Energy dependence of the fundamental parameters of the K^0-$\bar{K}^0$ system. II. Theoretical formalism

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(Received 2 December 1982)

We present a detailed analysis of the previously reported anomalous energy dependence of the fundamental K^0-$\bar{K}^0$ parameters $\Delta m = m_L - m_S$, τ_S, $|\eta_{+-}|$, and $\tan\phi_{+-}$. Such variations with energy can arise from the interaction of the kaons with an external field or medium. A phenomenological formalism is introduced to describe such energy-dependent influences on the K^0-$\bar{K}^0$ system. Using this formalism we demonstrate that effects of the type suggested by the data cannot be ascribed to an interaction of the kaons with an electromagnetic, hypercharge, or gravitational field, or to the scattering of the kaons from stray charges or cosmological neutrinos. The data are, however, compatible with an interaction which is even under charge conjugation, and models of such an interaction are discussed. We also consider the possibility that such effects may arise from a fundamental violation of Lorentz invariance. All of the mechanisms which appear capable of describing the data also suggest that similar effects could arise in neutrino oscillations, and some of the consequences of such a possibility are outlined.

I. INTRODUCTION

In a recent series of papers[1–3] we have reported evidence suggesting that several of the fundamental parameters of the K^0-$\bar{K}^0$ system may have an anomalous energy dependence. The data, which were obtained from a series of regeneration experiments at Fermilab,[4–6] specifically indicate that the values of $\Delta m = m_L - m_S$, τ_S, $|\eta_{+-}|$, and $\tan\phi_{+-}$ as determined in the K^0-$\bar{K}^0$ rest frame depend on the velocity of this frame with respect to the laboratory. If we let x denote the value of any of these four parameters in the proper frame of the kaons, then the anomalous behavior is manifested through nonzero values for the slope parameters $b_x^{(N)}$ defined by

$$x = x_0(1 + b_x^{(N)}\gamma^N), \quad \gamma = E_K/m, \quad N = 1,2 \; . \tag{1.1}$$

The object of the present paper is to study the $b_x^{(N)}$ theoretically with the aim of formulating a detailed model of the slope parameters.

An energy dependence of the neutral-kaon parameters in the kaon rest frame, such as that represented by Eq. (1.1), could (but need not necessarily) arise from the interaction of the K^0-$\bar{K}^0$ system with an external field or medium. (We will henceforth use the term "field" generically to denote *any* external influence on the K^0-$\bar{K}^0$ system, such as an electromagnetic, hypercharge, or gravitational field, the neutrino sea, or any other hypothetical medium permeating space.) Previous work along these lines has been aimed at setting limits on the effective couplings of various fields to the K^0-$\bar{K}^0$ system using the available low-energy data. Good[7] was the first to note that if the gravitational field has a component which is odd under charge conjugation (C), then the long-lived neutral kaon would decay rapidly into 2π. From the known limits on this decay mode, he was able to infer a limit on the strength of such a coupling to kaons. Following the actual observation of this (CP-violating) mode,[8] the idea of a C-odd field coupling to kaons was revived,[9] this time in the form of a long-range hypercharge interaction between the kaons and our galaxy. It was shown[9] that the coupling constant for this interaction could be chosen to account for the experimental value of $|\eta_{+-}|$, while at the same time remaining consistent with the limits implied[10] by the Eötvös-Dicke-Braginskii experiments.[11] However, a C-odd interaction mediated by a field with spin J leads to the prediction that $|\eta_{+-}| \propto \gamma^J$, from which $J>0$ could be ruled out even by the early low-energy data.[12] The remaining possibility, a C-odd $J=0$ field,[13,14] predicts the wrong value for ϕ_{+-} (at least in some models) and hence may also be ruled out.[12] A more detailed analysis of the effects of various cosmological fields on the K^0-$\bar{K}^0$ system was subsequently given by Nachtmann,[15] who considered the influence of particular choices of scalar, vector, and tensor fields on Δm as well as on η_{+-}. As can be seen from his Table III, however, none of the cases Nachtmann considers describes the data of Refs. 1–3. For example, for his scalar, vector, and tensor fields $|\eta_{+-}|$ is always directly proportional to γ^J, where $J=0$, 1, or 2, respectively. This contrasts with the behavior found in Refs. 1–3 which is described by Eq. (1.1). Nonetheless, Nachtmann's analysis is important both for its methodology and for the limits it sets on couplings of various C-odd fields to neutral kaons. These limits can, however, be significantly improved using the new Fermilab data,[1–3] as we proceed to discuss.

For later purposes we will need the specific numerical values of the slope parameters $b_x^{(N)}$, and hence we begin in Sec. II by reviewing the data of Refs. 1—3. Section III develops the formalism for describing an energy dependence of the neutral-kaon parameters. We assume that these effects can be accounted for by a set of complex γ-dependent functions u_a $(a=0,x,y,z)$ which, when added to the internal Hamiltonian for the K^0-$\bar{K}^0$ system, make Δm, $(\Gamma_L-\Gamma_S)$, and η_{+-} γ-dependent [see Eqs. (3.22) and (3.35)]. If a single u_a gives the dominant contribution to the observed γ dependence of the kaon parameters, then its real and imaginary parts can be fixed by using two of the four slope parameters $b_\Delta^{(N)}$, $b_\Gamma^{(N)}$, $b_\eta^{(N)}$, and $b_\phi^{(N)}$. It then follows that each u_a leads to two nontrivial relations among the four slope parameters, and these are given in Eqs. (3.54), (3.60), and (3.62) for u_x, u_y, and u_z, respectively. It should be emphasized that this treatment, in contrast to Nachtmann's,[15] is purely phenomenological in that it makes no assumptions concerning the origins of the u_a. Section III contains, in addition, a discussion of the γ dependence of the kaon parameters in the high-energy regime, where the effects of the u_a would be "large" in contrast to the present energy range where they are "small."

The slope relations derived in Sec. III are used in Sec. IV to demonstrate that several specific models of the u_a do not provide a natural description of effects of the type suggested by the data of Refs. 1—3. These include the hypotheses that the observed γ-dependent effects arise from an external electromagnetic or hypercharge field, or from the scattering of the kaons from stray charges or cosmological neutrinos. (Gravitational fields are considered separately as we discuss below.)

Section V discusses several theoretical models of the u_a which may be compatible with the experimental data. Consideration is given to the possibility that the u_a originate from some interaction which would also manifest itself elsewhere, such as in neutrino oscillations. The phenomenology of neutrino oscillations in the presence of such an interaction is discussed briefly.

In Appendix A, we review the kinematics of the regeneration process. Finally, in Appendix B, we present a detailed discussion of the behavior of the K^0-$\bar{K}^0$ system in a gravitational field. As we have noted previously in Ref. 2, care must be taken in describing the *observable* effects of a gravitational field, which affects not only the K^0-$\bar{K}^0$ system, but also the clocks and measuring rods that are used in studying it. We demonstrate that the experimental results of Refs. 1—3 cannot be explained in terms of any known gravitational effect. When combined with the results of Sec. IV, this leads to the conclusion that the experimental results, if correct, cannot be naturally explained in terms of any known interaction.

II. REVIEW OF THE DATA

We review in this section the salient features of the data presented in Refs. 1—3. From a theoretical point of view, the quantities of direct interest are the slope parameters $b_\Delta^{(N)}$, $b_\Gamma^{(N)}$, $b_\eta^{(N)}$, and $b_\phi^{(N)}$ $(N=1,2)$, which give the energy variation of Δm, $\Gamma_L-\Gamma_S$, $|\eta_{+-}|$, and $\tan\phi_{+-}$, respectively. As described in Refs. 1 and 3, we have extracted the slope parameters from the data under several different assumptions and these results are reproduced in Table I. For later purposes the following observations will be helpful.

(1) We begin by emphasizing that what we have determined experimentally is $b_{\Gamma S}$ $(\cong -b_{\tau S})$, and not b_Γ, because the individual widths $\Gamma_{L,S}$ appear in I(2.11) and not their difference. Even though $\Gamma_S \gg \Gamma_L$, b_Γ cannot be inferred from $b_{\Gamma S}$ without additional experimental or theoretical input, as we discuss in Sec. III. On the other hand, it is b_Γ, and not $b_{\Gamma S}$ which is simply related to the remaining slope parameters b_Δ, b_η, and b_ϕ. To determine b_Γ, the energy dependence of Γ_L must be measured, and a discussion of ways to do this is given in Ref. 3. We note in passing that the approximation $b_{\Gamma S}\cong -b_{\tau S}$ which we use repeatedly can be checked by actually fitting the data for $\Gamma_S=\hbar/\tau_S$. The agreement between the result so obtained and the approximate expression $b_{\Gamma S}\cong -b_{\tau S}$ is sufficiently good for our purposes.

(2) The single most important experimental result is the sign of b_Δ, which is *negative*. This observation by itself is

TABLE I. Summary of the data from Ref. 3. Results shown are for method A of Ref. 3. (1) Internal fit. (2) External fit, with low-energy values at $E_K \cong 5$ GeV: $\Delta m=(0.5349\pm0.0022)\times10^{10}$ $\hbar\,\mathrm{sec}^{-1}$, $\tau_S=(0.8923\pm0.0022)\times10^{-10}$ sec, $|\eta_{+-}|=(2.274\pm0.022)\times10^{-3}$, $\tan\phi_{+-}=0.986\pm0.041$. (3) As in (2) above, except $|\eta_{+-}|=(1.95\pm0.03)\times10^{-3}$.

Parameter		Fits of the form $x=x_0(1+b_x^{(N)}\gamma^N)$ x_0	$10^6 b_x^{(2)}$	χ^2/dof	x_0	$10^4 b_x^{(1)}$	χ^2/dof	Energy-independent fit x_0	χ^2/dof
$10^{-10}\Delta m$	(1)	0.557 ± 0.036	-8.48 ± 2.89	521/484	0.620 ± 0.066	-18.2 ± 6.05	522/484	0.482 ± 0.014	536/488
($\hbar\,\mathrm{sec}^{-1}$)	(2)	0.535 ± 0.002	-7.43 ± 1.48	533/488	0.535 ± 0.002	-9.07 ± 2.03	526/488	0.534 ± 0.002	604/492
	(3)	0.534 ± 0.002	-6.30 ± 1.46	550/488	0.535 ± 0.002	-8.49 ± 2.04	548/488	0.532 ± 0.002	573/492
$10^{10}\tau_S$	(1)	0.880 ± 0.015	$+1.77\pm0.90$	521/484	0.859 ± 0.029	$+4.35\pm2.58$	522/484	0.905 ± 0.007	536/488
(sec)	(2)	0.892 ± 0.002	$+1.27\pm0.38$	533/488	0.892 ± 0.002	$+1.47\pm0.56$	526/488	0.895 ± 0.002	604/492
	(3)	0.892 ± 0.002	$+0.99\pm0.38$	550/488	0.892 ± 0.002	$+1.27\pm0.57$	548/488	0.893 ± 0.002	573/492
$10^3\lvert\eta_{+-}\rvert$	(1)	2.14 ± 0.04	-2.01 ± 0.86	521/484	2.21 ± 0.07	-4.80 ± 2.15	522/484	2.09 ± 0.02	536/488
	(2)	2.23 ± 0.02	-3.60 ± 0.52	533/488	2.26 ± 0.02	-6.26 ± 0.84	526/488	2.14 ± 0.01	604/492
	(3)	2.07 ± 0.02	-0.20 ± 0.62	550/488	2.03 ± 0.03	$+1.78\pm1.14$	548/488	2.07 ± 0.01	573/492
$\tan\phi_{+-}$	(1)	1.276 ± 0.499	-33.7 ± 12.3	521/484	2.071 ± 1.840	-99.5 ± 33.3	522/484	0.709 ± 0.102	536/488
	(2)	0.954 ± 0.048	-21.5 ± 7.0	533/488	0.966 ± 0.052	-26.3 ± 10.1	526/488	1.009 ± 0.036	604/492
	(3)	1.033 ± 0.052	-22.3 ± 6.7	550/488	1.009 ± 0.054	-30.1 ± 10.0	548/488	1.081 ± 0.040	573/492

sufficient to rule out a number of possible sources for the observed effects, in particular an electromagnetic field, hypercharge field, or stray charges, as we discuss below.

(3) In Ref. 3, we also examined m_S for a possible energy variation and found none. As we discuss in Sec. III, the individual masses $m_{L,S}$ can have a different energy variation from that of $\Delta m = m_L - m_S$. Hence it is perfectly consistent to have the slope parameter $b_{\mu S}$ for m_S zero, while b_Δ, $b_{\tau S}$, b_η, and b_ϕ are nonzero. However, m_S is determined in a manner that is fundamentally different from that used for the other parameters. It is thus possible that $b_{\mu S}$ is in fact comparable to b_Δ, but nonetheless appears to be zero when analyzed as we have. This has important consequences for the construction of models of the b's, as we discuss elsewhere.

III. DESCRIPTION OF THE K^0-$\bar{K}^0$ SYSTEM IN AN EXTERNAL FIELD

We present in this section a systematic description of the K^0-$\bar{K}^0$ system in an arbitrary external field. Our objective is to provide a general framework for understanding the experimental results of Refs. 1—3 in terms of which specific theoretical models can later be formulated. It should be reemphasized at the outset that the term field will be used generically to denote *any* external influence on the K^0-$\bar{K}^0$ system, such as an electromagnetic or gravitational field, the neutrino sea, or any other hypothetical medium permeating space. We assume that in the absence of such a field the proper-time evolution of the K^0-$\bar{K}^0$ wave function $\Psi(t)$ is given by

$$-\frac{\partial\Psi(t)}{\partial t} = iH_0\Psi \ , \tag{3.1}$$

where H_0 is a 2×2 matrix. (We take $\hbar = c = 1$ in this section.) For various purposes it is convenient to express iH_0 in a number of equivalent forms:

$$iH_0 = \Gamma + iM \tag{3.2a}$$

$$= h_0 1 + h_x\sigma_x + h_y\sigma_y + h_z\sigma_z \tag{3.2b}$$

$$\equiv \begin{bmatrix} id & p^2 \\ q^2 & i\bar{d} \end{bmatrix} \ . \tag{3.2c}$$

Here $\Gamma = \Gamma^\dagger$ and $M = M^\dagger$ are 2×2 matrices, the σ's are the usual Pauli matrices, and $h_0, h_x, \ldots, d, \bar{d}, p^2$, and q^2 are complex numbers. In Table II, we summarize the restrictions imposed by charge conjugation (C), parity (P), and time reversal (T)[16] on the matrix elements of iH_0 using any of the forms (3.2a)—(3.2c). The eigenvalues $\lambda^\pm$ of iH_0 are given by

$$\lambda^\pm = \frac{i}{2}(d+\bar{d}) \pm \tfrac{1}{2}[4p^2q^2 - (d-\bar{d})^2]^{1/2}$$
$$\equiv \tfrac{1}{2}\Gamma^\pm + im^\pm \ , \tag{3.3}$$

and the corresponding eigenvectors $\Psi^\pm$ are

$$\Psi^\pm = \begin{bmatrix} a^\pm \\ b^\pm \end{bmatrix} \ , \tag{3.4a}$$

$$\frac{a^\pm}{b^\pm} = \frac{p^2}{\lambda^\pm - id} = \frac{\lambda^\pm - i\bar{d}}{q^2} \ . \tag{3.4b}$$

We choose the phases of $|K^0\rangle$ and $|\bar{K}^0\rangle$ such that

$$CP\,|K^0\rangle = -\,|\bar{K}^0\rangle \ , \tag{3.5}$$

in which case the CP eigenfunctions $|K_1^0\rangle$ and $|K_2^0\rangle$ are given by

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad (CP=+1) \ ,$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (CP=-1) \ . \tag{3.6}$$

If CP is not conserved (but CPT is), then the eigenfunctions in (3.6) are replaced by

$$\Psi^- = |K_S\rangle = (|p|^2 + |q|^2)^{-1/2}(p\,|K^0\rangle - q\,|\bar{K}^0\rangle) \ ,$$
$$\Psi^+ = |K_L\rangle = (|p|^2 + |q|^2)^{-1/2}(p\,|K^0\rangle + q\,|\bar{K}^0\rangle) \ . \tag{3.7}$$

The states $\Psi^\pm$ evolve in time according to

$$\Psi^\pm(t) = (e^{-\lambda^\pm t})\Psi^\pm(0) = (e^{-\Gamma^\pm t/2}e^{-im^\pm t})\Psi^\pm(0) \ , \tag{3.8}$$

where $\Gamma^+ = \Gamma_L$ and $\Gamma^- = \Gamma_S$ are the widths of K_L and K_S, respectively, and $m^\pm$ are the corresponding masses. It follows that

$$\lambda^+ - \lambda^- = 2pq = \tfrac{1}{2}(\Gamma_L - \Gamma_S) + i(m_L - m_S)$$
$$= \tfrac{1}{2}(\Gamma_L - \Gamma_S) + i\,\Delta m$$
$$\cong -\tfrac{1}{2}\Gamma_S + i\,\Delta m \ . \tag{3.9}$$

To describe the effects of an external field, we write

$$iH_0 = \Gamma + iM \to iH = \Gamma + iM + iF \ ,$$
$$F = u_0 1 + u_x\sigma_x + u_y\sigma_y + u_z\sigma_z \ . \tag{3.10}$$

The u's are complex numbers which are functions of $\gamma = E_K/m = (1-\beta^2)^{-1/2}$ and of position, in contrast to the h's in Eq. (3.2) which are constants. We can decompose

TABLE II. Restrictions imposed by discrete space-time symmetries on the matrix elements of iH_0 in Eq. (3.2).

Form of iH_0	CP	T	CPT
(3.2a)	$M_{12}=M_{12}^*=M_{21}=M_{21}^*$ $\Gamma_{12}=\Gamma_{12}^*=\Gamma_{21}=\Gamma_{21}^*$ $M_{11}=M_{22}$; $\Gamma_{11}=\Gamma_{22}$	$M_{12}=M_{12}^*=M_{21}=M_{21}^*$ $\Gamma_{12}=\Gamma_{12}^*=\Gamma_{21}=\Gamma_{21}^*$	$M_{11}=M_{22}$ $\Gamma_{11}=\Gamma_{22}$
(3.2b)	$h_y=h_z=0$	$h_y=0$	$h_z=0$
(3.2c)	$p^2=q^2$ $d=\bar{d}$	$p^2=q^2$	$d=\bar{d}$

the u_a ($a=x,y,z$) into their real and imaginary parts,

$$u_a=\xi_a+i\zeta_a \ , \tag{3.11a}$$

$$\xi_a=\xi_a^{(0)}+\xi_a^{(1)}\gamma+\xi_a^{(2)}\gamma^2+\cdots \ , \tag{3.11b}$$

$$\zeta_a=\zeta_a^{(0)}+\zeta_a^{(1)}\gamma+\zeta_a^{(2)}\gamma^2+\cdots \ . \tag{3.11c}$$

$\xi_a^{(N)}$ and $\zeta_a^{(N)}$ ($N=0,1,2,\ldots$) are now real functions, which can be directly related to the experimentally determined slope parameters b_Δ, b_Γ, b_η, and b_ϕ as we discuss below. Although $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ can, in principle, depend on $\vec{x}$, the present experiments are insensitive to $\vec{\nabla}\xi_a^{(N)}$ and $\vec{\nabla}\zeta_a^{(N)}$, and hence the ξ's and ζ's can be treated as if they were independent of position. (This point is discussed in greater detail in Appendix B.) However, in order to fully describe the u_a it will be necessary in the future to determine $\vec{\nabla}u_a$. Such measurements could also distinguish between the intrinsic contributions to H from d, $\bar{d}$, p^2, and q^2, and from the external (but velocity-independent) contributions from $\xi_a^{(0)}$ and $\zeta_a^{(0)}$. It should also be noted that in some models $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ can themselves be proportional to $\beta=v/c$, but since $\beta\cong 1$ in the high-energy regeneration experiments we are considering, we can take $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ to be constants. From a theoretical point of view, however, it may be potentially important to be able to distinguish between a dependence of u_a on γ^N or $\beta^N\gamma^N$, for example.

It will be useful in the ensuing discussion to have in mind a specific example of one of the u's. Suppose there existed a long-range field which coupled to the hypercharge Y, whose source was our own galaxy. Since K^0 and $\bar{K}^0$ have opposite values of Y, their coupling to this field would produce an energy difference which would manifest itself as an apparent breakdown of CP conservation.[7,9] Let A_0 denote the static hypercharge potential of a K^0 due to its interaction with the galaxy,

$$A_0=f^2\frac{Y_G}{R_G} \ , \tag{3.12}$$

where Y_G and R_G are the hypercharge and effective radius of the galaxy, and f^2 is an appropriate coupling constant. If A_0 is assumed to be the fourth component of a four-vector A_μ, then the potential seen by a kaon moving with velocity β is $A_0\gamma$. Since such a field produces equal and opposite energy shifts for K^0 and $\bar{K}^0$, its effects are represented by a contribution to F of the form

$$\begin{aligned} &u_z=A_0\gamma \ , \\ &\xi_z^{(1)}=A_0, \ \ \xi_z^{(N)}=0 \text{ for } N\neq 1 \ , \\ &\zeta_z^{(N)}=0, \ \text{ for all } N \ . \end{aligned} \tag{3.13}$$

It will be shown below that the observed energy dependence of Δm and η_{+-} cannot in fact be accounted for by such an interaction.

