

Lepton-number violation with quasi-Dirac neutrinos

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We investigate lepton-number violation in weak interactions with massive Dirac neutrinos. In the framework of a simple $SU(3)_L \times U(1)$ electroweak model we find that neutrinos naturally acquire only Dirac masses at tree level, and for an odd number of lepton families, one neutrino remains massless. After a spontaneous breakdown of symmetry we find that the $SU(2) \times U(1)$ effective theory has lepton-number-violating currents which couple to the standard gauge bosons. Flavor-conserving $|\Delta I| = 2$ processes such as neutrinoless double- β decay are forbidden in the simplest model, but processes such as $\mu^- + (A, Z) \rightarrow e^+ + (A, Z - 2)$ and $K^- \rightarrow \pi^+ e^- \mu^-$ are allowed.

I. INTRODUCTION

In the standard $SU(2) \times U(1)$ model of weak and electromagnetic interactions,¹ neutrinos are assumed to be massless since they are described by two-component van der Waerden spinors, and since the simplest Higgs structure for the standard model forbids lepton-number-violating interactions. Neutrino masses are generated by (a) the addition of a complex Higgs triplet, or (b) the addition of right-handed neutrino fields, or both.² Case (a) generates a lepton-number-violating Majorana mass. So does case (b), since in this case a bare Majorana mass term involving right-handed neutrinos is allowed by gauge invariance even in the absence of extra Higgs scalars. Majorana neutrino masses can thus be considered a natural consequence of gauge theories. Lepton-number-violating effects, such as neutrino-produced antileptons and neutrinoless double- β decay, are taken as evidence of a Majorana mass term for neutrinos.

It is now realized^{2,3} that theories with Majorana masses are much more complex in structure than theories with pure Dirac masses. It is therefore interesting to explore the idea of lepton-number-violating effects without Majorana masses: lepton-number violation arises from a lepton-number-violating current in the effective weak interaction, and the neutrinos naturally acquire only Dirac masses at tree level. It is shown here how this can be achieved within the framework of a previously proposed $SU(3)_L \times U(1)$ model.⁴ The novel feature here is that lepton number is a local gauge symmetry whose spontaneous breakdown occurs primarily in the interactions: to lowest order the free Lagrangian naturally conserves lepton number while the low-energy weak interactions do not. This scenario contrasts with the usual one in which lepton number is broken at tree level by Majorana-type masses.

The theory incorporates three generations of quarks and leptons, it contains no right-handed singlet neutrino fields, and with the minimal set of three Higgs triplets can give Dirac masses to *all* the fermions. Since the right-handed neutrinos are not singlets, they undergo gauge interactions

at the new lepton-number-violating mass scale. Lepton-number-violating mass terms can arise from calculable radiative corrections. Since these corrections are very small, lepton-number-violating effects arise predominantly from gauge-boson interactions, and neutrinos can be described to a good approximation as Dirac particles.

The plan of this paper is as follows. Section II gives the bare ingredients of the model, including all the fermions and Higgs scalars needed. Section III describes the gauge-boson masses, fermion masses, and mixing angles, and Sec. IV describes some neutrino phenomenology.

II. MODEL

We consider the gauge group $SU(3)_L \times U(1)$ as our model of the weak and electromagnetic interactions.⁴ The three families of left-handed leptons are assumed to belong to three antitriplets as follows:

$$\psi_{(a)} = \begin{bmatrix} E_\alpha \\ N_\alpha \\ N'_\alpha \end{bmatrix}_L, \quad \alpha = 1, 2, 3. \quad (2.1)$$

The first two entries in each antitriplet are the usually defined particles while the third is a neutral two-component lepton field. In the next section we shall be able to identify the primed fields as the left-handed charge conjugates of the unprimed neutrino fields. The cancellation of γ_5 anomalies⁵ in models of this type can be achieved by requiring an equal number of triplets and antitriplets, and requiring that the sum of the electric charges on all the fermions vanishes.⁶ We thus put the first two families of left-handed quarks into six (counting color) triplets

$$\begin{bmatrix} u \\ d \\ d' \end{bmatrix}_L, \quad \begin{bmatrix} c \\ s \\ s' \end{bmatrix}_L. \quad (2.2)$$

The d' and s' are two new heavy $-\frac{1}{3}$ charged quarks. The members of the third family of quarks are placed in three antitriplets:

$$\begin{pmatrix} b \\ t \\ t' \end{pmatrix}_L, \quad (2.3)$$

where the t and t' are two heavy $+\frac{2}{3}$ charged quarks. In a model of this kind the triangle anomalies are not canceled generation by generation. One cannot arbitrarily remove any one generation of quarks and leptons and still have an anomaly free model. Finally, all the right-handed components of the charged fields are placed in singlets. There are no neutral right-handed singlets.

