Quantum-chromodynamic corrections to meson-photon transition form factors

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We calculate corrections to the hard-scattering amplitude for meson-photon transition form factors using dimensional regularization. Special attention is paid to the pseudoscalar case in which there is an ambiguity associated with γ_5. We resolve this ambiguity by appealing to axial-vector Ward identities and check the answer by using a four-dimensional regularization.

I. INTRODUCTION

It has been shown in recent years that perturbative QCD gives predictions for certain exclusive processes involving strong interacting particles.[1,2] The simplest such process is the electromagnetic form factor for the meson-photon transition. The prediction of QCD is that, for large momentum transfer Q^2, the form factor is the convolution of a calculable hard-scattering amplitude with a nonperturbative distribution amplitude whose Q^2 dependence is calculable.[1]

If this prediction is to be used as a test for QCD, it is important to know the size of the radiative corrections to the lowest-order prediction. To obtain the complete prediction for the form factor to order α_s, we need to calculate the corrections to both the hard-scattering amplitude and the evolution kernel for the distribution amplitude. In this paper we calculate the correction to the hard-scattering amplitude using dimensional regularization. In the case of a pseudoscalar meson, the calculation is complicated by the γ_5 ambiguity of dimensional regularization. We resolve this ambiguity by two different methods: (1) comparing with a calculation using a four-dimensional regularization, and (2) demanding that the axial-vector Ward identities be preserved in dimensional regularization. The two methods are consistent and the result is in agreement with a recent calculation of the hard-scattering amplitude by del Aguila and Chase.[3]

In Sec. II we review the lowest-order prediction for the transition form factor, specializing to the case of pseudoscalar mesons and real photons. In Sec. III, we calculate corrections to the hard-scattering amplitude using dimensional regularization and exhibiting the γ_5 ambiguity explicitly. In Sec. IV, we discuss the corrections from the point of view of the operator-product expansion. We show that the γ_5 ambiguity can be resolved either by appealing to axial-vector Ward identities in dimensional regularization or by comparing with a four-dimensional regularization method. In Sec. V, we extend the calculation to include the cases of virtual photons and vector mesons. Conclusions are given in Sec. VI.

II. LOWEST-ORDER FORM FACTOR

The meson-photon transition form factor $F_{M\gamma}(Q)$ for a pseudoscalar meson M is defined in terms of the amplitude $\Gamma_{\mu\nu}$ for $\gamma^*\gamma\to M$:

$$\Gamma_{\mu\nu}=e^2F_{M\gamma}(Q)\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta \,, \tag{2.1}$$

where P and q are the momenta of the meson and virtual photon, and $Q^2=-q^2>0$. For large Q^2, this form factor is the convolution of a "hard-scattering amplitude" $T(x,Q,\mu)$, which can be calculated in perturbation theory, with a nonperturbative "distribution amplitude" $\phi(x,\mu)$ (Ref. 1):

$$F_{M\gamma}(Q)=\int_0^1 dx\, \phi(x,\mu)T(x,Q,\mu) \,. \tag{2.2}$$

The momentum scale μ is an arbitrary separation between "hard" and "soft" momenta, and we shall for simplicity take it to be $\mu=Q$. The distribution amplitude is universal, occurring also in other exclusive processes involving this meson such as its electromagnetic form factor. Its intuitive interpretation is that $\phi(x,Q)$ is the amplitude for the meson to consist of a $q\bar{q}$ pair, with the quark and antiquark collinear and on-shell relative to the momentum scale Q and sharing fractions x and $1-x$ of the meson's momentum.

Although ϕ cannot be calculated perturbatively, it satisfies an evolution equation of the form

$$\tfrac{1}{2}Q\frac{\partial}{\partial Q}\phi(x,Q)=\int_0^1 dy\; V(x,y,Q)\phi(y,Q) \tag{2.3}$$

in which the kernel V is calculable in perturbation theory:

$$V(x,y,Q)=C_F\frac{\alpha_s(Q)}{2\pi}\left[V_0(x,y)+\frac{\alpha_s}{2\pi}V_1(x,y)+\cdots\right]. \tag{2.4}$$

Here $\alpha_s(Q)$ is the QCD coupling constant and $C_F=\frac{4}{3}$ is a color factor. The Q^2 dependence of the running coupling constant is governed by the β function:

$$\frac{1}{2}Q\frac{\partial}{\partial Q}\alpha_s(Q)=\beta(\alpha_s(Q))=-\frac{\alpha_s(Q)^2}{2\pi}\left[\beta_0+\frac{\alpha_s}{2\pi}\beta_1+\cdots\right], \tag{2.5}$$

where $\beta_0=11-\frac{2}{3}f$, and f is the number of light-quark flavors. The lowest-order evolution kernel $V_0(x,y)$, which corresponds to single-gluon exchange between the quark and antiquark, was first calculated by Brodsky and Lepage[1,4]:

$$\begin{aligned}V_0(x,y)&=V_{\rm BL}(x,y)-\delta(x-y)\int_0^1 dz\, V_{\rm BL}(z,y)\,,\\ V_{\rm BL}(x,y)&=\frac{1-x}{1-y}\left[1+\frac{1}{x-y}\right]\theta(x-y)+\frac{x}{y}\left[1+\frac{1}{y-x}\right]\theta(y-x)\,.\end{aligned} \tag{2.6}$$

Its eigenfunctions $G_n(x)$ are Gegenbauer polynomials multiplied by the weight function $x(1-x)$:

$$\begin{aligned}&\int_0^1 dy\, V_0(x,y)G_n(y)=-\gamma_n G_n(x)\,,\\ &G_n(x)\equiv x(1-x)C_n^{(3/2)}(2x-1)\,,\\ &\gamma_n=\frac{1}{2}\left[1+4\sum_{j=2}^{n+1}\frac{1}{j}-\frac{2}{(n+1)(n+2)}\right].\end{aligned} \tag{2.7}$$

