Nonuniversal neutral currents of ν_e and ν_μ

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(Received 19 July 1982)

The present status and the general consequences of a violation of ν_e-ν_μ universality in weak neutral currents are examined from a phenomenological point of view. If ξ is the strength of the ν_e coupling relative to the ν_μ coupling, an analysis of the available data gives two possible values: $\xi=1.45\pm0.2$ and -0.6 ± 0.2. A measurement of the total cross section $\sigma(\nu_e e)$ would reveal the deviations of ξ from unity much more easily than the measurement of $\sigma(\bar{\nu}_e e)$. The effects of nonuniversality on neutrino cross sections when the ν_e undergoes flavor oscillations in vacuum are studied. A sizable nonuniversality will facilitate the observation of matter effects on the neutrino oscillations.

I. INTRODUCTION

The experimental data using ν_μ and $\bar{\nu}_\mu$ beams reveal that the structures of both the hadronic and electronic parts of the weak neutral current are in remarkable agreement with the minimal electroweak gauge model.[1] The corresponding data with ν_e and $\bar{\nu}_e$ beams are, however, not so extensive, because these beams are not available with sufficiently high intensity. Thus it remains an open question whether the coupling of the ν_e to the neutral current is indeed equal to that of the ν_μ, as required by the standard model. Besides the need to verify the model further, it is obviously very important to check the data on various neutrino scattering processes for any violations of the neutrino flavor universality.

In the following we would like to examine the consequences of a neutral-current interaction which violates the hypothesis of ν_e-ν_μ universality in a minimal way, but otherwise conforms to the customary view. That is, we assume the validity of the conventional $(V-A)$ theory with e-μ universality for the charged-current interactions. For the neutral-current processes also we shall take the usual current $\times$ current interaction with V and A currents, except that the ν_e current has a coupling strength ξ relative to the ν_μ current;

$$H_W=\frac{G}{\sqrt{2}}(J_\lambda^\dagger J^\lambda+N_\lambda N^\lambda)$$

$$N_\lambda=N_\lambda^{(h)}+\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_\mu+\bar{e}\gamma_\lambda(g_V+g_A\gamma_5)e+\xi\,\bar{\nu}_e\gamma_\lambda(1+\gamma_5)\nu_e+\cdots\ , \quad (1)$$

where G is the Fermi coupling constant, J_λ is the charged current, $N_\lambda^{(h)}$ is the hadronic part of the neutral current, and the dots refer to the currents $\bar{\mu}\mu$, $\bar{\nu}_\tau\nu_\tau$, etc., with which we will not be concerned here. The real parameter ξ will be equal to unity if the usually assumed ν_e-ν_μ universality holds.

In this paper, we present a phenomenological study of the consequences of $\xi\neq1$ for a variety of experiments that can be done using ν_e and $\bar{\nu}_e$ beams at reactors, meson factories, muon rings, and accelerators. Of special interest would be the effect of ν_e-ν_μ nonuniversality on the oscillations of neutrinos passing through vacuum and through a material medium, as relevant for the deep-mine experiments.

II. NEUTRINO-HADRON SCATTERING

From the definition of ξ in Eq. (1), it is obvious that the differential cross sections for a neutral-current process (either exclusive or inclusive) on a nuclear target induced by a ν_e and by a ν_μ of the same energy are in the ratio of ξ^2 to 1,

$$d\sigma(\nu_e N\to\nu_e+\cdots)=\xi^2 d\sigma(\nu_\mu N\to\nu_\mu+\cdots)\ . \quad (2)$$

This relation, which may be taken as the definition of ξ, holds for the corresponding antineutrinos as well. A special case which is of experimental interest is the deuteron disintegration by reactor antineutrinos,

$$d\sigma(\bar{\nu}_e d\to\bar{\nu}_e np)=\xi^2 d\sigma(\bar{\nu}_\mu d\to\bar{\nu}_\mu np)\ . \quad (3)$$