In order to generate masses for all the charged fermions, we require at least three triplets of Higgs fields $f^{(1)}$, $f^{(2)}$, and $f^{(3)}$. Their electric charges are

$$f^{(1)} \rightarrow \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}, \quad f^{(2)}, f^{(3)} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (2.4)$$

and their vacuum expectation values are

$$\langle f^{(1)} \rangle_0 = \begin{pmatrix} k_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle f^{(2)} \rangle_0 = \begin{pmatrix} 0 \\ k_2 \\ n \end{pmatrix}, \quad (2.5)$$

$$\langle f^{(3)} \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ k_3 \end{pmatrix}.$$

These are the most general vacuum expectation values consistent with electromagnetic gauge invariance.

The eight gauge bosons of SU(3) are denoted by

$$W_{a\mu}^b; \quad \sum_a W_{a\mu}^a = 0, \quad a, b = 1, 2, 3$$

and the singlet field is denoted by B_μ . At this point we can separate the nine gauge bosons into three distinct groups denoted by charge and CP transformations. We thus obtain

$$(i) \quad W_1^2, W_1^3; \quad W_3^1, W_2^1;$$

$$(ii) \quad B,$$

$$F = -\left(\frac{3}{2}\right)^{1/2} W_1^1,$$

$$H = \frac{1}{\sqrt{2}} (W_2^2 - W_3^3), \quad (2.6)$$

$$X_1 = \frac{1}{\sqrt{2}} (W_2^3 + W_3^2);$$

$$(iii) \quad X_2 = \frac{1}{\sqrt{2}} i (W_2^3 - W_3^2);$$

where we have dropped the Lorentz indices for ease of notation. In the absence of CP violation there is no mixing of X_2 with any of the fields in (ii). Linear combinations of B and F above are defined through

$$\begin{pmatrix} F \\ B \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A \\ X_0 \end{pmatrix}, \quad (2.7)$$

where A is the photon field. The fields X_0 , X_1 , and H can mix with each other to form heavy neutral gauge fields Z , $Z^{(1)}$, and $Z^{(2)}$.

Finally, the SU(3) coupling constant g and the U(1) cou-

pling constant g' are specified by defining the gauge-covariant derivative of the Higgs scalar $f^{(3)}$ as

$$(D_\mu f^{(3)})_a = \partial_\mu f_a^{(3)} - \frac{i}{\sqrt{2}} g W_{a\mu}^b f_b^{(3)} + i \frac{1}{3} g' B_\mu f_a^{(3)}. \quad (2.8)$$

The coupling constants are related to the magnitude of the electron charge $|e|$ by

$$g' \sin\phi = \frac{\sqrt{3}}{2} g \cos\phi = -|e|. \quad (2.9)$$

The convenient parameter⁴

$$x = \frac{3}{4} \cos^2\phi \quad (2.10)$$

has the same significance as the standard $x = \sin^2\theta_W$, but with the limit $0 < x \leq \frac{3}{4}$.

III. BOSON AND FERMION MASSES

The terms in the Lagrangian

$$- \sum_{j=1}^3 (D_\mu f^{(j)})^\dagger (D_\mu f^{(j)}) \quad (3.1)$$

along with the vacuum expectation values in Eq. (2.5) generate the gauge-boson masses. We shall assume that $k_3 \gg (k_1, k_2, n)$ so that, essentially, our model will break down to the standard low-energy (100 GeV) theory.⁷ The three light intermediate vector bosons W^+ , W^- , and Z are

$$W^+ = W_1^2 + \epsilon W_1^3,$$

$$W^- = W_2^1 + \epsilon W_3^1, \quad (3.2)$$

$$Z_0 = X_0 \left[\frac{\sqrt{3} \sin\phi + \alpha}{(3 \sin^2\phi + 1)^{1/2}} \right] + H \left[\frac{1 - \alpha\sqrt{3} \sin\phi}{(3 \sin^2\phi + 1)^{1/2}} \right]$$