Returning to Eq. (3.10), we see that in the presence of external interactions iH has the form

$$\begin{aligned} iH&=i\begin{bmatrix} d+u_0+u_z & -ip^2+u_x-iu_y \\ -iq^2+u_x+iu_y & \bar{d}+u_0-u_z \end{bmatrix} \\ &\equiv i\begin{bmatrix} d_u & -ip_u{}^2 \\ -iq_u{}^2 & \bar{d}_u \end{bmatrix} \ . \end{aligned} \tag{3.14}$$

The eigenvalues $\lambda_u^{\pm}$ of iH can be obtained immediately from Eq. (3.3) by simply replacing d, $\bar{d}$, p^2, and q^2 by the corresponding u-dependent parameters d_u, $\bar{d}_u$, $p_u{}^2$, and $q_u{}^2$, respectively. From Eq. (3.9), we then find

$$\begin{aligned} \lambda_u^{+}-\lambda_u^{-}&\equiv\tfrac{1}{2}(\Gamma_L-\Gamma_S)_u+i(\Delta m)_u \\ &=2pq\{1+(pq)^{-2}[iu_x(p^2+q^2)-u_y(p^2-q^2)-(u_x{}^2+u_y{}^2+u_z{}^2)]\}^{1/2} \ . \end{aligned} \tag{3.15}$$

When the right-hand side of Eq. (3.15) is separated into its real and imaginary parts, the dependence of $(\Gamma_L-\Gamma_S)_u$ and $(\Delta m)_u$ on the u_a can be inferred. In principle, Eq. (3.15) thus generates an exact, but complicated, expression for the γ dependence of the experimentally determined quantities $(\Gamma_L-\Gamma_S)_u$ and $(\Delta m)_u$ once the dependence of the u_a on γ is specified. The complete expression for $\lambda_u^+-\lambda_u^-$ will be used below when we discuss the behavior of the K^0-$\bar{K}^0$ system at very high energies. However, given the limited statistics and lower energies of the available data, the best we can hope to do at the present time is to recast Eq. (3.15) in the same form as that used in Refs. 1–3 to parametrize $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$, namely,

$$(\Delta m)_u=(\Delta m)(1+b_\Delta^{(N)}\gamma^N), \ \ N=1,2 \ , \tag{3.16a}$$

$$(\Gamma_L-\Gamma_S)_u=(\Gamma_L-\Gamma_S)(1+b_\Gamma^{(N)}\gamma^N), \ \ N=1,2 \ . \tag{3.16b}$$

Here Δm and $\Gamma_{S,L}$ are the values for the K_L-K_S mass difference and $K_{S,L}$ decay rates that would obtain in a world in which the u_a were zero, and $b_\Delta^{(N)}$, $b_\Gamma^{(N)}$ are functions of $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ as we discuss below. Since b_Δ and b_Γ are experimentally very small, Δm and $\Gamma_L-\Gamma_S$ can be identified with the low-energy ($\gamma\approx 10$) Particle Data Group values of these parameters. (This becomes an exact statement in models where $b_\Delta^{(N)}$ and $b_\Gamma^{(N)}$ are themselves proportional to β.)

Similar remarks apply to the γ dependence of $\eta_{+-}=A(K_L\to\pi^+\pi^-)/A(K_S\to\pi^+\pi^-)$ which is determined by the analog of Eq. (3.4),

$$\frac{a_u^{\pm}}{b_u^{\pm}}=\frac{p_u{}^2}{\lambda_u^{\pm}-id_u}=\frac{\lambda_u^{\pm}-i\bar{d}_u}{q_u{}^2} \ . \tag{3.17}$$

As we demonstrate below, Eq. (3.17) leads to the relations

$$|\eta_{+-}|_u=|\eta_{+-}|(1+b_\eta^{(N)}\gamma^N), \ \ N=1,2 \ , \tag{3.18a}$$

$$(\tan\phi_{+-})_u=(\tan\phi_{+-})(1+b_\phi^{(N)}\gamma^N), \ \ N=1,2 \ , \tag{3.18b}$$

where again $b_\eta^{(N)}$ and $b_\phi^{(N)}$ are functions of $\xi_a^{(N)}$ and $\zeta_a^{(N)}$. $|\eta_{+-}|_u$ and $|\eta_{+-}|$ are interpreted in exactly the same way as were $(\Delta m)_u$ and Δm, and similarly for $(\tan\phi_{+-})_u$ and $\tan\phi_{+-}$.

To proceed, we introduce a number of simplifications into Eqs. (3.15) and (3.17).

(1) *CPT* will be presumed to hold for the intrinsic Hamiltonian H_0 so that $d=\bar{d}$. Note, however, that $d_u \neq \bar{d}_u$ if $u_z \neq 0$.

(2) We assume that at the energies of the current experiments $|u_a|$ $(a=0,x,y,z)$ is small compared to $|p^2|$ or $|q^2|$, but not necessarily compared to $\epsilon \equiv 1-q/p$. If, for example, we examine the experimental results for Δm in Sec. II, we observe that a typical value of the momentum-dependent factor $b_\Delta^{(2)}\gamma^2$ is 0.2 at $p_K=70$ GeV/c. Since $b_\Delta^{(2)}\gamma^2$ arises from the terms in square brackets in Eq. (3.15), which typically are of the form u_x/p^2, or $u_x{}^2/p^2q^2$, etc., it follows that such terms must be small (barring accidental cancellations). Equation (3.15) can then be expanded in powers of $u_a/p^2,\ldots$, etc.

(3) Since the u-dependent terms are in fact small, we can approximate the denominators which arise in the expansion of Eq. (3.15) as follows. From Eq. (3.9),

$$2pq \simeq \Delta m(i-\Gamma_S/2\Delta m)=\Delta m(i-1.05) \simeq \Delta m(i-1) \ . \tag{3.19}$$

Using $q=p(1-\epsilon)$, we then have

$$2p^2 \cong \Delta m(i-1)(1+\epsilon) \ , \tag{3.20a}$$

$$2q^2 \cong \Delta m(i-1)(1-\epsilon) \ , \tag{3.20b}$$

$$p^2-q^2 \cong 2p^2\epsilon \cong \Delta m(i-1)\epsilon \ , \tag{3.20c}$$

$$p^2+q^2 \cong 2p^2(1-\epsilon) \cong \Delta m(i-1) \ , \tag{3.20d}$$

$$(p^2-q^2)^2 = O(\epsilon^2) \cong 0 \ , \tag{3.20e}$$

$$(p^2+q^2)^2 \cong (2p^2)^2(1-2\epsilon) \cong [\Delta m(i-1)]^2 = -2i(\Delta m)^2 \ , \tag{3.20f}$$

$$\frac{(p^2+q^2)^2}{4p^2q^2} \cong 1 \ , \tag{3.20g}$$

$$\frac{(p^2+q^2)(p^2-q^2)}{2p^2q^2} \cong 2\epsilon \ . \tag{3.20h}$$

Combining Eqs. (3.15) and (3.20), we find

$$\tfrac{1}{2}(\Gamma_L-\Gamma_S)_u + i(\Delta m)_u \cong [\tfrac{1}{2}(\Gamma_L-\Gamma_S)+i\,\Delta m]\left[1+\frac{u_x}{\Delta m}(1-i)+\frac{u_y}{\Delta m}\epsilon(1+i)-\frac{i}{(\Delta m)^2}[u_y{}^2+u_z{}^2-2i\epsilon u_x u_y]\right] \ . \tag{3.21}$$

We have retained in Eq. (3.21) all contributions of order u_a, $u_a{}^2$, $u_a\epsilon$, $u_a u_b$, and $u_a u_b \epsilon$, where u_a and u_b denote any of the terms u_0, u_x, u_y, or u_z. There are several reasons for keeping the (presumably) small higher-order terms in Eq. (3.21).

(a) To start with we see that the leading contribution from u_z is in fact proportional to $u_z{}^2$, for reasons detailed below, and hence consistency demands that we retain $u_y{}^2$ and $u_y\epsilon$ which could in principle be comparably large. The reason why there is no contribution linear in u_z is that $u_z\sigma_z$ is odd under C, which means that it can contribute to the (complex) mass difference only in second (or higher) orders.

(b) The only term in Eq. (3.21) which is (nominally) of leading order in small quantities is that proportional to u_x. If we assume that this term has a simple γ dependence such as γ^N with $N=1$, or 2, then Eq. (3.21) can be used to relate $\xi_x^{(N)}$ and $\zeta_x^{(N)}$ to the observed slope parameters b_Δ and b_Γ as we discuss shortly. Once $\xi_x^{(N)}$ and $\zeta_x^{(N)}$ are determined, however, the slopes of $|\eta_{+-}|$ and $\tan\phi_{+-}$ are also determined. As we point out below, it is not clear at the present time whether the experimental results of Sec. II are in fact consistent with u_x giving the dominant contribution to the various slope parameters. If they are not, then other (necessarily higher-order) terms must be included in Eq. (3.21), which is part of the reason why such terms have been retained. Among these $\epsilon u_x u_y$ is (nominally) third order in small quantities, but since this is the only such term it can be included with little additional effort.

We proceed to separate Eq. (3.21) into its real and imaginary parts in order to recast it in the form of Eqs. (3.16). Using Eq. (3.11a), we have

$$(\Gamma_L-\Gamma_S)_u=(\Gamma_L-\Gamma_S)\left[1+\frac{2\xi_x+\sqrt{2}\,|\epsilon|\,(\xi_y-\zeta_y)}{\Delta m} + \frac{[(-\xi_y{}^2+\zeta_y{}^2+2\xi_y\zeta_y)+(-\xi_z{}^2+\zeta_z{}^2+2\xi_z\zeta_z)-2\sqrt{2}\,|\epsilon|\,(\xi_x\xi_y-\zeta_x\zeta_y)]}{(\Delta m)^2}\right] \ , \tag{3.22a}$$

$$(\Delta m)_u=(\Delta m)\left[1+\frac{2\zeta_x-\sqrt{2}\,|\epsilon|\,(\xi_y+\zeta_y)}{\Delta m} - \frac{[(-\xi_y{}^2+\zeta_y{}^2-2\xi_y\zeta_y)+(-\xi_z{}^2+\zeta_z{}^2-2\xi_z\zeta_z)-2\sqrt{2}\,|\epsilon|\,(\xi_x\zeta_y+\xi_y\zeta_x)]}{(\Delta m)^2}\right] \ . \tag{3.22b}$$

In going from Eq. (3.21) to Eq. (3.22), we take $2\Delta m/(\Gamma_L-\Gamma_S) \cong -1$, and

$$\epsilon \cong |\epsilon|\frac{(1+i)}{\sqrt{2}} \ , \tag{3.23}$$

as we explain below. We will return to Eqs. (3.22) shortly after the analogous expressions for $|\eta_{+-}|$ and $\tan\phi_{+-}$ are derived.

Turning next to η_{+-} (and η_{00}), we introduce the following notation[17]:

$$A(K^0\to\pi^+\pi^-)=c',\quad A(\bar{K}^0\to\pi^+\pi^-)=\bar{c}'\ ,$$
$$A(K^0\to\pi^0\pi^0)=d',\quad A(\bar{K}^0\to\pi^0\pi^0)=\bar{d}'\ ,\tag{3.24}$$
$$\frac{\bar{c}'}{c'}=\epsilon'-1,\quad \frac{\bar{d}'}{d'}=-(1+2\epsilon'')\ .$$

Hence, in the absence of external fields, we have

$$\begin{aligned}\eta_{+-}&=\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=\frac{pc'+q\bar{c}'}{pc'-q\bar{c}'}\\&=\frac{1+(1-\epsilon)(\epsilon'-1)}{1-(1-\epsilon)(\epsilon'-1)}\\&\cong\tfrac{1}{2}(\epsilon+\epsilon')\ ,\end{aligned}\tag{3.25}$$

and similarly,

$$\eta_{00}\cong\tfrac{1}{2}(\epsilon-2\epsilon'')\ .\tag{3.26}$$

If we choose the phase of K^0 so that the stationary $I=0$ $K^0\to 2\pi$ amplitude is real,[18] then $\epsilon'=\epsilon''$. The current experimental values for η_{+-} and η_{00} at low energies are[19]

$$\begin{aligned}|\eta_{+-}|&=(2.274\pm0.022)\times10^{-3}\ ,\\ \phi_{+-}&=(44.6\pm1.2)^\circ\ ,\\ |\eta_{00}|&=(2.33\pm0.08)\times10^{-3}\ ,\\ \phi_{00}&=(54\pm5)^\circ\ .\end{aligned}\tag{3.27}$$

Equation (3.27) suggests that $\eta_{+-}\cong\eta_{00}$ and hence that $\epsilon'=\epsilon''\cong 0$, an assumption which we will henceforth make in the u-dependent terms. From Eqs. (3.25) and (3.26), it then follows that $\eta_{+-}=\eta_{00}=\epsilon/2$, in the absence of external fields. ϵ thus has the same phase ($\simeq 45^\circ$) as η_{+-} which leads immediately to Eq. (3.23).

In the presence of external fields p, q, d, and $\bar{d}$ are replaced by p_u, q_u, d_u, and $\bar{d}_u$, respectively. We continue to assume that $|u_a|/|p^2|<1$, etc., but now greater care must be taken in retaining various terms which are nominally higher order in the small parameters u_a and ϵ. The reason for this can be seen by noting that the momentum dependence of $|\eta_{+-}|$ in Eq. (3.18a),

$$|\eta_{+-}|_u=|\eta_{+-}|+|\eta_{+-}|b_\eta^{(N)}\gamma^N\ ,\tag{3.28}$$

arises from a term of the form $|\eta_{+-}|u_a\cong|\epsilon/2|u_a$ which is thus nominally of second order. It follows that we must retain at least *some* second-order terms in the expression for η_{+-}. However, not all such terms can be retained since the resulting expression would be too cumbersome to be of practical use. In order for $(\eta_{+-})_u$ to contain enough structure to describe the current data, but not too much to render it useless, we will invoke the following additional approximations

(4) All terms which are higher than second order are dropped.

(5) All terms of order ϵ^2, ϵ'^2, or $\epsilon\epsilon'$ are dropped.

(6) All terms of the form $u_a{}^2$ are also dropped. The justification for this assumption is that each such term is smaller than the corresponding one linear in u_a by a factor of order $|u_a|/\Delta m<1$, and leads to no new physics if included. However, terms of the form u_au_b $(a\neq b)$ will be retained.

(7) We set $u_a(1\pm\epsilon)\simeq u_a$, for essentially the same reasons as in (6) above. Although these approximations do not constitute a formal expansion of $(\eta_{+-})_u$ in small quantities, they do generate an expression for $(\eta_{+-})_u$ which is sufficiently accurate for our present purposes.

Returning to Eq. (3.4), the eigenfunctions in the presence of an external field are

$$\Psi_u^\pm=\begin{bmatrix}a_u^\pm\\ b_u^\pm\end{bmatrix}\ ,\tag{3.29}$$

where $a_u^\pm/b_u^\pm$ is given by Eqs. (3.14) and (3.17). Invoking the approximations in (1)—(7) above we find after some algebra

$$\frac{a_u^\pm}{b_u^\pm}\equiv\rho^\pm\cong\frac{u_z}{\Delta m}(1-i)\left[1-\frac{(u_x+iu_y)}{\Delta m}(1-i)\right]\pm\left[1+\epsilon-\left[\frac{u_x\epsilon(1-i)+u_y(1+i)}{\Delta m}\right]+\frac{2u_xu_y}{(\Delta m)^2}\right]\ .\tag{3.30}$$

Using Eq. (3.30), the eigenfunctions $\Psi_u^\pm$ can be written in the form

$$\begin{aligned}\Psi_u^+&=|K_L\rangle_u=N^+(\rho^+|K^0\rangle+|\bar{K}^0\rangle)\ ,\\ \Psi_u^-&=|K_S\rangle_u=-N^-(\rho^-|K^0\rangle+|\bar{K}^0\rangle)\ ,\\ N^\pm&=(1+|\rho^\pm|^2)^{-1/2}\ ,\end{aligned}\tag{3.31}$$

and hence,

$$(\eta_{+-})_u=\frac{N^+(\rho^+c'+\bar{c}')}{-N^-(\rho^-c'+\bar{c}')}\ .\tag{3.32}$$

Equation (3.32) can be simplified by use of the approximations in (1)—(7) above. We find

$$(\eta_{+-})_u\cong\frac{N^+}{N^-}\left[\tfrac{1}{2}(\epsilon+\epsilon')+\left[\frac{-u_x\epsilon(1-i)-u_y(1+i)+u_z(1-i)}{2\Delta m}\right]+\left[\frac{u_xu_y+iu_z(u_x+iu_y)}{(\Delta m)^2}\right]\right]\ .\tag{3.33}$$

To the required accuracy we can write

$$\frac{N^+}{N^-}\cong 1-\mathrm{Re}\left[\frac{u_z}{\Delta m}(1-i)\right] , \tag{3.34}$$

and hence,

$$(\eta_{+-})_u \cong \tfrac{1}{2}(\epsilon+\epsilon')+\frac{1}{2\,\Delta m}[-u_x\epsilon(1-i)-u_y(1+i)+u_z(1-i)]+\frac{1}{(\Delta m)^2}\left[u_xu_y+iu_xu_z-\tfrac{1}{2}u_yu_z+\frac{i}{2}u_yu_z^*\right] , \tag{3.35}$$

where the asterisk indicates complex conjugation. In a similar fashion we have for η_{00},

$$(\eta_{00})_u \simeq \tfrac{1}{2}(\epsilon-2\epsilon'')+\frac{1}{2\,\Delta m}[\cdots] + \frac{1}{(\Delta m)^2}[\cdots] , \tag{3.36}$$

where the expressions in square brackets in Eq. (3.36) are identical to the corresponding ones in (3.35). To recapitulate, Eqs. (3.35) and (3.36) contain all terms of the form ϵ, ϵ', u_a, $u_a\epsilon$, and u_au_b. All terms of third and higher order in small quantities have been dropped, as have the second-order terms proportional to ϵ^2, ϵ'^2, ϵ''^2, $\epsilon\epsilon'$, $\epsilon\epsilon''$, and $u_a{}^2$. Equation (3.35) can now be separated into its real and imaginary parts to obtain expressions for $|\eta_{+-}|_u$ and $(\tan\phi_{+-})_u$ as was done for $(\Gamma_L-\Gamma_S)_u$ and $(\Delta m)_u$. However, since the resulting expressions in the general case are cumbersome, we will quote the results for $|\eta_{+-}|_u$ and $(\tan\phi_{+-})_u$ only for the special cases that we consider below.

We note in passing that the linear contribution to $(\eta_{+-})_u$ from u_x appears with a coefficient ϵ, whereas the contributions linear in u_y and u_z do not. This observation, which has important phenomenological consequences, can be understood by examining the behavior of the terms proportional to u_x, u_y, and u_z under charge conjugation. In the conventions of Eq. (3.5), C is given by

$$C=\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} , \tag{3.37}$$

from which it follows that $u_0 1$ and $u_x\sigma_x$ are even under C, whereas $u_y\sigma_y$ and $u_z\sigma_z$ are odd. Such C-odd terms contribute differently to K^0 and $\bar{K}^0$ and thus lead directly to processes which manifest CP violation. By contrast, the term proportional to u_x cannot by itself lead to CP violation, but it can impart a momentum-dependence to a preexisting CP-violating term which would otherwise be constant. In the presence of the coupling $u_x\sigma_x$, Eq. (3.17) becomes

$$\frac{a_u^\pm}{b_u^\pm}=\pm\frac{p_u}{q_u}=\pm\frac{(p^2+iu_x)^{1/2}}{(q^2+iu_x)^{1/2}} . \tag{3.38}$$

We see from Eq. (3.38) that if $p^2=q^2$, so that there is no intrinsic CP violation, then $a_u^\pm/b_u^\pm=\pm 1$ and the eigenvectors $\Psi_u^\pm$ are just K_2^0 and K_1^0, independent of the form of u_x. However, when $p^2\neq q^2$, $a_u^\pm/b_u^\pm$ will depend on u_x and hence will be γ-dependent in general. It follows that a u_x-dependent term is manifest only when there is a preexisting (or intrinsic) CP violation, and hence that the contribution to $(\eta_{+-})_u$ from u_x must be of the form

$$\begin{aligned}(\eta_{+-})_u &= \eta_{+-}+(\text{constant})\epsilon u_x/2\\ &\cong \eta_{+-}[1+(\text{constant})u_x] ,\end{aligned} \tag{3.39}$$

in agreement with Eq. (3.35). Moreover, Eqs. (3.35) and (3.39) can be recast in the form of Eqs. (3.18) with $u_x\propto\gamma^N$, and could thus provide a phenomenological description of the data of Sec. II. In summary, then, the C-odd terms $u_y\sigma_y$ and $u_z\sigma_z$ lead to "large" γ-dependent contributions to $(\eta_{+-})_u$ whereas the C-even term $u_x\sigma_x$ leads to a "small" γ-dependent contribution. For later purposes it is worth noting that for $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$, the roles of the "large" and "small" terms are interchanged, as we see from Eqs. (3.22). We will return to quantify "large" and "small" more precisely below.

Another CP-violating parameter which is accessible experimentally, in addition to η_{+-} and η_{00}, is $\mathrm{Re}\epsilon$ which can be extracted from the charge asymmetry δ in the decays $K_L\to\pi^\pm l^\mp\nu(l=e$ or $\mu)$. In the absence of external fields (and assuming CPT), we have[20]

$$\delta=\frac{\Gamma(K_L\to\pi^-l^+\nu)-\Gamma(K_L\to\pi^+l^-\bar{\nu})}{\Gamma(K_L\to\pi^-l^+\nu)+\Gamma(K_L\to\pi^+l^-\bar{\nu})} = \frac{(1-|x_+|^2)}{|1+x_+|^2}\mathrm{Re}\epsilon , \tag{3.40}$$

where $x_+=A(\bar{K}^0\to\pi^-l^+\nu)/A(K^0\to\pi^-l^+\nu)$ measures the relative magnitudes of the $\Delta Q=\mp\Delta S$ semileptonic amplitudes. Even though data for the $K_{\mu 3}^0$ decay modes were collected in Ref. 4, as we have already noted in I, the cuts imposed in analyzing these data were such as to preclude a determination of δ.[21] Nonetheless, a measurement of δ at high energy can (and should) be carried out in the future, as we discuss in I.[22] If we take the current experimental limits[19] on x_+,

$$\begin{aligned}\mathrm{Re}x_+ &= -0.009\pm 0.020 ,\\ \mathrm{Im}x_+ &= 0.004\pm 0.026 ,\end{aligned} \tag{3.41}$$

to indicate that $x_+\equiv 0$, then the dependence of the u_a on γ can be determined directly from a measurement of $\delta=\delta(\gamma)$. (Even though $|x_+|$ could in principle be comparable to $|u_a|/\Delta m$, retaining x_+ merely serves to needlessly complicate the analysis of the γ dependence of δ. From a quantitative point of view the effect of x_+ on the slope of δ is entirely negligible, hence x_+ will hereafter be set equal to zero.) From Eq. (3.31) we find immediately that $\delta\to\delta_u$,

$$\delta_u\cong\frac{|\rho^+|^2-1}{|\rho^+|^2+1} , \tag{3.42}$$

where ρ^+ is given by Eq. (3.30). Making the same approximations as in (1)—(7) above we find

$$\delta_u \cong \mathrm{Re}\epsilon + \mathrm{Re}(\tilde{\rho}^+) - \tfrac{1}{2}\,\mathrm{Re}(\tilde{\rho}^+)^2 \ , \qquad \tilde{\rho}^+ \equiv \rho^+ - (1+\epsilon) \ . \tag{3.43}$$

It is understood that when writing out the explicit form of $(\tilde{\rho}^+)^2$ only the terms allowed by approximations (1)—(7) are retained. For later purposes we exhibit the expression for δ_u in the approximation when only the terms linear in the u_a are retained:

$$\delta_u \cong \mathrm{Re}\epsilon - \frac{1}{\Delta m}(\sqrt{2}\,|\epsilon|\,\xi_x + \xi_y - \zeta_y - \xi_z - \zeta_z) \ . \tag{3.44}$$

We will return to Eqs. (3.43) and (3.44) below where we relate the slope parameters b_δ and b_ϵ of δ_u and $(\mathrm{Re}\epsilon)_u$,

$$\delta_u = \delta(1 + b_\delta^{(N)}\gamma^N) \ , \tag{3.45}$$

$$(\mathrm{Re}\epsilon)_u = \mathrm{Re}\epsilon(1 + b_\epsilon^{(N)}\gamma^N) \tag{3.46}$$

to b_Δ, b_Γ, b_η, and b_ϕ.