The hard-scattering amplitude has a perturbative expansion of the form

$$T(x,Q)=\frac{N}{Q^2}\left[T_0(x)+C_F\frac{\alpha_s}{2\pi}T_1(x)+\cdots\right], \tag{2.8}$$

where N is a normalization constant. In lowest order, it is simply the transition form factor for the state $|q\bar{q},x\rangle$ consisting of a quark and an antiquark in a pseudoscalar color singlet state with collinear on-shell momenta xP and $(1-x)P$, where $P^2=0$. The Feynman diagrams for the amplitude $\gamma^*\gamma\to q\bar{q}$ are shown in Fig. 1. The $q\bar{q}$ pair is projected into a pseudoscalar state by multiplying the amplitude by the Dirac matrix $\not{P}\gamma_5$ and taking the trace. It is projected into a color singlet by tracing in the color indices. Factoring out the Lorentz structure as in Eq. (2.1) and absorbing constant factors into N, we obtain

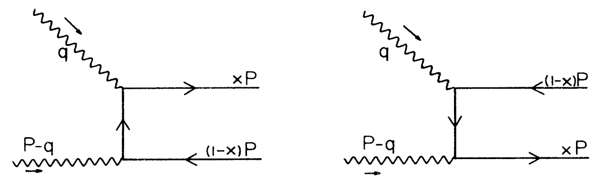

FIG. 1. Lowest-order Feynman diagrams for the amplitude $\gamma^*\gamma\to q\bar{q}$.

$$T_0(x)=\frac{1}{1-x}+\frac{1}{x}\,. \tag{2.9}$$

The normalization constant N depends on the quark content of the meson. For the pion, whose quark wave function is $u\bar{u}-d\bar{d}$, the normalization constant is

$$N=\sqrt{12}(e_u{}^2-e_d{}^2)\,,$$

where e_q is the fractional charge of quark q.

Using the lowest-order expression for the hard-scattering amplitude, the meson-photon transition form factor in Eq. (2.2) reduces to

$$F_{M\gamma}(Q)=\frac{N}{Q^2}\int_0^1 dx\,\phi(x,Q)\left[\frac{1}{1-x}+\frac{1}{x}\right]. \tag{2.10}$$

If the lowest-order evolution kernel $V_0(x,y)$ is used in Eq. (2.3) to determine the Q^2 evolution of ϕ, then this expression includes all the leading logarithms of Q.

It is convenient to expand the distribution amplitude in terms of the eigenfunctions of V_0:

$$\begin{aligned}\phi(x,Q)&=\sum_{n=0}^{\infty}\phi_n(Q)G_n(x)\,,\\ \phi_n(Q)&=\frac{4(2n+3)}{(n+1)(n+2)}\times\int_0^1 dx\, C_n^{(3/2)}(2x-1)\phi(x,Q)\,.\end{aligned} \tag{2.11}$$

For these Gegenbauer moments of ϕ, the evolution equation simplifies in lowest order to

$$\frac{1}{2}Q\frac{\partial}{\partial Q}\phi_n(Q)=-C_F\frac{\alpha_s(Q)}{2\pi}\gamma_n\phi_n(Q)\,. \tag{2.12}$$

The lowest-order hard-scattering amplitude $T_0(x)$ has the following expansion in Gegenbauer polynomials:

$$T_0(x)=\sum_n \frac{4(4n+3)}{(2n+1)(2n+3)} C_{2n}^{(3/2)}(2x-1) \ . \tag{2.13}$$

Inserting this into Eq. (10), we find that the form factor is just the sum of the even Gegenbauer moments of ϕ:

$$F_{M\gamma}(Q)=\frac{N}{Q^2}\sum_n \phi_{2n}(Q) \ . \tag{2.14}$$

We note that since the lowest eigenvalue of the evolution kernel is $\gamma_0=0$, the corresponding eigenfunction $G_0(x)=x(1-x)$ has no Q^2 evolution. Therefore, if the distribution amplitude has this special form, the Q^2 dependence of the form factor is completely determined by the hard-scattering amplitude.

III. CORRECTION TO THE HARD-SCATTERING AMPLITUDE

We now consider the calculation of QCD corrections to the transition form factor. To obtain an answer which includes all the next-to-leading logarithms of Q, three ingredients are needed: (1) $T_1(x)$, the order-α_s correction to the hard-scattering amplitude, (2) $V_1(x,y)$ the second term in the expansion of the evolution kernel, and (3) β_1, the second coefficient of the β function. The coefficent β_1 is already known.[5] In this paper, we will calculate only the correction to the hard-scattering amplitude. $V_1(x,y)$ is also required in order to obtain an expression for the form factor which is independent of regularization and factorization schemes. We can however obtain a scheme-independent answer for the special case of the distribution amplitude $G_0(x)=x(1-x)$, which is a 0 eigenfunction of the evolution kernel.

