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Bethe-Salpeter basis for quark-pair-creation model: Understanding of VPP, $VP\gamma$, and $P\gamma\gamma$ couplings

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A Bethe-Salpeter harmonic-oscillator framework, developed recently for $q\bar{q}$ mesons in the instantaneous (null-plane) approximation, is employed for the construction of MMM couplings ($M=P$ or V) through quark-loop diagrams. The resulting structure of the matrix elements exhibits the central feature of the quark-pair-creation model, that is, a multiplicative structure for the orbital and spin wave functions of all three $q\bar{q}$ states involved, together with a couple of essential kinematical factors (but only in the numerator) as part of the field-theoretic result. The model is illustrated through two typical meson couplings involving equal-mass quarks, $\rho\to\pi\pi$ and $\omega\to\rho\pi$, as well as through the two electromagnetic processes $\omega\to\gamma\pi^0$ and $\pi^0\to\gamma\gamma$. The $\rho\to\pi\pi$ and $\omega\to\gamma\pi^0$ widths (via direct γ coupling) come out as 142.7 MeV and 888 keV, respectively, with no adjustable parameters. The $\omega\to\gamma\pi^0$ matrix element also agrees to within 1%, with that obtained from the corresponding $\omega\rho\pi$ coupling through vector-meson-dominance (VMD) substitution. The $\pi^0\to\gamma\gamma$ width (7.7 eV) agrees excellently with experiment when it is deduced from $\omega\rho\pi$ coupling via double VMD substitution, but the corresponding amplitude obtained from direct γ coupling is only about half the experimental value. Theoretical reasons are given for this apparent paradox.

I. INTRODUCTION

Quark or duality diagrams[1,2] not only provide a topological insight into hadronic processes in terms of their quark content, but also bring out the spin structure of the matrix elements, in particular the $^3P_0$ form of a $q\bar{q}$ pair created out of the vacuum,[3–5] as an essential ingredient for the mesonic transition $M_A\to M_B+M_C$ and the similar baryonic process $B\to B'+M$. A more dynamical exploitation of these ideas to include the orbital structure of the matrix elements, was made by LeYaouanc *et al.*[6] through the suggestion that the complete matrix element for $M_A\to M_B+M_C$ be given by a simultaneous overlap integral of the form

$$\int \langle \psi_B\psi_C \,|\, T_{\text{vac}} \,|\, \psi_A \rangle d\tau \;, \tag{1.1}$$

where T_{vac} ($\sim\vec{\sigma}\cdot\vec{q}$) is essentially a $^3P_0$ operator representing the $J=0$ structure of the $q\bar{q}$ pair lifted out of the vacuum. The model, hereafter termed the quark-pair-creation model (QPCM), was originally nonrelativistic in content and clothed in harmonic-oscillator wave functions, but was soon extended to the relativistic level,[7–10] with several kinds of applications.[10]

Despite its phenomenological character, the remarkable predictive powers of QPCM soon became apparent from its capacity to incorporate several unifying principles, such as V-meson universality, quark additivity, and $SU(6)_W$, under a single broad canvas.[6] As a result its applications have been extensive, from nonrelativistic[11] to relativistic.[12,13] On the other hand, a formal theoretical basis for its structure, preferably from field-theoretic premises has so far been lacking at the *quantitative* level, as distinct from a qualitative understanding.[14] The nearest candidate was perhaps the four-dimensional formulation of Bohm, Joos, and Krammer[15] for expressing transition matrix elements in terms of Bethe-Salpeter (BS) amplitudes for the states concerned. Their QPCM-like structures are fairly explicit in the limit of infinitely large quark masses.[13] However, apart from large quark masses the formulation of Bohm, Joos, and Krammer[15] had other "problems" caused by their Euclidean formulation for all timelike internal momenta, leading to less desirable consequences.

The purpose of this paper is to offer an alternative field-theoretic basis for the QPCM in terms of the instantaneous approximation to the BS equation, which is generally regarded as an acceptable mathematical device for eliminating the (unwanted) timelike momenta, so as to pave the way for a probabilistic interpretation for the resulting wave functions. In some recent publications,[16–18] the original instantaneous approximation to the BS equations[19] has been generalized from $q\bar{q}$ to qqq (Ref. 16) and $qq\bar{q}\bar{q}$ (Ref. 18) systems, and employed for a calculation of their spectra.[17,18] The null-plane approximation is an alternative device for reducing the BS equation to an effective three-dimensional form.[20–22] The resulting equation seems to exhibit a strong formal resemblance to the corresponding instantaneous-approximation structure,[22] a fact which can be usefully exploited for doing instantaneous-approximation calculations in the (simpler) null-plane language.

In this paper we shall examine the mathematical structure for hadron couplings within an instantaneous-approximation-oriented BS formalism,[22] and exhibit its strong similarity to QPCM-like structures,[6] including the effect of spin which now appears as a well-defined factor in the numerator of the matrix element in the form of a trace. The essential details of the procedure are illustrated in terms of the realistic cases of $\rho\to\pi\pi$ and $\omega\to\rho\pi$ couplings, in each of which the spin effects play an important role. Another related case considered in this paper is the electromagnetic $\omega\to\gamma\pi^0$ coupling, together with its possible link with the hadronic $\omega\to\rho\pi$ coupling expected on a vector-meson-dominance (VMD) basis.[23] Finally we consider the matrix element for $\pi^0\to\gamma\gamma$ in two different ways: (i) a direct BS amplitude for $\pi^0\to\gamma\gamma$ decay through the standard quark triangle diagram and (ii) a QPCM simulated virtual $\omega\rho^0\pi^0$ coupling supplemented by double electromagnetic substitution on the lines of Schwinger.[23] The only additional ingredient needed for making absolute predictions in all these cases is an ansatz relating to a certain Lorentz-invariant adaptation, *vide* ansatz (3.3), of an otherwise three-dimensional (Gaussian) form factor in each of these cases on lines suggested some time ago[24,25] for a relativistic adaptation of QPCM matrix elements.