$$+ \beta X_1.$$

The small mixing parameters α, β, ϵ are given by

$$\alpha = \frac{\sqrt{3} \sin\phi}{(3 \sin^2\phi + 1)^2} \left[4 \frac{k_1^2}{k_3^2} + 2(1 - 3 \sin^2\phi) \frac{k_2^2}{k_3^2} \right], \quad (3.3)$$

$$\epsilon = (1 - x)^{1/2} \beta = -\frac{k_2 n}{k_3^2}. \quad (3.4)$$

The masses also obey the standard mass relation

$$m^2(W^\pm) = m^2(Z)(1 - x) = \frac{g^2}{2} (k_1^2 + k_2^2). \quad (3.5)$$

The small parameter ϵ gives an effective coupling between the primed and unprimed fermions, and will describe the flavor-changing weak interactions of these fermions. The parameter α is a small correction to the flavor-conserving neutral currents. All the other massive gauge bosons will have masses proportional to $(k_3)^2$, and will not contribute significantly to the effective interaction Lagrangian. For completeness, we have, neglecting quadratic terms in the small parameters,

$$\begin{aligned}
W'^+ &= W_1^3 - \epsilon W_1^2, \\
W'^- &= W_3^1 - \epsilon W_2^1, \\
Z^{(1)} &= -\frac{-1 + \alpha\sqrt{3}\sin\phi}{(3\sin^2\phi + 1)^{1/2}} X_0 - \frac{\sqrt{3}\sin\phi + \alpha}{(3\sin^2\phi + 1)^{1/2}} H \\
&\quad + \sqrt{3}\sin\phi\beta X_1, \\
Z^{(2)} &= \frac{-2\beta\sqrt{3}\sin\phi}{(3\sin^2\phi + 1)^{1/2}} X_0 - \frac{\beta(3\sin^2\phi - 1)}{(3\sin^2\phi + 1)^{1/2}} H \\
&\quad + X_1,
\end{aligned} \tag{3.6}$$

and the masses of all the heavy gauge bosons are

$$\begin{aligned}
m^2(W'^{\pm}) &= m^2(X_2) = m^2(Z^{(2)}) \\
&= \frac{3\sin^2\phi}{3\sin^2\phi + 1} m^2(Z^{(1)}) \\
&= \frac{g^2}{2} (k_3)^2.
\end{aligned} \tag{3.7}$$

The three sets of Higgs scalars $f_{(n)}^a$ also generate fermion masses. If we designate $(\psi_L^{(m)})^a$ as a typical fermion triplet and $S_R^{(p)}$ as a right-handed charged fermion [which transforms as a singlet under $SU(3)_L$], then we see that Yukawa terms like

$$\sum_{m,n,p} g_{mnp} (\bar{\psi}_L^{(m)})^a f_{(n)}^a S_R^{(p)} + \text{H.c.}, \tag{3.8}$$

will generate masses for all the charged fermions. The constants g_{mnp} are the appropriate couplings.

We shall assume negligible mixing between primed and unprimed quarks. This is perhaps an inelegant feature of the model, but it is not unreasonable considering the fact that the primed quarks are very massive.⁸

In addition to (3.8), we can construct another Yukawa term for the leptons of the form

$$\frac{1}{2} \sum_{\alpha,\beta} \epsilon^{abc} g_{\alpha\beta} \psi_{(\alpha)}^{T a} C^{-1} \psi_{(\beta)}^b f_{(1)}^{c*} + \text{H.c.}, \tag{3.9}$$

where C is the charge-conjugation matrix and $g_{\alpha\beta} = -g_{\beta\alpha}$ are the coupling constants. This term couples the primed and unprimed neutrino fields together. Equation (3.9) creates Dirac masses for the neutrinos when we identify

$$(N'_{\alpha L}) \equiv (N_{\alpha}^c)_L = (N_{\alpha R})^c, \tag{3.10}$$

where N^c is the charge conjugate of N

$$N^c = C\bar{N}^T.$$

The mass matrix for the neutrinos then becomes

$$-\mathcal{L}_{\text{mass}} = \sum_{\alpha,\beta} (\bar{N}_{\alpha})_R M_{\alpha\beta} (N_{\beta})_L + \text{H.c.}, \tag{3.11}$$