The order-α_s correction $T_1(x)$ to the hard-scattering amplitude can be obtained from the calculation of the transition form factor for the state $|q\bar{q},x\rangle$ consisting of a pseudoscalar color-singlet $q\bar{q}$ pair with collinear on-shell momenta. The correction to this form factor is infrared divergent, but the divergence must have the form[1]

$$\begin{aligned}[T_1(x)]_{\text{divergent}} &= c\int_0^1 dy\, T_0(y)V_0(y,x) \\ &= c\left[\frac{1}{1-x}[\tfrac{3}{2}+\ln(1-x)] + \frac{1}{x}[\tfrac{3}{2}+\ln(x)]\right] ,\end{aligned} \tag{3.1}$$

where c is a divergent constant. It can therefore be eliminated by the following redefinition of the uncalculable distribution amplitude:

$$\phi(x)\rightarrow\phi(x)+cC_F\frac{\alpha_s}{2\pi}\int_0^1 dy\, V_0(x,y)\phi(y) \ . \tag{3.2}$$

With a finite change in c, extra finite terms can also be absorbed into ϕ, but this amounts to a simple change in the scale μ in Eq. (2.2). The remaining finite terms depend on the infrared regularization method and, in general, on the choice of gauge. The regularization-dependent terms represent "soft" effects and should therefore be absorbed into the distribution amplitude along with the divergences. The remaining terms belong to the hard-scattering amplitude. However, the separation of the soft terms from the hard-scattering terms is not always trivial. The operator-product expansion, which will be discussed in Sec. IV, provides a way of accomplishing this separation so as to obtain a gauge-invariant hard-scattering amplitude which can be calculated consistently in different regularization schemes.

There is a method of calculation which avoids this complication and allows all the finite terms associated with the divergence to be absorbed into the scale μ by the redefinition in Eq. (3.2). The method is to use dimensional regularization with massless on-shell quarks to handle the infrared divergences. The infrared poles are absorbed into the distribution amplitude, and all finite terms are assigned to the hard-scattering amplitude. This method was used in a recent calculation of the correction to the hard-scattering amplitude for the electromagnetic form factor of the meson.[6]

Dimensional regularization leads to an ambiguity in the calculation of the transition form factor for a pseudoscalar meson because of the presence of the pseudoscalar Dirac matrix γ_5.[7] The root of the problem is the lack of an unambiguous generalization of γ_5 in N dimensions. In practice, the ambiguity arises in the evaluation of a trace containing a γ_5 and a pair of contracted γ matrices. One of the properties of γ_5 in four dimensions is

$$\mathrm{Tr}(\gamma_5 \not{a} \not{b})=0 \ . \tag{3.3}$$

Using this property and the standard N-dimensional Dirac algebra, we can simplify the following trace:

$$\mathrm{Tr}(\gamma_5 \not{a}\gamma_\mu \not{b}\not{c}\not{d}\gamma^\mu)=(N-6)\mathrm{Tr}(\gamma_5\not{a}\not{b}\not{c}\not{d}) \ . \tag{3.4}$$

If we instead use the anticommutation property of γ_5,

$$\gamma_5\gamma_\mu=-\gamma_\mu\gamma_5, \quad \mu=1,\ldots,N \tag{3.5}$$

and then contrast the γ matrices, we obtain instead

$$\mathrm{Tr}(\gamma_5 \not{a} \gamma_\mu \not{b} \not{c} \not{d} \gamma^\mu) = (2-N)\mathrm{Tr}(\gamma_5 \not{a} \not{b} \not{c} \not{d}) \ . \tag{3.6}$$

The difference is proportional to $N-4$, so if this trace multiplies a pole in $N-4$, there is a finite ambiguity in the answer.

The ambiguity can be resolved by appealing to Ward identities. If a trace contains an even number of γ_5's, the Ward identities are preserved if the anticommutation property Eq. (3.5) and the property $\gamma_5{}^2=1$ are used to eliminate the γ_5's from the trace.[7] The dimensional regularization method can then be applied without difficulty. This approach was used in the calculation of the hard-scattering amplitude for the meson electromagnetic form factor.[6]

If there is only a single γ_5 in the trace as in this problem, then there is no simple prescription which will preserve the Ward identities. One solution is to adopt a definite prescription for γ_5 which violates the Ward identities, and to add, order by order in perturbation theory, finite counterterms which restore them.[8] An example of such a prescription is the original one of 't Hooft and Veltman[9]:

$$\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4 \ . \tag{3.7}$$

With this prescription γ_5 has the cumbersome commutation rules

$$\gamma_5\gamma_\mu = \begin{cases} -\gamma_\mu\gamma_5, & \mu=1,\ldots,4 \\ +\gamma_\mu\gamma_5, & 4<\mu\leq N \ . \end{cases} \tag{3.8}$$

It is tempting to try to avoid the ambiguity by never specifying the commutation properties of γ_5. If the problem contains only four independent vectors, property (3.3) is sufficient to reduce each trace to the form $\mathrm{Tr}(\gamma_5 \not{a} \not{b} \not{c} \not{d})$ and one can then apply the four-dimensional identity

$$\mathrm{Tr}(\gamma_5 \not{a} \not{b} \not{c} \not{d}) = 4i\epsilon_{\mu\nu\lambda\sigma} a^\mu b^\nu c^\lambda d^\sigma \ . \tag{3.9}$$

However, this prescription is in fact equivalent to that of implementing the prescription of Eq. (3.7).

Another possible approach is to determine for each individual diagram how the γ matrices should be manipulated so as to respect the Ward identities. This is essentially the method that was used by del Aguila and Chase in their calculation of the transition form factor.[3] It is less straightforward to apply than the counterterm method, but if it can be applied, it simplifies the calculation. We will verify in Sec. IV that the results of del Aguila and Chase are indeed correct.

