Chiral bag models and the $\pi N \rightarrow \pi\pi N$ reaction

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We explore alternative formulations of bag models preserving chiral symmetry, in which the pion is treated as a separate local field that is allowed to penetrate into the bag interior. Particular emphasis is placed on the cubic terms in the pion field. These are essential for a description of the process $\pi N \rightarrow \pi\pi N$. We find that the phenomenology of that reaction indicates a clear preference amongst various possible formulations of chiral bag models with pion penetration of the bag.

I. INTRODUCTION

The MIT bag model[1,2] has achieved reasonable success in accounting for the properties of hadrons as built up from quarks and gluons by providing a phenomenological framework that embodies confinement and asymptotic freedom. More recently, the model has been extended[3–7] in order to include chiral symmetry or the principle of partially conserved axial-vector currents (PCAC). These treatments have dealt with the pion as a separate, local field, rather than trying to develop it as a quark-antiquark system with its own bag. Although this may represent a failing at the ideological level, it has allowed for rather detailed theoretical treatments[5–7] of various phenomena pertinent to nuclear physics at intermediate energies where the reasonableness of descriptions involving pion exchange has been fairly well established.

In particular, the so-called cloudy bag model (CBM),[5,6] in which the pion is permitted to penetrate into the bag interior, has proved successful in describing (3,3)-resonance phenomena through the linear coupling of the pion field to the quarks in the bag which represent the nucleon or Δ states. Failure to exclude the pion from the bag interior may violate intuitive pictures of asymptotic freedom, and efforts have been made[7] to construct chiral bag models that do not permit pion penetration; nonetheless, the cloudy-bag approach possesses an apparent simplicity which has allowed for its straightforward application at least to effects of lowest order in the pion field.

The chiral bag models also contain terms of higher order in the pion field, and only relatively recently has detailed attention been given to the possible role that these terms may play in processes involving several physical pions. In particular, the quadratic pion term has been examined[8] in the CBM in order to determine that it fulfills the appropriate soft-pion conditions[9] and yields the well-established result for the πN s-wave scattering amplitude. The verification that higher-order pion terms in chiral bag models yield correct phenomenology seems to us to be an important test of the overall validity of these models. We shall here provide a related examination of the third-order terms in the pion field which govern the process $\pi N \rightarrow \pi\pi N$. This is very likely the highest-order pionic effect for which direct comparison with phenomenology is possible. It is distinctly nontrivial since it is able to establish that, of the many possible forms that may be reached from the nonlinear Lagrangian of the CBM, certain selections may yield consistency with pion-production experiments in a particularly straightforward manner. Naturally, the various transformations of the original nonlinear chiral bag model are all equivalent representations of the same basic theory. However, given the need to truncate the nonlinear fields at some order and to identify the appropriately transformed field with the physical particle in question, particular versions of effective Lagrangians may allow for relatively direct application to phenomenology. Such indeed was the purpose of effective Lagrangians constructed in the context of soft-pion theories[9–11] even before the advent of quark models of hadron structure.

II. GENERAL FORMALISM

We start by developing a chiral bag model which is deliberately constructed in order to correctly incorporate the phenomenology of soft pions.[9–11] The chiral-invariant Lagrangian with which we deal is[3,10]

$$\mathscr{L}=\sum_a\left[\frac{i}{2}\bar{q}_a\overleftrightarrow{\partial}q_a-B\right]\theta_V-\frac{1}{2f}\sum_a\bar{q}_a(\sigma+i\underline{\tau}\cdot\underline{\pi}\gamma_5)q_a\Delta_S$$
$$+\tfrac{1}{2}(\partial^\mu\underline{\pi}\cdot\partial_\mu\underline{\pi}+\partial^\mu\sigma\partial_\mu\sigma)-\tfrac{1}{2}m^2\underline{\pi}^2\,, \qquad (1)$$

where q_a is the quark field in the baryon bag carrying flavor and color a, θ_V is a step function for the bag volume, Δ_S is the surface δ function, $\underline{\pi}$ is the pionic field, and m is the pion mass; the pion mass term is, of course, to be ignored in considerations relating to strict chiral symmetry. The σ field is here restricted such that[10]