In Sec. II we write down the complete matrix element for a $\rho\to\pi\pi$ transition in terms of the instantaneous approximation to the BS amplitudes for $q\bar{q}$ states as developed in Ref. 22 for a direct adaptation to four-dimensional Feynman diagrams, and integrate over the timelike part of the relevant four-momentum of integration to exhibit the resulting structure in a three-dimensional form, together with its explicit similarity to the usual QPCM matrix elements.[6] Corresponding matrix elements for $\omega\rho\pi$, $\omega\gamma\pi^0$, and $\pi^0\gamma\gamma$ couplings are also listed for comparison. Numerical results are presented in Sec. III on the basis of the ansatz (3.3),[24,25] noted in the foregoing, for the Gaussian form factors involved. The $\rho\to\pi\pi$ and direct electromagnetic $\omega\to\gamma\pi^0$ widths work out as 142.7 and 0.888 MeV, respectively, with no adjustable parameters. The $\omega\to\gamma\pi^0$ amplitude also agrees with its VMD counterpart deduced[23] from $\omega\to\rho\pi$. The double VMD version[23] of the $\pi^0\to\gamma\gamma$ width (7.7 eV) likewise agrees with data (7.9 eV),[26] but the $\pi^0\to\gamma\gamma$ amplitude based on direct photon coupling is only half the observed amplitude. Theoretical reasons for this apparent anomaly are discussed. Finally, in keeping with the illustrative applicational scope of this paper, more comprehensive applications including unequal-mass kinematics ($m_1\neq m_2$), couplings of L-excited states, three-body decays, etc., are relegated to a future communication.

II. QPCM FOR MMM AND $MM\gamma$ COUPLINGS

In a recent paper[22] we have described the structure of the $\bar{q}qM$ vertex in the instantaneous approximation. We shall draw freely from the results of Sec. II of Ref. 22, including notations, definitions and phase conventions of these vertices, to derive most of the results of this section. According to the results of Ref. 22, the three-dimensional $\bar{q}qM$ vertex function $\Gamma(\vec{q})$ in the $m_1=m_2$ case is given by

$$\Gamma(\vec{q})=A_M\Gamma D(\vec{q})\phi(\vec{q})/2\pi i\ , \tag{2.1}$$

$$D(\vec{q})=2M(m_q{}^2+\vec{q}\,^2-\tfrac{1}{4}M^2)\ , \tag{2.2}$$

$$\phi=\phi_0(\vec{q})=(\pi\beta^2)^{-3/4}\exp(-\tfrac{1}{2}\vec{q}\,^2\beta^{-2})\ , \tag{2.3}$$

$$\begin{gathered}(2A_M)^{-2}(2\pi)^{-3}=M(\lambda\beta^2+m_q{}^2)\ ,\\ \lambda=\tfrac{3}{2},\quad \Gamma=\gamma_5\quad (P\text{ mesons})\ ,\\ \lambda=1,\quad \Gamma=i\gamma\cdot\hat{\epsilon}\quad (V\text{ mesons})\ .\end{gathered} \tag{2.4}$$

In terms of these vertices, the four-dimensional BS amplitude is expressible as[22]

$$\Psi(p_1,p_2)=i^2S_F(p_1)\Gamma(\vec{q})S_F(-p_2)\ . \tag{2.5}$$

Consider first Figs. 1(a) and 1(b) where the indicated four-momenta of the quarks may be expressed in terms of total and relative four-momenta as[16]

$$\begin{gathered}p_{1,2}=\tfrac{1}{2}P\pm q,\quad p'_{1,2}=\tfrac{1}{2}P'\pm q'\ ,\\ p''_{1,2}=\tfrac{1}{2}P''\pm q''\ .\end{gathered} \tag{2.6}$$

Further, the equalities

$$p_1=p'_1,\quad p''_1=-p'_2,\quad p''_2=p_2 \tag{2.7}$$

give rise to the connections

$$q'=q+\tfrac{1}{2}p'',\quad q''=q-\tfrac{1}{2}P' \tag{2.8}$$

and the hadronic conservation is checked as

$$P=P'+P''\ . \tag{2.9}$$

The color indices in Figs. 1(a) and 1(b), in an obvious notation, give rise to the overall factor

$$3^{-1/2}\delta_{ij}(\rho)3^{-1/2}\delta_{jk}(\pi')3^{-1/2}\delta_{Ki}(\pi'')=3^{-1/2}\ . \tag{2.10}$$