$$M_{\alpha\beta} = g_{\alpha\beta} k_1. \tag{3.12}$$

Since the mass matrix is an antisymmetric 3×3 matrix, one of the mass eigenvalues must be zero and the other two are degenerate. In fact, given any $N \times N$ antisymmetric matrix M , we have

$$\begin{aligned}
\text{Det}(M - \lambda I) &= \text{Det}(M - \lambda I)^T \\
&= \text{Det}(-M - \lambda I) \\
&= (-1)^N \text{Det}(M + \lambda I)
\end{aligned} \tag{3.13}$$

so that the nonzero eigenvalues of M can be arranged in pairs, $W, -W$. If N is odd, one of the eigenvalues must be zero. Signs here are unphysical and one may choose phases in the diagonalizing matrices in such a way that all eigenvalues are real and positive. It follows then that for an odd number of lepton families one neutrino is massless while the others are doubly degenerate.

Since we have placed both particle and antiparticle in the same multiplet, radiative corrections can induce lepton-number-violating Majorana mass terms as first noticed by Wolfenstein.⁹ A massless Dirac neutrino at tree level will not remain massless. As we shall show in Sec. IV, these radiative corrections are extremely small, and we are perfectly justified in ignoring them at this time.

The weak neutrino eigenstates N_{α} in (2.1) are related to the (Dirac) mass eigenstates ν_{α} by a unitary transformation¹⁰ $N_L = U_L \nu_L$ ($\alpha = 1, 2, 3$).

Substituting this into (3.11) we have

$$(U_R)^{\dagger} M (U_L) = D, \tag{3.14}$$

where the diagonal matrix D is of the special form

$$D = \text{diagonal}(0, m, m) \tag{3.15}$$

as a result of the skew symmetric nature of the Yukawa coupling in Eq. (3.9).

Notice that the diagonalizing matrices U_L and U_R are not unique since one can always perform an arbitrary rotation on the eigenstates of the two degenerate mass eigenvalues. This freedom will be used later in order to simplify the parametrization (4.3) for the leptonic mixing matrix, Eq. (4.2). In addition the antisymmetric form of M in Eq. (3.12) lets us always write

$$U_L = U_R^* S = U_R^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \tag{3.16}$$

for any parametrization of U_L .

IV. NEUTRINO PHENOMENOLOGY

Since we are dealing with massive neutrinos we expect leptonic mixing to be nontrivial. Indeed, the usual charged-current weak interactions are described by a 3×3 unitary matrix K ;

$$\mathcal{L}_{\text{CC}}(\Delta I = 0) = \frac{ig}{\sqrt{2}} W_{\mu} \bar{e}_L \gamma_{\mu} K \nu_L + \text{H.c.} \tag{4.1}$$

given by

$$K = \Omega_L^{\dagger} U_L, \tag{4.2}$$

where Ω_L is a unitary matrix which defines the physical mass eigenstate fields e_L in terms of the weak eigenstates E_L , Eq. (2.1),

$$E_L = \Omega_L e_L,$$

and U_L is a similar transformation on the left-handed neutrino fields [Eq. (3.14)].

Due to the special form of the neutrino mass matrix in this model, the unitary matrix K is in fact much simpler than the Cabibbo-Kobayashi-Maskawa matrix for the quarks. In addition to the possibility of performing diagonal phase transformations, the degeneracy between ν_2 and

ν_3 allows one to perform an arbitrary unitary transformation in the ν_2 - ν_3 subspace so as to bring K to a canonical form of the type

$$K = \begin{pmatrix} cc' & s & s'c \\ -sc' & c & -ss' \\ -s' & 0 & c' \end{pmatrix}, \quad (4.3)$$

where $c \equiv \cos\psi$, $c' \equiv \cos\psi'$, etc. Notice that K is real (CP conservations) and depends on only two mixing angles ψ and ψ' . This is considerably simpler than the corresponding situation for massive Majorana neutrinos.^{2,3}

We now turn to the new lepton-number-violating gauge interactions. Since in our original assignments for the lepton fields, Eqs. (2.1) and (3.10), we have placed both particle and antiparticle in the same multiple, we expect lepton number to be violated in the gauge sector in a natural way. The effective $\Delta I=2$ charged-current interaction Lagrangian will be

$$\mathcal{L}_{CC}(\Delta I=2) = \frac{ig}{\sqrt{2}} \epsilon W_\mu \bar{e}_L \gamma_\mu K' (\nu^c)_L + \text{H.c.} \quad (4.4)$$

Notice it is proportional to the small parameter ϵ of Eq. (3.4). The matrix K' is given by

$$K' = \Omega_L^\dagger U_R^*, \quad (4.5)$$

where U_R is the unitary matrix in (3.14) which transforms the right-handed neutrinos. From (4.2), (4.5), and (3.16) one has the relation

$$KS^T = K'. \quad (4.6)$$

The above equations have interesting phenomenological consequences.