The remaining parameters whose γ dependence can be determined are $m_{L,S}$ and $\Gamma_{L,S}$. From Eqs. (3.3), (3.14), and (3.15) we have

$$\begin{aligned}(m^\pm)_u &= \tfrac{1}{2}\,\mathrm{Im}\{i(d_u + \bar{d}_u) \\ &\qquad \pm [4p_u{}^2 q_u{}^2 - (d_u - \bar{d}_u)^2]^{1/2}\} \\ &= \mathrm{Re}d + \xi_0 \pm \tfrac{1}{2}(\Delta m)_u \ ,\end{aligned} \tag{3.47}$$

where $(\Delta m)_u$ is given by Eqs. (3.15) and (3.22), $m_u^+ = (m_L)_u$, and $m_u^- = (m_S)_u$. Combining Eq. (3.47) with Eq. (3.22b), and noting that $m_S = \mathrm{Re}d - \Delta m/2$, we find

$$(m_S)_u \cong m_S \left\{ 1 + \frac{\Delta m}{2m}\left[\frac{2\xi_0 - 2\xi_x + \sqrt{2}\,|\epsilon|\,\xi_y + \sqrt{2}\,|\epsilon|\,\zeta_y}{\Delta m} + O(u_a{}^2) \right] \right\} , \tag{3.48a}$$

$$(m_L)_u \cong m_L \left\{ 1 + \frac{\Delta m}{2m}\left[\frac{2\xi_0 + 2\xi_x - \sqrt{2}\,|\epsilon|\,\xi_y - \sqrt{2}\,|\epsilon|\,\zeta_y}{\Delta m} + O(u_a{}^2) \right] \right\} , \tag{3.48b}$$

where $m \cong m_S \cong m_L$. The expressions in square brackets (apart from ξ_0) determine the γ dependence of $(\Delta m)_u$, as we see from Eq. (3.22b). Hence, unless ξ_0 is much larger than ξ_a or ζ_a (for $a = x,y,z$), the slopes of $(m_S)_u$ and $(m_L)_u$ will be smaller than that of $(\Delta m)_u$ by a factor of order $\Delta m/m = 7.07\times 10^{-15}$. We hasten to add that in some models ξ_0/ξ_x is in fact of order $m/\Delta m$ so that the slope parameters for m_S and Δm are comparable. As noted in Sec. II, the apparent absence of any energy dependence in m_S cannot at present be taken as evidence against such a model due to the different procedures used to study m_S and Δm. Proceeding in the same way we find for $(\Gamma_{L,S})_u$

$$\begin{aligned}(\Gamma_S)_u \cong \Gamma_S \Bigg\{ & 1 + \frac{1}{\Delta m}\left[-\zeta_0 + \zeta_x + \frac{|\epsilon|}{\sqrt{2}}(\xi_y - \zeta_y) \right] \\ & + \frac{1}{2(\Delta m)^2}[(-\xi_y{}^2 + \zeta_y{}^2 + 2\xi_y\zeta_y) + (-\xi_z{}^2 + \zeta_z{}^2 + 2\xi_z\zeta_z) - 2\sqrt{2}\,|\epsilon|\,(\xi_x\xi_y - \zeta_x\zeta_y)] \Bigg\} ,\end{aligned} \tag{3.49a}$$

$$\begin{aligned}(\Gamma_L)_u \cong \Gamma_L \Bigg\{ & 1 - \frac{1}{\Delta m}\frac{\Gamma_S}{\Gamma_L}\left[\zeta_0 + \zeta_x + \frac{|\epsilon|}{\sqrt{2}}(\xi_y - \zeta_y) \right] \\ & - \frac{1}{2(\Delta m)^2}\frac{\Gamma_S}{\Gamma_L}[(-\xi_y{}^2 + \zeta_y{}^2 + 2\xi_y\zeta_y) + (-\xi_z{}^2 + \zeta_z{}^2 + 2\xi_z\zeta_z) - 2\sqrt{2}\,|\epsilon|\,(\xi_x\xi_y - \zeta_x\zeta_y)] \Bigg\} ,\end{aligned} \tag{3.49b}$$

where we have used the approximation $\Gamma_S \cong 2\Delta m$. We note immediately that the slope of Γ_S is of order $u_a/\Delta m$ in contrast to m_S where it is of order u_a/m. This means that the slope of $(\Gamma_S)_u$ would be expected *a priori* to be comparable to that of Δm, $|\eta_{+-}|$, and $\tan\phi_{+-}$. Note, however, that $\Gamma_{S,L}$ depend on ζ_0, which Δm, $|\eta_{+-}|$, and $\tan\phi_{+-}$ do not. It follows that if we wish to eliminate this dependence on ζ_0, so as to relate $\Gamma_{L,S}$ to the other parameters, we must deal with the combination $(\Gamma_L - \Gamma_S)_u$, as in Eq. (3.22a). As we have already noted in Sec. II, we cannot extract $(\Gamma_L)_u$ from the present analysis, and hence b_Γ can be inferred from $b_{\Gamma S}$ only if we make some additional assumption about ζ_0. Finally, we note that the slope of $(\Gamma_L)_u$ is nominally of order $(\Gamma_S/\Gamma_L)\times(u_a/\Delta m)$, and hence could (but need not) be substantially larger than that of $(\Gamma_S)_u$.

Equations (3.22), (3.35), (3.43), (3.48), and (3.49) are the principal phenomenological results of this section. We now demonstrate how these results can be used to relate the physically measurable slope parameters b_Δ, b_Γ, b_η, b_ϕ, and b_δ, to one another in various cases of interest. (Since m_S and Γ_S depend on u_0, their slopes cannot be related to those of the other variables without invoking additional theoretical assumptions, as we have already noted.) If we assume that the experimental results of Sec. II arise from the existence of a single nonvanishing u_a which is proportional to γ^N, then the measured slopes are determined by the two unknown parameters $\xi_a^{(N)}$ and $\zeta_a^{(N)}$. Since four independent slope parameters can be determined (b_Δ, b_Γ, b_η, and b_ϕ), it follows that there exist in such a case two nontrivial predictions which can then be used to test whether a single u_a can, in fact, account for the data. From a theoretical point of view, this is the most interesting possibility to consider, both on grounds of simplicity and also because each u_a corresponds to an interaction with well-defined charge conjugation. Moreover, several specific models can be cast in such a form including the previously described hypercharge field whose effects are characterized by Eq. (3.13).

Consider, for example, the case of a pure u_x coupling. Setting $u_y = u_z = 0$ in Eqs. (3.22), (3.35), and (3.44), we find

$$(\Gamma_L-\Gamma_S)_u \cong (\Gamma_L-\Gamma_S)\left[1+\frac{2\zeta_x}{\Delta m}\right] , \tag{3.50a}$$

$$(\Delta m)_u \cong (\Delta m)\left[1+\frac{2\xi_x}{\Delta m}\right] , \tag{3.50b}$$

$$(\eta_{+-})_u \cong \frac{\epsilon}{2}-\frac{1}{2\Delta m}u_x\epsilon(1-i) , \tag{3.50c}$$

$$\delta_u \cong \mathrm{Re}\epsilon - \frac{\sqrt{2}\,|\epsilon|\,\xi_x}{\Delta m} , \tag{3.50d}$$

where we have set $\epsilon'=0$ in Eq. (3.50c). The expression for $(\eta_{+-})_u$ can now be decomposed into its real and imaginary parts which then give

$$|\eta_{+-}|_u = |\eta_{+-}|\left[1-\frac{(\xi_x+\zeta_x)}{\Delta m}\right] , \tag{3.51a}$$

$$(\tan\phi_{+-})_u = (\tan\phi_{+-})\left[1+\frac{2(\xi_x-\zeta_x)}{\Delta m}\right] . \tag{3.51b}$$

If we write

$$\xi_x = \xi_x^{(N)}\gamma^N,\quad \zeta_x = \zeta_x^{(N)}\gamma^N , \tag{3.52}$$

then the various slope parameters are given by

$$b_\Gamma^{(N)} = \frac{2\zeta_x^{(N)}}{\Delta m} ,\quad b_\Delta^{(N)} = \frac{2\xi_x^{(N)}}{\Delta m} , \tag{3.53a}$$

$$b_\eta^{(N)} = \frac{-(\xi_x^{(N)}+\zeta_x^{(N)})}{\Delta m} ,$$

$$b_\phi^{(N)} = \frac{2(\xi_x^{(N)}-\zeta_x^{(N)})}{\Delta m} , \tag{3.53b}$$

$$b_\delta^{(N)} = \frac{-2\xi_x^{(N)}}{\Delta m} = -b_\Delta^{(N)} . \tag{3.53c}$$

Combining Eqs. (3.53), we find for a pure u_x coupling

$$b_\phi^{(N)} = b_\Delta^{(N)} - b_\Gamma^{(N)} , \tag{3.54a}$$

$$b_\eta^{(N)} = -\tfrac{1}{2}(b_\Delta^{(N)} + b_\Gamma^{(N)}) , \tag{3.54b}$$

$$b_\delta^{(N)} = b_\eta^{(N)} - \tfrac{1}{2}b_\phi^{(N)} . \tag{3.54c}$$

Note that Eqs. (3.54) are independent of N: The relationships among the measured slope parameters are thus independent of how the γ dependence is parametrized, provided that ξ_x and ζ_x vary in the same way with N. This is useful to know because there is at present no compelling reason why the K^0-$\bar{K}^0$ parameters must vary either with γ or with γ^2, as we have assumed for simplicity in Sec. II.

Equations (3.54a) and (3.54b) have a simple geometric interpretation which we will exploit in Sec. IV to analyze the experimental data. Define a set of variables x_1, x_2, x_1' and x_2' as follows:

$$b_\Delta^{(N)} = \frac{x_1}{\sqrt{2}},\quad b_\Gamma^{(N)} = \frac{-x_2}{\sqrt{2}} , \tag{3.55a}$$

$$b_\eta^{(N)} = \frac{-x_1'}{2},\quad b_\phi^{(N)} = x_2' . \tag{3.55b}$$

Equations (3.54a) and (3.54b) then assume the form

$$x_1' = \frac{1}{\sqrt{2}}(x_1-x_2) , \tag{3.56a}$$

$$x_2' = \frac{1}{\sqrt{2}}(x_1+x_2) . \tag{3.56b}$$

An interaction which is pure u_x is thus defined by a point in the x_1-x_2 plane which is obtained by specifying $b_\Delta^{(N)}$ and $b_\Gamma^{(N)}$, or equivalently x_1 and x_2. The same theory could also be defined by specifying $b_\eta^{(N)}$ and $b_\phi^{(N)}$ which generates a point in the x_1'-x_2' plane. The content of Eqs. (3.56) is that the x_1'-x_2' coordinate system is obtained from the x_1-x_2 system by a clockwise rotation through 45°. It follows that if the interaction is pure u_x, the experimental points in the x_1-x_2 and x_1'-x_2' planes should coincide. We will call a representation of $b_\Delta^{(N)}$, $b_\Gamma^{(N)}$, $b_\eta^{(N)}$, and $b_\phi^{(N)}$ a "simultaneous slope plot" (SSP) for obvious reasons. The SSP for a pure u_x theory will be discussed in Sec. IV, using the data of Sec. II. As we will show in the ensuing discussion, the slope relations in Eqs. (3.54a) and (3.54b) are unique to a pure u_x coupling, and hence serve to distinguish it from a pure u_y or pure u_z coupling.

The remaining slope relation [Eq. (3.54c)] is, by contrast, the same for all couplings, and hence merely serves as a consistency check on the experimental data. This can be seen by noting that since we have assumed throughout this section that

$$(\epsilon)_u = 2(\eta_{+-})_u = 2\,|\eta_{+-}|_u e^{i(\phi_{+-})_u} , \tag{3.57}$$

any change in $(\mathrm{Re}\epsilon)_u$ induced by the u_a is determined by the corresponding changes induced in $|\eta_{+-}|_u$ and $(\phi_{+-})_u$. This can be quantified by differentiating Eq. (3.57) with respect to γ^N: Using the relation

$$d(\tan\phi_{+-})/d\gamma^N = \sec^2\phi_{+-}\,d\phi_{+-}/d\gamma^N ,$$

we find

$$\frac{d(\mathrm{Re}\epsilon)_u}{d\gamma^N} = \mathrm{Re}\epsilon\left[-\sin(\phi_{+-})_u\cos(\phi_{+-})_u\frac{d(\tan\phi_{+-})_u}{d\gamma^N} + \frac{d\ln|\eta_{+-}|_u}{d\gamma^N}\right] . \tag{3.58}$$

From Eqs. (3.18) and (3.46) we then find, to lowest order in various small quantities,

$$b_\delta^{(N)} \cong b_\epsilon^{(N)} \cong b_\eta^{(N)} - \sin^2(\phi_{+-})_u b_\phi^{(N)} \cong b_\eta^{(N)} - \tfrac{1}{2}b_\phi^{(N)} , \tag{3.59}$$

in agreement with Eq. (3.54c). Using Eqs. (3.35) and (3.44), the interested reader can verify Eq. (3.59) explicitly for the case of a pure u_y or pure u_z coupling, by proceeding in analogy with the pure u_x case considered above.

We turn next to the pure u_y case. If the terms linear u_y dominate in Eqs. (3.22), then the analogs of Eqs. (3.54a) and (3.54b) become

$$b_\phi^{(N)}=\frac{1}{|\epsilon|^2}(b_\Delta^{(N)}+b_\Gamma^{(N)}) , \tag{3.60a}$$

$$b_\eta^{(N)}=\frac{1}{2|\epsilon|^2}(b_\Delta^{(N)}-b_\Gamma^{(N)}) . \tag{3.60b}$$

However, since the linear contributions are suppressed by a factor of $|\epsilon|\cong 4\times 10^{-3}$, it remains an open (experimental) question as to whether they are actually larger than the quadratic contributions. In fact, an analysis of the exact expression for $\lambda_u^+-\lambda_u^-$ in Eq. (3.15) suggests that for the u_y case the quadratic terms do indeed give the dominant contributions to $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$, as we discuss below. If we thus drop the terms linear in u_y, the pure u_y case becomes effectively the same as the pure u_z case to which we now turn.

The case of pure u_z coupling must be treated differently from pure u_x because there is no contribution to $\lambda_u^+-\lambda_u^-$ linear in u_z, as we have previously noted. This means that the analogs of the relations in Eqs. (3.54) depend on how each of the slopes is assumed to vary with N as we now show. Let us assume that $(\Delta m)_u$, $(\Gamma_L-\Gamma_S)_u$, $|\eta_{+-}|_u$, and $(\tan\phi_{+-})_u$ vary with γ as γ^M, $\gamma^{M'}$, γ^N, and $\gamma^{N'}$, respectively, with coefficients $b_\Delta^{(M)}$, $b_\Gamma^{(M')}$, $b_\eta^{(N)}$, and $b_\phi^{(N')}$. From Eqs. (3.22) and (3.35) we then find

$$\frac{|\epsilon|^2}{2}[(b_\eta^{(N)})^2\gamma^{2N}-\tfrac{1}{4}(b_\phi^{(N')})^2\gamma^{2N'} -b_\eta^{(N)}b_\phi^{(N')}\gamma^{N+N'}]=b_\Gamma^{(M')}\gamma^{M'} , \tag{3.61a}$$

$$-\frac{|\epsilon|^2}{2}[(b_\eta^{(N)})^2\gamma^{2N}-\tfrac{1}{4}(b_\phi^{(N')})^2\gamma^{2N'} +b_\eta^{(N)}b_\phi^{(N')}\gamma^{N+N'}]=b_\Delta^{(M)}\gamma^{M} . \tag{3.61b}$$

For a pure u_z coupling to lead to a consistent description of the various slope parameters, we must set $2N=2N'=M=M'$, which gives

$$b_\Gamma^{(2N)}=\frac{|\epsilon|^2}{2}[(b_\eta^{(N)})^2-\tfrac{1}{4}(b_\phi^{(N)})^2-b_\eta^{(N)}b_\phi^{(N)}] , \tag{3.62a}$$

$$b_\Delta^{(2N)}=\frac{-|\epsilon|^2}{2}[(b_\eta^{(N)})^2-\tfrac{1}{4}(b_\phi^{(N)})^2+b_\eta^{(N)}b_\phi^{(N)}] . \tag{3.62b}$$

The simplest nontrivial application of Eqs. (3.62), which corresponds to $N=1$, will be analyzed in detail in Sec. IV. For present purposes we simply note that for a pure u_z coupling $b_\eta^{(N)}$ or $b_\phi^{(N)}$ will be larger than $b_\Gamma^{(2N)}$ or $b_\Delta^{(2N)}$ by a factor of order 10^5, due to the coefficient $|\epsilon|^2$ in Eqs. (3.62). This is what was meant previously by the observation that a term proportional to $u_x(u_z)$ produces a "small" ("large") γ-dependent contribution to $(\eta_{+-})_u$: For a u_x coupling $b_\eta^{(N)}$ and $b_\phi^{(N)}$ will be comparable to $b_\Delta^{(N)}$ and $b_\Gamma^{(N)}$, whereas for a pure u_z coupling theory they will be $\sim 10^5$ larger. This provides a clear experimental distinction between the C-even u_x coupling and the C-odd u_z and u_y couplings.

Up to this point we have examined the K^0-$\bar{K}^0$ parameters in the energy regime where $|u_a|/\Delta m<1$. (As we have already noted, $|u_a|/\Delta m\cong 0.2$ at a typical energy of 70 GeV.) We observe, however, that as γ increases a regime will be reached for which $|u_a|/\Delta m>1$. In this regime the eigenfunctions $\Psi_u^\pm$ and the eigenvalues $\lambda_u^\pm$ in Eq. (3.15) are determined by the characteristics of the external field, rather than by the internal dynamics of the K^0-$\bar{K}^0$ system. For values of γ sufficiently large that p^2 and q^2 are negligible compared to $|u_a|$, we find

$$\lambda_u^+-\lambda_u^-\to 2i({u_x}^2+{u_y}^2+{u_z}^2)^{1/2} . \tag{3.63}$$

Hence, if a single u_a gives the dominant contribution at high energy, then

$$\lambda_u^+-\lambda_u^-=2iu_a=2i(\xi_a+i\zeta_a),\quad a=x,y,z \tag{3.64a}$$

$$(\Delta m)_u=2\xi_a\sim 2\xi_a^{(N)}\gamma^N , \tag{3.64b}$$

$$\tfrac{1}{2}(\Gamma_L-\Gamma_S)_u=-2\zeta_a\sim -2\zeta_a^{(M)}\gamma^M , \tag{3.64c}$$

and the eigenfunctions $\Psi_u^\pm$ become

$$|K_1^0\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix},\quad |K_2^0\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix},\quad \text{pure } u_x , \tag{3.65a}$$

$$|K_3^0\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix},\quad |K_4^0\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix},\quad \text{pure } u_y , \tag{3.65b}$$

$$|K^0\rangle=\begin{bmatrix}1\\0\end{bmatrix},\quad |\bar{K}^0\rangle=\begin{bmatrix}0\\1\end{bmatrix},\quad \text{pure } u_z . \tag{3.65c}$$

It follows that if the external coupling is pure u_x, then CP conservation is restored at high energy, and the eigenfunctions become the familiar CP eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$. By contrast, if the coupling is pure u_z, then the eigenstates are $|K^0\rangle$ and $|\bar{K}^0\rangle$ which have well defined hypercharge. However, should the external coupling become pure u_y at high energy, then the eigenstates are $|K_3^0\rangle$ and $|K_4^0\rangle$ which have neither well defined CP nor well defined hypercharge. Since neither of these states is forbidden by CP from decaying into 2π, we expect that their lifetimes (τ_3 and τ_4) should be given by

$$\tau_3\cong\tau_4\cong\tau_S . \tag{3.66}$$

We conclude this discussion with an analysis of the intermediate energy regime where $|u_a|/\Delta m\approx 1$. In this regime the dependence of $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$ on γ is given by the exact expression in Eq. (3.15), which is in general some complicated function. If we continue to assume that a single u_a gives the dominant contribution to $\lambda_u^+-\lambda_u^-$, then the behavior of $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$ at *any* energy is determined by the two parameters $\xi_a^{(N)}$ and $\zeta_a^{(M)}$ (where M and N may or may not be the same). Hence given sufficiently good data at lower energies, $\xi_a^{(N)}$ and $\zeta_a^{(M)}$ could be extracted from a simultaneous fit to $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$, and these results used to extrapolate to higher energies.

