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D^+-D^0 mass difference

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The hadronic component of SU(2) mass splitting of heavy pseudoscalar mesons is calculated from QCD sum rules. The electromagnetic component is obtained from a generalization of the Dashen sum rule. Our result for the D^+-D^0 mass difference is in very good agreement with the experimental value.

According to conventional ideas, isospin symmetry SU(2) is broken by electromagnetism and by the u-d quark mass difference. The mass splitting between members of an SU(2) multiplet is, correspondingly, made up of electromagnetic and hadronic contributions. The hadronic contribution to the mass differences of light pseudoscalar mesons is usually calculated[1] by the method of current algebra. As is well known, this method fails for heavy pseudoscalar mesons like the D and the B.

In the present paper, we use the technique of Shifman, Vainshtein, and Zakharov[2] for quantum-chromodynamic (QCD) sum rules, which is applicable to both light and heavy pseudoscalar mesons. For the hadronic component of mass differences of light pseudoscalar mesons, we recover the results of current algebra. The corresponding result for heavy pseudoscalar mesons is new. Furthermore, the electromagnetic component of the mass difference is usually calculated only for light pseudoscalar mesons. For the π^+-π^0 mass difference, this is calculated[3] from first principles using current-algebra techniques. For the electromagnetic piece of the K^+-K^0 mass difference, one uses the Dashen sum rule[4] which relates it to the π^+-π^0 mass difference. In the present work, we also extend Dashen's sum rule to the D mesons, so that the electromagnetic component of the D^+-D^0 mass difference is also obtained in terms of the π^+-π^0 mass difference. This allows us to calculate the total D^+-D^0 mass difference,[5] and our result is in very good agreement with the value obtained by experiments.

Consider the polarization tensor for axial-vector currents

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq\cdot x}\langle 0|T^*(A_\mu(x)A_\nu^\dagger(0))|0\rangle = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_t(Q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_l(Q^2) \quad , \tag{1}$$

where

$$A_\mu(x) = \bar{q}_2(x)\gamma_\mu\gamma_5 q_1(x), \quad Q^2 = -q^2 \quad ,$$

and $\Pi_{t,l}$ are the transverse and longitudinal invariant functions. For pseudoscalar mesons we are interested only in Π_l. We now use the operator-product expansion (OPE) to write

$$\Pi_l(Q^2) = C_l^I(Q^2) + \sum_{i=1}^{2} C_l^{m_i}(Q^2)\langle 0|m_i\bar{q}_i q_i|0\rangle + C_l^G(Q^2)\langle 0|G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle + \cdots \quad , \tag{2}$$

where $G_{\mu\nu}^a$ is the gluon field tensor with color index a, and only a few of the low-dimensional operator terms have been displayed. In perturbative QCD, only the $C_{\mu\nu}^I(Q^2)$ term—the coefficient of the unit operator—survives, since all other vacuum expectation values vanish. Shifman, Vainshtein, and Zakharov have argued that as a consequence of the nonperturbative structure of the QCD vacuum, these vacuum expectation values need not vanish, and may be treated as phenomenological parameters. For large Q^2, the coefficients in the expansion (2) can be calculated using perturbative QCD. Explicit calculations[2,6,7] show that the gluon-condensate term makes a negligible contribution to Π_l. In the case of light quarks, this is consistent with the current-algebra result that the dominant contribution to Π_l comes from the quark-pair condensate which describes chiral-symmetry breaking. We shall see later that in this case the quark-condensate contribution also dominates the perturbative term in (2). On the other hand, when a heavy quark is involved, we will see that the perturbative term dominates all others.