$$\sigma=(f^2-\underline{\pi}^2)^{1/2} . \tag{2}$$

The chiral transformations that leave the Lagrangian of Eq. (1) invariant are

$$q_a \to T(\underline{\pi})q_a=(1+i\underline{\tau}\cdot\underline{\epsilon}\gamma_5)q_a , \tag{3a}$$

$$\underline{\pi} \to U(\underline{\pi})=\underline{\pi}-2\underline{\epsilon}\sigma , \tag{3b}$$

and the corresponding axial-vector current is

$$\underline{A}_\mu=\tfrac{1}{2}\sum_a \bar{q}_a\underline{\tau}\gamma_5\gamma_\mu q_a\theta_V-\underline{\pi}\partial_\mu\sigma+\sigma\partial_\mu\underline{\pi} , \tag{4}$$

which is fully conserved in the soft-pion limit and otherwise fulfills PCAC. To zero order in the surface interaction, $\mathcal{L}$ of Eq. (1) leads, of course, to the MIT bag solution[1,2] for the baryon, together with a free pion.

In order to link $\mathcal{L}$ conveniently with soft-pion properties, we apply a transformation motivated by Weinberg's work[11]

$$\begin{aligned} q_a &\to S(\underline{\pi})q_a \\ &=\frac{1}{[2f(f+\sigma)]^{1/2}}(f+\sigma+i\gamma_5\underline{\tau}\cdot\underline{\pi})q_a , \end{aligned} \tag{5a}$$

$$\underline{\pi} \to V(\underline{\pi})=\frac{2f}{f+\sigma}\underline{\pi} . \tag{5b}$$

For the quark terms in Eq. (1) this yields

$$\begin{aligned} \bar{q}\overleftrightarrow{\partial}q &\to \bar{q}\overleftrightarrow{\partial}q+2\bar{q}\gamma^\mu S(\partial_\mu S^\dagger)q \\ &=\bar{q}\overleftrightarrow{\partial}q+\frac{i}{f}\bar{q}\left\{\gamma_5\underline{\tau}\cdot\partial\!\!\!/\underline{\pi}-i\gamma_5\frac{\underline{\tau}\cdot\underline{\pi}\underline{\pi}\cdot\partial\!\!\!/\underline{\pi}}{\sigma(f+\sigma)}\right. \\ &\qquad\left.-\frac{\underline{\tau}\cdot\underline{\pi}\times\partial\!\!\!/\underline{\pi}}{f+\sigma}\right\}q , \end{aligned} \tag{6a}$$

$$\bar{q}\left[\sigma+\frac{i}{f}\underline{\tau}\cdot\underline{\pi}\gamma_5\right]q \to \bar{q}q , \tag{6b}$$

and the pionic part of the Lagrangian becomes

$$\begin{aligned} &\tfrac{1}{2}(\partial_\mu\underline{\pi}\cdot\partial^\mu\underline{\pi}+\partial_\mu\sigma\partial^\mu\sigma)-\tfrac{1}{2}m^2\underline{\pi}^2 \\ &\qquad\to\frac{1}{[1+(\underline{\pi}^2/4f^2)]^2}\partial^\mu\underline{\pi}\cdot\partial_\mu\underline{\pi} \\ &\qquad-\frac{1}{2}\frac{1}{[1+(\underline{\pi}^2/4f^2)]}m^2\underline{\pi}^2 , \end{aligned} \tag{7}$$

while the second term in the large parentheses in Eq. (6a) is exactly canceled. By this procedure we have transformed the interaction from a surface δ coupling to a volume interaction. We obtain finally the simple form

$$\begin{aligned} \mathcal{L}=&\sum_a\left[\frac{i}{2}\bar{q}_a\overleftrightarrow{\partial}q_a-B\right]\theta_V-\tfrac{1}{2}\sum_a\bar{q}_aq_a\Delta_S \\ &+\sum_a\bar{q}_a\gamma^\mu\frac{1}{1+g^2\underline{\pi}^2}[g\gamma_5\underline{\tau}\cdot\partial_\mu\underline{\pi}-g^2\underline{\tau}\cdot(\underline{\pi}\times\partial_\mu\underline{\pi})]q_a\theta_V+\frac{1}{2}\frac{1}{(1+g^2\underline{\pi}^2)^2}\partial^\mu\pi\cdot\partial_\mu\underline{\pi}-\frac{1}{2}\frac{1}{1+g^2\underline{\pi}^2}m^2\underline{\pi}^2 , \end{aligned} \tag{8}$$