A. Neutrino oscillations

Since leptonic mixing, given in (4.1)–(4.3) is nontrivial, this model has neutrino oscillations.^{10–12} If at time $t=0$ a neutrino is produced in association with a type “ α ” electron, the probability amplitude for it to produce, after traveling a distance $L \approx t$, a type- β , electron is

$$A(\alpha \rightarrow \beta; t) \propto \sum_\gamma K_{\alpha\gamma} K_{\beta\gamma} e^{-iE_\gamma t}, \quad (4.7)$$

where the usual approximation is

$$E_\gamma \approx E + \frac{m_\gamma^2}{2E} \quad (4.8)$$

which holds when $m_\gamma \ll E$ where E is the neutrino energy. Due to the unitarity of K , Eq. (4.7) can be rewritten as

$$A(\alpha \rightarrow \beta; L) \propto \delta_{\alpha\beta} + R(L) \sum_{\gamma=2,3} K_{\alpha\gamma} K_{\beta\gamma}, \quad (4.9)$$

where the oscillating factor R is, in our case,

$$R(L) = \exp \left[-im^2 \frac{L}{2E} \right] - 1, \quad m \ll |\vec{p}|. \quad (4.10)$$

Thus, ordinary neutrino oscillations are described by a single mass parameter and only two mixing angles. It is easy to see then that the oscillations between ν_μ and ν_τ are suppressed (by small mixing angles) relative to ν_e - ν_μ and ν_e - ν_τ oscillations. This is consistent with experiment.¹³

Similar features were discussed in a phenomenological model by Petcov.⁹

B. Neutrino masses from β decay

A very sensitive method of measuring the mass of the electron neutrino has been to observe the deviations from a straight-line Kurie plot near the end point for the β decay of tritium.¹⁴ The Kurie plot, in the presence of neutrino mixing, depends on the mixing angles and masses of all the neutrino mass eigenstates which couple to the electron. The end point of the Kurie spectrum is determined by the lightest neutrino, but the shape is determined by all the neutrinos.¹⁵

When a neutrino of mass m_α is emitted in β decay, the Kurie function K takes the form

$$K_\alpha^2 = F^2 \Delta (\Delta^2 - m_\alpha^2)^{1/2} \theta(\Delta - m_\alpha), \quad (4.11)$$

where $\Delta = E_0 - E_\beta$. Here E_0 is the maximum allowed electron kinetic energy and E_β is the kinetic energy of the electron. F is the nuclear Coulomb factor.

When there is neutrino mixing, the neutrino which couples to the electron is a linear combination of mass eigenstates ν_α . The Kurie function then becomes

$$K^2 = \sum_\alpha P_\alpha K_\alpha^2 = \sum_\alpha P_\alpha \Delta (\Delta^2 - m_\alpha^2)^{1/2} \theta(\Delta - m_\alpha) F^2, \quad (4.12)$$

where P_α is the probability that the neutrino ν_α is emitted in β decay. From Eqs. (4.1) and (4.3) one has

$$\frac{K^2}{F^2} = (1 - c^2 c'^2) \Delta (\Delta^2 - m^2)^{1/2} \theta(\Delta - m) + c^2 c'^2 \Delta^2. \quad (4.13)$$

As we see the deviation from a straight line near the end point of the Kurie plot denotes a massive neutrino, but the poor statistics at the end point and the finite resolution of any experiment make it quite hard to see.

We also mention that in the present model the lepton-number-violating β decay $n \rightarrow pe\nu_e$ would also occur due to Eq. (4.4), but with a smaller rate (see next section). The effect of including this mode in our calculation is to multiply the right-hand side of Eq. (4.13) by $(1 + \epsilon^2)$ (see next section).