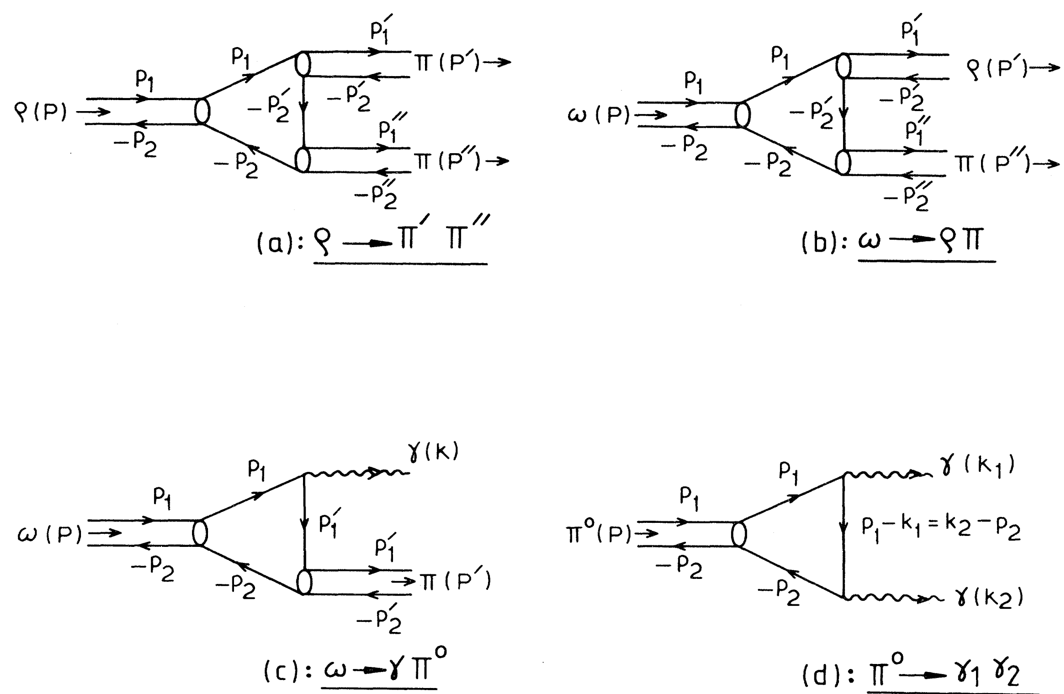

FIG. 1. Triangle diagrams for (a) $\rho\to\pi'\pi''$, (b) $\omega\to\rho\pi$, (c) $\omega\to\gamma\pi^0$, and (d) $\pi^0\to\gamma_1\gamma_2$ matrix elements. For the four-momentum label on the various quark lines, see text.

The isospin factor for both $\rho\to\pi\pi$ and $\omega\to\rho\pi$ works out as $(2^{1/2})$.

$\rho\pi\pi$ coupling

The invariant matrix element for $\rho\to\pi\pi$, Fig. 1(a), is now

$$\mathscr{F}(\rho\to\pi\pi)=(2\pi)^8(\tfrac{2}{3})^{1/2}\mathrm{Tr}\int d^4q\,\bar{\Psi}_\pi(p_1',p_2')(m_q-i\gamma\cdot p_2')\bar{\Psi}_\pi(p_1'',p_2'')(m_q-i\gamma\cdot p_2'')\Psi_\rho(p_1,p_2)(m_q+i\gamma\cdot p_1)\,. \tag{2.11}$$

Using (2.5)—(2.8), (2.11) simplifies to

$$\mathscr{F}(\rho\to\pi\pi)=(2\pi)^8(\tfrac{2}{3})^{1/2}A_\rho A_\pi^{\,2}\int d^4q\,\Delta^{-1}[\mathrm{Tr}(\rho\pi\pi)][D\phi(\rho\pi\pi)]\,, \tag{2.12}$$

$$\begin{aligned}[\mathrm{Tr}(\rho\pi\pi)]&=\mathrm{Tr}(m_q-i\gamma\cdot p_1')(m_q+i\gamma\cdot p_1'')(m_q+i\gamma\cdot p_2)i\gamma\cdot\epsilon\\&=4\epsilon\cdot q(m_q^{\,2}+q^2-2\cdot Q-\tfrac{1}{4}M_\rho^{\,2})+\epsilon\cdot Q(m_q^{\,2}+q^2+\tfrac{1}{4}M_\rho^{\,2})\;\;(2Q=P'-P'',\;\;A\cdot B=\vec{\mathrm{A}}\cdot\vec{\mathrm{B}}-A_0B_0),\end{aligned} \tag{2.13}$$

$$\Delta=(m_q^{\,2}+p_1'^{\,2})(m_q^{\,2}+p_1''^{\,2})(m_q^{\,2}+p_2^{\,2})\,, \tag{2.14}$$

$$(2\pi i)^3[D\phi(\rho\pi\pi)]=D_\rho(\vec{q})\phi_\rho(\vec{q})D_\pi(\vec{q}\,')\phi_\pi(\vec{q}\,')D_\pi(\vec{q}\,'')\phi_\pi(\vec{q}\,'')\,, \tag{2.15}$$

where the D and ϕ functions for the indicated composite are given by (2.2) and (2.3), respectively. The crucial step lies in doing the q_0 integration in (2.12) which, in view of the instantaneous approximation, involves q_0 only in the denominators and at most in the Tr factor (2.13). Now it greatly simplifies the algebra if the equivalent of the q_0 integration is carried out in the null-plane language[22] (without of course claiming to go beyond the physical limits of the instantaneous approximation). In particular if p_{2-} (not p_{1-}) is chosen[21,22] as the integration variable in (2.12), with the correspondence[22] $\frac{1}{2}dp_{2-}\Longleftrightarrow dq_0$, then a simple pole appears only in the factor $(m_q^{\,2}+p_2^{\,2})^{-1}$. Now if the value of p_{2-} at this pole, viz., $(p_{2\perp}^{\,2}+m_q^{\,2})/p_{2+}$, is substituted in the remaining two denominators, the following simple results emerge:

$$(m_q{}^2+p_1'{}^2)^{-1}D_\pi(\vec{q}\,')\Longrightarrow 2p_{2+}' ,\quad (m_q{}^2+p_1''{}^2)^{-1}D_\pi(\vec{q}\,'')\Longrightarrow 2p_{2+}'' , \tag{2.16}$$

where[21]

$$p_{2+}'=\tfrac{1}{2}P_+'-q_+' ,\quad p_{2+}''=\tfrac{1}{2}P_+''-q_+'' \tag{2.17}$$

together with the connections (2.7) which hold for the (+) components in particular. Taking account of the factor $(2p_{2+})^{-1}$ as a part of the overall residue arising out of the p_2 propagator, and using the relation $p_{2+}''=p_{2+}$, Eq. (2.7), the matrix element (2.12) reduces to

$$\mathscr{F}(\rho\rightarrow\pi\pi)=i^{-2}(2\pi)^6(\tfrac{2}{3})^{1/2}A_\rho A_\pi{}^2\int d^3q\,\phi_{\pi'}\phi_{\pi''}D_\rho\phi_\rho(2p_{2+}')[\mathrm{Tr}(\rho\pi\pi)] . \tag{2.18}$$

Equation (2.18) reveals rather clearly the QPCM structure of the matrix element with every relevant factor appearing in the *numerator.* The main ingredients, viz., the three-dimensional wave functions of the three-hadrons involved,[6] are already explicit. The effects of the γ matrices due to the BS structures of the vertices, as well as those arising out of the propagators are collectively incorporated in the trace factor. This is a precise substitute for the simulation of such effects through certain piecemeal assumptions (e.g., partial symmetry), followed by relativistic boost prescriptions in a relativistic formulation of QPCM.[12] Finally, the factors $(2p_{2+}')$ and $D_\rho(q)$ in (2.18) have no obvious counterpart in QPCM, nonrelativistic[6] or relativistic,[12] but must be regarded as parts of an overall relativistic package implied in a three-dimensional reduction of a standard BS-oriented Feynman amplitude. Nevertheless the similarity to QPCM is striking enough to warrant the conclusion that the three-dimensional BS formulation not only gives a good theoretical support to QPCM but also provides certain nontrivial kinematical factors to lend a field-theoretic precision to the otherwise intuitive QPCM concept.

$\omega\rho\pi$ coupling

For $\omega\rho\pi$ coupling, Fig. 1(b), a very similar mechanism is operative, with identical color and isospin factors. The BS matrix element, which is directly adaptable from (2.12) with the replacements $\rho\rightarrow\omega$, $\pi'\rightarrow\rho$, and $\pi''\rightarrow\pi$, reduces to the form

$$\mathscr{F}(\omega\rightarrow\rho\pi)=(2\pi)^8(\tfrac{2}{3})^{1/2}A_\omega A_\rho A_\pi\int d^4q\,\Delta^{-1}[\mathrm{Tr}(\omega\rho\pi)][D\phi(\omega\rho\pi)] , \tag{2.19}$$

where Δ is given by (2.14) and $[D\phi]$ by an expression similar to (2.15), but

$$[\mathrm{Tr}(\omega\rho\pi)]=-4m_q\epsilon_{\mu\nu\lambda\sigma}\rho_\mu\omega_\sigma P_\lambda Q_\nu . \tag{2.20}$$

Integration over dq_0 in a similar manner to $\rho\pi\pi$ and use of (2.16) gives

$$\mathscr{F}(\omega\rightarrow\rho\pi)=i^{-2}(2\pi)^6(\tfrac{2}{3})^{1/2}A_\omega A_\rho A_\pi\int d^3q\,\phi_\rho\phi_\pi D_\omega\phi_\omega[\mathrm{Tr}(\omega\rho\pi)](2p_{2+}') \tag{2.21}$$

which once again reveals a relativistic QPCM structure admitting of an identical interpretation to (2.18), except for a simpler structure (q-independent) for the trace factor.

$\omega\rightarrow\gamma\pi^0$ and $\pi^0\rightarrow\gamma\gamma$ couplings

To illustrate the working of the BS formalism for direct electromagnetic couplings, we have chosen two typical cases, viz., $\omega\rightarrow\gamma\pi^0$ and $\pi^0\rightarrow\gamma\gamma$ transitions corresponding to Figs. 1(c) and 1(d), respectively. The color factor in $\omega\rightarrow\gamma\pi^0$ is now

$$3^{-1/2}\delta_{ij}(\omega)3^{-1/2}\delta_{ij}(\pi^0)=1 \tag{2.22}$$

while in the $\pi^0\rightarrow\gamma\gamma$ case, it is

$$3^{-1/2}\delta_{ij}(\pi^0)\delta_{ij}=3^{1/2} . \tag{2.23}$$

Thus, as expected, the color factors for $\omega\rightarrow\gamma\pi^0$ and $\pi^0\rightarrow\gamma\gamma$ transitions are, respectively, $3^{1/2}$ and 3 times the color factor relevant for a purely hadronic coupling. (This is merely a check on the consistency of our use of the Kronecker δ's for color wave functions.) For the electromagnetic coupling of a quark we take

$$\mathscr{L}(\bar{q}\gamma q)=\tfrac{1}{2}e\bar{q}i\gamma_\mu A_\mu(\tau_3+\tfrac{1}{3})q \tag{2.24}$$

whence the entire charge factor for a $\omega\to\gamma\pi^0$ transition works out as e only, while for $\pi^0\to\gamma\gamma$ it is