Likewise $\xi_a^{(N)}$ and $\zeta_a^{(M)}$ could also be extracted from a fit to $|\eta_{+-}|_u$ and $(\tan\phi_{+-})_u$. This is in fact what we have done since, as noted previously, we have no results for $(\Gamma_L-\Gamma_S)_u$. Instead of formally fitting to the data for $|\eta_{+-}|_u$ and $(\tan\phi_{+-})_u$, we have simply deduced by trial and error the values of ξ_a and ζ_a which, when inserted into the complete expression for $(\eta_{+-})_u$,

$$(\eta_{+-})_u=\frac{-N^+(\rho^+-1)}{N^-(\rho^--1)} , \tag{3.67}$$

reproduce the experimental data for $35 \leq E_K \leq 105$ GeV. $\rho^{\pm}$ and $N^{\pm}$ are defined in Eqs. (3.30) and (3.31), respectively, and we have set $\bar{c}'/c' = -1$ (i.e., $\epsilon' = 0$) in Eq. (3.32). The resulting values of ξ_a and ζ_a (for each of the cases $a = x,y,z$) can then be inserted into the exact expression for $\lambda_u^+ - \lambda_u^-$ in Eq. (3.15) to deduce the high-energy behavior of $(\Delta m)_u$ and $(\Gamma_L - \Gamma_S)_u$. For later purposes it is convenient to use Eq. (3.15) to express $(\Delta m)_u$ and $(\Gamma_L - \Gamma_S)_u$ in the form

$$\lambda_u^+ - \lambda_u^- = i\,\Delta m\,(P_u + iQ_u)^{1/2}$$

$$= \tfrac{1}{2}(\Gamma_L - \Gamma_S)_u + i(\Delta m)_u \;, \qquad (3.68a)$$

$$(\Delta m)_u = \Delta m\,(P_u{}^2 + Q_u{}^2)^{1/4} \cos\frac{\theta}{2} \;, \qquad (3.68b)$$

$$\tfrac{1}{2}(\Gamma_L - \Gamma_S)_u = -\Delta m\,(P_u{}^2 + Q_u{}^2)^{1/4} \sin\frac{\theta}{2} \;, \qquad (3.68c)$$

$$\theta = \arctan(Q_u / P_u) \;. \qquad (3.68d)$$

The exact expressions for P_u and Q_u for each of the cases $a = x,y,z$ are given in Table III, and our results are presented in Tables IV and V, and in Figs. 1–3. The columns labeled $|\eta_{+-}|_u$, $(\tan\phi_{+-})_u$, and $(\Delta m)_u$ give the experimental values of the corresponding variables in the energy range $E_K \leq 105$ GeV, obtained using the slope parameters from the external fits (lines 2 in Table I). The fits to $(\Delta m)_u$ for the u_x, u_y, and u_z cases (using the parameters in Table III) are denoted by $(\Delta m)_x$, $(\Delta m)_y$, and $(\Delta m)_z$, respectively, and similarly for the other variables. The salient features which emerge from these results can be summarized as follows.

(a) For the u_x case the fits to $(\Delta m)_u$ and $|\eta_{+-}|_u$ for $E_K \leq 105$ GeV are reasonably good, but $(\tan\phi_{+-})_u$ is not well reproduced above $E_K = 75$ GeV. This is a consequence of the fact that the slope parameter $b_\phi^{(2)}$ for $(\tan\phi_{+-})_u$ is sufficiently large that the terms in $(\tan\phi_{+-})_u$ which are bilinear or quadratic in ξ_x and ζ_x give a noticeable contribution for $E_K \leq 105$ GeV. In particular, since $b_\phi^{(2)}\gamma^2 \cong (b_\phi^{(2)}\gamma^2)^2 \cong 1$ at 105 GeV, the simple parametrization of the data used in Refs. 1–3 and Table I, although convenient from an experimental point of view, does not accurately represent the expected γ dependence.

(b) For the u_y and u_z cases the fits to $|\eta_{+-}|_u$ are both quite good, but $(\Delta m)_u$ is predicted to be a constant for $E_K \leq 105$ GeV, in disagreement with the data. Additionally, the fit to $(\tan\phi_{+-})_u$ suffers from the same problem as in the u_x case. In the next section, we will analyze u_y and u_z predictions for $(\Delta m)_u$ in greater detail, where we will elaborate on the conclusion drawn from Table IV that the data for $(\Delta m)_u$ cannot be accounted for by either a u_y or u_z coupling.

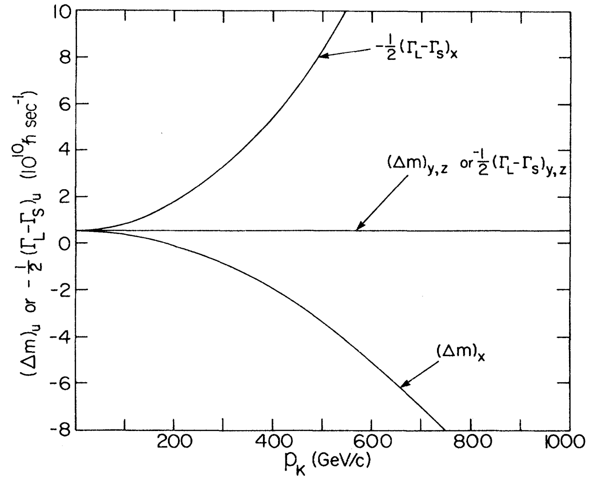

FIG. 1. Plot of $(\Delta m)_u$ and $-\frac{1}{2}(\Gamma_L - \Gamma_S)_u$ in Eq. (3.68) as a function of p_K for the u_x, u_y, and u_z cases. The input values of the various parameters are given in Table III, and we have taken $|\epsilon| = 4.548 \times 10^{-3}$ in u_y. As can be seen from Table IV, the u_y and u_z cases are indistinguishable. Note that $(\Delta m)_x$ vanishes at $E_K = 186$ GeV. See text and Table IV for further details.

(c) We return now to the u_x coupling which both the preceding discussion and the following analysis in Sec. IV suggest could account for all the present data. We see from Table IV and Fig. 1 that for the u_x case $(\Delta m)_x$ goes through zero at $E_K \cong 186$ GeV and then changes sign as the energy is increased. This is an extremely interesting effect, particularly since it occurs in an energy regime which is readily accessible at Fermilab. To understand how this comes about we note from Eq. (3.68b) that $(\Delta m)_u{}^2$ [and hence $(\Delta m)_u$] vanishes when

$$\cos^2\frac{\theta}{2} = \tfrac{1}{2}(1 + \cos\theta)$$

$$= \tfrac{1}{2}\left[\frac{(P_u{}^2 + Q_u{}^2)^{1/2} + P_u}{(P_u{}^2 + Q_u{}^2)^{1/2}}\right] = 0 \;. \qquad (3.69)$$

We see from Table III that for the indicated values of $\xi_x^{(2)}$ and $\zeta_x^{(2)}$, P_u is always negative, and hence $\cos^2\theta/2$ vanishes when $Q_u = 0$. Using the specific expression for Q_u in the u_x case, and taking $\xi_x = \xi_x^{(2)}\gamma^2$, we find that Q_u vanishes when $\gamma = 374$, which corresponds to $E_K = 186$ GeV. The vanishing of the K_L-K_S mass difference as γ^2 (and hence the field strength) increases, is thus analogous to the

TABLE III. Explicit form of $\lambda_u^+ - \lambda_u^-$ in Eq. (3.68). For each of u_x, u_y, and u_z, the entries give the complete expressions for P_u and Q_u, as well as the numerical values of the parameters $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ for $a = x,y,z$. We have taken $|\epsilon| = 4.548 \times 10^{-3}$ in u_y.

	P_u	Q_u	$\frac{\xi_a^{(N)}}{\Delta m}$	$\frac{\zeta_a^{(N)}}{\Delta m}$
u_x	$\frac{4}{\Delta m}(\xi_x - \zeta_x) + \frac{4}{(\Delta m)^2}(\xi_x{}^2 - \zeta_x{}^2)$	$2 + 4\frac{(\xi_x + \zeta_x)}{\Delta m} + \frac{8\xi_x\zeta_x}{(\Delta m)^2}$	$\frac{\xi_x^{(2)}}{\Delta m} = -3.58 \times 10^{-6}$	$\frac{\zeta_x^{(2)}}{\Delta m} = +7.18 \times 10^{-6}$
u_y	$-4\sqrt{2}\,\lvert\epsilon\rvert\,\frac{\xi_y}{\Delta m} + \frac{4(\xi_y{}^2 - \zeta_y{}^2)}{(\Delta m)^2}$	$2 - 4\sqrt{2}\,\lvert\epsilon\rvert\,\frac{\zeta_y}{\Delta m} + \frac{8\xi_y\zeta_y}{(\Delta m)^2}$	$\frac{\xi_y^{(1)}}{\Delta m} = +3.64 \times 10^{-6}$	$\frac{\zeta_y^{(1)}}{\Delta m} = +4.82 \times 10^{-6}$
u_z	$\frac{4(\xi_z{}^2 - \zeta_z{}^2)}{(\Delta m)^2}$	$2 + 8\frac{\xi_z\zeta_z}{(\Delta m)^2}$	$\frac{\xi_z^{(1)}}{\Delta m} = \frac{\zeta_y^{(1)}}{\Delta m}$	$\frac{\zeta_z^{(1)}}{\Delta m} = \frac{-\xi_y^{(1)}}{\Delta m}$

TABLE IV. Dependence of Δm and $-\frac{1}{2}(\Gamma_L-\Gamma_S)$ on E_K for the case of a pure coupling ($a=x, y$, or z). The column labeled $(\Delta m)_u$ gives the data for Δm in the range $35 \le E_K \le 105$ GeV obtained from lines 2 of Table I. The remaining columns give the predictions for $\Delta m_a = \Delta m(u_a)$ and $-\frac{1}{2}(\Gamma_L-\Gamma_S)_a$ in units of $10^{10}\,\hbar\,\mathrm{sec}^{-1}$ obtained from Eqs. (3.68), using the parameters given in Table III.

E_K (GeV)	$(\Delta m)_u$	$(\Delta m)_x$	$(\Delta m)_y$	$(\Delta m)_z$	$-\frac{1}{2}(\Gamma_L-\Gamma_S)_x$	$-\frac{1}{2}(\Gamma_L-\Gamma_S)_y$	$-\frac{1}{2}(\Gamma_L-\Gamma_S)_z$
5	0.535	0.535	0.535	0.535	0.536	0.535	0.535
35	0.515	0.516	0.535	0.535	0.573	0.535	0.535
45	0.502	0.504	0.535	0.535	0.598	0.535	0.535
55	0.486	0.488	0.535	0.535	0.629	0.535	0.535
65	0.467	0.470	0.535	0.535	0.666	0.535	0.535
75	0.444	0.448	0.535	0.535	0.709	0.535	0.535
85	0.419	0.423	0.535	0.535	0.759	0.535	0.535
95	0.390	0.396	0.535	0.535	0.815	0.535	0.535
105	0.358	0.365	0.535	0.535	0.877	0.535	0.535
150		0.188	0.535	0.535	1.232	0.535	0.535
200		−0.083	0.535	0.535	1.775	0.535	0.535
250		−0.430	0.535	0.535	2.472	0.535	0.535
300		−0.855	0.535	0.535	3.324	0.535	0.535
350		−1.357	0.535	0.535	4.331	0.535	0.535
400		−1.936	0.535	0.535	5.493	0.535	0.535
450		−2.592	0.535	0.535	6.811	0.535	0.535
500		−3.325	0.535	0.535	8.283	0.535	0.535
600		−5.024	0.535	0.535	11.69	0.535	0.535
700		−7.031	0.535	0.535	15.72	0.535	0.535
800		−9.348	0.535	0.535	20.37	0.535	0.535
900		−11.97	0.535	0.535	25.64	0.535	0.535
1000		−14.91	0.535	0.535	31.53	0.535	0.535

crossing of the e and β levels in $n=2$ hydrogen[23] when the external field reaches 553 G.

Before leaving the discussion of $(\Delta m)_u$, we note that the preceding analysis of the u_z case does *not* apply to the case of an external hypercharge or electromagnetic field, even though both are examples of a u_z coupling. The reason for this is that an arbitrary u_z coupling is described by two independent parameters $\xi_z^{(N)}$ and $\zeta_z^{(N)}$, which can be chosen by fitting to the data for b_Δ and b_Γ. By contrast, the hypercharge interaction in Eq. (3.13) is described by a single

TABLE V. Dependence of $|\eta_{+-}|$ and $\tan\phi_{+-}$ on E_K for the case of a pure coupling ($a=x, y$, or z). The columns labeled $|\eta_{+-}|_u$ and $(\tan\phi_{+-})_u$ give the data for $|\eta_{+-}|$ and $\tan\phi_{+-}$ in the range $35 \le E_K \le 105$ GeV obtained from lines 2 of Table I. The remaining columns give the predictions for $|\eta_{+-}|_u$ (in units of 10^{-3}) and $(\tan\phi_{+-})_u$ obtained from Eq. (3.67), using the parameters given in Table III.

E_K (GeV)	$\|\eta_{+-}\|_u$	$\|\eta_{+-}\|_x$	$\|\eta_{+-}\|_y$	$\|\eta_{+-}\|_z$	$(\tan\phi_{+-})_u$	$(\tan\phi_{+-})_x$	$(\tan\phi_{+-})_y$	$(\tan\phi_{+-})_z$
5	2.23	2.27	2.25	2.25	0.95	1.00	0.97	0.97
35	2.19	2.23	2.11	2.11	0.85	0.90	0.79	0.79
45	2.16	2.20	2.06	2.06	0.79	0.84	0.74	0.74
55	2.13	2.16	2.03	2.03	0.70	0.78	0.68	0.68
65	2.09	2.11	1.99	1.99	0.60	0.71	0.63	0.63
75	2.05	2.05	1.96	1.96	0.49	0.63	0.57	0.57
85	1.99	1.98	1.93	1.93	0.35	0.56	0.52	0.52
95	1.94	1.90	1.90	1.90	0.21	0.49	0.47	0.46
105	1.87	1.81	1.87	1.87	0.04	0.42	0.41	0.41
150		1.38	1.82	1.82		0.15	0.19	0.18
200		0.97	1.85	1.85		−0.05	−0.05	−0.05
250		0.69	1.97	1.97		−0.17	−0.27	−0.28
300		0.50	2.18	2.18		−0.26	−0.48	−0.49
350		0.38	2.44	2.44		−0.31	−0.68	−0.68
400		0.30	2.75	2.75		−0.35	−0.86	−0.87
450		0.24	3.08	3.08		−0.38	−1.04	−1.05
500		0.19	3.44	3.44		−0.40	−1.20	−1.22
600		0.14	4.19	4.19		−0.43	−1.51	−1.54
700		0.10	4.98	4.98		−0.45	−1.78	−1.83
800		0.08	5.79	5.79		−0.46	−2.03	−2.10
900		0.06	6.61	6.61		−0.47	−2.26	−2.36
1000		0.05	7.44	7.44		−0.47	−2.47	−2.59

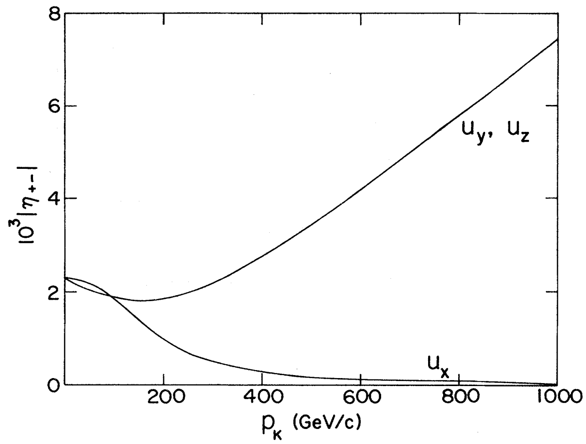

FIG. 2. Plot of $|\eta_{+-}|_u$ from Eq. (3.67) as a function of p_K. The values of $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ for $a=x$, y, and z are given in Table III.

parameter A_0, which means that in this case b_Δ and b_Γ are constrained to be related to each other in a particular way [see Eq. (4.1) below]. For this reason the effect of a hypercharge or electromagnetic field should be studied separately. Combining Eqs. (3.13) and (3.15), we find immediately

$$(\Delta m)_u{}^2=[(\Delta m)^4+4A_0{}^4\gamma^4]^{1/2}+2A_0{}^2\gamma^2\ , \tag{3.70a}$$

$$[\tfrac{1}{2}(\Gamma_L-\Gamma_S)_u]^2=[(\Delta m)^4+4A_0{}^4\gamma^4]^{1/2}-2A_0{}^2\gamma^2\ . \tag{3.70b}$$

We see that $(\Delta m)_u{}^2$ [and hence $(\Delta m)_u$] is a monotonically *increasing* function of γ, which contrasts with both the data and with the results for the general u_z case shown in Fig. 1. We will return to this point in Sec. IV.

Thus far we have avoided discussing the energy dependence of specific decay modes of K_L or K_S because these introduce additional slope parameters which are not directly related to those we have already considered. However, should future experiments confirm that Γ_S is energy-dependent, it would follow that at least some of its partial decay modes must also be energy dependent, and hence some consideration should be given to individual modes as well. In fact, as we have already noted in I, the data from which b_η is extracted really measure the energy dependence of $\Gamma(K_L\to\pi^+\pi^-)/\Gamma(K_L\to\pi\mu\nu)$. Moreover, experiment E-617 at Fermilab, which is currently in progress, will be capable in principle of measuring the energy dependence of this and other ratios, as well as that of various individual decay modes. It is thus interesting to study the energy dependence of some specific decay modes, and to compare these to the results we have found for the other K^0-$\bar{K}^0$ parameters.

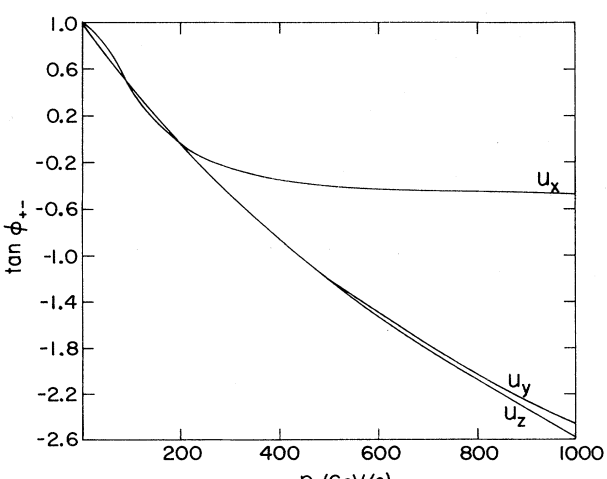

FIG. 3. Plot of $(\tan\phi_{+-})_u$ from Eq. (3.67) as a function of p_K. The values of $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ for $a=x$, y, and z are given in Table III.

Let $\Gamma_{L,S}^j$ denote the decay rate for $K_{L,S}\to j$, where $j=\pi^+\pi^-,\pi\mu\nu,\ldots$, so that

$$\Gamma_{L,S}=\sum_j \Gamma_{L,S}^j\ . \tag{3.71}$$

The energy dependence of $\Gamma_{L,S}^j$ is parametrized by writing

$$(\Gamma_S^j)_u=\Gamma_S^j[1+b_{\Gamma S}^{(N)}(j)\gamma^N]\ , \tag{3.72}$$

and similarly for $(\Gamma_L^j)_u$. If we combine Eqs. (3.71) and (3.72) with the corresponding expressions for $(\Gamma_{L,S})_u$,

$$(\Gamma_L)_u=\Gamma_L(1+b_{\Gamma L}^{(N)}\gamma^N)\ , \tag{3.73a}$$

$$(\Gamma_S)_u=\Gamma_S(1+b_{\Gamma S}^{(N)}\gamma^N)\ , \tag{3.73b}$$

we find immediately

$$b_{\Gamma S}^{(N)}=\sum_j B_S^j b_{\Gamma S}^{(N)}(j)\ ,\qquad B_S^j=\Gamma_S^j/\Gamma_S\ , \tag{3.74}$$

and similarly for $b_{\Gamma L}^{(N)}$. For Γ_S the branching ratios B_S^{+-} and B_S^{00} for decay into $\pi^+\pi^-$ and $\pi^0\pi^0$, respectively, are

$$B_S^{+-}=0.6861\pm0.0024\ ,\qquad B_S^{00}=0.3139\pm0.0024\ , \tag{3.75}$$

and hence using Eq. (3.74),

$$b_{\Gamma S}^{(N)}\cong0.69b_{\Gamma S}^{(N)}(+-)+0.31b_{\Gamma S}^{(N)}(00)\ . \tag{3.76}$$

In Sec. IV, we will demonstrate that the sign of $b_\Delta^{(2)}$, among other things, suggests that the observed energy dependence arises from a C-even field. If this is the case, then

$$b_{\Gamma S}^{(N)}(+-)\cong b_{\Gamma S}^{(N)}(00)\ , \tag{3.77}$$

neglecting effects due to the $\pi^\pm-\pi^0$ mass difference. From Eqs. (3.76) and (3.77) we then have

$$b_{\Gamma S}^{(N)}\cong b_{\Gamma S}^{(N)}(+-)\cong b_{\Gamma S}^{(N)}(00)\ , \tag{3.78}$$

so that the experimental result for $b_{\Gamma S}^{(N)}$ actually measures the energy dependence of $\Gamma(K_S\to\pi^+\pi^-)$ and $\Gamma(K_S\to\pi^0\pi^0)$ as well.

