left[\frac{\omega_1 r}{R}\right]\right]\right.$$
$$\left.+j_1(kr)\left[j_0\left[\frac{\omega_0 r}{R}\right]j_1\left[\frac{\omega_1 r}{R}\right]+j_1\left[\frac{\omega_0 r}{R}\right]j_2\left[\frac{\omega_1 r}{R}\right]\right]\right\} \tag{25a}$$

and

$$c(k)=\int_0^R j_1(\omega_\gamma r)j_1(kr)\left[j_0{}^2\left[\frac{\omega_0 r}{R}\right]-\tfrac{1}{5}j_1{}^2\left[\frac{\omega_0 r}{R}\right]\right]r^2dr\ , \tag{25b}$$

$$\langle E2\rangle=-\frac{N_0N_1}{10\sqrt{10\pi}}\int_0^R j_1(\omega_\gamma r)\left[j_2\left[\frac{\omega_1 r}{R}\right]j_1\left[\frac{\omega_0 r}{R}\right]-5j_1\left[\frac{\omega_1 r}{R}\right]j_0\left[\frac{\omega_0 r}{R}\right]\right]r^2dr\ . \tag{26}$$

After numerical evaluation of these integrals, we arrive at the result

$$\frac{E2}{M1}=-0.0092 \tag{27}$$

for $R=1$ fm. Reduction to $R=0.9$ fm yields a change of $<5\%$ in Eq. (27). Experimentally[14–18] the value of this ratio ranges from[18] 0 to[14] -0.05; the former value is obtained by dispersion-relation analysis, and the latter by a fit to the amplitudes using the πN scattering phase shifts and the Watson theorem. The analysis based on the isobar model yields a value of -0.012 ± 0.013; this very large range of uncertainty also encompasses almost all the other results (a mean value using all the analyses would give ≈ -0.02). The fits to the cross sections and the resonant multipole amplitudes obtained by Metcalf and Walker[17] are rather better than the dispersion-relation approaches,[18] and we thus view our result of Eq. (27) as being in accord with the available information.

TABLE II. Functions in the integrands of Eq. (24).

i	F_i
1	$\omega_k-\omega_\gamma+\delta$
2	$\omega_k-\omega_\gamma$
3	$\omega_k+\delta$
4	ω_k
5	$\omega_k+\delta+\Delta E-\omega_\gamma$
6	$k^{-1}(\omega_k+\delta)(\omega_k+\delta+\Delta E-\omega_\gamma)$
7	$k^{-1}(\omega_k+\delta)(\omega_k+\Delta E-\omega_\gamma)$
8	$k^{-1}(\Delta E+\omega_k)(\omega_k+\delta-\omega_\gamma)$
9	$k^{-1}(\omega_k+\delta+\Delta E)(\omega_k+\delta-\omega_\gamma)$
10	$k^{-1}(\delta+\Delta E+\omega_k)(\omega_k-\omega_\gamma)$
11	$\delta^{-1}(k+\delta)a(k)k/c(k)$
12	$(\omega_k-\omega_\gamma)a(k)/c(k)$
13	$(\omega_k+\delta-\omega_\gamma)a(k)/c(k)$

In a previous study Vento *et al.*[11] found that a better fit for g_A is obtained within their model when a significant d-state admixture in the ground state of the hadron ($\approx 45\%$ for N, $\approx 60\%$ for Δ) is included. Using their values we would obtain an $|E2/M1|$ ratio of $\gtrsim 2\%$ which is somewhat larger than our result. Nevertheless, the contribution of the d-state admixture of the $\Delta(1232)$ to the $|E2/M1|$ ratio obtained from Fig. 1(a)—to which the considerations of Ref. 11 are restricted—is only $\lesssim 0.2$ of the value of Eq. (27), i.e., about 0.17% which is much smaller than the 2% obtainable through the approach of Vento *et al.*[11] Of course this implies that our specific radial matrix elements do not allow for a sufficiently large correction to g_A through the mechanism of d-state hadronic admixture (here 1.5% for N, 2% for Δ). On the other hand, the very large admixtures of Ref. 11 lead to an $M1$ amplitude which is reduced by 30% from our CBM value.

The value for the $E2/M1$ ratio that we have obtained in Eq. (27) is almost energy independent and thus should be taken as relevant through the $\Delta(1232)$ range, say 300 to 400 MeV photon laboratory energy. A major part of the energy dependence of the $E2/M1$ ratio stems from the diagrams of Fig. 2 as was seen in the CGLN (Ref. 20) calculation. The fact that these diagrams become important at photon energies less than 200 MeV is due to their dependence on $\cos\delta_{33}$ which causes them to vanish at resonance in the treatment of CGLN. At the same energy the main contribution to the $M1$ multipole (the magnetic-spin excitation) diminishes significantly for $\omega_\gamma<200$ MeV and the ratio can reach -0.2. Our result is to be seen as a net contribution to the ratio to be added to the well-known amplitude of CGLN. The energy dependence which then results for the $E2/M1$ ratio is in reasonable agreement with experiment.[17]

We conclude that the pionic effects contained in the cloudy-bag approach provide a consistent and reasonable picture for the $M1$ amplitude of $\gamma N \rightarrow \pi N$ in the $\Delta(1232)$ region; this complements the results there for elastic πN scattering. In the particular model based on the effective Lagrangian of Eq. (4) we obtain a quark-quark tensor force mediated by pion exchange which yields a d-state admixture for the nucleon and Δ isobar. This admixture in turn leads to an $E2$ multipole component for $\gamma N \rightarrow \pi N$ that is consistent with the experimental data, which are unfortunately still somewhat ambiguous as regards the $E2$ amplitude. In spite of the smallness of the $E2/M1$ ratio at resonance we feel that it gives a direct and valuable check on the predictions of models with coupling to $l=2$ excited states such as the present approach.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge useful discussions in this subject with Professors F. Cannata and S. Nussinov. This work was supported in part by the U.S.-Israel Binational Science Foundation and by the Israel Academy of Sciences and Humanities—Basic Research Foundation.