$$2^{-1/2}e^2[(\tfrac{2}{3})^2-(-\tfrac{1}{3})^2]=e^2 6^{-1/2}\ . \tag{2.25}$$

The resultant matrix element, Fig. 1(c), for $\omega\to\gamma\pi^0$ is now

$$\mathscr{F}(\omega\to\gamma\pi^0)=(2\pi)^4 A_\pi A_\omega e\ \mathrm{Tr}\int d^4q[i\gamma_\mu A_\mu\bar{\Psi}_\pi(p_1',p_2')(m_q-i\gamma\cdot p_2)\Psi_\omega(p_1,p_2)]\ , \tag{2.26}$$

which reduces to the somewhat simpler form

$$\mathscr{F}(\omega\to\gamma\pi^0)=(2\pi)^4 A_\pi A_\omega e\int d^4q(2\pi i)^{-2}\Delta_0^{-1}D_\omega\phi_\omega D_\pi\phi_\pi[\mathrm{Tr}(\omega\gamma\pi^0)]\ , \tag{2.27}$$

where the trace factor is formally the same as (2.20), but with $\rho_\mu\to A_\mu$, and Δ_0 is given by (2.14) with $p_1''\to p_1$. The integration over q_0 in the null-plane language[21] and use of (2.16) gives

$$\mathscr{F}(\omega\to\gamma\pi^0)=i^{-1}(2\pi)^3 A_\pi A_\omega e\int d^3q\ \phi_\omega\phi_\pi[\mathrm{Tr}(\omega\gamma\pi)](2p_{2+}')\ . \tag{2.28}$$

The equation for $\omega\to\gamma\pi^0$ is simpler than the QPCM form (2.18) for its purely hadronic counterpart $\omega\to\rho\pi$ just to the extent expected, viz., ρ mesons's $q\bar{q}$ wave function and the D_ω function have together given place to the charge factor e characterizing the electromagnetic coupling of the *elementary* photon. Finally we record the matrix element for $\pi^0\to\gamma\gamma$, Fig. 1(d), in the form

$$\mathscr{F}(\pi^0\to\gamma\gamma)=6^{-1/2}e^2\mathrm{Tr}\int d^4q[\Psi_\pi(p_1,p_2)\gamma\cdot\epsilon_1 iS_F(q-Q)\gamma\cdot\epsilon_2+(1\rightleftarrows 2)]\ , \tag{2.29}$$

where $Q=k_1-k_2$ and the second term amounts to a reversal of the direction of the quark lines. Despite its apparent simplicity, the integrand has two q_0 poles, so that its structure is not amenable to the null-plane variable integration technique described prior to Eq. (2.16) and is best performed directly in terms of the q_0 variable, taking account of all its relevant poles. The calculation in this case is straightforward but messy. The final result may be conveniently given in terms of an *effective*, dimensionless coupling constant G_π, defined by the Lagrangian

$$\mathscr{L}(\pi^0\to\gamma\gamma)=6^{-1/2}e^2G_\pi\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu^{(1)}\epsilon_\nu^{(2)}P_\rho Q_\sigma/4\pi^2M\ , \tag{2.30}$$

and normalized to the experimental value

$$G_\pi(\mathrm{expt})=3.65 \tag{2.31}$$

to fit the observed $\pi^0\to\gamma\gamma$ width of 7.9 eV. The result for G_π after integration over q_0 in (2.27) is given by the formula

$$G_\pi/4\pi^2=\tfrac{5}{3}Mm_qA_\pi\int d^3q\ \phi_\pi\vec{q}^{\,2}\omega_q^{-2}(\omega_q^2-\tfrac{1}{3}\vec{q}^{\,2})^{-1}\ . \tag{2.32}$$

III. RESULTS AND DISCUSSION

We are now in a position to present the numerical results for the different transitions considered in the last section. The $\vec{q}$ integrations are all elementary, since only Gaussian functions and certain (positive) powers of the $\vec{q}$ variable are involved in each case. The $2p'_+$ factor needs a little care since, unlike all other factors it involves the q_0 variable whose "pole" value for the appropriate case must be substituted. For example, for $\rho\pi\pi$ coupling, this factor eventually simplifies to

$$2p_{2+}'=P_+'-P_+''-2q_+=2Q_3-2q_3+2\omega_q-M \tag{3.1}$$

and similarly for others. In all these cases a suitable *translation* in the $\vec{q}$ variable, designed to bring the $\vec{q}$ dependence of the Gaussian wave functions to the standard form is implied and its effect taken into account in all the other $\vec{q}$-dependent factors including $2p_{2+}'$, in each case. Further, for a hadronic (three-meson coupling) transition $M_1\to M_2+M_3$, the matrix element would have the following multiplicative Gaussian factor, arising out of the $\vec{q}$ translation:

$$F(\vec{P}_i^{\,2})\equiv\exp\left[-\tfrac{1}{16}\sum_{1,2,3}\vec{P}_1^{\,2}\beta_0^2\beta_2^{-2}\beta_3^{-2}\right]\ ,\qquad 2\beta_0^{-2}=\beta_1^{-2}+\beta_2^{-2}+\beta_3^{-2}\ , \tag{3.2}$$

where β_i $(i=1,2,3)$ is the inverse radius of the meson M_i of three-momentum $\vec{P}_i$, in accordance with Eq. (2.3). This formula covers both $\rho\to\pi\pi$ (real) and $\omega\to\rho\pi$ (virtual) transitions. Now if the coupling is calculated on this (three-dimensional) basis the predicted strength comes out too small, as measured, e.g., by a $\rho\to\pi\pi$ width of a mere 42.8