We proceed to calculate the order-α_s correction to the $q\bar{q}$-photon transition form factor. The relevant diagrams are those shown in Fig. 2, together with the corresponding crossed diagrams obtained by interchanging the two photon vertices. The individual diagrams are gauge dependent, and we calculate them in the Feynman gauge. The diagrams contain ultraviolet (UV) as well as infrared (IR) divergences. We use dimensional regularization in $N=4-2\epsilon$ dimensions to regularize both types of divergences, distinguishing the poles $1/\epsilon$ by the subscripts UV and IR. To calculate the momentum integrals, we use the analytic continuation

$$\int \frac{d^4k}{(2\pi)^4} \to \pi^\epsilon \Gamma(1-\epsilon)\mu^{2\epsilon} \int \frac{d^Nk}{(2\pi)^4} \ , \tag{3.10}$$

where μ is an arbitrary mass parameter. This eliminates the usual constant $\ln(4\pi)-\gamma$ which is associ-

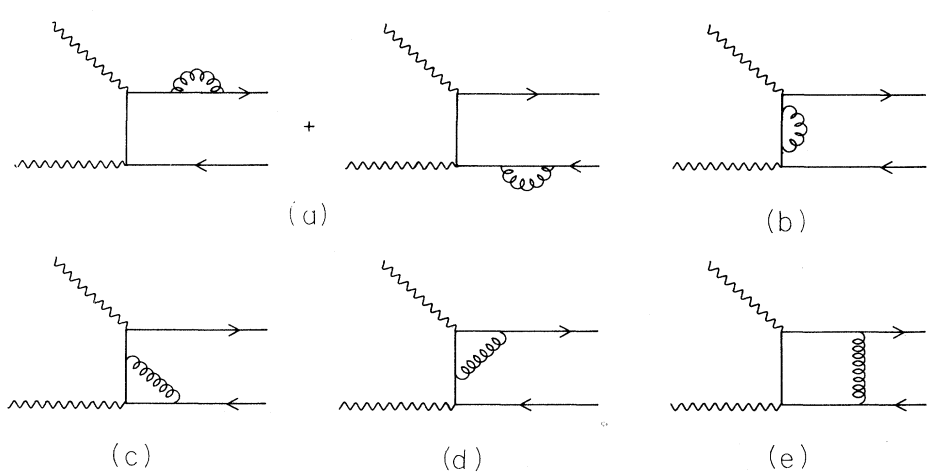

FIG. 2. Order-α_s corrections to the amplitude $\gamma^*\gamma \to q\bar{q}$.

ated with each pole $1/\epsilon$. Subtraction of only the poles in ϵ thus corresponds to the modified minimal-subtraction scheme $\overline{\text{MS}}$. To display the γ_5 ambiguity, we use a parameter δ which is 1 if pairs of γ matrices are contracted together through the γ_5 as in Eq. (3.6) and 0 if they are contracted together in the other direction as in Eq. (3.4). The 't Hooft—Veltman prescription for γ_5 corresponds to $\delta=0$.

The contribution to $T_1(x)$ from each of the diagrams in Fig. (2) is

$$
\begin{aligned}
T_1^{2(\text{a})}(x)&=\frac{1}{1-x}\left[-\frac{1}{2}\left[\frac{1}{\epsilon_{\text{UV}}}+2\delta\right]+\frac{1}{2}\left[\frac{1}{\epsilon_{\text{IR}}}+2\delta\right]\right],\\
T_1^{2(\text{b})}(x)&=\frac{1}{1-x}\left[-\frac{1}{2}\left[\frac{1}{\epsilon_{\text{UV}}}+2\delta\right]+\tfrac{1}{2}\ln(1-x)-\tfrac{1}{2}\right],\\
T_1^{2(\text{c})}(x)&=\frac{1}{1-x}\left[\frac{1}{2}\left[\frac{1}{\epsilon_{\text{UV}}}+2\delta\right]-\frac{1}{\epsilon_{\text{IR}}}+\tfrac{1}{2}\ln(1-x)-2\right],\\
T_1^{2(\text{d})}(x)&=\frac{1}{1-x}\left[\frac{1}{2}\left[\frac{1}{\epsilon_{\text{UV}}}+2\delta\right]-\frac{1}{\epsilon_{\text{IR}}}\left[1+\frac{1}{x}\ln(1-x)\right]+\frac{1}{2x}\ln^2(1-x)-\frac{3-x}{2x}\ln(1-x)-2\right],\\
T_1^{2(\text{e})}(x)&=\frac{1}{1-x}\left[\frac{1-x}{x}\ln(1-x)\left[\frac{1}{\epsilon_{\text{IR}}}-4\delta\right]-\frac{1-x}{2x}\ln^2(1-x)+5\frac{1-x}{x}\ln(1-x)\right].
\end{aligned}
\tag{3.11}
$$

The contributions from the corresponding crossed diagrams are obtained by interchanging x and $(1-x)$. For simplicity, we have set $\mu=Q$; the μ dependence can be recovered by replacing each pole by $1/\epsilon-\ln(Q^2/\mu^2)$. The contribution T_1^a comes from propagator corrections to on-shell quark lines. This is 0 using dimensional regularization, since the correction must be proportional to $(P^2)^{-\epsilon}$ and therefore vanishes for $P^2=0$. We have represented this 0 as a pair of canceling UV and IR poles.

There are γ_5 ambiguities associated with the UV poles in the quark propagator corrections [diagrams 2(a) and 2(b)] and the photon vertex corrections [diagrams 2(c) and 2(d)]. These propagator and vertex corrections are related by the Ward identity of QED. They should therefore be calculated just as they would be if they were not part of a trace with γ_5. This determines the choice $\delta=0$ in diagrams (a) through (d). The only remaining ambiguity is that associated with the infrared pole in the gluon exchange diagram 2(e). We will show in Sec. IV that the correct answer is obtained by taking $\delta=1$ in $T_1{}^{2e}(x)$, which corresponds to contracting pairs of γ matrices together through the γ_5.