where $g=(2f)^{-1}$. The equations of motion and boundary conditions for this Lagrangian are then, for the quarks,

$$i n\!\!\!/ q_a(x)=q_a(x), \quad \text{for } x\in S , \tag{9}$$

$$i\partial\!\!\!/ q_a(x)=-\gamma^\mu\frac{1}{1+g^2\underline{\pi}^2}[g\gamma_5\underline{\tau}\cdot\partial_\mu\underline{\pi}-g^2\underline{\tau}\cdot(\underline{\pi}\times\partial_\mu\underline{\pi})]q_a(x), \quad \text{for } x\in V , \tag{10}$$

where n_μ is the normal to the bag surface, and for the pion

$$(\partial^2+m^2)\underline{\pi}(x)=-ig\sum_a\bar{q}_a\gamma_5\underline{\tau}q_a\Delta_S+O(\underline{\pi}^3) . \tag{11}$$

The above equations ascribe different dynamics to the quarks in which the coupling to the pion field persists inside the bag volume. This coupling within the bag was already encountered in Ref. 8; it has been suggested[6] that the existence of a pionic field for $x\in V$ is to be interpreted as pertaining to quark-antiquark excitations in the bag interior.

The chiral invariance of the new Lagrangian in Eq. (8) is ensured by the fact that it was obtained from the old one of Eq. (1) by the chiral transformations, Eqs. (5a) and (5b). In order to obtain the new infinitesimal chiral transformation we proceed by calling T' the new quark chiral transformation, $T'=STS^{\dagger}$. Analogously, the new pion chiral transformation is $V'(\underline{\pi})=V(U(V^{-1}(\underline{\pi})))$, where V^{-1} is the inverse transformation of Eq. (5b), $V^{-1}(\underline{\pi})=(1+\underline{\pi}^2/4f^2)^{-1}\underline{\pi}$. The new transformations are then

$$q_a \to T'(q_a)=(1-ig\underline{\tau}\cdot\underline{\pi}\times\underline{\epsilon})q_a \ , \tag{12a}$$

$$\underline{\pi}\to V'(\underline{\pi})=\underline{\pi}-2g\underline{\pi}\cdot\underline{\epsilon}\underline{\pi}-\frac{\underline{\epsilon}}{g}(1-g^2\underline{\pi}^2) \ , \tag{12b}$$

with conserved axial-vector current

$$\begin{aligned}\underline{A}_{\mu}=&\frac{1}{1+g^2\underline{\pi}^2}\sum_a \bar{q}_a[2g\underline{\tau}\times\underline{\pi}\gamma_5+2g^2\underline{\tau}\cdot\underline{\pi}\,\underline{\pi}+\underline{\tau}(1-g^2\underline{\pi}^2)]\gamma_5\gamma_\mu q_a\theta_V\\ &-\frac{1}{(1+g^2\underline{\pi}^2)^2}\left[2g\underline{\pi}\cdot(\partial_\mu\underline{\pi})\underline{\pi}+\frac{1}{g}(1-g^2\underline{\pi}^2)\partial_\mu\underline{\pi}\right] .\end{aligned} \tag{13}$$

For our subsequent purposes we shall wish to consider terms linear, quadratic, and cubic in pion-baryon interactions, and quartic in the pion field alone,

$$\mathcal{L}_{\pi qq}=g\sum_a \bar{q}_a\gamma^\mu\gamma_5\underline{\tau}q_a\cdot(\partial_\mu\underline{\pi})\theta_V \ , \tag{14}$$

$$\mathcal{L}_{\pi\pi qq}=-g^2\sum_a \bar{q}_a\gamma^\mu\underline{\tau}q_a\cdot(\underline{\pi}\times\partial_\mu\underline{\pi})\theta_V \ , \tag{15}$$

$$\mathcal{L}_{\pi\pi\pi qq}=-g^3\sum_a \bar{q}_a\gamma^\mu\gamma_5\underline{\tau}q_a\cdot(\partial_\mu\underline{\pi})\underline{\pi}^2\theta_V \ , \tag{16}$$

$$\mathcal{L}_{\pi\pi\pi\pi}=-g^2\underline{\pi}^2(\partial^\mu\underline{\pi})\cdot(\partial_\mu\underline{\pi})+\tfrac{1}{2}g^2m^2\underline{\pi}^4 \ . \tag{17}$$