MeV. This shortcoming of the three-dimensional BS formalism seems to have a precise counterpart in the nonrelativistic version of QPCM, as reflected in the corresponding Gaussian form factor in terms of the three-momenta of the hadrons concerned.[6,11] A practical solution to this QPCM malady that was suggested some time ago[24,25] consists in a four-dimensional Lorentz-invariant adaptation from the three-dimensional QPCM counterpart of (3.2) to its four-dimensional QPCM counterpart, via the replacements[24]

$$\vec{P}_i^{\,2} \Longrightarrow P_{\mu i}^{\,2} = -M_i^{\,2} \ . \tag{3.3}$$

As explained earlier,[25] this extrapolation is not meaningful *except* for modest values of M^2 ($\lesssim 1$ GeV^2). Further, as a compensation for this effect one needs a renormalization with respect to a reference coupling,[8,25] say $\rho\rho\rho$, at an unphysical point. In view of the unqualified success of this prescription at the QPCM level[24,25] we are inclined to adopt the same point of view in the present case as well. However, for the reference $\rho\rho\rho$ coupling we now choose the more symmetrical point $(-m_\rho^{\,2}, -m_\rho^{\,2}, -m_\rho^{\,2})$ for each $P_\mu^{\,2}$, instead of the less symmetrical QPCM choice[8,24] $(-m_\rho^{\,2}, -m_\rho^{\,2}, 0)$ based on a preferential treatment to the exchanged ρ quantum. This renormalization amounts to a multiplication of (3.2) by the factor

$$Z = \exp(-\tfrac{3}{16} m_\rho^{\,2} \beta_\rho^{\,-2}) = [\exp(1.179)]^{-1} \tag{3.4}$$

after substitution of the four-momenta (3.3) for the various three-momenta. Adopting this prescription, the standard $\rho\pi\pi$ coupling constant,[27] which may be identified through

$$\mathscr{F}(\rho \to \pi\pi) \equiv g_{\rho\pi\pi}(2\vec{\rho}\cdot\vec{Q}) \ , \tag{3.5}$$

is expressible as

$$g_{\rho\pi\pi} = 2(\tfrac{2}{3})^{1/2}(2\pi)^6 A_\rho A_\pi^{\,2}(\pi^3 \beta_\rho^{\,2} \beta_\pi^{\,4})^{-3/4} \mathscr{I}_\rho Z F(P_{\mu i}^{\,2}) \ , \tag{3.6}$$

$$\vec{\rho}\cdot\vec{Q}\mathscr{I}_\rho = \int d^3q \exp(-\vec{q}^{\,2}/\tilde{\beta}^{\,2}) \times \{\tfrac{1}{4}[\mathrm{Tr}(\rho\pi\pi)] D_\rho(2p'_{2+})\} \ , \tag{3.7}$$

the last three factors in (3.7), all defined earlier, to be understood to have been given the requisite $\vec{q}$ translation, and

$$\tilde{\beta}^{\,-2} = \tfrac{1}{2}\beta_\rho^{\,-2} + \tfrac{1}{2}\beta_\pi^{\,-2} + \tfrac{1}{2}\beta_\pi^{\,-2} \ . \tag{3.8}$$

The expression (3.6) may be compared with the formal expression for g_ρ, the coupling strength for ρ annihilation into the vacuum, given in this BS model by[22]

$$g_\rho^{\,-1} = \frac{6^{1/2} m_\rho^{\,-5/2}(\beta_\rho^{\,2}/\pi)^{3/4}(m_q^{\,2} + \beta_\rho^{\,2} + \frac{1}{4} m_\rho^{\,2})}{(m_q^{\,2} + \beta_\rho^{\,2})^{1/2}} \ . \tag{3.9}$$

An idea of their relative numerical values is obtained through the comparison

$$g_{\rho\pi\pi} = 1.126 g_\rho, \quad g_\rho^{\,2}/4\pi = 2.14 \ . \tag{3.10}$$

This value of $g_{\rho\pi\pi}$ corresponds to a $\rho\pi\pi$ width of 142.7 MeV, in reasonable accord with the observed value[26] of 158 ± 5 MeV. On the other hand, the parameter g_ρ ($< g_{\rho\pi\pi}$), as discussed in Ref. 22 gives the correct value of the $\rho \to e^+e^-$ width at 6.43 keV (6.54 ± 0.5). This twin capacity to provide a basically correct pattern of values for both these fundamental parameters must be regarded as a welcome feature of the proposed BS dynamics.[28]

We state, without further explanation the analogous result for $g_{\omega\rho\pi}$ defined[27] by

$$\mathscr{F}(\omega \to \rho\pi) = 2m_\omega^{\,-1} g_{\omega\rho\pi} \epsilon_{\mu\nu\lambda\sigma} P_\mu \omega_\sigma P_\lambda P'_\nu \tag{3.11}$$

which may be expressed by a formula similar to (3.6) obtained with the same ansatz (3.3) on an expression analogous to (3.2), and renormalized by the same Z factor (3.4). The numerical value now works out as

$$g_{\omega\rho\pi} = (1.011) g_\rho \approx g_\rho \ . \tag{3.12}$$

As the first example of electromagnetic decays we consider Eq. (2.28) for $\omega \to \gamma\pi^0$, whose matrix element, among other things will now contain the simpler Gaussian factor (involving only the photon momentum $\vec{k}$)

$$F_\gamma(\vec{k}^{\,2}) = \exp(-\vec{k}^{\,2}\beta_0^{\,2}/16\beta_\pi^{\,2}\beta_\omega^{\,2}) \tag{3.13}$$

as a substitute for (3.2), where

$$\beta_0^{\,-2} = \tfrac{1}{2}\beta_\omega^{\,-2} + \tfrac{1}{2}\beta_\pi^{\,-2} \ . \tag{3.14}$$