We now wish to relate $b_{\Gamma S}^{(N)}(+-)$ to various theoretical parameters such as the u_a, just as we did for b_Δ, b_Γ, b_η, and b_ϕ when we related them to u_x in Eq. (3.54). Using Eq. (3.31), we have immediately,

$$(\Gamma_S^{+-})_u=|-N^-(\rho^-\langle\pi^+\pi^-|H_w|K^0\rangle+\langle\pi^+\pi^-|H_w|\bar{K}^0\rangle)|^2\times(\text{PS})\ , \tag{3.79}$$

where ρ^- and N^- are defined in Eqs. (3.30) and (3.31), and where (PS) denotes the appropriate phase-space factor. This can be specified more precisely by writing the effective $K^0 \to \pi^+\pi^-$ coupling in the form

$$\mathscr{L}(x)=c'mK^0(x)\phi(x)\phi^\dagger(x)+\text{H.c.}\ , \tag{3.80}$$

where m is the K^0 mass, $\phi(x)$ is the π^- field operator, and c' is the constant appearing in Eq. (3.24). The K^0 decay rate Γ_K would then be

$$\Gamma_K=\frac{|c'|^2}{16\pi}(m^2-4m_\pi{}^2)^{1/2}\ , \tag{3.81}$$

where $(m^2-4m_\pi{}^2)^{1/2}$ is the phase-space factor. From Eqs. (3.79) and (3.81), we see that an external field can give rise to an energy dependence of Γ_S in three distinct ways: (i) It can change ρ^- and hence the relative admixtures of K^0 and $\bar{K}^0$ in K_S. This is the effect that gives rise to the energy dependence of Δm and $(\Gamma_L-\Gamma_S)$. (ii) It can induce an energy dependence in the weak matrix element $\langle\pi^+\pi^-\,|\,H_w\,|\,K^0\rangle$ by changing c' to c'_u and similarly $\bar{c}'$ to $\bar{c}'_u$. (iii) The external field can also make m and m_π energy dependent: $m\to m_u$ and $m_\pi\to(m_\pi)_u$. Taking all of these effects into account, we find, after some algebra (and assuming as before that $\bar{c}'/c'=-1$),

$$(\Gamma_S^{+-})_u=\Gamma_S^{+-}\left[\frac{|c'_u|^2}{|c'|^2}[1-\tfrac{3}{4}\,|\alpha^-|^2-\tfrac{1}{2}\text{Re}(\alpha^-)^2+\tfrac{3}{2}\text{Re}(\epsilon^*\alpha^-)+\text{Re}(\epsilon\alpha^-)]\frac{[m_u{}^2-4(m_\pi)_u{}^2]^{1/2}}{(m^2-4m_\pi{}^2)^{1/2}}\right]\ , \tag{3.82a}$$

$$\alpha^\pm=\rho^\pm\mp(1+\epsilon)\ . \tag{3.82b}$$

We note that there is no term in Eq. (3.82) linear in α^- and hence there is no contribution to Γ_S^{+-} from terms of order u_x, u_y, u_z, or $u_x\epsilon$. If we again anticipate the results of Sec. IV and set $u_y=u_z=0$, we see that the leading contribution arising from α^- is $O(u_x\epsilon^2)$, which is far too small to account for the observed value of $b_{\Gamma S}$. This is an important result in unraveling the origin of the observed energy dependence: It indicates that the external field changes not only p^2 and q^2 (which we learn from the energy variation of Δm), but also at least one of the other parameters c', m, or m_π. It may be possible to go further and disentangle the energy variation of c' from that of m and m_π if, as is suggested by some models, all masses have a common γ dependence,

$$\begin{aligned} m_u&=m\,(1+b_\mu^{(2)}\gamma^2)\ ,\\ (m_\pi)_u&=m_\pi(1+b_\mu^{(2)}\gamma^2)\ , \end{aligned} \tag{3.83}$$

characterized by a universal slope parameter $b_\mu^{(2)}$. Using energy conservation, γ for the π and K are related by

$$\gamma_\pi=\frac{m}{2m_\pi}\gamma_K\equiv\frac{m}{2m_\pi}\gamma\ . \tag{3.84}$$

Combining Eqs. (3.83) and (3.84), we find

$$\frac{[m_u{}^2-4(m_\pi)_u{}^2]^{1/2}}{(m^2-4m_\pi{}^2)^{1/2}}\cong\left\{1-\frac{m^2}{4m_\pi{}^2}[b_\mu^{(2)}]^2\gamma^4\right\}^{1/2}\ . \tag{3.85}$$

We note that in such a picture there is no contribution to the phase-space γ dependence which is of order $b_\mu^{(2)}\gamma^2$. Hence by measuring the γ dependence of $\Gamma(K_S\to\pi^+\pi^-)$ at sufficiently small values of γ such that $b_\mu^{(2)}\gamma^2\ll 1$, one can isolate the γ dependence of c'_u for which the term proportional to γ^2 should be the leading contribution.

Proceeding in an analogous way for $K_L\to\pi^+\pi^-$, the analogs of Eqs. (3.79) and (3.82) are

$$\begin{aligned}(\Gamma_L^{+-})_u&=|N^+(\rho^+\langle\pi^+\pi^-\,|\,H_w\,|\,K^0\rangle\\ &\quad+\langle\pi^+\pi^-\,|\,H_w\,|\,\bar{K}^0\rangle)\,|^2\times(\text{PS})\\ &\cong\Gamma_L^{+-}\frac{|c'_u|^2}{|c'|^2}\left[1+2\,\text{Re}\left[\frac{\alpha^+}{\epsilon}\right]\right]\\ &\quad\times\frac{[m_u{}^2-4(m_\pi)_u{}^2]^{1/2}}{(m^2-4m_\pi{}^2)^{1/2}}\ .\end{aligned} \tag{3.86}$$

We have retained only the leading contribution from α^+, but for the pure u_x case this is now of order $u_x/\Delta m$ and hence is *a priori* comparable to the other contributions. If we divide Eq. (3.86) by Eq. (3.82), the factors containing c'_u, m_u, and $(m_\pi)_u$ all cancel, and the remaining contributions from $\alpha^\pm$ reproduce the result of Eq. (3.35) for $(\eta_{+-})_u$ with $\epsilon'=0$.

We conclude by considering the energy dependence of $\Gamma_L^{\mu 3}$,

$$\begin{aligned}\Gamma_L^{\mu 3}&\equiv\Gamma(K_L\to\pi\mu\nu)\\ &=\Gamma(K_L\to\pi^+\mu^-\bar{\nu})+\Gamma(K_L\to\pi^-\mu^+\nu)\ .\end{aligned} \tag{3.87}$$

As has already been noted, the present results for $|\eta_{+-}|^2$ actually determine the energy dependence of $\Gamma_L^{+-}/\Gamma_L^{\mu 3}$. Hence by combining Eq. (3.86) with the analogous result for $\Gamma_L^{\mu 3}$, we can see to what extent the energy dependence of this ratio actually reproduces that of $|\eta_{+-}|^2$. Although $\Gamma_L^{\mu 3}$ appears in the denominator of the expression for δ in Eq. (3.40), we must be careful not to simply take over Eq. (3.42) in which various common factors have been canceled from the numerator and denominator. Assuming CPT and the $\Delta Q=\Delta S$ rule (i.e., $x_+=0$), the $K_L\to\pi\mu\nu$ amplitudes are[20]

$$\begin{aligned}A\,(K_L\to\pi^-l^+\nu)&=N^+\rho^+f_+\ ,\\ A\,(K_L\to\pi^+l^-\bar{\nu})&=-N^+f_+^*\ .\end{aligned} \tag{3.88}$$

N^+ and ρ^+ are as given in Eqs. (3.30) and (3.31) and f_+ is a form factor defined by

$$\begin{aligned}&\langle\pi^-(p_\pi)\,|\,J_\lambda(0)\,|\,K^0(p_K)\rangle\\ &\quad=(4E_\pi E_K V^2)^{-1/2}[(p_K+p_\pi)_\lambda f_++(p_K-p_\pi)_\lambda f_-]\ .\end{aligned} \tag{3.89}$$

We have assumed for simplicity that $f_-/f_+ \ll 1$, which is consistent with both SU(3) and experiment. Using Eq. (3.88), we find

$$\Gamma_L^{\mu 3} = |f_+|^2 |N^+|^2 [1+|\rho^+|^2] \times (\text{PS}) = |f_+|^2 \times (\text{PS}) , \quad (3.90)$$

where (PS) denotes the appropriate phase-space factor. We see that $\Gamma_L^{\mu 3}$ is completely independent of ρ^+ and hence of the u_a. It follows that if $\Gamma_L^{\mu 3}$ is found to be energy dependent, then this will be an indication (as in the case of Γ_S^{+-}) that either f_+ and/or the phase-space factor are energy dependent. Should it turn out, however, that $\Gamma_L^{\mu 3}$ is a constant, then the observed energy dependence of $\Gamma_L^{+-}/\Gamma_L^{\mu 3}$ comes entirely from the numerator. Using Eq. (3.86) we then see that the slope of Γ_L^{+-} is given by $(-2)(\xi_x+\zeta_x)/\Delta m$, which is exactly the result expected for $|\eta_{+-}|^2$. The consistency of our results can then be checked by noting that since

$$|\eta_{+-}|_u^2 = \left[\frac{\Gamma_L^{+-}}{\Gamma_L^{\mu 3}} \right]_u \left[\frac{\Gamma_L^{\mu 3}}{\Gamma_S^{+-}} \right]_u , \quad (3.91)$$

it must follow that $(\Gamma_S^{+-})_u \cong$constant to $O(u_a)$. That this is indeed the case has already been noted in the discussion following Eq. (3.82), where we observed that there was no term linear in α^- in that equation.

We conclude with a discussion of the limitations of the present analysis.

(a) The formalism developed in this section does *not* apply to the case of an external gravitational field, which has been treated separately in Appendix B. The reason for this is that a conventional gravitational field affects not only the K^0-$\bar{K}^0$ system itself but also the clocks and measuring rods that are used in studying it. Hence, care must be taken in describing the *observable* effects of an external gravitational field, as we discuss in detail in Appendix B.

(b) We have assumed throughout that the effects of external (nongravitational) fields can be described by the field matrix F in Eq. (3.10). This is certainly the case for $(\Gamma_L-\Gamma_S)_u$ and $(\Delta m)_u$ which are simply the real and imaginary parts of the eigenvalue difference $\lambda_u^+ - \lambda_u^-$ in Eq. (3.15). However, as we see from Eq. (3.25), an external field can in principle affect η_{+-} not only by changing p and q to p_u and q_u, respectively, but also by modifying c' and $\bar{c}'$ as well. This means that in the presence of a field Eq. (3.25) should be replaced by

$$(\eta_{+-})_u = \tfrac{1}{2}(\epsilon_u + \epsilon_u') , \quad (3.92)$$

where ϵ_u' accounts for the effects of the field on c' and $\bar{c}'$. Since the experimental data on η_{+-} and η_{00} suggest that $\epsilon' = \epsilon'' \cong 0$, as we have already noted, we have set $\epsilon_u' = \epsilon_u'' = 0$ as well on the presumption that a small modification of an already small contribution to η_{+-} can be neglected. This assumption can, of course, be tested by studying η_{+-}/η_{00} as a function of energy. If the real and/or imaginary parts of this ratio are seen to vary with energy, this would be an indication that ϵ_u' or ϵ_u'' or both could not be neglected. In fact, an experiment is already in progress (E-617 at Fermilab) which will measure the energy dependence of $|\eta_{+-}|/|\eta_{00}|$.

(c) If we return to the expression for the $\pi^+\pi^-$ rate $I_{+-}(t)$ in Eq. I(2.11), we see that Δm and τ_S enter in several places: There are, of course, the explicit contributions exhibited in I(2.11), but there are also the implicit contributions to ρ from $\alpha(L/\Lambda_S)$ as we can see from Eq. I(2.8). Since the latter correspond to the values of these parameters inside the target, it is in principle possible that these could differ from the corresponding values in free space. Such a difference could arise, for example, if the K^0-$\bar{K}^0$ system were totally or partially "shielded" from the external field by the target. To take such a contingency into account we should in principle allow $(\Delta m)_u$ to be a different function inside and outside the target. We have not done so, and hence have neglected the possibility of shielding, for the following reasons. (1) From a practical point of view it would be impossible to realistically carry out such an analysis given the data available to us. (2) For the carbon and lead targets, which are short compared to hydrogen, $\alpha(L/\Lambda_S)$ is given to lowest order by the length L of the target, and is thus independent of Δm and Γ_S anyway. Thus, shielding is not a consideration for the carbon data which give the dominant contribution to τ_S and more than half of the contribution to Δm. (3) An external field whose coupling to K^0-$\bar{K}^0$ is proportional to σ_x would be even under charge conjugation, and hence could not be shielded in any case. Even a field which was odd under C could be shielded only if it coupled to ordinary matter (electrons, protons, and neutrons) and to the K^0-$\bar{K}^0$ parameters Δm and Γ_S with comparable strengths.

The question of whether the external interaction is shielded in matter can be addressed experimentally by comparing the energy dependence of the K^0 parameters in vacuum and in a material medium. One approach would be to exploit the dependence of the regeneration amplitude ρ on Δm and τ_S [see Eqs. I(2.7) and I(2.8)]. A direct comparison of the 2π rates behind thick and thin regenerators of the same material as a function of energy would isolate the energy dependence of $\alpha(L/\Lambda_S)$, which depends on the mass difference and K_S lifetime *inside the regenerator.* The "double-beam" technique used in Ref. 5 should allow a relatively systematic error-free comparison of the 2π rates behind side-by-side regenerators of different lengths.

IV. PHENOMENOLOGICAL ANALYSIS OF THE EXPERIMENTAL RESULTS

The purpose of this section is to compare the experimental results of Sec. II for the slope parameters b_Δ, $b_{\Gamma S}$, b_η, and b_ϕ to the corresponding theoretical expressions derived in Sec. III. We will show that effects of the type suggested by the data cannot be naturally accounted for by the interaction of the K^0-$\bar{K}^0$ system with an external electromagnetic or hypercharge field. Furthermore, it will be demonstrated more generally that such effects cannot be attributed to *any* C-odd field which transforms as either u_y or u_z. By contrast, a pure u_x coupling, which is even under C, may be able to account for such effects as we discuss in more detail below. Such a coupling cannot, however, be due to an external gravitational field for the reasons elaborated upon in Appendix B. We will finally demonstrate that, in the standard SU(2)×U(1) model, an energy dependence of Δm and η_{+-} cannot be due to the scattering of K^0 and $\bar{K}^0$ from the cosmological neutrino sea which is presumed to permeate space. Having thus eliminated some of the more obvious explanations for these effects, we will thus be led to consider the possibility that they arise from a new interaction. The properties of this interaction will be described phenomenologically in this section, and a specific example of such an interaction will be discussed briefly in Sec. V.

Consider first the case of an external hypercharge field, which is characterized by Eqs. (3.12) and (3.13). Such a coupling depends on a single real parameter $\xi_z^{(1)}=A_0$, and hence leads to several nontrivial relations among the four observable slope parameters b_Δ, b_Γ, b_η, and b_ϕ. We proceed to show that the predictions of such a coupling disagree with the suggestions of the data. Returning to Eqs. (3.22), we find

$$(\Delta m)_u=(\Delta m)\left[1+\left[\frac{\xi_z^{(1)}}{\Delta m}\right]^2\gamma^2\right], \tag{4.1}$$

$$(\Gamma_L-\Gamma_S)_u=(\Gamma_L-\Gamma_S)\left[1-\left[\frac{\xi_z^{(1)}}{\Delta m}\right]^2\gamma^2\right]. \tag{4.2}$$

We note immediately that for such a coupling $b_\Delta^{(2)}=(\xi_z^{(1)}/\Delta m)^2$ is necessarily *positive*, whereas $b_\Delta^{(2)}$ is experimentally observed to be *negative*. One can verify that $b_\Delta^{(2)}$ is positive for a hypercharge field, by noting from Eqs. (3.13)–(3.15) that as γ increases the effective mass difference between K^0 and $\bar{K}^0$ (and hence between K_L and K_S) also increases. [As we have noted in Sec. III, $(\Delta m)_u$ is a monotonically increasing function of γ at all energies for a hypercharge field.]

Secondly, using Eq. (3.35) and setting $\epsilon'=0$, $(\eta_{+-})_u$ is given by

$$(\eta_{+-})_u\cong\tfrac{1}{2}\epsilon+\frac{1}{2\,\Delta m}u_z(1-i)\ , \tag{4.3}$$

from which it follows that

$$b_\phi^{(1)}=-\frac{\sqrt{8}}{|\epsilon|}\frac{\xi_z^{(1)}}{\Delta m}\ . \tag{4.4}$$

Combining Eqs. (4.4) and (4.1), we find

$$|b_\Delta^{(2)}|=\frac{|\epsilon|^2}{8}|b_\phi^{(1)}|^2\ . \tag{4.5}$$

Using lines 2 of Table I, and taking $|\epsilon|=2|\eta_{+-}|=(4.548\pm0.044)\times10^{-3}$, the right-hand side of Eq. (4.5) is numerically equal to $(1.8\pm1.4)\times10^{-11}$, whereas the left-hand side is equal to $(7.43\pm1.48)\times10^{-6}$. Alternatively, if we fix $|b_\Delta^{(2)}|$ using Table I, then $|b_\phi^{(1)}|$ is predicted on the basis of Eq. (4.5) to be 1.70 ± 0.17, which is substantially larger than the value $(2.6\pm1.0)\times10^{-3}$ that is experimentally observed. (We note in passing that our results would be essentially unchanged had we used the internal fit values from lines 1 of Table I, rather than the external values.) This comparison quantifies our previous remarks to the effect that the u_z (and similarly u_y) coupling gives rise to a large contribution to η_{+-}: The same calculation for the u_x case leads to a small value of b_ϕ, one which is more compatible with experiment, as we discuss below. Of course, *any* relation among the slope parameters involving b_Δ must come within $\sim$3–5 standard deviations of agreeing with experiment, since at this level b_Δ is consistent with zero. Hence, when we speak of the compatibility of a given prediction with experiment, we intend to compare the relative orders of magnitude of predicted and measured quantities, and not to suggest that a particular coupling (such as u_z) is actually ruled out by the data. From this point of view one can say that, at the very least, an external hypercharge field does not provide a natural explanation for the effects of the type that have been suggested by the data.

We can generalize this conclusion to an arbitrary u_z coupling by using the slope relations in Eqs. (3.62), taking $N=1$. Inserting into Eq. (3.62) the values for $b_\phi^{(1)}$ and $b_\eta^{(1)}$ from Table I (again using the results in lines 2), we find

$$b_\Gamma^{(2)}\ [\text{Eq. (3.62)}]=-(3.1\pm1.5)\times10^{-11}\ , \tag{4.6a}$$

$$b_\Delta^{(2)}\ [\text{Eq. (3.62)}]=-(0.3\pm1.5)\times10^{-11}\ . \tag{4.6b}$$

We note from Eq. (4.6b) that $b_\Delta^{(2)}$ is substantially smaller than the experimental value given in Table I, for any choice of parameters in the u_z case. Hence, even if the couplings in Eqs. (3.12) and (3.13) are generalized to allow $\zeta_z^{(1)}\neq0$, the extra freedom so obtained is not sufficient to avoid the difficulty that arises from Eq. (4.6b). We emphasize again that for a general u_z coupling b_ϕ and b_η are independent parameters, whereas for an external hypercharge field they are both related to $\xi_z^{(1)}$ and hence to each other. It follows that a hypercharge field should not be viewed as simply a special case of an arbitrary u_z coupling, but should be treated separately as we have done. Finally, we note that if we set $\epsilon=0$ in Eq. (4.3), then we recover the model of Bell and Perring,[9] and of Bernstein, Cabibbo, and Lee[9] in which η_{+-} arises *entirely* from an external hypercharge field. From Eq. (4.3), such a model can be ruled out immediately on the grounds that (a) $\phi_{+-}\cong-45°$ (rather than $+45°$) and is moreover a constant as a function of energy, and (b) $|\eta_{+-}|$ is directly proportional to u_z and hence to γ, contrary to the results of experiment.

The preceding analysis can also be used to demonstrate that the observed energy dependence of Δm and η_{+-} cannot be due to an interaction of the K^0-$\bar{K}^0$ system with stray electromagnetic fields or charges. Since the electromagnetic interaction is odd under C, it follows that its coupling to K^0-$\bar{K}^0$ must be of the form $u_z\sigma_z$, just as for a hypercharge field. This can also be seen by noting from the Gell-Mann–Nishijima relation that the electric charge is the same as the hypercharge, up to an additive constant $\pm\frac{1}{2}$ for K^0 and $\bar{K}^0$, respectively. Moreover, since K^0 and $\bar{K}^0$ are electrically neutral, it follows that the $K^0K^0\gamma$ and $\bar{K}^0\bar{K}^0\gamma$ vertices vanish at $q^2=0$, where q is the four-momentum of the photon. Consequently, the only electromagnetic contribution to u_z at $q^2=0$ arises from a contact (i.e., δ-function) interaction of a kaon with an electromagnetic charge.[24] However, even if stray charges were present in the otherwise empty space between the regenerator and detector, the density of such charges that would be required to produce any detectable effect in the present experiments is unphysically large.[25] We thus conclude that an energy dependence of Δm and η_{+-} of the type suggested by the data cannot be attributed to electromagnetic effects, both on the basis of an analysis of the general u_z case and also a consequence of the electromagnetic properties of K^0 and $\bar{K}^0$.

We turn next to the u_y case, which for all practical purposes is identical to u_z: As we have seen in Sec. III, the terms linear in ξ_y and ζ_y make a negligible contribution to $(\Delta m)_u$ and $(\Gamma_L-\Gamma_S)_u$ at currently available energies. If these terms are then dropped the resulting expressions for $(\Delta m)_u$, $(\Gamma_L-\Gamma_S)_u$, $|\eta_{+-}|_u$, and $(\tan\phi_{+-})_u$ in the u_y and u_z cases are effectively the same (up to some signs), as can be seen from Eqs. (3.22) and (3.35). Hence, we can immediately take over the preceding analysis of the general u_z case to demonstrate that the relative magnitudes of $b_\Delta^{(2)}$, $b_\eta^{(1)}$, and $b_\phi^{(1)}$ cannot be naturally accommodated by a pure u_y coupling either. It must be emphasized, however, that even though pure u_y *or* pure u_z does not work, a combination of u_y *and* u_z (both of which are *C*-odd) will work, since then four parameters are available to fit to the four measurable slope parameters.