The sum of the infrared poles have exactly the form predicted in Eq. (3.1) with the constant $c=-1/\epsilon_{\text{IR}}$. The poles can therefore be absorbed into a redefinition of the distribution amplitude as in Eq. (3.2). Subtracting them from the contributions to $T_1(x)$ in Eq. (3.11) and setting $\delta=0$ in diagrams 2(a) through 2(d), we find that the hard-scattering amplitude to order α_s, is

$$
\begin{aligned}
T(x,Q)=\frac{N}{Q^2}\frac{1}{1-x}\Bigg\{1+C_F\frac{\alpha_s(Q)}{2\pi}\Bigg[&\frac{1}{2}\ln^2(1-x)+(7-8\delta)\frac{1-x}{2x}\ln(1-x)\\
&-\frac{9}{2}+\left[\frac{3}{2}+\ln(1-x)\right]\ln(Q^2/\mu^2)\Bigg]\Bigg\}+[x\leftrightarrow(1-x)]\,.
\end{aligned}
\tag{3.12}
$$

The correction $V_1(x,y)$ to the evolution kernel must be calculated in the same scheme and combined with this calculation to obtain order-α_s corrections which are scheme-independent. The exception is if the distribution amplitude has the form $G_0(x)=x(1-x)$, which has no Q^2 evolution. Convoluting G_0 with the hard-scattering amplitude in Eq. (3.12), we obtain the scheme-independent result

$$
\int_0^1 dx\, G_0(x)T(x,Q)=\frac{N}{Q^2}\left[1-\frac{9-4\delta}{2}C_F\frac{\alpha_s(Q)}{2\pi}\right]. \tag{3.13}
$$

Note that the logarithms of Q^2/μ^2 have canceled out. We will use this scheme-independent result in the next section to show that the correct answer is given by $\delta=1$.

IV. RESOLUTION OF THE γ_5 AMBIGUITY

One way to resolve the γ_5 ambiguity in the hard-scattering amplitude $T(x,Q)$ is to repeat the calculation using a regularization in which there is no ambiguity. The convolution of $T(x,Q)$ with $G_0(x)$ in Eq. (3.13) is scheme independent, so by calculating it with a different regularization scheme, we can determine the correct value for the ambiguity parameter δ. However, the calculation of $T(x,Q)$ is more complicated in other regularization methods, since the $q\bar{q}$ form factor contains finite soft terms which should be absorbed into the distribution amplitude along with the divergences. The operator-product expansion provides an unambiguous way of separating these terms from those which belong to the hard-scattering amplitude.

The amplitude for $\gamma^*\gamma\rightarrow M$ can be expressed in terms of the time-ordered product of electromagnetic currents:

$$\Gamma_{\mu\nu}=\int d^4x\, e^{iq\cdot x}\langle M\mid TJ_\mu(x)J_\nu(0)\mid 0\rangle \ . \tag{4.1}$$

For large values of $Q^2=-q^2$, we can apply the operator-product expansion for two currents near the light cone.[10] With this expansion, the transition form factor can be written as[3]

$$F_{M\gamma}(Q)=\sum_n C_n(Q)M_n \ . \tag{4.2}$$

The coefficients C_n are universal, independent of the state M. The M_n's are matrix elements of local operators between M and the vacuum.

$$M_nP_{\mu_1}\cdots P_{\mu_{n+1}} =\langle M\mid \bar{\psi}\gamma_5\gamma_{\mu_1}(\vec{\partial}+\overleftarrow{\partial})^n C_n^{(3/2)}\left[\frac{\vec{D}-\overleftarrow{D}}{\vec{\partial}+\overleftarrow{\partial}}\right]\psi\mid 0\rangle \ , \tag{4.3}$$

where P is the momentum of the state M, ψ is a quark field, and D is the gauge-covariant derivative. We have suppressed the Lorentz indices $\mu_2,\ldots,\mu_{n+1}$ on the right side of this equation.

Since the coefficients C_n are universal, they can be calculated by replacing the meson state M by the state $\mid q\bar{q},x\rangle$ in which the quark and antiquark have momenta xP and $(1-x)P$. The form factor for this state was calculated to lowest order in Sec. II:

$$F_{q\bar{q}\gamma}(x,Q)=\frac{N}{Q^2}T_0(x) \ . \tag{4.4}$$

The matrix element $M_n(x)$ is defined in Eq. (4.3) and is proportional in lowest order to the Gegenbauer polynomial $C_n^{(3/2)}(2x-1)$. The operator-product-expansion equation (4.2) therefore has the form

$$\frac{N}{Q^2}T_0(x)=\sum_n C_nC_n^{(3/2)}(2x-1) \ . \tag{4.5}$$

Thus, in lowest order, the C_n's are proportional to the coefficients in the Gegenbauer expansion of the hard-scattering amplitude.

To obtain the order-α_s corrections to the coefficients C_n, we must calculate the corrections to the $q\bar{q}$ form factor and the matrix elements $M_n(x)$. The same infrared divergences arise in both calculations and they cancel when we extract the coefficients C_n. The finite regularization-dependent soft terms cancel as well, so the C_n's are independent of the infrared regularization scheme. Thus the identification of the $C_n(Q)$'s as expansion coefficients of $T(x,Q)$ provides a definition of the hard-scattering amplitude which is independent of the infrared regularization.