In this case the elaborate procedure (3.2)—(3.4) is no longer necessary but may be replaced by the simpler prescription[22]

$$\vec{k}^{\,2} \Longrightarrow k_\mu^{\,2} = 0$$

so that (3.13) merely reduces to unity. The amplitude (2.28) then works out

$$\mathscr{F}(\omega \to \gamma\pi^0) = i^{-1}(2\pi)^3 e A_\pi A_\omega [\mathrm{Tr}(\omega\gamma\pi)] \times \langle 2p'_{2+} \rangle (\beta_0^{\,4} \beta_\pi^{\,-2} \beta_\omega^{\,-2})^{3/4} \ , \tag{3.15}$$

where the factor $\langle 2p'_{2+} \rangle$ represents the expectation value of that quantity (after $\vec{q}$ translation) with respect to the $\phi_\pi \phi_\omega$ distribution. The numerical effect is best seen by writing (3.15) in the form (3.11) with the replacement

$$g_{\omega\rho\pi} \rightarrow g_{\omega\gamma\pi} = ef, \quad f = 1.01 , \tag{3.16}$$

which corresponds to a $\omega \rightarrow \gamma\pi^0$ width of 0.888 MeV, in almost exact agreement with experiment[26] (0.89±0.04).

It is of interest to ask if (and to what extent) our formalism checks with the VMD ansatz expressed in the Schwinger[23] language of electromagnetic substitution, viz.,

$$\rho_\mu \rightarrow e g_\rho{}^{-1} A_\mu, \quad \omega_\mu \rightarrow \tfrac{1}{3} e g_\rho{}^{-1} A_\mu . \tag{3.17}$$

A comparison of (3.16) obtained with direct photon coupling, Fig. 1(c), with Schwinger's corresponding expression for $\omega\gamma\pi^0$ coupling (obtained from $\omega\rho\pi$ via VMD), shows only a 1% difference. On the other hand, (3.12) shows that if we had tried to simulate $\omega\gamma\pi^0$ coupling from Eq. (3.11), via the VMD substitution (3.17), we would once again have obtained only a 1% increase over Schwinger's $\omega\gamma\pi^0$ amplitude. We thus arrive at a rather pleasant VMD consistency check on our formalism at least for the process $\omega \rightarrow \gamma\pi^0$.

However, for the $\pi^0 \rightarrow \gamma\gamma$ amplitude we encounter the following problem. A direct computation of the integral (3.32) leads to the estimate

$$G_\pi = 1.746 \tag{3.18}$$

which is just about half the experimental value 3.65, Eq. (2.31). On the other hand if, emboldened by the above success of VMD for the $\omega \rightarrow \gamma\pi^0$ case, we had, following Schwinger,[23] attempted the double VMD substitution (3.17) in the $\omega \rightarrow \rho\pi$ coupling to obtain the $\pi^0 \rightarrow \gamma\gamma$ amplitude, the results (3.11) and (3.12) would show that our $\pi^0 \rightarrow \gamma\gamma$ amplitude would have been only about 2% higher than Schwinger's VMD value, and would therefore have led to an estimated $\pi^0 \rightarrow \gamma\gamma$ width of 7.7 eV to be compared with Schwinger's 7.4 eV and the observed 7.9 eV.[26]

We are thus faced with a twofold problem viz., (a) the $\pi^0 \rightarrow \gamma\gamma$ amplitude via direct photon coupling is merely half of the observed value, while (b) the double VMD, substitution in the $\pi^0 \rightarrow \omega\rho$ amplitude following Schwinger[23] gives the observed strength, thus exhibiting a factor of 2 discrepancy with the direct photon coupling mechanism. An understanding of problem (a) is facilitated by recognizing the role of the *valence* contribution to the $\gamma\gamma$ decay of π^0 as a $q\bar{q}$ state, which by gauge invariance[29] is expected to be half the total decay amplitude for $q^2 \rightarrow 0$.[30] Now, in our BS formalism we have not considered anything but the valence quarks, so the result (3.18) which is just in accord with the above theoretical expectation must be interpreted as just the valence-quark contribution. Such a low amplitude for the valence-quark contribution to $\pi^0 \rightarrow \gamma\gamma$ decay via direct photon coupling is a reflection of the severity of the low-energy constraints on the pion's wave function.[30]

On the other hand, problem (b) remains: why should the double electromagnetic substitution in $\pi^0\rho_\omega$ coupling yield the desired strength for $\pi^0 \rightarrow \gamma\gamma$? A possible suggestion is that the *valence* contributions to the ω and ρ wave functions in the $\pi^0\rho\omega$ vertex are a more reliable index of their *total* contributions than is the case for the pion. And since the contribution of the pion wave function to the $\omega \rightarrow \rho\pi^0$ overlap integral is relatively small, the low-energy constraint on its valence wave function is much less effective in reducing the strength of the $\pi^0\rho_\omega$ vertex than is the case with the $\pi^0 \rightarrow \gamma\gamma$ amplitude via direct photon coupling where no other hadronic state is available to cushion the pionic attenuation effect. Since no further reduction in strength is caused by the double VMD substitution in $\pi^0\rho_\omega$, it need not be surprising if the $\pi^0 \rightarrow \gamma\gamma$ amplitude based on the empirical VMD mechanism (numerically tested in many situations) happens to provide the correct answer by thus circumventing the problem of pionic attenuation.