APPENDIX: BARYON WAVE FUNCTIONS

We here display nucleon and Δ-isobar $l=2$ wave functions for one quark in the $d_{3/2}$ orbital. We have omitted the color degree of freedom and deal, of course, with u and d quarks only. We give the expressions for the quark content of the wave functions for positive projections of spin of the baryon; expressions for negative spin projections are obtained from

$$N_d^{-m}(\uparrow\downarrow,(\tfrac{3}{2},\nu))=N_d^{m}(\downarrow\uparrow,(\tfrac{3}{2},-\nu)) , \qquad \text{(A1)}$$

and

$$\Delta_d^{-m}(\uparrow\downarrow,(\tfrac{3}{2},\nu))=-\Delta_d^{m}(\downarrow\uparrow,(\tfrac{3}{2}-\nu)) . \qquad \text{(A2)}$$

The negative isospin projections are obtained from the positive ones by merely changing $u \rightarrow d$, $d \rightarrow u$ and reversing the overall sign of the wave function in the nucleon case.

For nucleons we have ($\underline{P}$ denotes permutations)

$$p_{1/2}=\frac{1}{\sqrt{3}}\Big[\tfrac{1}{3}(u_\uparrow u_\uparrow d_{(3/2)(-1/2)}+p)-\tfrac{1}{6}(u_\uparrow d_\uparrow u_{(3/2)(-1/2)}+\underline{P})+\tfrac{1}{6}(u_\downarrow d_\uparrow u_{(3/2)(1/2)}+\underline{P})+\tfrac{1}{6}(u_\uparrow d_\downarrow u_{(3/2)(1/2)}+\underline{P})$$
$$-\tfrac{1}{3}(u_\uparrow u_\downarrow d_{(3/2)(1/2)}+\underline{P})+\frac{1}{\sqrt{3}}(u_\downarrow u_\downarrow d_{(3/2)(3/2)}+\underline{P})-\frac{1}{2\sqrt{3}}(u_\downarrow d_\downarrow u_{(3/2)(3/2)}+\underline{P})\Big] ,$$

while for Δ isobars,

$$\Delta_{3/2}^{++}=\frac{1}{\sqrt{3}}[(\tfrac{2}{5})^{1/2}(u_\uparrow u_\uparrow u_{(3/2)(1/2)}+\underline{P})-(\tfrac{3}{10})^{1/2}(u_\uparrow u_\downarrow u_{(3/2)(3/2)}+\underline{P})] ,$$

$$\Delta_{1/2}^{++}=\frac{1}{\sqrt{3}}\left[2(\tfrac{2}{15})^{1/2}(u_\uparrow d_\uparrow u_{(3/2)(-1/2)}+\underline{P})+\frac{1}{\sqrt{30}}(u_\uparrow u_\downarrow u_{(3/2)(1/2)}+\underline{P})-(\tfrac{2}{5})^{1/2}(u_\downarrow u_\downarrow u_{(3/2)(3/2)}+\underline{P})\right] ,$$

$$\Delta_{3/2}^{+}=\frac{1}{\sqrt{3}}\Big[(\tfrac{2}{15})^{1/2}(u_\uparrow u_\uparrow d_{(3/2)(1/2)}+\underline{P})+\left[\tfrac{2}{15}\right]^{1/2}(u_\uparrow u_{(3/2)(1/2)}d_\uparrow+\underline{P})$$
$$-\frac{1}{\sqrt{10}}(u_\downarrow u_\uparrow d_{(3/2)(3/2)}+\underline{P})-\frac{1}{\sqrt{10}}(u_{(3/2)(3/2)}u_\uparrow d_\downarrow+\underline{P})-\frac{1}{\sqrt{10}}(u_{(3/2)(3/2)}d_\uparrow u_\downarrow+\underline{P})\Big] ,$$

$$\Delta_{1/2}^{+}=\frac{1}{3}\Big[2(\tfrac{2}{15})^{1/2}(u_\uparrow u_\uparrow d_{(3/2)(-1/2)}+\underline{P})+2(\tfrac{2}{15})^{1/2}(u_\uparrow d_\uparrow u_{(3/2)(-1/2)}+\underline{P})$$
$$-\frac{1}{\sqrt{30}}(u_\uparrow d_\downarrow u_{(3/2)(1/2)}+\underline{P})-\frac{1}{\sqrt{30}}(u_\uparrow u_\downarrow d_{(3/2)(1/2)}+\underline{P})$$
$$-\frac{1}{\sqrt{30}}(d_\uparrow u_\downarrow u_{(3/2)(1/2)}+\underline{P})-(\tfrac{2}{5})^{1/2}(u_\downarrow u_\downarrow d_{(3/2)(3/2)}+\underline{P})-(\tfrac{2}{5})^{1/2}(u_\downarrow d_\downarrow u_{(3/2)(3/2)}+\underline{P})\Big] .$$

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