In order to compare these terms with the development of the Weinberg pion-nucleon Lagrangian,[11] one must solve the coupled equations (9)—(11) and obtain a modified quark wave function to be used in Eqs. (14)—(16). These higher-order corrections have thus far generally been ignored,[8] as we do here at this stage (see the Appendix, where we briefly examine the validity of this approximation). Then the MIT bag solution[2] applies,

$$q_a=\frac{N}{\sqrt{4\pi}}\begin{bmatrix} j_0(\Omega r)\\ i\vec{\sigma}\cdot\hat{r}j_1(\Omega r)\end{bmatrix}v \ , \tag{18}$$

where a static bag surface and recoilless baryon have been used; v is the spin-isospin function. With these restrictions, the use of Eq. (18) then leads to[5,8]

$$\begin{aligned}\mathcal{L}_{\pi qq}=&\frac{N^2}{4\pi}\frac{5}{3}[j_0{}^2(\Omega r)-\tfrac{1}{3}j_1{}^2(\Omega r)]\\ &\times\theta(R-r)\mathcal{L}_{\pi NN} \ ,\end{aligned} \tag{19}$$

where the Lagrangian $\mathcal{L}_{\pi NN}$ refers to the bag representation of the nucleon and is thus to be identified with the Lagrangian of Weinberg which deals, of course, with nucleon fields. An identical result, with the same factor as in Eq. (19), obtains for the relationship between $\mathcal{L}_{\pi\pi\pi qq}$ of Eq. (16) and the corresponding $\mathcal{L}_{\pi\pi\pi NN}$ of Weinberg since the structure of the terms linear and cubic in $\underline{\pi}$ is identical in the quark space.

Now the result of Eq. (19) for the pion-baryon interaction linear in the pion field is the same as that of the cloud-bag model[5] except for a small change[8] in the relevant vertex form factor arising from a tensor contribution. This tensor part is negligible at small momenta, as here, but does represent a specific feature of the model in that it allows the coupling of the quarks to non-spherically-symmetric states which may be admixed in the baryons. As pointed out by Vento *et al.*,[12] such states may contribute significantly to static properties of the baryon such as the axial-vector coupling constant q_A, or to dynamic features, for instance, in the pion photoproduction process.

Ignoring this tensor term, all the results of the CBM that are based on the p-wave πNN vertex, such as the description of the 3,3 resonance, will go through essentially unchanged. For quadratic terms in the pion field, which will be required in order to treat s-wave πN scattering near threshold, the result of Eq. (15) is the same as that found by Thomas[8] to second order by transforming the quark field in the CBM. As Thomas has shown, this interaction immediately yields the Weinberg amplitudes for threshold πN scattering. In fact, his result may be obtained from Eq. (8) if the pion field there is taken as $\underline{\pi}=2f\hat{\underline{\phi}}\tan(\phi/2f)=\underline{\phi}+O(\phi^3)$, where $\underline{\phi}=\phi\hat{\underline{\phi}}$ is the pion field of Ref. 8, and so his linear and quadratic terms are identical to our Eqs. (14) and (15), differences first being encountered at the next order and higher.