The QCD sum rule can be obtained from Eq. (2) by writing a dispersion representation for $\Pi_l(Q^2)$ and taking the Borel transform through the operation

$$L_M = \lim_{\substack{n\to\infty\\ Q^2\to\infty\\ Q^2/n\to M^2}} \frac{1}{n!}(Q^2)^{n+1}\left(-\frac{\partial}{\partial Q^2}\right)^n \quad , \tag{3}$$

where M is a mass parameter. If we retain only the perturbative and the quark-condensate terms on the right-hand side of Eq. (2), and calculate the coefficients of these terms to lowest order in QCD coupling, we obtain[8]

$$\frac{1}{\pi}\int \mathrm{Im}\Pi_l(s)e^{-s/M^2}ds = \frac{3}{4\pi^2}M^2(m_1+m_2)\int_0^1 dx[(m_1-m_2)x+m_2]\exp[-s(x)/M^2] - \left[(m_1+m_2)(\langle 0|\bar{q}_1q_1|0\rangle + \langle 0|\bar{q}_2q_2|0\rangle) + O\left(\frac{1}{M^2}\right)\right] \quad , \tag{4}$$

where

$$s(x) = \frac{{m_1}^2}{1-x} + \frac{{m_2}^2}{x} \quad ,$$

and m_1 and m_2 are the quark masses. Furthermore, we have chosen $M >> m_1, m_2$. If we separate out the pseudoscalar-meson-pole contribution to the dispersion integral, and assume[2,7] that the continuum contribution is effectively given by the

$q_1\bar{q}_2$ quark-pair state beyond a suitably high threshold $s \geq \Lambda_0^2$, the sum rule becomes

$$m_p^2 f_p^2 e^{-m_p^2/M^2} \simeq \frac{3}{4\pi^2} M^2(m_1+m_2)\int_{x_{0-}}^{x_{0+}} dx[(m_1-m_2)x+m_2](e^{-s(x)/M^2}-e^{-\Lambda_0^2/M^2}) - (m_1+m_2)(\langle 0|\bar{q}_1 q_1|0\rangle + \langle 0|\bar{q}_2 q_2|0\rangle) \ , \quad (5)$$

where $x_{0\pm}$ are the roots of the quadratic equation $s(x)=\Lambda_0^2$. Also, m_P and f_P are the mass and decay constant of the pseudoscalar meson P.

For light quarks, in the chiral limit $m_{1,2}, m_p \to 0$, the sum rule (5) reduces to the well-known current-algebra result[1]

$$m_P^2 f_P^2 = -(m_1+m_2)(\langle 0|\bar{q}_1 q_1|0\rangle + \langle 0|\bar{q}_2 q_2|0\rangle) \ . \quad (6)$$

From Eq. (6), the hadronic component of the mass differences π^+-π^0 and K^+-K^0 can be obtained in the standard manner.[1] Together with the Dashen sum rule for the electromagnetic component of the mass differences, Weinberg[1] has used these results to estimate the quark mass ratios.

If P is a heavy pseudoscalar meson, with one heavy quark (say, q_1) and the other, a light u or d quark, the first term on the right-hand side of Eq. (5) is no longer negligible. In fact it is easy to see that it dominates over the quark-condensate term. Assuming $f_{D^+} = f_{D^0}$, the hadronic component of the D^+-D^0 mass difference can then be obtained as

$$\frac{[m_{D^+}^2 - m_{D^0}^2]_h}{m_D^2} \simeq \frac{m_d - m_u}{m_c}\frac{I_0}{I_1} \ , \quad (7)$$

where the integrals $I_{0,1}$ can be simplified if we take M to be much larger than $m_{1,2}$ and Λ_0:

$$I_n = \int_{x_{0-}}^{x_{0+}} dx[-s(x)+\Lambda_0^2]x^n \quad (n=0,1) \ . \quad (8)$$

With m_1 $(=m_c)$ $>>$ m_2 $(=m_{u,d})$, and assuming $\Lambda_0 >> m_c$, we get

$$I_0/I_1 \simeq 2 \ , \quad (9)$$

independently of M and Λ_0, as long as they are chosen to be suitably large. We then obtain

$$\frac{(m_{D^+} - m_{D^0})_h}{m_D} \simeq \frac{m_d - m_u}{m_c} \ . \quad (10)$$