We arrive finally at the pure u_x case which, as we shall see, is the most natural choice to describe the present data. The characteristic feature of a u_x coupling is the prediction that $b_\phi^{(N)}$ and $b_\eta^{(N)}$ are comparable in magnitude to $b_\Delta^{(N)}$ and $b_\Gamma^{(N)}$, as can be seen from Eqs. (3.54). This prediction is compared with the data in Fig. 4, where a "simultaneous slope plot" (SSP) for the u_x case is exhibited [see Eqs. (3.55) and (3.56)]. The darkened region delineates the overlap of the values of $b_\Delta^{(2)}$, $b_\eta^{(2)}$, and $b_\phi^{(2)}$ obtained using the method B results from I, which give the most conservative version of our results. For a u_x theory to describe effects of the type suggested by the data in Sec. II, the band corresponding to b_Γ must pass through this region. Since we have at present no data on the energy dependence of Γ_L, and hence on $\Gamma_L-\Gamma_S$, we cannot infer b_Γ from the results of I. Using the SSP one can read off the allowed values of the slope parameters, and these can then be immediately translated into limits on the external couplings $\xi_x^{(2)}$ and $\zeta_x^{(2)}$ by using Eqs. (3.53). When analyzing the SSP in Fig. 4, one should bear in mind that there is

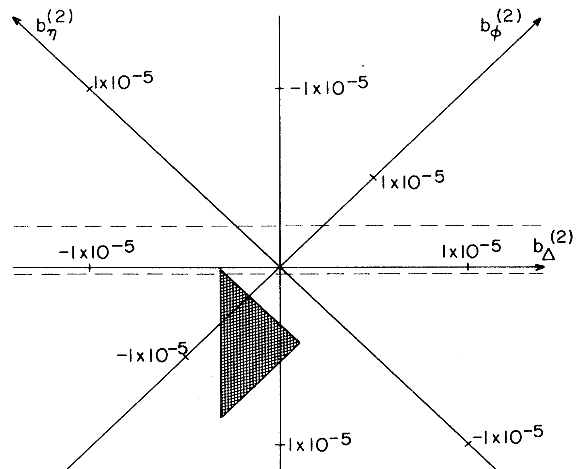

FIG. 4. Simultaneous slope plot for the $u_x\sigma_x$ case. Note that the scales on the different axes are in the ratio $b_\Delta^{(2)}:b_\Gamma^{(2)}:b_\eta^{(2)}:b_\phi^{(2)}=\sqrt{2}:\sqrt{2}:2:1$. The darkened region delineates the overlap of the values of $b_\Delta^{(2)}$, $b_\eta^{(2)}$, and $b_\phi^{(2)}$, and the dashed lines illustrate the overlap of this region with a hypothetical b_Γ band assuming $B_\Gamma=b_{\Gamma S}$. See text for further details.

a potential built-in uncertainty of $\pm10\%$ in Eqs. (3.54)–(3.56) which define the SSP. This arises from the fact that in deriving these relations we have consistently used $2\Delta m\,c^2/\hbar\Gamma_S=\hbar\Gamma_S/2\Delta m\,c^2=1$, whereas in reality $2\Delta m\,c^2/\hbar\Gamma_S=0.9546(46)$ and $\hbar\Gamma_S/2\Delta m\,c^2=1.0476(50)$.

It is useful to quantify the suggestion of the SSP in Fig. 4 that an interaction term of the form $u_x\sigma_x$ could account for the present data. This is expressed in the SSP by the fact that the vertical band corresponding to $b_\Delta^{(2)}$ passes through the intersection of the $b_\phi^{(2)}$ and $b_\eta^{(2)}$ bands, leading to the shaded region shown. The same conclusion can be drawn by eliminating $b_\Gamma^{(N)}$ in Eqs. (3.54a) and (3.54b) which leads to

$$b_\Delta^{(N)}+b_\eta^{(N)}-\tfrac{1}{2}b_\phi^{(N)}=0\ . \tag{4.7}$$

Numerically the left-hand side of Eq. (4.7) is equal to $(6.4\pm6.9)\times10^{-6}$ if we take $N=2$ and use the internal-fit results (Table I, lines 1), and is $(0.3\pm3.8)\times10^{-6}$ using the external fit, lines 2. The agreement between Eq. (4.7) and the data suggests that if a single interaction term can account for the present results it is $u_x\sigma_x$, but confirmation of this result must await a determination of b_Γ.

Using Eq. (3.54), we can derive another interesting result for the various K^0-$\bar{K}^0$ parameters. It is well known that the relation

$$\tan\phi_{+-}\cong\frac{2\,\Delta m}{-(\Gamma_L-\Gamma_S)}\cong\frac{2\,\Delta m}{\Gamma_S}\cong1\ , \tag{4.8}$$

which is suggested by the superweak theory,[26] holds quite well experimentally at low energies. It is then interesting to ask whether Eq. (4.8) holds at high energies as well, given the present data.[27] For this to be the case we must have

$$(\tan\phi_{+-})_u=\frac{2(\Delta m)_u}{-(\Gamma_L-\Gamma_S)_u}\ , \tag{4.9a}$$

where

$$(\tan\phi_{+-})_u=\tan\phi_{+-}(1+b_\phi^{(N)}\gamma^N)\ , \tag{4.9b}$$

$$(\Delta m)_u=\Delta m(1+b_\Delta^{(N)}\gamma^N)\ , \tag{4.9c}$$

$$(\Gamma_L-\Gamma_S)_u=(\Gamma_L-\Gamma_S)(1+b_\Gamma^{(N)}\gamma^N)\ . \tag{4.9d}$$

Combining Eqs. (4.9a)–(4.9d), and assuming that $b_x^{(N)}\gamma^N\ll1$ for $x=\phi,\Gamma,\Delta$, we find

$$b_\phi^{(N)}=b_\Delta^{(N)}-b_\Gamma^{(N)}\ , \tag{4.10}$$

which is just Eq. (3.54a). Hence the condition that the superweak relation in Eq. (4.8) hold at high energies is just that the energy dependence of these quantities originate from a term $u_x\sigma_x$, which as we have seen is the coupling favored by the data. [Although we have assumed for simplicity that $b_x^{(N)}\gamma^N\ll1$ in deriving (4.10), this assumption is unnecessary if we use the exact expressions for the various parameters given in Eqs. (3.15) and (3.32)].

Should it turn out when b_Γ is measured that u_x is not in fact compatible with the data, then we will have shown that *no* coupling transforming as a single u_a can account for an energy dependence of Δm and η_{+-} such as the data suggest. The next possibilities to consider are combinations of two of the u_a, namely, u_x and u_y, u_x and u_z, or u_y and u_z. As we have already noted, such couplings would be described by four real parameters, whose values could always be chosen to reproduce the experimental results for the four slope parameters b_Δ, b_Γ, b_η, and b_ϕ. Two general possibilities thus suggest themselves. (a) A combination of u_y and u_z would be odd under C and hence could arise from a C-odd field which interacts in some complicated manner with the K^0-$\bar{K}^0$ system. (b) By contrast, a combination of u_x and u_y or u_x and u_z would *not* correspond to a coupling with well-defined C. Such an interaction might arise, for example, from an external long-range field whose quanta themselves carried a quantum number such as hypercharge. Note that whichever of the two options (a) or (b) is realized, at least one member of each pair will be odd under C. Having thus detected an energy-dependent influence on the K^0-$\bar{K}^0$ system which has a C-odd component, we might be led to inquire whether there is an associated component which does not depend on energy. Such a field (whatever it was) could be responsible for all or part of the "intrinsic" contribution to η_{+-}, a possibility which may be worth reconsidering,[13,14] should the analysis of future experiments point in this direction.

We conclude this section with a brief discussion of kaon scattering from the "neutrino sea." If we suppose that space is filled with a sea of neutrinos,[28,29] which may be relics of the early stages of our universe, then regeneration of K_S from K_L can also occur in "free" space via the interactions

$$K^0+\nu\to K^0+\nu\ , \tag{4.11a}$$

$$\bar{K}^0+\nu\to\bar{K}^0+\nu\ . \tag{4.11b}$$

We will denote the corresponding forward-scattering amplitudes by $f^\nu(0)$ and $\bar{f}^\nu(0)$, respectively, where ν generically represents any of the species ν_e, ν_μ, ν_τ, $\bar{\nu}_e,\ldots,$. The weak neutral-current interactions in (4.11a) and (4.11b) can be mediated by Z^0 exchange and, since only the C-odd polar-vector current of the kaons can contribute $[f^\nu(0)-\bar{f}^\nu(0)]$ will in general be nonzero for elastic scattering from the neutrino sea. It turns out, however, that in the standard SU(2)$\times$U(1) model $f^\nu(0)$ and $\bar{f}^\nu(0)$ separately vanish. This can be seen by noting that the conventional assignments for the u,d,s,c quarks are

$$\begin{bmatrix} u \\ d_\theta \end{bmatrix}_L,\ \begin{bmatrix} c \\ s_\theta \end{bmatrix}_L, u_R,\ c_R,\ d_R,\ s_R\ ,$$

$$d_\theta=d\cos\theta_C+s\sin\theta_C, \tag{4.12}$$

$$s_\theta=-d\sin\theta_C+s\cos\theta_C\ ,$$

where R and L denote the right- and left-handed components, respectively, and θ_C is the Cabibbo angle. It follows that the $Z^0K^0K^0$ and $Z^0\bar{K}^0\bar{K}^0$ couplings, which in the forward direction are proportional to the weak "charges" Q_3 of K^0 and $\bar{K}^0$,

$$Q_3=Q_3^L+Q_3^R\ , \tag{4.13}$$

are exactly zero for $K^0=d\bar{s}$ and $\bar{K}^0=\bar{d}s$. However, this will not be the case for models with unconventional couplings, such as one in which c_R and s_R are assigned to doublets (along with some unspecified higher mass quarks), rather than to individual singlets.[30] However, even in such a model the regeneration parameter ρ in Eq. I(2.7) is many orders of magnitude too small to produce effects of the type suggested by the data, provided that the neutrino number density N is not unexpectedly large.

The principal conclusions of this section are that effects of the type suggested by the data cannot be accounted for by an external hypercharge or electromagnetic field, or by scattering from stray charges or cosmological neutrinos. It is worth emphasizing that these conclusions, as well as the argument of Appendix B which rules out gravity as an explanation for these effects, do not depend on the assumption that we are in a regime where $|u_a|/\Delta m$ is small. Recall from Eq. (3.70a) that for a hypercharge or electromagnetic interaction $(\Delta m)_u{}^2$ is a monotonically increasing function of energy at all energies, contrary to what is seen in the data. The neutrino sea can be ruled out simply because there are too few neutrinos to produce a detectable effect. Taken together these conclusions already suggest that the observed effects arise from a new interaction. The assumption that $|u_a|/\Delta m$ is small is, however, necessary in order to argue that the only u_a capable of describing the present results is u_x, which leads directly to Eq. (4.7). Using this result we will focus in the next section on specific models which give rise to an interaction term $u_x\sigma_x$.

Note added. It is worth emphasizing that the limits which derive from the present results on the nonexistence of certain cosmological fields are far more stringent than those obtainable by any other current methods. For example, the Eötvös-Dicke-Braginskii experiments[11] imply a limit on the coupling strength f of the hypercharge field in Eq. (3.12) given by [using the results of Roll *et al.* (Ref. 11)]

$$\frac{f^2}{Gm_p{}^2}<6\times10^{-8}\ ,\ \ 95\%\ \text{C.L.}\ , \tag{4.14}$$

where G is the Newtonian gravitational constant, and m_p is the proton mass. By way of comparison we note from Eqs. (3.35) and (3.12) that for the hypercharge case A_0 and $b_\eta^{(2)}$ are related by

$$b_\eta^{(2)}=\frac{A_0{}^2}{|\epsilon|^2(\Delta m)^2}=\frac{1}{|\epsilon|^2(\Delta m)^2}\left[\frac{f^2Y_G}{R_G}\right]^2, \tag{4.15}$$

where we have assumed that A_0 is of galactic origin. Since $b_\eta^{(2)}$ is necessarily positive for a hypercharge field (as we have previously noted), whereas experimentally $b_\eta^{(2)}=-(2.01\pm0.86)\times10^{-6}$ from Table I, it follows that at the 3σ (99.7%) confidence level, our result for $b_\eta^{(2)}$ excludes the value $b_\eta^{(2)}\geq+0.57\times10^{-6}$. This limit when combined with Eq. (4.15) then implies (using the galactic mass and radius quoted in Sec. V below)

$$\frac{f^2}{Gm_p{}^2}<1\times10^{-14}\ ,\ \ 99.7\%\ \text{C.L.}\ , \tag{4.16}$$

which is more stringent than the limit obtained from the Eötvös-Dicke-Braginskii experiments. Future experiments presently being contemplated at the Fermilab Tevatron should improve on the present limit by at least an order of magnitude. This analysis also suggests that the sensitivity of the Fermilab experiments is such that any mechanism which could account for the observed energy dependences of the K^0-$\bar{K}^0$ parameters could very well not manifest itself in other current experiments.

V. MODELS OF THE u_a

We examine in this section some models of the u_a in an attempt to see whether effects of the type suggested by the experimental data of Sec. II can be understood in a simple way. Since a detailed discussion of the experimental and theoretical constraints on such models will be presented elsewhere, we will confine our attention here to some examples which suggest new experiments, such as those considered in I. Of particular interest is the possibility that the u_a originate from some interaction which would also manifest itself elsewhere, such as in neutrino oscillations, for which there are some experimental suggestions at the present time.[31] Should this turn out to be the case, then failure to take account of the possible effects of such an interaction might lead to inconsistencies in interpreting the data, as we discuss shortly.

We have shown in Sec. IV that the only pure u_a coupling capable of accounting in a natural way for the experimental results of Sec. II is u_x, which is even under charge conjugation. One possibility is that a term proportional to u_x arises from an interaction of the K^0-$\bar{K}^0$ system with a mass-energy distribution, with this interaction being mediated by a C-even tensor (or scalar-tensor) field. This possibility becomes all the more attractive if we assume that the source of this field is an already known "charge". Since the only known "charges" that a macroscopic object carries with which it can couple to K^0 and $\bar{K}^0$ are mass-energy and hypercharge, it follows that, having eliminated hypercharge in Sec. IV, we are again led to mass-energy as the source of the unknown field. This field cannot, however, be a metric gravitational field, in particular the field of general relativity, for the reasons discussed in Appendix B. If the source of this field is indeed mass-energy, the implication would be that it couples not only to K^0 and $\bar{K}^0$ but to all particles and fields, including neutrinos. The same conclusion would also follow if the "field" were some material medium permeating space. In this view the effects of this field are manifest in the K^0-$\bar{K}^0$ system only because they enter in the combination $u_a/\Delta m$, where Δm is a small quantity. If this is the case, then such effects may also show up in neutrino oscillations where comparably small mass scales could exist. Before returning to discuss models of the external interaction, we comment briefly on the phenomenology of neutrino oscillations in the presence of such an interaction.

Following Ref. 31, we will assume that for a two-neutrino system $\Delta m_{12}{}^2 = m_2{}^2 - m_1{}^2 \cong 1\ \mathrm{eV}^2$. If we also assume that $m_{1,2}$ are each of order 1 eV, we see that for all relevant experiments $\gamma_{1,2} = E_{1,2}/m_{1,2}$ (where $E_{1,2}$ are the energies of neutrino species 1 and 2, respectively) are much higher than in the present kaon experiments. For example, $\gamma_{1,2} = 10^6$, 10^7, 10^9, and 10^{11} for a neutrino with an energy of 1 MeV, 10 MeV, 1 GeV, and 100 GeV, respectively. These values compare to $\gamma \cong 260$ for the highest-energy kaons in Refs. 4–6. The net effect of the new interaction, which is proportional to the analogs of $\xi_a^{(N)}\gamma^N/\Delta m$ and $\zeta_a^{(N)}\gamma^N/\Delta m$, could nonetheless be small if $\xi_a^{(N)}$ and $\zeta_a^{(N)}$ are suppressed for some dynamical reason. However, the possibility that $\gamma_{1,2}$ are large points up some potential differences between a system of oscillating neutrinos and the K^0-$\bar{K}^0$ system. Since $m_{1,2}$ and Δm_{12} could be comparable for neutrinos, in contrast to kaons where $\Delta m/m = 7.1\times 10^{-15}$, it may happen that $m_{1,2}$ as well as Δm_{12} will appear to vary with energy. From Eqs. (3.48) we see that this depends not only on $m_{1,2}$ and their difference, but also on the analogs of $\xi_0, \xi_x, \ldots$, etc. A second potential difference between these systems is the possibility that at current neutrino energies $|\Delta m_{12}|$ may be *increasing* with increasing energy, rather than decreasing as is the case for kaons at present energies. This is suggested by Fig. 1 which indicates that for kaons $|\Delta m|$ increases for $\gamma > 374$. Should $m_{1,2}$ and/or Δm_{12} depend on energy, then failure to take this into account might lead to the conclusion that the value of Δm_{12} obtained, say, in a high-energy experiment at Fermilab would be different from that obtained in a low-energy reactor or beam-dump experiment. Clearly neutrino experiments designed to look for such effects would be of great interest. A discussion of such experiments, and an analysis of neutrino oscillations in the presence of external fields, will be presented elsewhere.

Let us return to pursue the possibility that the energy-dependence of Δm, τ_S, and η_{+-} originates from a field, hereafter called the U field, which couples to mass-energy but which is nonetheless different from gravity. To account for such a γ dependence it is natural to suppose that U is a tensor field. If the quanta of this field were massless, we would be led inexorably[32] to general relativity, which we have already shown cannot account for the effects observed (see Appendix B). This suggests that the U-field quanta have a nonzero rest mass m_u, from which several important consequences follow. (1) The interaction mediated by the U field would have a finite range. (2) Given the weakness of the coupling of the U field to kaons (and presumably to other matter as well), the U quanta could conceivably be present in our galaxy (and elsewhere in our universe) in large numbers, and yet go undetected by conventional means. If this were the case, the U quanta could be partially responsible for the "missing mass" of the universe.[33]

The preceding discussion suggests a number of possible mechanisms for generating the U field. For example, starting with the expression in Eq. (B20), we deliberately construct a nonuniversal (and hence nongravitational) interaction by allowing γ_{PPN} to be different for K_L and K_S: $\gamma_{\mathrm{PPN}} \to \xi_L$ or ξ_S with $\xi_L \neq \xi_S$. To implement the requirement that the U field have a finite range, we assume that the only contributions to $\Phi(r)$ come from matter within our galaxy. Starting from Eq. (B20) and setting $E_L' \cong E_S'$ for K_L and K_S in regeneration (see Appendix A), the K_L-K_S phase difference is given by

$$\text{phase difference} = \frac{i}{\hbar}\int [p_L'(1+\xi_L\Phi) - p_S'(1+\xi_S\Phi)]dz'$$

$$\cong -\frac{i}{\hbar}\int \left[\Delta m\left[1-(\xi_L-\xi_S)\frac{m}{\Delta m}\Phi\beta^2\gamma^2\right]\right]\frac{mc^2}{p'}[1+\tfrac{1}{2}(\xi_L+\xi_S)\Phi]dz' , \tag{5.1}$$

where use has been made of Eqs. (A12). In the limit $\xi_L=\xi_S=\gamma_{\rm PPN}$, Eq. (5.1) goes over into the standard results given in Eq. (B34). However, when $\xi_L\neq\xi_S$, Δm becomes γ-dependent,

$$\Delta m\to\Delta m\left[1-(\xi_L-\xi_S)\frac{m}{\Delta m}\Phi\beta^2\gamma^2\right]$$
$$\equiv\Delta m(1+b_\Delta^{(2)}\gamma^2)\ . \tag{5.2}$$

We notice immediately that in such a model $b_\Delta^{(2)}$ itself depends on β^2, a possibility which we anticipated in Sec. III. This is of little practical consequence in the present case, since $\beta^2\gamma^2=\gamma^2-1$, which means that Δm will appear to be varying linearly when plotted against either $\beta^2\gamma^2$ or γ^2. However, $\beta\gamma$ and γ could be distinguished, at least in principle. Since $(\xi_L-\xi_S)$ represents an off-diagonal ($\Delta S=2$) contribution from u_x, it is reasonable to suppose that $|\xi_L-\xi_S|\sim|\Delta m/m|$, in which case

$$|b_\Delta^{(2)}|\cong\Phi\ , \tag{5.3}$$

where we have set $\beta^2=1$. Φ is now the gravitational potential due to the galaxy. We take our galaxy[34] to be a homogeneous two-dimensional disc with a mass $1.4\times10^{11}\,M_\odot$ and a diameter of 25 kpc$=7.7\times10^{22}$ cm, with the Sun located at a distance of 10 kpc from the galactic center. Numerical evaluation of Φ then gives (assuming that $\hbar/m_uc\gg$ galactic radius)

$$|b_\Delta^{(2)}|\cong\Phi\cong0.9\times10^{-6}\ , \tag{5.4}$$

in surprisingly good agreement[35] with the experimental value $|b_\Delta^{(2)}|=(7.4\pm1.5)\times10^{-6}$. Inclusion of the contributions from the local group of galaxies raises the result in Eq. (5.4) to about 1×10^{-6}. It is, of course, difficult at this point to understand why so crude an estimate of $b_\Delta^{(2)}$ should come anywhere near the observed value. If this is not entirely due to coincidence, it may indicate that the origin of the observed effects is a small breakdown of universality in the gravitational interaction, which perhaps manifests itself only at the quantum level.

One can extend the preceding model to understand qualitatively the relation between b_Δ and b_Γ. Suppose we examine the effects of the U field in a basis given by the CP eigenfunctions K_1^0 and K_2^0. In analogy to Nachtmann,[15] we take $K_{1,2}^0$ to be initially degenerate, and treat the weak interaction and the U field as perturbations. The analog of iH in Eq. (3.10) can be written in the form

$$iH\to i\begin{bmatrix}d_{11} & d_{12}\\ d_{21} & d_{22}\end{bmatrix}\ , \tag{5.5}$$

where $d_{11},d_{12},\ldots,$ are complex numbers. In terms of the unperturbed mass m_0 of $K_{1,2}^0$, $d_{11,22}$ can be written as

$$\begin{aligned}d_{11}&=m_0-\tfrac{1}{2}(\Delta m+i\Gamma_S)+u_{11}\ ,\\ d_{22}&=m_0+\tfrac{1}{2}(\Delta m-i\Gamma_L)+u_{22}\ ,\end{aligned} \tag{5.6}$$

where u_{11} and u_{22} represent the effects of the U field in the K_1^0-K_2^0 basis. When transformed back into the K^0-$\bar{K}^0$ basis, the Hamiltonian in Eq. (5.5) gives the following contributions to the various parameters in Eq. (3.14):

$$\begin{aligned}u_0+\tfrac{1}{2}(d+\bar{d})&=\tfrac{1}{2}(d_{11}+d_{22})\\ &=m_0-\frac{i}{4}(\Gamma_L+\Gamma_S)+\tfrac{1}{2}(u_{11}+u_{22})\ ,\\ \tfrac{1}{2}(p^2+q^2)+iu_x&=\frac{i}{2}(d_{22}-d_{11})\\ &=\frac{i}{2}\left[\Delta m-\frac{i}{2}(\Gamma_L-\Gamma_S)+(u_{22}-u_{11})\right]\ ,\\ u_z+\tfrac{1}{2}(d-\bar{d})&=\tfrac{1}{2}(d_{12}+d_{21})\ ,\\ \tfrac{1}{2}(p^2-q^2)+u_y&=\frac{i}{2}(d_{12}-d_{21})\ .\end{aligned} \tag{5.7}$$

Hence, if u_{11} and u_{22} are real, which means that $K_{1,2}^0$ act simply as if they had different masses in the presence of the U field (even though they were initially degenerate), then there arises a contribution to u_x of the form

$$u_x=\tfrac{1}{2}(u_{22}-u_{11})\ , \tag{5.8}$$

which is real. From Eqs. (3.11) and (3.53) this means that $b_\Gamma=0$ but $b_\Delta\neq0$. Conversely, in order for b_Γ to be different from zero the U field must affect the lifetimes of $K_{1,2}^0$ as well as their masses. In such a model once b_Δ and b_Γ are determined, b_η and b_ϕ can then be obtained by using Eqs. (3.54).