III. RESULTS FOR THE $\pi N\to\pi\pi N$ PROCESS

We focus now on the $\pi N\to\pi\pi N$ process in order to study explicitly the consequences of third-order pionic terms. For terms cubic or quartic in the pion field, the CBM with Thomas's transformation yields, in place of Eqs. (16) and (17),

$$\mathscr{L}^{\rm CBM}_{\pi\pi qq} = -\tfrac{2}{3}g^3 \sum_a \bar{q}_a \gamma^\mu \gamma_5 \underline{\tau} q_a \cdot[(\partial_\mu \underline{\phi})\underline{\phi}^2 - \underline{\phi}\,\underline{\phi}\cdot\partial_\mu\underline{\phi}] \tag{20}$$

and

$$\mathscr{L}^{\rm CBM}_{\pi\pi\pi\pi} = \tfrac{1}{3}g^2[(\underline{\phi}\cdot\partial^\mu\underline{\phi})(\underline{\phi}\cdot\partial_\mu\underline{\phi}) - \underline{\phi}^2(\partial^\mu\underline{\phi})\cdot(\partial_\mu\underline{\phi})] \ . \tag{21}$$

These are precisely the terms that enter into the calculation of the pion-production process $\pi N \rightarrow \pi\pi N$ at threshold, which is known to be well described by the diagrams in Fig. 1 as they are calculated with Weinberg's Lagrangian.[13–15] Thus, the Lagrangian

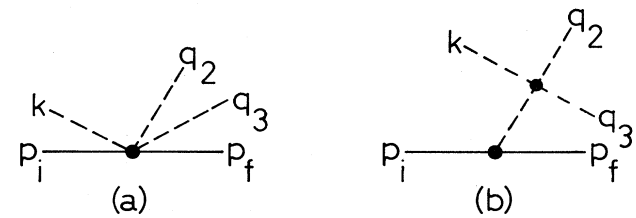

FIG. 1. Diagrams that enter in the calculation of $\pi N \rightarrow \pi\pi N$. (a) The contact term involving $\mathscr{L}_{\pi\pi\pi NN}$. (b) The $\pi\pi$ scattering term containing $\mathscr{L}_{\pi\pi\pi\pi}$ and $\mathscr{L}_{\pi NN}$.

of Eq. (8) will directly yield this phenomenology by construction, that is, it takes $g_0 = 1/2f$ and $h_0 = -1/4f^2$ or $\xi \equiv 2f(g_0 + 2h_0 f) = 0$ in the conventional parametrization,[13–15]

$$\mathscr{L}_{\pi\pi\pi NN} = -g^3 \frac{g_A}{g_V}(\bar{N}\gamma^\mu\gamma_5 \underline{\tau} N)\cdot[2g_0 f(\partial_\mu \underline{\pi})\underline{\pi}^2 + 2(2g_0 f - 1)\underline{\pi}\,\underline{\pi}\cdot\partial_\mu\underline{\pi}] \tag{22}$$

and

$$\mathscr{L}_{\pi\pi\pi\pi} = g^2[2(1-2g_0 f)(\underline{\pi}\cdot\partial^\mu\underline{\pi})(\underline{\pi}\cdot\partial_\mu\underline{\pi}) - 2g_0 f \underline{\pi}^2 \partial^\mu \underline{\pi}\cdot\partial_\mu\underline{\pi} + \tfrac{1}{2}(3g_0 f - 2h_0 f - 1)m^2(\underline{\pi}^2)^2] \ , \tag{23}$$

while the conventional CBM requires $g_0 = 1/3f$ and $h_0 = 0$ or $\xi = \frac{2}{3}$.

Since Weinberg's Lagrangian refers only to nucleons, and not to Δ-isobar contributions, it is clear that if we wish to reproduce the same phenomenology in the context of the CBM we must also restrict ourselves to nucleon states. In practice, the comparison of the theoretical approach based on nucleons only with experimental results, which contain Δ contributions as well, has been made[13–15] by means of careful partial-wave treatment and extrapolation to threshold; nonetheless the Δ contributions are reflected in the rather large error with which ξ is determined. The experimental result is[14] $\xi = 0.05 \pm 0.26$, or after more detailed analysis,[15] $\xi = -0.2 \pm 0.3$; within quantum chromodynamics, ξ is expected[15] to be zero. Note that for $\xi = 1$ the $\pi N \rightarrow \pi\pi N$ cross section is known[13] to be underestimated by a factor of three near threshold for $g_0 = 1/2f$. For the conventional CBM we obtain a cross section for the $(\pi, 2\pi)$ process at threshold which is less by a factor of 0.44 than that obtained in the present approach.