It is interesting that this result is what one would naively expect on the basis of the constituent quark model, except for the important difference that the quark masses here refer to the current-algebra masses. Using Weinberg's[1] estimate $m_d \simeq 7$ MeV, $m_u \simeq 4$ MeV, and the results of Shifman, Vainshtein, and Zakharov,[2] $m_c \simeq 1.26$ GeV, we obtain numerically

$$(m_{D^+} - m_{D^0})_h \simeq 4.4 \text{ MeV} \ . \quad (11)$$

To calculate the electromagnetic component of the D^+-D^0 mass difference, we use P-spin[9] $(c \leftrightarrow u)$ invariance.[10] Straightforward algebra gives

$$m_\gamma^2(D^+) = m_\gamma^2(\pi^+) \ ,$$

$$m_\gamma^2(D^0) = m_\gamma^2(\pi^0) + (\tfrac{1}{3})^{1/2}\langle \pi^0|H_{\rm EM}^{\rm eff}|\eta_8\rangle + (\tfrac{2}{3})^{1/2}\langle \pi^0|H_{\rm EM}^{\rm eff}|\eta_{15}\rangle \ . \quad (12)$$

It is easy to see that the transition matrix elements of the effective electromagnetic interaction vanish in the soft-pion limit, so we obtain the analog of the Dashen sum rule

$$(m_{D^+}^2 - m_{D^0}^2)_\gamma = (m_{\pi^+}^2 - m_{\pi^0}^2)_\gamma \ . \quad (13)$$

Since the π^+-π^0 mass difference has no hadronic component,[1] we get

$$(m_{D^+} - m_{D^0})_\gamma = (m_{\pi^+} - m_{\pi^0})\frac{m_\pi}{m_D} \simeq 0.34 \text{ MeV} \ . \quad (14)$$

From Eqs. (11) and (14) we obtain the total D^+-D^0 mass difference

$$m_{D^+} - m_{D^0} \simeq 4.7 \text{ MeV} \ ,$$

to be compared with the experimental value[11] $m_{D^+} - m_{D^0} = 4.7 \pm 0.3$ MeV.

It should be pointed out that several different estimates of $m_d - m_u$ exist in the literature,[12] so that the precise agreement of our result with the experimental value may be somewhat fortuitous. To settle this question, the QCD sum rule should be applied to the calculation of other pseudoscalar mass differences (e.g., K mesons). At the same time, a more detailed analysis of the sum rule (5) for a range of M values should be carried out. These and related questions will be discussed in detail elsewhere.[7]

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[1]S. Weinberg, in *Festschrift for Rabi* (New York Academy of Sciences, New York, 1977), and references therein.

[2]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

[3]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Lett. 18, 759 (1967).

[4]R. Dashen, Phys. Rev. 183, 1245 (1969).

[5]For completely different attempts at calculating the D^+-D^0 mass difference, see, for example: R. Dutt and S. N. Sinha, Phys. Lett. 70B, 103 (1977); T. Fukuda, Prog. Theor. Phys. 59, 2179 (1978); B. K. Barik and S. N. Jena, Phys. Rev. D 24, 2905 (1981).

[6]L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Lett. 97B, 257 (1980).

[7]V. S. Mathur and M. T. Yamawaki (unpublished).

[8]V. S. Mathur and M. T. Yamawaki, Phys. Lett. 107B, 127 (1981). See also, L. J. Reinders, S. Yazaki, and H. R. Rubinstein, Phys. Lett. 104B, 305 (1981); E. V. Shuryak, Nucl. Phys. B198, 83 (1982).

[9]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[10]H. Harari [Phys. Rev. Lett. 17, 1303 (1966)] showed some time ago that the $I=1$ electromagnetic mass differences as in K^+-K^0, D^+-D^0, etc., are dominated by large loop energies, so one might expect quark masses to be unimportant in these calculations. In any case, since we find the electromagnetic component of the D^+-D^0 mass difference to be less than 10% of the hadronic component, the use of P-spin invariance in estimating the former is not crucial to our final result.

[11]Particle Data Group, Phys. Lett. 111B, 1 (1982).

[12]See, for instance, H. Pagels and S. Stokar, Phys. Rev. D 22, 2876 (1980).