Although the foregoing discussion suggests that a nonuniversal gravitational interaction (with $\xi_L\neq\xi_S$) could account for the observed energy dependence of the K^0-$\bar{K}^0$ parameters, we know of no specific theory which would in fact give $\xi_L\neq\xi_S$. This is a major limitation of such a picture, since in the absence of a detailed theory we are unable to use the kaon data to study other phenomena such as neutrino oscillations. For this reason we have focused[1,2,36] on a Lagrangian model based on a massive tensor field $U_{\mu\nu}$ which appears capable not only of explaining the present data, but also of making predictions for other processes as well. Since a detailed description of this model will be presented elsewhere,[36] we will limit the present discussion to a brief summary of its salient features. As is well known, a tensor field $U_{\mu\nu}$ whose quanta have a nonzero mass m_u gives rise to a theory lacking the general coordinate invariance of general relativity (GR). Hence an external U field behaves in much the same way as any other external field, and motion with respect to this field is detectable just as it would be for an external electromagnetic field. In the limit $m_u\to0$, general coordinate invariance is restored, but the resulting theory is not GR, but rather a theory characterized by the exchange of both scalar and tensor massless gravitons. For this reason, the phenomenological consequences of such a theory are quite different from those of GR: For example, the deflection of light by the Sun (or radar time delay) is predicted to be $\frac{3}{4}$ of the GR value, while the precession of the perihelion of Mercury is $\frac{2}{3}$ of that predicted by GR.[37] Since the current experimental data, particularly for the radar time delay,[38] strongly support GR it follows that gravitational forces cannot arise *exclusively* from a massive tensor field $U_{\mu\nu}$, irrespective of how small m_u is. In the weak-field limit, however, the data are consistent with a two-tensor theory in which the usual metric tensor $g_{\mu\nu}$ is replaced by $f_{\mu\nu}$,

$$f_{\mu\nu}=\cos^2\theta_u g_{\mu\nu}+\sin^2\theta_u U_{\mu\nu} , \quad (5.9)$$

where θ_u is a mixing angle. It can be shown[2,36] that the data for radar time delay[38] imply that $\sin^2\theta_u \lesssim 8\times 10^{-3}$, and hence that the coupling of $U_{\mu\nu}$ to matter must be much weaker than that of gravity. Even so, such a two-tensor theory faces other formidable challenges, most notably from the Hughes-Drever experiments,[39] as we discuss in greater detail in Ref. 36. At present it appears that such a model can be made consistent with all available data on a variety of systems, but additional experiments are clearly needed.[36] If this model does indeed prove to be viable, then it could have important implications for the study of neutrino oscillations as well, since $U_{\mu\nu}$ couples universally to the energy momentum tensor $T_{\mu\nu}$.

The preceding picture of the U field is based on the assumption that the dominant contribution to F in Eq. (3.10) comes from u_x. Should the data, particularly the magnitude and sign of b_Γ, indicate that this is not the case, we would then be forced to consider models involving combinations of two or more of the u_a. As we have noted in Sec. IV, one interesting possibility is that the U field manifests itself as some combination of u_y and u_z both of which are odd under C. The four parameters $\xi_y^{(N)}$, $\zeta_y^{(N)}$, $\xi_z^{(N)}$, and $\zeta_z^{(N)}$ could then be phenomenologically chosen to fit the four slope parameters b_Δ, b_Γ, b_η, and b_ϕ. However, there would still remain the question of whether the fitted values of $\xi_y^{(N)}, \ldots,$ could be obtained from a consistent dynamical model. The same remarks apply, of course, to other combinations of the u_a, such as u_x and u_y and u_z.

Thus far we have considered models in which the U field is an external influence on the K^0-$\bar{K}^0$ system, either in the form of a field in the usual sense or an external medium such as the neutrino sea. There remains the possibility that the u_a represent instead the effects of a breakdown in the usual Wigner-Weisskopf description of the K^0-$\bar{K}^0$ system due, for example, to a small nonlinear term in the Schrödinger equation. A particularly attractive choice for such a term is[40] $-b\ln|\Psi|^2$, where b is a constant, and limits on b have been set by recent experiments.[41] It is not clear at present whether such a term could, in fact, account for the effects of the type reported in Refs. 1—3. However, if this did turn out to be the case, it would again be natural to look for similar effects in neutrino oscillations.

We conclude this section with a discussion of what is perhaps the most direct (if least popular) interpretation of the present data, namely, that they represent a fundamental breakdown of Lorentz invariance. Taken on their face value, the data imply that observers comoving with K_L and K_S can discern how fast they are traveling with respect to ostensibly empty space, simply by measuring Δm or η_{+-}. In attributing such effects to the presence of an external field or medium, we are arguing in effect that the laws of physics are Lorentz invariant, but that space is not really empty. There is, however, an alternative view which is that the fundamental laws of physics themselves violate Lorentz invariance at some level. Such a possibility has been discussed, prior to the present work, by a number of authors,[42] and has been the subject of renewed interest in the context of unified gauge theories.[43—46] Ellis *et al.*,[44] and also Zee,[45] have considered possible violations of Lorentz invariance in proton decay which take place on a scale $a_{\text{decay}}\sim 1/M_X$, where M_X is the mass of the superheavy gauge boson expected in grand unified theories. In such a model one expects Lorentz-noninvariant (LNI) effects only on distance scales of order 10^{-28} cm. It is thus difficult to understand what relevance, if any, such a model would have for the present kaon data. On the other hand, Nielsen and Picek[46] have recently considered the possibility of LNI effects arising on a scale of $10^{-16}\text{cm}\sim 1/M_W$, which could in principle lead to effects of the type suggested by the data of Refs. 1—3.

In the preceding discussion of LNI effects in proton decay the starting point is typically an assumed noncausal behavior of the X-boson propagator. Still another way to introduce LNI effects is to take seriously recent work on lattice gauge theories to the extent of supposing that space-time is really a lattice.[47] In such a case the usual relation between the energy $E(\vec{k})$ and the momentum $\vec{k}$ of a free particle,

$$E^2=\vec{k}^2+m^2 , \quad (5.10)$$

would get modified to

$$E^2=\vec{k}^2+m^2-\frac{\vec{k}^4}{M^2} , \quad (5.11)$$

where M^{-1} is determined by the lattice spacing. In such a model the K_L-K_S mass difference Δm would appear to be energy dependent,

$$\Delta m\to\Delta m\left[1+\frac{\vec{k}^2}{M^2}\gamma^2\right] , \quad \gamma=E_K/m \quad (5.12)$$

with a coefficient $b_\Delta^{(2)}=\vec{k}^2/M^2$ which was itself energy dependent. Fitting Eq. (5.12) to the data of Refs. 1—3, we find $M\cong 3\times 10^4$ GeV, which means that at cosmic-ray energies other anomalies could appear.

Given the crudeness of all existing models of LNI effects, it is difficult to tell whether any such model is relevant to the effects described in Refs. 1—3. Clearly any complete theory of LNI effects must be able to account for the data not only on τ_S, but also for Δm and η_{+-}. If such a theory can in fact be constructed, the question would arise as to whether its predictions would differ from those of a model based on an external field $U_{\mu\nu}$. Although this is a difficult question to answer at present, one possible approach would be to study experiments in which the kaons traveled in the *vertical* direction, which would thus be sensitive to the Earth's contribution to the *gradient* of $U_{\mu\nu}$. If the Earth is the source of all or part of the anomalous energy dependence of Refs. 1—3, then such experiments should reveal effects which depend in a well-defined way on $\cos\alpha$, where α is the azimuthal angle. A detailed analysis of such experiments is currently in progress, and will be presented elsewhere.

VI. CONCLUSIONS

We have developed in this paper a general theoretical framework for describing energy-dependent effects in the proper frame of the K^0-$\bar{K}^0$ system. This framework allows us to analyze the K^0-$\bar{K}^0$ system not only at relatively low energies, where $|u_a|/\Delta m < 1$, but also at very high energies ($|u_a|/\Delta m \gg 1$), where the properties of the kaons are determined primarily by the external influence which gives rise to the u_a. We emphasize that this treatment is purely phenomenological in that it makes no assumption concerning the origin of the u_a in Eq. (3.10).

Using this formalism we demonstrated in Sec. IV that effects of the type suggested by the data of Refs. 1–3 (see Sec. II) cannot be ascribed to an external electromagnetic or hypercharge field, or to the scattering of kaons from stray charges or cosmological neutrinos. When taken together with the analysis of Appendix B, which indicates that such effects cannot arise from gravitational interactions either, we are led to conclude that the anomalous energy dependence of the K^0-$\bar{K}^0$ parameters may be the signature of a new interaction. As we have emphasized in Sec. IV, this conclusion does not depend on the approximation $|u_a|/\Delta m < 1$. However, if this approximation is invoked, we can proceed further and demonstrate that a term of the form $u_x\sigma_x$ in Eq. (3.10) can account for the present data, but $u_y\sigma_y$ or $u_z\sigma_z$ cannot. Since $u_x\sigma_x$ is even under C and CP, this suggests that the origin of the anomalous energy dependence of the neutral-kaon parameters may be a C-even external field or medium.

In Sec. V, we considered some specific models of a $u_x\sigma_x$ interaction, including a C-even massive tensor field $U_{\mu\nu}$. We also discussed the possibility that such effects may arise from an interaction which is intrinsically Lorentz-noninvariant. As we noted, it may be possible to distinguish between effects due to a LNI interaction and those due to an external tensor field by experiments which measure $\vec{\nabla}\xi_a(\vec{x})$ and $\vec{\nabla}\zeta_a(\vec{x})$. It is clear, however, that if the data of Refs. 1–3 are correct, then the source of these effects will represent a new and hitherto unexplored realm of physics.

ACKNOWLEDGMENTS

We are deeply indebted to our many colleagues who contributed to this work in various ways. We are particularly grateful to Professor Bruce Winstein for many valuable suggestions. Two of the authors (H.Y.C. and E.F.) wish to thank Professor Chen Ning Yang for the hospitality of the Institute for Theoretical Physics at Stony Brook where this work was begun. This work was supported in part by the U.S. Department of Energy.

APPENDIX A: KINEMATICS FOR K_L-K_S INTERFERENCE

In the absence of external fields, interference effects in K_S regeneration arise from the difference between the free-particle wave functions $\exp[(i/\hbar)P_L\cdot x]$ and $\exp[(i/\hbar)P_S\cdot x]$ describing K_L and K_S, respectively. It is this difference which gives rise to the oscillatory factor proportional to $\cos(\Delta m\, c^2 t/\hbar)$ in Eq. I(2.11), as is well known. In the presence of external fields the oscillatory factor becomes more complicated, particularly in the case of various phenomenological theories that we want to consider. It is thus worthwhile to reexamine the seemingly trivial derivation of the oscillatory factor from the point of view of generalizing it to the external field case. From the preceding discussion it follows that in the laboratory frame the matrix element for $K_L + T \to K_S + T$ contains the factor

$$e^{i(\vec{p}_L-\vec{p}_S)\cdot\vec{x}/\hbar}e^{-i(E_L-E_S)t/\hbar} \underset{E_L=E_S}{\longrightarrow} e^{i\Delta p z/\hbar}\,, \tag{A1}$$

for motion in the z direction. Along the classical trajectory

$$z = vt = \frac{c^2 p}{E}t\,, \tag{A2}$$

as discussed in Appendix B below. Combining Eqs. I(A3) and (A2) above allows the oscillatory factor in Eq. (A1) to be written as

$$e^{i\Delta p z/\hbar} = e^{-i\Delta m c^2 t/\hbar\gamma}\,, \tag{A3}$$

which then leads immediately to the expression in Eq. I(2.11) in the kaon rest frame.

For later purposes it is instructive to rederive Eq. (A3) in another way. Starting with the free-particle wave function for K_L we again use (A3) to write

$$e^{iP_L\cdot x/\hbar} = e^{ip_L z/\hbar - iE_L t/\hbar} = e^{-im_L c^2 t/\hbar\gamma_L}\,, \tag{A4}$$

and similarly for K_S. Using Eq. (A12b), we see that the matrix element for $K_L + T \to K_S + T$ contains the factor

$$\exp[-i(c^2 t/\hbar)(m_L/\gamma_L - m_S/\gamma_S)] = e^{-2i\Delta m c^2 t/\hbar\gamma}\,, \tag{A5}$$

in apparent contradiction with Eq. (A3). The discrepancy between Eqs. (A3) and (A5) can be resolved by noting that Eq. (A4), and its analog for K_S, builds in the fact that $v_L = c^2 p_L/E \neq v_S = c^2 p_S/E$. It follows that during the time t that a K_S travels the distance D between the regenerator and the detector, the K_L will travel a distance of only $(v_L/v_S)D < D$, as shown in Fig. 5. Hence, the accumulated K_L-K_S phase difference $(\Delta\phi_1)$ between the regenerator and detector obtained by using Eq. (A5),

$$\Delta\phi_1 = \frac{-2i\,\Delta m\, c^2 t}{\hbar\gamma}\,, \tag{A6}$$

gives only part of the result. There is an additional contribution $\Delta\phi_2$, shown by the dashed line in Fig. 5, which is

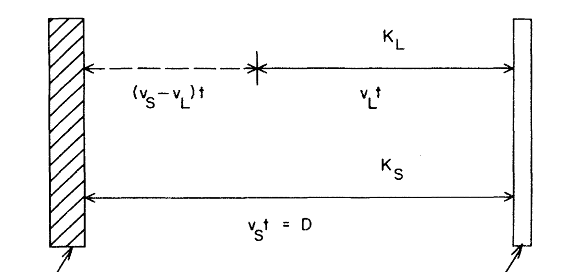

FIG. 5. Phase correction in K_L-K_S interference. During a time interval, t, the (faster) K_S travels a distance D, whereas the (slower) K_L travels a distance $v_L t < D$. The phase correction for K_L is $ip_L\Delta z/\hbar$, where $\Delta z = (v_S - v_L)t$ as indicated by the dashed line.

given by

$$\Delta\phi_2 = \frac{i}{\hbar} p\,\Delta z = \frac{i}{\hbar} p(v_S - v_L)t = +\frac{i\,\Delta m\,c^2 t}{\hbar\gamma} , \tag{A7}$$

where again use has been made of Eq. I(A3). The total phase difference $\Delta\phi$ is then given by

$$\Delta\phi = \Delta\phi_1 + \Delta\phi_2 = \frac{-i\,\Delta m\,c^2 t}{\hbar\gamma} , \tag{A8}$$

in complete agreement with Eq. (A3). Thus, when it is convenient for some purpose to start with the covariant wave function in Eq. (A3), or with its analog in the presence of an external field as in Appendix B, then a phase correction must be made as in Eq. (A7). Although the net effect of this correction in the free-particle case is to simply replace $m_{L,S}/\gamma_{L,S}$ in the covariant expression (A4) by $\Delta m/\gamma$, the effect in other cases is more complicated.

Thus far we have considered the kinematics which govern interference effects in the regeneration process for which $E_L \simeq E_S$ but $\vec{p}_L \neq \vec{p}_S$. It is interesting to note that interference effects can also be studied in another class of processes where just the opposite conditions prevail, namely, $E_L \neq E_S$ but $\vec{p}_L = \vec{p}_S$. Consider, for example,

$$e^+ + e^- \rightarrow \phi(1020) \rightarrow K_L + K_S . \tag{A9}$$

If the ϕ is produced at rest in the laboratory, then conservation of three-momentum requires that $\vec{p}_L = \vec{p}_S$. It then follows that $E_L \neq E_S$ owing to the K_L-K_S mass difference. Processes whose kinematics complement those of regeneration may provide an important tool in studying the velocity dependence of the parameters of the K^0-$\bar{K}^0$ system. This can be seen by noting that in the presence of an external gravitational field, for example, the free-particle wave function in Eq. (A4) generalizes to

$$\exp\left[(i/\hbar)\int g_{\mu\nu} P_L^{\mu} dx^{\nu}\right] = \exp\left[(i/\hbar)\int g_{\mu\nu}(m_L dx^{\mu}/d\tau) dx^{\nu}\right] , \tag{A10}$$

as we show in Eq. (B27). It follows that in the presence of a static gravitational field, the matrix element for $K_L + T \rightarrow K_S + T$ contains the factor

$$\exp\left[(i/\hbar)\int g_{ij}(P_L^i - P_S^i)dx^j + (i/\hbar)\int g_{00}(P_L^0 - P_S^0)dx^0\right] , \tag{A11}$$

which is the generalization of (A1). We thus see that experiments in which either the generalized momenta $P_{L,S}^i$ or the generalized energies $P_{L,S}^0$ are unequal would allow for a separate determination of g_{ij} and g_{00}, respectively. Similar remarks hold for the space and time components of other possible long-range fields which couple to K_L and K_S.

We will present elsewhere a detailed analysis of the influence of long-range fields on interference phenomena in $\phi \rightarrow K_L + K_S$. We note that since $\beta = 0.217$ and $\gamma = 1.024$ for K_L and K_S in ϕ decay, the kaons are sufficiently relativistic for velocity-dependent effects to be studied in a high-statistics experiment. Although $\psi/J \rightarrow K_L + K_S$ would produce kaons which were even more relativistic ($\gamma = 3.1$), this mode is highly suppressed and in fact has not yet been seen. Another mode which may be potentially interesting is $\psi(3770) \rightarrow D + \bar{D}$, where $\gamma = 1.011$.

We conclude by citing some kinematic relations for regeneration which follow from Eq. I(A3) and $E_L \cong E_S$:

$$m_L\gamma_L - m_S\gamma_S = 0 , \tag{A12a}$$

$$\frac{m_L}{\gamma_L} - \frac{m_S}{\gamma_S} \cong \frac{2\,\Delta m}{\gamma} , \tag{A12b}$$

$$\beta_L{}^2\xi_L - \beta_S{}^2\xi_S = (\xi_L - \xi_S)\beta^2 - \frac{\Delta m}{m\gamma^2}(\xi_L + \xi_S) , \tag{A12c}$$

$$E_L\beta_L{}^2\xi_L - E_S\beta_S{}^2\xi_S = \frac{mc^2}{\gamma}\left[\beta^2\gamma^2(\xi_L - \xi_S) - \frac{\Delta m}{m}(\xi_L + \xi_S)\right] . \tag{A12d}$$

In Eqs. (A12), $\beta_{L,S} = v_{L,S}/c$, $\gamma_{L,S} = (1 - \beta_{L,S}{}^2)^{-1/2}$, $\beta = \frac{1}{2}(\beta_L + \beta_S)$, $\gamma = \frac{1}{2}(\gamma_L + \gamma_S)$, and $\xi_{L,S}$ are two arbitrary constants which may or may not be the same for K_L and K_S. Equations (A12) are useful in analyzing the effects of gravity and other external interactions on the K^0-$\bar{K}^0$ system. The analogs of Eqs. (A12) for the case $\vec{p}_L = \vec{p}_S$, but $E_L \neq E_S$ are

$$m_L\gamma_L - m_S\gamma_S \cong \frac{\Delta m}{\gamma} , \tag{A13a}$$

$$\frac{m_L}{\gamma_L} - \frac{m_S}{\gamma_S} \cong \frac{\Delta m}{\gamma}(1 + \beta^2) , \tag{A13b}$$

$$m_L\gamma_L\beta_L - m_S\gamma_S\beta_S = 0 , \tag{A13c}$$

$$\beta_L{}^2\xi_L - \beta_S{}^2\xi_S \cong (\xi_L - \xi_S)\beta^2 - \frac{\Delta m}{m}\frac{\beta^2}{\gamma^2}(\xi_L + \xi_S) , \tag{A13d}$$

$$E_L\beta_L{}^2\xi_L - E_S\beta_S{}^2\xi_S = \frac{mc^2\beta^2}{\gamma}\left[\gamma^2(\xi_L - \xi_S) - \frac{\Delta m}{2m}(\xi_L + \xi_S)\right] . \tag{A13e}$$

APPENDIX B: THE K^0-$\bar{K}^0$ SYSTEM IN A GRAVITATIONAL FIELD

We present in this appendix a detailed description of the behavior of the K^0-$\bar{K}^0$ system in a gravitational field. The main purpose of this discussion is to establish that, in all known theories of gravity, no *observable* effects (of the type described in Sec. II) arise from the motion of the K^0-$\bar{K}^0$ system with respect to a static gravitational field. The primary reason for this is that the experiments under consideration are insensitive to the *gradient* of the gravitational potential, and hence may be considered as "local" experiments for present purposes. After a general description of metric and nonmetric theories of gravity, and their implications for the present experiments, we derive the wave function for a kaon in a weak static spherically symmetric (SSS) gravitational field. This discussion serves both to illustrate the more general arguments, and to set the stage for a subsequent description of *nonlocal* experiments, ones in which gravity-induced interference effects in the K^0-$\bar{K}^0$ system could in principle be studied.