C. $\Delta I=2$ interactions

From Eq. (4.4) we see that lepton number is violated in charged-gauge-boson exchange. Thus, in addition to the usual weak decays there will be lepton-number-violating decays which proceed with a strength proportional to the small parameter ϵ of Eq. (3.4). For example, in the case of π^+ decay in Figs. (1a) and (1b) we have

$$\frac{R(\pi^+ \rightarrow e^+ \bar{\nu}_\alpha)}{R(\pi^+ \rightarrow e^+ \nu_e)} \approx \epsilon^2. \quad (4.14)$$

By the same mechanism one obtains the decay $\mu^+ \rightarrow e^+ \nu_\alpha \nu_\beta$, $e^+ \bar{\nu}_\alpha \bar{\nu}_\beta$ in addition to the usual muon decay. Both of these rates are also proportional to ϵ^2 . A maximum value of $\epsilon \approx 0.1$ is consistent with experiment.¹⁶ New contributions to the muon decay width proportional

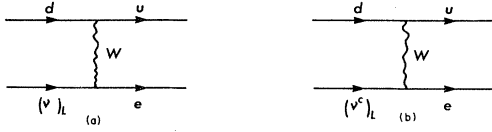


FIG. 1. (a) Lepton-number-conserving interaction. (b) Lepton-number-violating interaction.

to ϵ^4 would also be present.

In addition to decays, one can see lepton-number-violating effects in neutrino charged-current events. A neutrino produced in association with a lepton can also trigger a charged-current reaction off hadrons to produce an antilepton. The amplitude is proportional to the parameter ϵ . This process will be distance independent in contrast with ordinary $\Delta l=2$ oscillation process generated by a bare Majorana mass.⁹ In the present case the neutrino in flight does not oscillate into an antineutrino. Notice also that this lepton-number-changing interaction will always change the lepton flavor. By this we mean that an incoming ν_e , for example, can produce an outgoing μ^+ or τ^+ , but never an e^+ . We can see this from the neutrino mass term in Eq. (3.9). The antisymmetric structure of the flavor indices shows that the N_α and N_α^c for neutrinos of the same flavor never couple. For this reason the model with the mass term in Eq. (3.9) cannot produce neutrinoless double- β decay since all lepton-number-violating terms must also change flavor. Neither can one have rare $\Delta l=2$ K decays such as $K^- \rightarrow \pi^+ e^- e^-$ or $K^- \rightarrow \pi^+ \mu^- \mu^-$. What is allowed, however, is $K^- \rightarrow \pi^+ e^- \mu^-$, since this violates both lepton number and flavor. One such diagram is shown in Fig. 2. From (3.2), and (4.4) we can estimate very crudely,

$$\frac{\Gamma(K^- \rightarrow \pi^+ e^- \mu^-)}{\Gamma(K^- \rightarrow \pi^0 \bar{\nu}_\mu \mu^-)} \approx (\epsilon G_F m f_\pi \sin \psi')^2, \quad (4.15)$$

where f_π is the pseudoscalar decay constant [$f_\pi \simeq m(\pi)$]. If we take $\epsilon \simeq 0.1$ and $m \simeq 0.5$ MeV (the upper bound on the muon neutrino mass) then (4.15) becomes proportional to 10^{-18} , well below the experimental limit¹⁷ of 10^{-7} .

Diagrams such as the one in Fig. 2 also lead to rare $\Delta l=2$ τ decays, such as $\tau^- \rightarrow e^+ \pi^- \pi^-$ and $\tau^- \rightarrow \mu^+ \pi^- \pi^-$, where

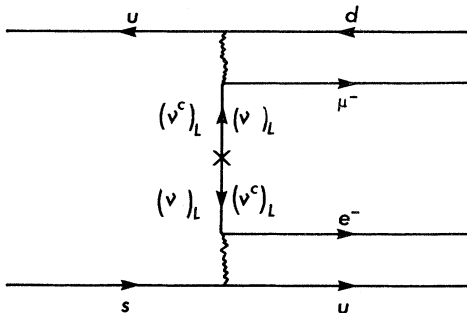


FIG. 2. Contribution to $K^- \rightarrow \pi^+ e^- \mu^-$ decay.

$$\frac{\Gamma(\tau^- \rightarrow e^+ \pi^- \pi^-)}{\Gamma(\tau^- \rightarrow \mu^+ \pi^- \pi^-)} \approx t g^2 \psi.$$