In summary, we see that the correct phenomenological treatment of the pion-production process $\pi N \rightarrow \pi\pi N$ strongly motivates the use of a particular effective Lagrangian within the context of chiral bag models. Identifying the physically pertinent fields for the quarks and the pions as those which arise in this effective Lagrangian, we find that a specific and unconventional form for the dynamics has been dictated, namely, one in which the pion field interacts with the quarks within the bag. Since such pionic effects are small for bag radii $\gtrsim 0.7$ fm, the effective CBM Lagrangian proposed here does not appreciably alter previous results. It does, however, allow for a uniform treatment of pion-baryon interactions including third-order terms in the pion field. Such a treatment may be particularly useful for the handling of pion-nucleus interactions in the medium-energy domain.

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APPENDIX

We here study the modifications of the bag eigenenergies in the framework of the present model and static-cavity solutions.[1,2] We display an approximate, lowest-order procedure for renormalizing the single-quark eigenenergies. We do not present a more complete[4,6,16] renormalization of the hadron mass, which would include two-quark interaction terms, since our main purpose here is to establish that the dynamics represented by Eqs. (8)—(10) do not significantly alter the size of the pion correction as compared with previous approaches.[6,16] By a procedure analogous to that of Jaffe,[4] we obtain from Eq. (11) a pion field in the form

$$\underline{\pi}=\frac{g\omega_0}{12\pi(\omega_0-1)R^3}\sum_{\alpha\beta\gamma\delta} v^{\dagger}_{\alpha\beta}\underline{\tau}\vec{\sigma}\cdot\hat{r}v_{\gamma\delta}a^{\dagger}_{\alpha\beta}a_{\gamma\delta}\ , \tag{A1}$$

where $\alpha\beta,\gamma\delta$ are spin-isospin indices, $a^{\dagger},a$ are quark creation and annihilation operators, and $\omega_0=2.04$ is the lowest eigenmode of the MIT bag model.[1–4] Inserting Eq. (A1) in Eq. (10), retaining the lowest order in the interaction, and assuming q_a to have the functional dependence

$$q(\underline{r},t)=\begin{bmatrix}F(r)+iG(r)\vec{\sigma}\cdot\hat{r}\\ H(r)+iK(r)\vec{\sigma}\cdot\hat{r}\end{bmatrix}e^{-i\omega t}\ , \tag{A2}$$

we obtain (restricting to the one-quark space)

$$i\partial q=b\sum_{\gamma\delta}\begin{bmatrix}-3F(r)+iG(r)\vec{\sigma}\cdot\hat{r}\\ H(r)+iK(r)\vec{\sigma}\cdot\hat{r}\end{bmatrix}v_{\gamma\delta}a_{\gamma\delta}e^{-i\omega t}\ , \tag{A3}$$

where

$$b=\frac{g^2\omega_0}{12\pi(\omega_0-1)R^3}\ . \tag{A4}$$

In order that the solution obey the boundary condition of Eq. (9), we have

$$G(r)=H(r)=0 \tag{A5}$$

and

$$F(r)=Aj_0(cr)\ ,\qquad K(r)=A\eta j_1(cr)\ , \tag{A6}$$

where

$$c=[(\omega+3b)(\omega-b)]^{1/2}\ ,\qquad \eta=[(\omega+3b)/(\omega-b)]^{1/2}\ , \tag{A7}$$

and A has the same form as in the MIT-bag solution.[1,2] The allowed frequencies ω obey

$$j_0(cR)=\eta j_1(cR)\ , \tag{A8}$$

which reduces to the original MIT-bag condition[1,2] when $b \ll \omega$. Table I shows the allowed values for ω as a function of R. It is readily seen that for values of R near 1 fm the modification of the MIT eigenmodes is negligible. This result agrees with the previous calculations of the CBM.[4,5,16]

TABLE I. Bag eigenenergies including first-order pionic effects; Eq. (A8) ($\omega_0=2.04$).

Bag radius (fm)	ω
1.2	1.97
1.1	1.96
1.0	1.94
0.9	1.92
0.8	1.88
0.7	1.84
0.6	1.76
0.5	1.65

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