We have seen that the data of Refs. 1–3 suggest that $|\eta_{+-}|$, ϕ_{+-}, and $\Delta = 2\,\Delta m\, c^2 \tau_S/\hbar$ are all energy dependent. In anticipation of the ensuing discussion it should be noted that all of these parameters are dimensionless nongravitational quantities. We also note that the gravitational potential with which the K^0-$\bar{K}^0$ system interacts in the regeneration experiments may be taken to be a constant. This is clearly the case for the contributions from distant (e.g., galactic) sources, whose effects can be linearly added in the weak-field limit. The largest contribution to the gradient of the potential presumably comes from the Earth, but since the kaons travel essentially *horizontally* in these experiments, they again see only a constant potential.[48] It follows that these experiments are local in the sense that they do not probe the variation of the gravitational potential in the vicinity of the apparatus. The suggestion of the regeneration experiments is that they have detected an apparent velocity dependence of several dimensionless nongravitational parameters by means of a local measurement. The question is then whether such an effect can arise from a gravitational field. To proceed, we consider some of the results of the Caltech group[49–53] which has analyzed theories of gravity in a rather general framework. We caution the reader at the outset that these authors consider primarily classical (i.e., nonquantum) theories of gravity, and hence the applicability of their analysis to the K^0-$\bar{K}^0$ system remains somewhat of an open question. Central to their discussion are several versions of the equivalence principle, including the weak equivalence principle (WEP) and the Einstein equivalence principle (EEP). The WEP is what Misner, Thorne, and Wheeler[54] refer to as "uniqueness of free fall," and what some other authors term "equality of passive and inertial masses." The EEP subsumes the WEP and adds the requirement that[49] "the outcome of any local, nongravitational test experiment is independent of where and when in the Universe it is performed, and independent of the velocity of the (freely falling) apparatus." Thorne, Lee, and Lightman[49] note that one consequence of the EEP is that "dimensionless ratios of nongravitational physical constants must be independent of location, time, and velocity." It follows from the previous discussion that gravitational theories which embody the EEP cannot lead to the observed energy (or velocity) dependence of $|\eta_{+-}|$, ϕ_{+-}, or Δ. Moreover, Schiff[55,56] has conjectured that[49] "any complete and self-consistent gravitation theory that obeys WEP must also, unavoidably, obey EEP." If Schiff's conjecture is correct, then *no* self-consistent theory of gravity which embodies the WEP could lead to a velocity dependence of $|\eta_{+-}|$, ϕ_{+-}, or Δ. It can be shown[51,52] that Schiff's conjecture is equivalent to the statement that any complete and self-consistent theory of gravity, which is relativistic and which embodies the WEP, is necessarily a metric theory. Most of the familiar theories of gravity, including general relativity and the Brans-Dicke-Jordan[57] theory, are in fact metric theories. This simply means that they are characterized by a metric tensor $g_{\mu\nu}$ whose geodesics are the trajectories of freely falling test bodies. In addition, the nongravitational laws of physics in such theories assume their special relativistic forms in local freely falling frames. From the preceding discussion, it then follows that the observed velocity dependence of $|\eta_{+-}|$, ϕ_{+-}, and Δ cannot be accounted for in the framework of any metric theory of gravity.

There are, however, nonmetric theories of gravity as well. These are usually formulated in terms of some fundamental Lagrangian as is the case, for example, for the theory of Belinfante and Swihart.[58] If Schiff's conjecture is correct, then all such theories are either equivalent to metric theories or else are inconsistent with the WEP. Indeed, an analysis by Lee and Lightman[50] of a class of nonmetric theories, including that of Belinfante and Swihart, indicates that all of these are inconsistent with the current experimental limits for the Eötvös-Dicke-Braginskii (EDB) experiments,[11] and hence with the WEP. The class of theories for which their analysis is applicable are those in which the equations of motion of a charged particle in a SSS gravitational field can be derived from the Lagrangian

$$L=\int \mathscr{L}\,dt=-\int [mc^2(T-H\beta'^2)^{1/2} +(e/c)\vec{\beta}'\cdot\vec{A}]dt \;. \tag{B1}$$

Here T and H are arbitrary functions of the coordinates, $\vec{A}$ is the electromagnetic vector potential, and $\vec{\beta}'=\vec{v}'/c$ is the coordinate velocity (to be distinguished from the measured velocity defined below.) In addition, Maxwell's equations for this class of theories must assume the same form as in metric theories [see Eq. (B4) below]. Such theories are thus characterized by four arbitrary functions of the coordinates (T,H,ϵ, and μ) and hence this description of gravitational effects is known as the $TH\epsilon\mu$ formalism.[59] All metric theories fall into this class, as we discuss in more detail below, and these are distinguished from nonmetric theories by the fact that the former obey the "metric meshing law,"

$$\epsilon=\mu=(H/T)^{1/2}\;, \tag{B2}$$

while the latter do not. To the extent to which nonmetric theories agree with the EDB experiments they also tend to simulate metric theories. Further discussion of these points can be found in Refs. 49–53, which also deal with the limitations of the $TH\epsilon\mu$ formalism. A similar analysis for a gravitational theory with torsion has been given by Ni.[60] We note in passing that although we have not considered gravity theories with torsion[61] in any detail, the predictions of such theories tend to coincide with those of general relativity in situations such as ours, where the source and test particle have no spin. (We here neglect the spin of the Earth.)

To summarize the preceding arguments, there is no known viable complete and self-consistent relativistic theory of gravity which violates the EEP, and hence which could account for a velocity dependence of $|\eta_{+-}|$, ϕ_{+-}, or Δ. We stress that this conclusion derives from the analysis of a (necessarily) restricted class of theories, in which only gravitational and electromagnetic effects are considered, and at that only in a semiclassical manner. We have not considered supergravity theories,[62] nor more general types of nonmetric theories. An example of the latter would be one which was a metric theory with respect to electromagnetism, but not with respect to the weak interactions, a possibility which has been suggested by Haugan and Will.[53] Given the central role of the weak interactions in the K^0-$\bar{K}^0$ system, and the fact that the EDB experiments are relatively insensitive to the presence of these interactions in nuclei,[63] such a possibility may well be worth exploring if the experimental results of Refs. 1—3 are confirmed.

We turn next to a detailed examination of the wave function of a kaon in a SSS metric gravitational field. This will serve both to elaborate on the general arguments given above, and to develop the formalism needed for describing the nonlocal gravitational experiments referred to previously. To understand why there are no *observable* effects arising from the motion of a kaon with respect to a gravitational field, we focus on the difference between the *coordinate* velocity v' and the *measured* velocity v that an experimentalist would see in the laboratory. This difference is best illustrated by considering the propagation of light rays in a gravitational field where Maxwell's equations assume the form[64]

$$F^{\mu\nu}_{;\mu}=-J^{\nu}\,, \tag{B3a}$$

$$F_{\mu\nu;\lambda}+F_{\nu\lambda;\mu}+F_{\lambda\mu;\nu}=0\,. \tag{B3b}$$

In Eqs. (B3), $F^{\mu\nu}(x)$ is the electromagnetic field-strength tensor, J^{ν} is the source current, and the semicolon denotes covariant differentiation. It is relatively straightforward to show that Eqs. (B3) can be recast in the form

$$\vec{\nabla}\cdot(\epsilon\vec{E})=\rho,\quad \vec{\nabla}\times(\vec{B}/\mu)=\vec{J}+\partial(\epsilon\vec{E})/c\,\partial t\,, \tag{B4a}$$

$$\vec{\nabla}\cdot\vec{B}=0,\quad \vec{\nabla}\times\vec{E}+\partial\vec{B}/c\,\partial t=0\,, \tag{B4b}$$

where $\epsilon=\epsilon(x)$ and $\mu=\mu(x)$ can be expressed in terms of the components of the metric tensor $g_{\mu\nu}(x)$. For a SSS geometry $g_{\mu\nu}(x)$ is specified in isotropic coordinates by writing

$$\begin{gathered} ds^2=f(r)(dx^2+dy^2+dz^2)+g_{00}(r)c^2dt^2\,,\\ r=(x^2+y^2+z^2)^{1/2}\,, \end{gathered} \tag{B5}$$

in which case $n=(\epsilon\mu)^{1/2}$, where

$$\epsilon=\mu=n=[-f(r)/g_{00}(r)]^{1/2}\,. \tag{B6}$$

It is instructive to verify Eq. (B6) by rewriting Eq. (B5) for light in the form

$$0=ds^2=dx^2+dy^2+dz^2-[-g_{00}(r)/f(r)]c^2dt^2\,. \tag{B7}$$

A light ray can thus be viewed as propagating in a Minkowskian space-time, but with a local index of refraction given by Eq. (B6). In this picture the gravitational deflection of light arises from the *variation* of $n(r)$ from point to point along the light path, which then leads to the refraction of the light ray in accordance with Snell's law.[65] For a ray traveling, say, in the z direction we have

$$c=[-f(r)/g_{00}(r)]^{1/2}(dz/dt)=nv'\,, \tag{B8}$$

which relates the *measured* (or physical) *velocity* $c=2.997\,924\,58\ (1.2)\times10^{10}$ cm/sec to the *coordinate velocity* $v'=dz/dt$. We can define the measured velocity v more generally by writing[66]

$$v=\frac{dl}{(-g_{00})^{1/2}dt}=\frac{c\,dl}{(-g_{00})^{1/2}dx^0}\equiv\frac{dl}{dt_L}\,, \tag{B9a}$$

$$dl^2=\left[g_{ij}-\frac{g_{0i}g_{0j}}{g_{00}}\right]dx^i dx^j,\quad i,j=1,2,3\,. \tag{B9b}$$

Physically, the quantities v, dl, and $dt_L=(-g_{00})^{1/2}dt$ are those that an experimentalist would find in the laboratory if he measured velocities, lengths, and time intervals by using light signals which are defined to travel at the speed $c=2.99\cdots\times10^{10}$ cm/sec. We now focus on Δm and show that, when the K_L and K_S wave functions in a gravitational field are expressed in terms of these measured quantities, Δm is independent of the measured γ,

$$\gamma=[1-n^2(dz/dx^0)^2]^{-1/2}=[1-(v/c)^2]^{-1/2}\,. \tag{B10}$$

It then follows that the origin of any velocity dependence of Δm cannot be a coupling of the K^0-$\bar{K}^0$ system to an external metric gravitational field.

In the presence of gravity, the wave function $\Psi(\vec{x},t)$ for a scalar particle of mass m in a matter-free region of space is determined by

$$(-D_\mu D^\mu+\kappa^2)\Psi(\vec{x},t)=0\,, \tag{B11}$$

where $\kappa=mc/\hbar$ and D_μ is the covariant derivative. [In the presence of a matter distribution Eq. (B11) would contain an additional term proportional to the scalar curvature.] If we express D_μ in terms of the ordinary derivatives $\partial_\mu=\partial/\partial x^\mu$ and $\partial^\mu=g^{\mu\nu}\partial_\nu$, then Eq. (B11) becomes

$$-\partial_\mu\partial^\mu\Psi-\left[\frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}\right](\partial^\mu\Psi)+\kappa^2\Psi=0\,, \tag{B12}$$

where $g=g(x)=-\det g_{\mu\nu}(x)$. For a particle traveling in a weak SSS metric gravitational field, $g_{\mu\nu}(x)$ is given in terms of the functions $f(r)$ and $g_{00}(r)$ in Eq. (B5) by

$$\begin{gathered} f(r)\simeq1+2\gamma_{\rm PPN}\Phi,\quad -g_{00}(r)\simeq1-2\Phi\,,\\ \Phi=\frac{GM}{rc^2}\ll1,\quad g_\mu=c^2\frac{\partial\Phi}{\partial x^\mu}=(\vec{g},0)\,. \end{gathered} \tag{B13}$$

Here G is the Newtonian gravitational constant, $G=6.6720(41)\times10^{-8}$ cm^{3}gm^{-1}sec^{-2}, M is the mass of the source (which is located at $r=0$), and the constant $\gamma_{\rm PPN}$ is a parametrized-post-Newtonian (PPN) parameter[51] which distinguishes among different metric theories of gravity. Combining Eqs. (B12) and (B13), the differential equation for Ψ becomes

$$-(1-2\gamma_{\rm PPN}\Phi)\nabla^2\Psi+(1+2\Phi)\frac{\partial^2\Psi}{c^2\partial t^2}+(1-\gamma_{\rm PPN})\frac{\vec{g}\cdot\vec{\nabla}\Psi}{c^2}+\kappa^2\Psi=0\,, \tag{B14}$$

where we have neglected terms $O(\Phi^2)$ and higher. In cases of practical interest, the gravitational potential Φ is not only small but is also slowly varying compared to the de Broglie wavelength of the particles in question. Hence, if we take the particles to be moving initially in the z direction, we can solve Eq. (B14) in the WKB approximation by writing

$$\Psi(z,t)=Ae^{iS(z)/\hbar - iEt/\hbar}$$
$$S(z)=S_0(z)+\hbar S_1(z)+\cdots \,, \qquad \text{(B15)}$$

where A is an overall normalization constant. Combining Eqs. (B14) and (B15), and equating coefficients of $\hbar^0$ and $\hbar^1$, we find

$$\hbar^0 \to -(1+2\Phi)E^2/c^2+(1-2\gamma_{\rm PPN}\Phi)(S_0')^2 + m^2c^2=0 \,, \qquad \text{(B16a)}$$

$$\hbar^1 \to (1-2\gamma_{\rm PPN}\Phi)(2S_1'S_0' - iS_0'') + i(1-\gamma_{\rm PPN})S_0' g_z/c^2=0 \,, \qquad \text{(B16b)}$$

where $S_0'=\partial S_0/\partial z$, etc. Solving Eqs. (B16) for $S_0(z)$ and $S_1(z)$, we find for $\Psi(z,t)$

$$\Psi(z,t)=Ak_0^{-1/2}(z)\exp[(1/2)(1-\gamma_{\rm PPN})\Phi] \times \exp\left[i\int^z k_0(z')dz' - (i/\hbar)Et\right] ,$$
$$\hbar k_0(z)=\pm\left[\frac{(1+2\Phi)E^2/c^2-m^2c^2}{1-2\gamma_{\rm PPN}\Phi}\right]^{1/2} . \qquad \text{(B17)}$$

Equation (B17) can be simplified by noting that the total (conserved) energy E in the presence of a gravitational field is given by[66]

$$E=\frac{mc^2(-g_{00})^{1/2}}{(1-n^2\beta'^2)^{1/2}} \equiv E'(-g_{00})^{1/2} \cong E'(1-\Phi) \,. \qquad \text{(B18)}$$

Hence

$$\hbar k_0 \cong \pm p'(1+\gamma_{\rm PPN}\Phi) \simeq \pm p'(g_{33})^{1/2} \,,$$
$$p'=(E'^2/c^2-m^2c^2)^{1/2}=\frac{mv'n}{(1-n^2\beta'^2)^{1/2}} \,. \qquad \text{(B19)}$$

Combining Eqs. (B17) to (B19), we can write the oscillatory factor in $\Psi(z,t)$ in the form

$$\text{oscillatory factor} = \exp\left[(i/\hbar)\int^z p'(1+\gamma_{\rm PPN}\Phi)dz' - (i/\hbar)Et\right] \qquad \text{(B20)}$$

for a particle moving in the $+z$ direction. We will return to Eq. (B20) below.

The wave function $\Psi(z,t)$ in Eq. (B17) has the characteristic form of a WKB eigenfunction, namely, a product of an oscillatory factor and a slowly varying amplitude function. It is instructive to note that the oscillatory factor in Eqs. (B17) and (B20), which is what governs the various interference effects that we are considering, can be written in the following simple and useful form[67]:

$$\text{oscillatory factor}=\exp\left[(-i/\hbar)mc^2\int d\tau\right] ,$$
$$c^2d\tau^2=-ds^2=-g_{\mu\nu}dx^\mu dx^\nu \,. \qquad \text{(B21)}$$

In the absence of gravitational fields, $\int d\tau = t/\gamma$ in the laboratory frame, in which case the expression in Eq. (B21) reduces to the usual free-particle result given in Eq. (A4). For motion in the z direction we have

$$c^2d\tau^2=-g_{33}dz^2-g_{00}(dx^0)^2=-g_{00}c^2dt^2\left[1+\frac{g_{33}}{g_{00}}\beta'^2\right] = -g_{00}c^2dt^2(1-n^2\beta'^2) \,. \qquad \text{(B22)}$$

Hence, the phase of the oscillatory factor in Eq. (B21) is given by

$$-\frac{imc^2}{\hbar}\int d\tau = \frac{-imc^2}{\hbar}\int dt(-g_{00})^{1/2}(1-n^2\beta'^2)^{1/2} \equiv (-i/\hbar)\int \mathscr{L}\,dt \,. \qquad \text{(B23)}$$

Equation (B23) is both simple and exact, and we will return to it in the ensuing discussion. We note in passing that $\mathscr{L}$ can be rewritten in the form

$$\mathscr{L}=mc^2[-g_{00}-g_{33}\beta'^2]^{1/2} \,, \qquad \text{(B24)}$$

which is identical to the first term in Eq. (B1) if we identify $T=-g_{00}$ and $H=g_{33}$. The metric meshing law in Eq. (B2) then reads

$$\epsilon=\mu=(H/T)^{1/2}=(-g_{33}/g_{00})^{1/2} \,, \qquad \text{(B25)}$$

which is identical to Eq. (B6) with $g_{33}=f(r)$. To establish the equivalence of Eqs. (B20) and (B21) we write

$$d\tau=\frac{d\tau^2}{d\tau}=-\frac{g_{\mu\nu}dx^\mu dx^\nu}{c^2d\tau} \,, \qquad \text{(B26)}$$

$$\text{oscillatory factor} =\exp\left[(i/\hbar)\int g_{\mu\nu}(m\,dx^\mu/d\tau)dx^\nu\right] =\exp\left[(i/\hbar)\int g_{\mu\nu}P^\mu dx^\nu\right] . \qquad \text{(B27)}$$

Combining Eq. (B27) with the expression for $d\tau$ in Eq. (B23), we find

$$-\frac{i}{\hbar}mc^2\int d\tau = -\frac{i}{\hbar}\int\left[g_{33}m\frac{dz}{d\tau}dz+g_{00}m\frac{dx^0}{d\tau}dx^0\right] = -\frac{i}{\hbar}\int(-g_{00})^{1/2}\left[\frac{mc^2dt}{(1-n^2\beta'^2)^{1/2}} - \frac{mv'n^2dz}{(1-n^2\beta'^2)^{1/2}}\right] , \qquad \text{(B28)}$$

in agreement with Eqs. (B17)—(B20). This verifies Eq. (B21) and thus establishes the equivalence of (B20) and (B21). We note that the expression in (B23) builds in the information that the particle is moving along the classical trajectory $dz=v'dt$, whereas (B28) does not. It follows that when the K_L-K_S phase difference is calculated using (B23), account must be taken of the fact that $v_L'\neq v_S'$ by applying the phase correction described in Appendix A.

For a particle moving in a weak field ($\Phi \ll 1$), the exact expressions in (B20) or (B23) can be simplified by making use of Eq. (B13). We have

$$(-g_{00})^{1/2} \cong 1-\Phi, \quad n^2 \cong 1+2(1+\gamma_{\rm PPN})\Phi ,$$
$$(-g_{00})^{1/2}(1-n^2\beta'^2)^{1/2} \cong (1/\gamma)[1-\gamma^2(1+\beta'^2\gamma_{\rm PPN})\Phi] , \tag{B29}$$

and hence,

oscillatory factor

$$=\exp\left[\frac{-imc^2}{\hbar}\int \frac{dt}{\gamma}[1-\gamma^2(1+\beta'^2\gamma_{\rm PPN})\Phi]\right] . \tag{B30}$$

Equation (B30) can be used to formulate another description of the gravitational deflection of light. We note that the second term in (B30) corresponds to an effective potential $V(r)$ given by

$$V(r) = -mc^2\gamma(1+\beta'^2\gamma_{\rm PPN})\Phi(r)$$
$$= -\frac{GM}{r}\left[\frac{E'}{c^2}\right](1+\beta'^2\gamma_{\rm PPN}) , \tag{B31}$$

where use has been made of Eq. (B18). Since this expression is already $O(G)$, the coordinates can be taken to be approximately Minkowskian (to leading order) and the primes can be dropped in Eq. (B31). For photons, or other relativistic particles with $\beta \cong 1$, Eq. (B31) gives an effective potential which is $(1+\gamma_{\rm PPN})$ times the "Newtonian" result for a photon with an effective mass E/c^2. Since $\gamma_{\rm PPN}=1$ in general relativity, this leads to the well known prediction that a photon is deflected by twice the Newtonian value, in excellent agreement with experiment.[68,38]

We are now in a position to demonstrate explicitly that the observed dependence of Δm on γ cannot be a metric gravitational effect. Returning to the expression for the oscillatory phase in Eq. (B23), and using Eqs. (B9) and (B10), we have

$$-\frac{imc^2}{\hbar}\int d\tau = -\frac{imc^2}{\hbar}\int \frac{dt_L}{\gamma} , \tag{B32}$$

where $dt_L = dt(-g_{00})^{1/2}$ is the measured time interval in the isotropic coordinate system, which coincides with the laboratory frame. We have argued previously that since the kaons travel essentially horizontally in the regeneration experiments discussed in Refs. 1–3, Φ [and hence $\gamma=\gamma(\Phi)$] are approximately constant as a function of position. It follows that the factor $1/\gamma$ in Eq. (B32) can be removed from under the integral in which case Eq. (B32) reduces to the usual free-particle result of Eq. (A4). (Recall that the time interval dt_K in the kaon rest frame is given by $dt_K = dt_L/\gamma$.)

We can summarize the preceding discussion as follows. Under the conditions of the regeneration experiments the gravitational potential experienced by the kaons is for practical purposes a constant. As is well known, such a constant can always be absorbed by redefining the coordinates in such a way as to make the metric Minkowskian over the dimensions of the apparatus. It follows that the phase of the oscillatory factor becomes

$$-\frac{imc^2}{\hbar}\int d\tau = -\frac{imc^2}{\gamma\hbar}\int dt_L = -\frac{imc^2}{\hbar}\int dt_K \tag{B33}$$

in the kaon rest frame, and hence all reference to the velocity of the kaon with respect to the gravitational field disappears. Since the oscillatory phase in (B33) is identical in form to that for a free particle, we can take over the kinematic analysis given in Appendix A to obtain the phase difference between K_L and K_S,

$$\text{phase difference} = -\frac{i\,\Delta m\, c^2 t_K}{\hbar} . \tag{B34}$$

[Note that it is misleading to try to obtain (B34) from (B33) by simply using the fact that $d\tau$ is an invariant, without taking account of the fact that $v_L \neq v_S$.] We see from (B34) that the phase difference between K_L and K_S in the regeneration experiments is determined by a *velocity-independent* mass difference Δm, even in the presence of a metric gravitational field. It must be emphasized however, that if appropriate interference experiments were carried out on kaons traveling in the *vertical* direction, then velocity-dependent gravitational effects could in principle be seen. For such experiments the variation in γ between the regenerator and detector cannot be ignored,[69] and hence γ cannot be removed from under the integral sign. In addition the kinematic correction described in Appendix A becomes more complicated. A detailed discussion of K_L-K_S interference experiments in the vertical direction will be presented elsewhere.

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