We consider the calculation of corrections to the coefficient C_0. To isolate this coefficient in the operator-product expansion equation (4.2), we convolute both sides with the distribution amplitude $G_0(x)=x(1-x)$:

$$\int_0^1 dx\, G_0(x)F_{q\bar{q}\gamma}(x,Q) =\sum_n C_n(Q)\int_0^1 dx\, G_0(x)M_n(x) \ . \tag{4.6}$$

In the lowest order the matrix element $M_n(x)$ is proportional to $C_n^{(3/2)}(2x-1)$ and the integral projects out the term $n=0$ in the sum. Unless there are x-dependent logarithms associated with the infrared divergences, the order-α_s corrections preserves the x-dependence of the lowest-order matrix element[11] and Eq. (4.6) reduces to

$$\int_0^1 dx\, G_0(x)F_{q\bar{q}\gamma}(x,Q) =C_0(Q)\int_0^1 dx\, G_0(x)M_0(x) \ . \tag{4.7}$$

In this case, only the corrections to the $q\bar{q}$ form factor and the single matrix element M_0 need be calculated in order to obtain the coefficient C_0.

In dimensional regularization, this property holds in a trivial way. The corrections to the matrix elements vanish since they must be proportional to $(P^2)^{-\epsilon}$ and $P^2=0$. The correction to the coefficient C_0 is therefore proportional to the right-hand side of Eq. (3.13). Normalizing C_0 to be 1 in lowest order, we have

$$C_0=1-\frac{9-4\delta}{2}C_F\frac{\alpha_s}{2\pi} . \tag{4.8}$$

Since this coefficient is scheme-independent, we can determine the ambiguity parameter δ by calculating C_0 using a different regularization.

With gluon mass regularization or off-shell quark regularization, there are x-dependent logarithms in the corrections to the matrix elements $M_n(x)$. Therefore all these matrix elements must be calculated in order to determine the coefficient C_0 using Eq. (4.6). These logarithms are absent in "equal-mass regularization," in which the quark and the gluon are both given the same small mass m.[11] Therefore Eq. (4.7) holds with this method and the only matrix element that we need to calculate is M_0.

We proceed to calculate the coefficient C_0 using equal-mass regularization. Infrared divergences appear as logarithms of the parameter $\rho_{\rm IR}=m^2/Q^2$. It is necessary to keep the mass m in the numerator of the fermion propagator, because an m^2 can combine with a pole $1/m^2$ to give a finite term. We use Pauli-Villars regularization to handle the ultraviolet divergences, which then appear as logarithms of $\rho_{\rm UV}=M^2/Q^2$, where M is the mass of the gluon regulator field. The corrections to the $q\bar{q}$ form factor are given by the diagram in Fig. 2, along with the corresponding crossed diagrams. With the coefficient C_0 normalized to be 1 in lowest order, the contributions of these diagrams are

$$\begin{aligned}
C_0^{2(\rm a)}&=C_F\frac{\alpha_s}{2\pi}(-\tfrac{1}{4}\ln\rho_{\rm UV}+\tfrac{1}{4}\ln\rho_{\rm IR}+\tfrac{1}{8}) ,\\
C_0^{2(\rm b)}&=C_F\frac{\alpha_s}{2\pi}(-\tfrac{1}{4}\ln\rho_{\rm UV}-\tfrac{3}{4}) ,\\
C_0^{2(\rm c)}&=C_F\frac{\alpha_s}{2\pi}(\tfrac{1}{4}\ln\rho_{\rm UV}-\tfrac{1}{2}\ln\rho_{\rm IR}-\tfrac{5}{4}) ,\\
C_0^{2(\rm d)}&=C_F\frac{\alpha_s}{2\pi}(\tfrac{1}{4}\ln\rho_{\rm UV}+\tfrac{1}{2}\ln\rho_{\rm IR}+\tfrac{5}{4}) ,\\
C_0^{2(\rm e)}&=C_F\frac{\alpha_s}{2\pi}(-\tfrac{1}{4}\ln\rho_{\rm IR}-\tfrac{9}{8}) .
\end{aligned} \tag{4.9}$$

The corresponding crossed diagrams give identical contributions.

We also need to calculate the correction to M_0, which is essentially the matrix element of the axial-vector current $J_\mu^5=\bar{\psi}\gamma_5\gamma_\mu\psi$ between the vacuum and the state $|q\bar{q},x\rangle$. The relevant diagrams are shown in Fig. 3. Their contributions must be substracted from the coefficient C_0:

$$\begin{aligned}
C_0^{3(\rm a)}&=-C_F\frac{\alpha_s}{2\pi}(-\tfrac{1}{2}\ln\rho_{\rm uv}+\tfrac{1}{2}\ln\rho_{\rm IR}+\tfrac{1}{4}) ,\\
C_0^{3(\rm b)}&=-C_F\frac{\alpha_s}{2\pi}(\tfrac{1}{2}\ln\rho_{\rm uv}-\tfrac{1}{2}\ln\rho_{\rm IR}-\tfrac{5}{4}) .
\end{aligned} \tag{4.10}$$

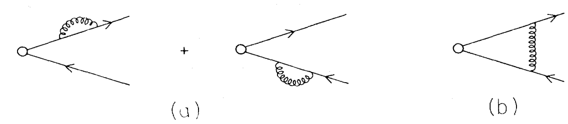

FIG. 3. Order-α_s corrections to the axial-vector current $J_\mu^5=\bar{\psi}\gamma_5\gamma_\mu\psi$.

Combining these contributions with those in Eq. (4.9), we obtain the coefficient C_0 to order α_s:

$$C_0=1-\tfrac{5}{2}C_F\frac{\alpha_s}{2\pi} . \tag{4.11}$$

Comparing this with the result of dimensional regularization in Eq. (4.8), we find that the correct value for the γ_5-ambiguity parameter is $\delta=1$.