We can generalize the mass term for neutrinos to allow for lepton-number-violating and flavor-conserving interactions by adding a completely symmetric complex sextet $\phi_{ab} = \phi_{ba}$ of the Higgs scalars to the theory. The covariant derivative of ϕ_{ab} is

$$D_\mu \phi_{ab} = \partial_\mu \phi_{ab} - \frac{ig}{\sqrt{2}} \left[\phi_{ac} W_{b\mu}^c + \phi_{cb} W_{a\mu}^c \right] + \frac{2i}{3} g' \phi_{ab} B_\mu. \quad (4.16)$$

If the only nonzero vacuum expectation value is $\langle \phi_{23} \rangle = \langle \phi_{32} \rangle = v/\sqrt{2}$. Then via a new Yukawa coupling term of the form

$$\frac{1}{\sqrt{2}} \sum_{\alpha, \beta} h_{\alpha\beta} \psi_{(\alpha)}^{T a} C^{-1} \phi_{ab} \psi_{(\beta)}^b + \text{H.c.} \quad (4.17)$$

(where $h_{\alpha\beta} = h_{\beta\alpha}$) a symmetric Dirac neutrino mass matrix is generated, in addition to Eq. (3.11). The complete Dirac mass matrix

$$M_{\alpha\beta} = g_{\alpha\beta} k_1 + h_{\alpha\beta} v,$$

is now totally arbitrary so that we can now have lepton-number-violating interactions that do not change flavor. In particular, we can have neutrinoless double- β decay¹⁸ mediated by a quasi-Dirac neutrino.¹⁹ Notice also that such a pattern of vacuum expectation values does not destroy the mass relationship in Eq. (3.5). This is unlike the situation in an $SU(2) \times U(1)$ model augmented by a symmetric Higgs triplet² where one is forced to have a bare Majorana neutrino mass in order to generate the $(\beta\beta)_{0\nu}$ decay by the standard diagram. This would in turn break the "canonical" W -to- Z mass ratio.

In such a generalized sextet scheme, however, a tree Dirac neutrino mass matrix may be argued not to be technically natural, to the extent that there is no compelling reason to set $\langle \phi_{22} \rangle = 0 \langle \phi_{33} \rangle$.²⁰

Finally, a word about radiative corrections. In models of the present type one in general expects Majorana masses to be generated radiatively.⁹ It is clearly so, if the model is to engender the $(\beta\beta)_{0\nu}$ process, as discussed in the first work in Ref. 9. Moreover, since the model is renormalizable and the free Lagrangian is invariant under lepton number, the radiatively induced Majorana mass has to be finite. Specifically, consider the diagram depicted in Fig. 3, which is present in the sextet model. It may be roughly estimated as

$$\frac{g^2}{4\pi^2} \frac{\epsilon m^3}{m^2(W)} \ln \frac{m^2}{m^2(Z)}. \quad (4.18)$$

If we take $m \simeq 0.5$ MeV, $m(W) \simeq m(Z) \simeq 100$ GeV, $\epsilon \simeq 10^{-1}$, and $g^2/4\pi^2 \simeq 10^{-3}$ then

$$\frac{M(\text{Majorana})}{M(\text{Dirac})} \simeq 10^{-13}. \quad (4.19)$$

Charged-gauge-boson-exchange contributions have been estimated by Wolfenstein and Pecov.⁹ Although typically much higher than Eq. (4.18), they are still rather small⁹ so that we are justified in ignoring Majorana terms and treating the neutrinos, to a good approximation, as Dirac particles. Of course one can imagine neutrino-oscillation ex-

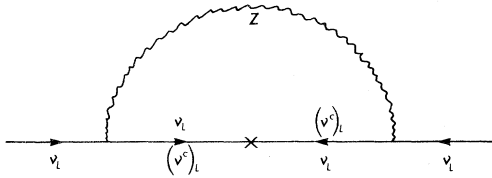


FIG. 3. Radiatively induced Majorana mass term.

periments accurate enough to be sensitive to such small mass splittings.

Noted added. A coupling similar to Eq. (3.9) was proposed by A. Zee [Phys. Lett. **93B**, 389 (1980)] in the context of an $SU(2) \times U(1)$ model. We also note that, as we

finished the work reported here, we learned of a paper by K. Enquist, J. Maalampi, and K. Mursula [Phys. Lett. **124B**, 89 (1983)], which discusses the possibility of lepton-number violation with Dirac neutrinos in a left-right-symmetric model.