We end this section with some comments on the theoretical status of the instantaneous approximation, the key ingredient of the BS formalism[16,22] which underlies the structure of hadronic matrix elements given in Sec. II. The main points of departure of our approach to the instantaneous approximation from the more conventional ones[31] (characterized by the standard reduction to $++$ and $--$ components[19] of the $q\bar{q}$ wave function) have been discussed more fully in Refs. 16 and 22, as well as in an unpublished report.[32] Although these questions strictly do not fall within the purview of the present (applicationally oriented) paper, we repeat a few salient points in defense of the structure of our matrix elements *vis-à-vis* those likely to arise from more orthodox instantaneous-approximation treatments[31] (which usually leave out the $+-$ and $-+$ components).

The entire issue boils down to two related questions:

(a) Why should one want to depart from the orthodox instantaneous-approximation formalism[19] at all, and how does one account for the difference so produced?

(b) Is such a variation permissible within the acceptable limits of the instantaneous-approximation definition?

As to question (a) the main motivation for our alternative approach to the instantaneous approximation is structural simplicity of the $q\bar{q}$ BS equation which arises out of a Gordon reduction of the γ matrices prior to the introduction of the instantaneous approximation.[16] The same step also results in a remarkable degree of transparency in the structure of the $\bar{q}Mq$ vertex functions[22,32] which thus becomes directly amenable to the language of Feynman diagrams. The corresponding structures obtained from the more orthodox instantaneous-approximation approach involving only the $(++,--)$ components are algebraically more complicated. The difference is traceable to certain $(+-,-+)$ components which are absent in the orthodox instantaneous-approximation treatment but seem to enter our formalism in definite proportions determined entirely by the Gordon reduction procedure (which is otherwise unambiguous in prescription). Now if the sole purpose of the BS method were to handle $q\bar{q}$ systems only, it would be hard to justify such a modification on conventional instantaneous-approximation treatment, despite its appeal of simplicity and transparency. However, it has been argued elsewhere[13] that the BS approach, to be physically meaningful, should be applicable to bigger N-quark systems $(N>2)$, such as demonstrated for qqq (Ref. 17) and $qq\bar{q}\bar{q}$ (Ref. 18) systems without excessive effort. And the differential advantages of the Gordon-reduction treatment over the $(\pm)$ component reduction for each quark, become increasingly apparent[17,18] as N increases beyond 2. As far as our experience goes, the above statement is certainly true for the harmonic kernel[17,18] and may well hold for other shapes, too.

As to question (b) regarding the permissibility of our version of the instantaneous approximation, the (surreptitious) entry of certain $(+-,-+)$ components, albeit in definite proportions, through the Gordon reduction discussed above, technically amounts to the introduction of noninstantaneous effects. In this connection it is good to recall that since the original proposal,[19] the definition of the instantaneous approximation itself has undergone several vicissitudes, inevitably bringing in noninstantaneous effects in their wake. A detailed discussion on the subject, including the precise mode of appearance of noninstantaneous effects, has been given in Fishbane and Namyslowski.[21] The nature and extent of the noninstantaneous effects brought into our formalism through the Gordon reduction seems to be just of the same order as discussed by these authors,[21] and to that extent constitutes a defense of the former, within the "acceptable" limits of the definition of the instantaneous approximation.

Finally our alternative approach to the instantaneous approximation has proved of some benefit from another angle, too, viz., in extending the algebraic correspondence between the instantaneous-approximation language and the null-plane language from the known case of scalar quarks[21] to that of spinor quarks.[22,32] (This contrasts with the situation with the conventional instantaneous-approximation formalism where, from the very nature of the reduction procedure, considerable departures from such similarity are expected.) This similarity has already been exploited in Sec. II in connection with the integration over the poles in the null-plane language.[33]

To summarize, we have tried to provide a field-theoretic rationale for QPCM using a three-dimensional BS formalism for $q\bar{q}$ states developed recently.[16,22] The central features of QPCM are all reproduced, together with a couple of kinematical factors appearing in the numerator of the resulting overlap integral. Numerical evaluation has had to be preceded by an ansatz for a Lorentz-invariant adaptation of the Gaussian form factor characterizing the hadronic matrix elements, similar to one which had been employed earlier at the phenomenological QPCM level[8,24,25] for a successful description of a number of hadronic and electromagnetic transitions. Not only are the results for $\rho\to\pi\pi$ (142.7 MeV) and $\omega\to\gamma\pi^0$ (888 keV) widths in excellent accord with experiment[26] without adjustable parameters, but the $\omega\to\gamma\pi^0$ amplitude also checks with the VMD hypothesis[23] on $\omega\to\rho\pi$ to within 1%. For the $\pi^0\to\gamma\gamma$ amplitude, too, our calculated 50% value based on direct photon coupling, compared with the full strength obtainable from $\omega\rho\pi$ coupling via double electromagnetic substitution,[23] harmonizes rather well with theoretical expectations.[29,30] These illustrative results should hopefully pave the way for a viable calculational program for many more related processes, including strange-meson couplings (involving unequal-mass kinematics), L-excited hadron decays, QPCM formulation of baryonic decays, and so on. Some of these applications are under way.

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