We now show how the same result can be obtained using dimensional regularization with the 't Hooft—Veltman prescription for γ_5. This corresponds to $\delta=0$ in Eq. (4.8), so the contribution to the coefficient C_0 from the $q\bar{q}$ form-factor diagrams in Fig. 2 is

$$C_0^{2(\rm a),(b),(c),(d),(e)}=1-\tfrac{9}{2}C_F\frac{\alpha_s}{2\pi} . \tag{4.12}$$

We now consider the matrix element of the axial-vector current $J_\mu^5=\bar{\psi}\gamma_5\gamma_\mu\psi$. The diagrams in Fig. 3 give no contribution to this matrix element, because in dimensional regularization the corrections must be proportional to $(P^2)^\epsilon$ and we have taken $P^2=0$. However, with the 't Hooft—Veltman prescription for γ_5, the Ward identity for the axial-vector current vertex is violated. A counterterm must be introduced to restore the Ward identity, and it can contribute an order-α_s correction to the matrix element. We must therefore calculate this counterterm.

The Ward indentity for the one-particle-irreducible axial-vector current vertex Γ_μ^5 is

$$q^\mu\Gamma_\mu^5=S^{-1}(p_1)\gamma_5+\gamma_5S^{-1}(p_2) , \tag{4.13}$$

where $q=p_2-p_1$ and $S^{-1}(p)=\not{p}-\Sigma(p)$ is the inverse quark propagator. The order-α_s corrections to Γ_μ^5 and Σ are given by the diagrams in Fig. 4(a) and 4(b). Using dimensional regularization and the commutation rules for γ_5 in Eq. (3.8), we find that to order α_s

$$\begin{aligned}
q^\mu\Gamma_\mu^5=&[\not{p}_1-\Sigma(p_1)]\gamma_5+\gamma_5[\not{p}_2-\Sigma(p_2)]\\
&+2C_F\frac{\alpha_s}{2\pi}(\not{p}_1\gamma_5+\gamma_5\not{p}_2) .
\end{aligned} \tag{4.14}$$

The Ward identity is violated, but it can be restored by a finite renormalization of the axial-vector current. The vertex for the renormalized current

$$[1-2C_F(\alpha_s/2\pi)]J_\mu^5$$

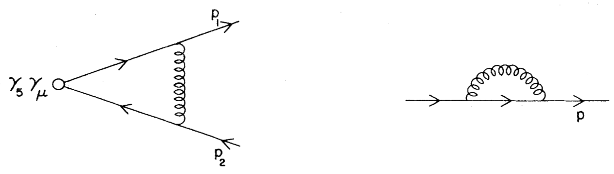

FIG. 4. Order-α_s contribution to the (a) one-particle-irreducible axial-vector-current vertex and (b) quark inverse propagator.

does satisfy the Ward identity (4.13). This renormalization of the axial-vector current is also needed if corrections to the triangle anomaly in dimensional regularization are to vanish as required by the Adler-Bardeen theorem.[12] The corresponding counterterm contributes an order-α_s correction to the matrix element M_0 which must be substracted from the other contributions to C_0:

$$C_0^{\text{counterterm}} = -\left[-2C_F\frac{\alpha_s}{2\pi}\right] . \tag{4.15}$$

Combining this with the form-factor contribution in Eq. (4.12), we find that the correction to the coefficient C_0 agrees with the result obtained by equal-mass regularization in Eq. (4.10).

We have thus verified by two different methods that the hard-scattering amplitude in dimensional regularization is correctly given by Eq. (3.12) with $\delta=1$. The trace manipulations which are consistent with the Ward identities are to contract pairs of γ matrices together through the γ_5 in the gluon-exchange diagram Fig. 2(e) and to contract them together in the other direction for the remaining diagrams.

V. VIRTUAL PHOTONS AND VECTOR MESONS

In Sec. III we calculated the corrections to the hard-scattering amplitude for the transition form factor in the case of a pseudoscalar meson and real photon. Having resolved the γ_5 ambiguity of dimensional regularization, we can easily extend the calculation to the case in which the photon is off-shell. We define the form factor $F_{M\gamma^*}$ in terms of the amplitude for $\gamma^*\gamma^*\to M$:

$$\Gamma_{\mu\nu} = e^2 F_{M\gamma^*}(Q,w)\epsilon_{\mu\nu\alpha\beta}P^\alpha\left[\frac{q_1-q_2}{2}\right]^\beta , \tag{5.1}$$

where q_1 and q_2 are the momenta of the two virtual photons, $Q^2=-({q_1}^2+{q_2}^2)>0$ and $w={q_1}^2/Q^2$. With this definition, $F_{M\gamma^*}(Q,w)$ approaches the form factor $F_{M\gamma}(Q)$ for real photons as $w\to 1$. For large Q^2, this form factor can be written in the form of a convolution as in Eq. (2.2), with the same distribution amplitude $\phi(x,Q)$ but a different hard-scattering amplitude $T(x,Q)$. The calculation of the order-α_s correction to $T(x,Q)$ using dimensional regularization proceeds exactly as in the real photon case. We omit all the details of the calculation and give only the final answers:

$$T(Q,w,x) = \frac{N}{Q^2}\frac{1}{(1-x)w+x(1-w)}\left[1+C_F\frac{\alpha_s(Q)}{2\pi}t(x,w)\right]+(x\leftrightarrow 1-x) ,$$

$$\begin{aligned} t(x,w) = {} & \left[\frac{w-x}{2w-1}-\left[\frac{z}{2w-1}\right]^2\right]\left[\left[\frac{1}{x}L_1+\frac{1}{1-x}L_2\right]L_3-\frac{1}{2}\left[\frac{1}{x}{L_1}^2+\frac{1}{1-x}{L_2}^2\right]\right] \\ & +\frac{1}{2}\frac{1-z}{2w-1}(L_1-L_2)(2L_3-L_1-L_2)+\tfrac{3}{2}(L_3-3) \\ & -\left[\frac{3}{2}\frac{w-x}{2w-1}-\left[\frac{z}{2w-1}\right]^2\right]\left[\frac{1}{x}L_1+\frac{1}{1-x}L_2\right]-\frac{1}{2}\frac{3-2z}{2w-1}(L_1-L_2) , \end{aligned} \tag{5.2}$$

where $z=(1-x)w+x(1-w)$ and L_1, L_2, and L_3 are logarithms:

$$L_1=\ln\frac{z}{w},\quad L_2=\ln\frac{z}{1-w},\quad L_3=\ln z+\ln(Q^2/\mu^2) . \tag{5.3}$$

In the limit $w\to 1$, we regain the expression for the real photon case given by Eq. (3.12) with $\delta=1$.