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¹S. L. Glashow, Nucl. Phys. **22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

²For a general analysis of neutrino masses in $SU(2) \times U(1)$ theories, see J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); T. P. Cheng and L.-F. Li, *ibid.* **22**, 2860 (1980).

³J. Schechter and J. W. F. Valle in Ref. 2; Phys. Rev. D **23**, 1666 (1981); **21**, 309 (1980); M. Doi *et al.*, Phys. Lett. **102B**, 323 (1981); S. M. Bilenky *et al.*, *ibid.* **94B**, 495 (1980); V. Barger *et al.*, Phys. Rev. Lett. **45B**, 692 (1980).

⁴H. Georgi and A. Pais, Phys. Rev. D **19**, 2746 (1979); M. Singer, J. W. F. Valle, and J. Schechter, *ibid.* **22**, 738 (1980); T. Brando and M. Singer, Syracuse report, 1980 (unpublished).

⁵S. L. Adler, Phys. Rev. **117**, 2426 (1967); J. S. Bell and R. Jackiw, Nuovo Cimento **51**, 47 (1969).

⁶H. Georgi and S. L. Glashow, Phys. Rev. D **6**, 429 (1972).

⁷B. W. Lee and R. Shrock, Phys. Rev. D **17**, 2410 (1978).

⁸For a general discussion of "vertical mixing," see Georgi and Pais (Ref. 4).

⁹J. W. F. Valle, Phys. Rev. D **27**, 1672 (1983); L. Wolfenstein, Nucl. Phys. **B186**, 147 (1981); S. T. Petcov, Phys. Lett. **110B**, 245 (1982). For early work, see J. M. Jauch, Helv. Phys. Acta **27**, 89 (1954); S. Kamefuchi, S. Tanaka Prog. Theor. Phys. **14**, 225 (1955); W. Pauli, Nuovo Cimento **6**, 204 (1957).

¹⁰B. Pontecorvo, Zh. Eksp. Teor. Fiz. **34**, 247 (1958), [Sov.

Phys. JETP **34**, 172 (1958)].

¹¹Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

¹²For a recent review, see B. Pontecorvo, Phys. Rep. **41C**, 225 (1978); V. Barger, K. Whisnant, and R. J. N. Phillips, Phys. Rev. D **22**, 1636 (1980); R. J. N. Phillips, Rutherford Report No. RL-82-100, 1982 (unpublished).

¹³For a review of neutrino oscillation experiments, see D. H. Perkins, in *Proceedings of the 1981 CERN-JINR School of Physics, Hanko, Finland*, edited by H. I. Miettinen (CERN, Phys. Lett. Geneva, 1982), p. 152.

¹⁴V. A. Lyubimov *et al.*, Phys. Lett. **94B**, 266 (1980); J. J. Simpson, Phys. Rev. D **23**, 649 (1981).

¹⁵B. H. J. McKellar, Phys. Lett. **97B**, 93 (1980).

¹⁶S. E. Willis, Phys. Rev. Lett. **44**, 522 (1980); **45**, 1370 (1980); P. Kalyniak and J. N. Ng, Phys. Rev. D **24**, 1874 (1981).

¹⁷Particle Data Group, Phys. Lett. **111B**, 3 (1982).

¹⁸For a recent analysis on $(\beta\beta)_{0\nu}$, see M. Doi *et al.*, Osaka Report No. OS-GE 82-43, 1982 (unpublished); W. C. Haxton, G. J. Stephenson, Jr., and D. Strotman, Phys. Rev. Lett. **47**, 153 (1981); Phys. Rev. D **25**, 2360 (1982); and Los Alamos Reports Nos. LA-UR-82-1286 and LA-UR-82-1627 (unpublished); P. Minkowski, Nucl. Phys. **B201**, 269 (1982).

¹⁹Such a possibility in the context of a pure $V-A$ theory is very interesting phenomenologically in connection with testing the neutrino mass from $(\beta\beta)_{0\nu}$ data. See Valle (Ref. 9).

²⁰Notice also that in the sextet scheme the simple predictions on the neutrino mass spectrum and mixing, Eq. (3.15), would be lost.