The transition form factors for vector mesons can also be treated using QCD perturbation theory.[1] The form factors for transversely polarized vector mesons are suppressed by an extra power of Q^2, so we only consider a longitudinally polarized vector meson M. The form factor can be defined in terms of the amplitude for $\gamma^*\gamma^*\to M$:

$$\Gamma_{\mu\nu} = e^2F_{M\gamma^*}(Q,w)P\cdot\left[\frac{q_1-q_2}{2}\right]\left[g^{\mu\nu}+\frac{1}{2w-1}\frac{(q_1-q_2)^\mu P^\nu+P^\mu(q_1-q_2)^\nu}{Q^2}-\frac{2}{(2w-1)^2}\frac{P^\mu P^\nu}{Q^2}\right] \tag{5.4}$$

which satisfies $q_1^{\mu}\Gamma_{\mu\nu}=q_2^{\nu}\Gamma_{\mu\nu}=0$. The calculation of corrections to the hard-scattering amplitude is the same as in the pseudoscalar case, except that the pseudoscalar projection matrix $P\gamma_5$ is replaced by P. There is therefore no γ_5 ambiguity, so the calculation is straightforward. The final answer is the same as in Eq. (5.2) except that it must be antisymmetrized under the exchange of x and $1-x$ instead of symmetrized, and the function $t(x,w)$ is replaced by

$$t'(x,w)=t(x,w)-\left[\frac{z}{2w-1}\right]^2\left[\frac{1}{x}L_1+\frac{1}{1-x}L_2\right]-\frac{z}{2w-1}(L_1-L_2)\ . \tag{5.5}$$

In the limit $w\to 1$, we obtain the hard-scattering amplitude for the case of a real photon:

$$T(x,Q)=\frac{N}{Q^2}\frac{1}{1-x}\left\{1+C_F\frac{\alpha_s(Q)}{2\pi}\left[\tfrac{1}{2}\ln^2(1-x)-3\frac{1-x}{2x}\ln(1-x)-\tfrac{9}{2}+[\tfrac{3}{2}+\ln(1-x)]\ln(Q^2/\mu^2)\right]\right\}$$
$$-[x\leftrightarrow(1-x)]\ . \tag{5.6}$$

These results are all in agreement with the calculations of del Aguila and Chase.[3]

VI. CONCLUSIONS

We have calculated the QCD corrections to the hard-scattering amplitudes for the meson-photon transition form factors using dimensional regularization. In the case of a pseudoscalar meson, the calculation was complicated by a finite ambiguity associated with γ_5. We resolved this ambiguity by comparing the answer in dimensional regularization with a calculation using the operator-product expansion and a completely four-dimensional regularization method. We then showed that the ambiguity could also be resolved by demanding that the axial-vector Ward identities be preserved in dimensional regularization. If a definite prescription for γ_5 is adopted, finite counterterms must be introduced to restore the Ward identities and these counterterms give finite corrections to the hard-scattering amplitude. The correct answer can also be obtained without the use of counterterms if one uses a different prescription for γ_5 in the gluon exchange diagram Fig. 2(e) from that used to calculate the other diagrams.

To obtain the complete order-α_s corrections to these transition form factors, we also need the correction to the evolution kernel $V(x,y)$ for the meson distribution amplitude. The calculation of this correction will soon be completed.[11] Only when it is available will we be able to discuss the complete phenomenological implications of the QCD corrections to these form factors. However there are some quantities for which we can make predictions to order α_s without knowing the corrections to the evolution kernel. For example, by taking an appropriate ratio of form factors, the order-α_s correction to V can be made to cancel. One such ratio is $|F_{M\gamma}|^2/F_M$, where $F_M(Q)$ is the meson electromagnetic form factor for which the order-α_s corrections are already available.[6]

For pseudoscalar mesons, we can also make a prediction to order α_s for the form factor $F_{M\gamma}(Q)$ at asymptotic values of Q. In this limit, only the $n=0$ Gegenbauer moment of the distribution amplitude $\phi(x,Q)$ survives, since all the higher moments fall off by negative powers of $\ln Q$. The asymptotic distribution amplitude therefore has the form $\phi(x)=Ax(1-x)$, which does not evolve with Q^2. Hence the correction to the evolution kernel does not contribute to the form factor $F_{M\gamma}(Q)$, and the only correction comes from the hard-scattering amplitude. In the case of pions, the normalization factor A in the asymptotic distribution amplitude is related to the pion decay constant[1]: $A=\sqrt{3}f_\pi$. The complete prediction to order α_s for the asymptotic form factor $F_{\pi\gamma}(Q)$ is therefore

$$F_{\pi\gamma}(Q)\to 6({e_u}^2-{e_d}^2)\frac{f_\pi}{Q^2}\left[1-\frac{5}{2}C_F\frac{\alpha_s(Q)}{2\pi}\right]\ . \tag{5.7}$$

However, since the contribution of the next highest Gegenbauer moment is suppressed only by a fractional power of $\ln Q$, this prediction may not be accurate until very large values of Q.

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