In order to estimate the value of ξ from the available information with reactor neutrinos, we shall substitute the calculated value for $\bar{\nu}_\mu d$ using the Weinberg-Salam model (which has been tested in the GeV range), and use the available experimental information on the left-hand side for $\bar{\nu}_e d$. In the notation of Reines *et al.*,[2] the measured value of $\sigma(\bar{\nu}_e d)$, expressed as the ratio

$$[\sigma(\bar{\nu}_e d \to e^+ nn)/\sigma(\bar{\nu}_e d \to \bar{\nu}_e np)]_{\rm expt} = 0.167 \pm 0.093 \ , \quad (4)$$

should therefore be compared with the theoretical value[2] which accounts for a possible nonuniversal neutral-current coupling, namely,

$$(\sigma_{{\rm CC}d}/\sigma_{{\rm NC}d})_{\rm theory} = (0.43 \pm 0.01)/\xi^2 \ . \quad (5)$$

We thus obtain the estimate

$$|\xi| = 1.6 \pm 0.5 \ . \quad (6)$$

The error on ξ may have to be increased to reflect the uncertainties inherent in the calculated reactor neutrino spectrum that enters Eq. (5), and also the complete neglect of any oscillations of reactor antineutrinos within a few meters.

III. ELASTIC νe SCATTERING

Denoting the mass of the electron by m, the laboratory energy of the incident neutrino by E, and the laboratory kinetic energy of the recoil electron by (yE), the y distributions for $\nu_\mu e$ and $\nu_e e$ scattering according to Eq. (1) are

$$\frac{d\sigma}{dy}(\nu_\mu e) = \hat{\sigma}\left[(g_V+g_A)^2 + (g_V-g_A)^2(1-y)^2 + \frac{my}{E}(g_A{}^2 - g_V{}^2)\right] , \quad (7)$$

$$\frac{d\sigma}{dy}(\nu_e e) = \hat{\sigma}\{[2+\xi(g_V+g_A)]^2 + \xi^2(g_V-g_A)^2(1-y)^2 + \frac{my}{E}[(1+\xi g_A)^2 - (1+\xi g_V)^2]\} , \quad (8)$$

$$\hat{\sigma} \equiv \frac{G^2 mE}{2\pi} \ . \quad (9)$$

The corresponding formulas for $\bar{\nu} e$ are obtained simply by interchanging the coefficients of the first two terms [the coefficient of the first (y-independent) term is exchanged with that of the $(1-y)^2$ term].

At high enough energies, where we can neglect the (m/E) terms, we obtain

$$\xi^2 = \frac{(1-y)^2 \frac{d\sigma}{dy}(\nu_e e) - \frac{d\sigma}{dy}(\bar{\nu}_e e)}{(1-y)^2 \frac{d\sigma}{dy}(\nu_\mu e) - \frac{d\sigma}{dy}(\bar{\nu}_\mu e)} \ . \quad (10)$$

This relation presumably will be more useful if we consider only those events in which the recoil electron energy is maximal ($y \simeq 1$), since

$$\xi^2 = \left[\frac{d\sigma}{dy}(\bar{\nu}_e e) \Big/ \frac{d\sigma}{dy}(\bar{\nu}_\mu e)\right]_{y=1} \ . \quad (11)$$

On the other hand, we may also integrate Eqs. (7) and (8) over y from 0 to 1 and obtain

$$\xi^2 = \frac{\sigma(\nu_e e) - 3\sigma(\bar{\nu}_e e)}{\sigma(\nu_\mu e) - 3\sigma(\bar{\nu}_\mu e)} \ . \quad (12)$$

This relation for the case of $\xi = 1$ was derived first by Sehgal,[3] and was also found to be valid in a more general context[4–6] such as S,P,T couplings and the presence of nondiagonal V,A,S,P,T couplings.

For fixing the sign of ξ we look at the sign of the interference term in Eq. (8) between the charged-current contribution 2 and the neutral-current contribution $\xi(g_V+g_A)$. If we set $\xi = 1$ by assuming ν_e-ν_μ universality, then experiments can test for the destructive interference between the charged and neutral currents expected in the Weinberg-Salam model, a point well emphasized by Kayser *et al.*[6] On the other hand, treating ξ as unknown but assuming the validity of the Weinberg-Salam model for $\nu_\mu e$ scattering, we can fix the sign of ξ. In any case it is obvious that the sign of the interference term gives only the overall sign of the combination $\xi(g_V+g_A)$.

In order to see the sensitivity of the total cross sections to the parameter ξ, we shall integrate the expressions (7) and (8) over y from 0 to 1, and substitute the couplings of the Weinberg-Salam model,

$$g_V = -\tfrac{1}{2} + 2\sin^2\theta_W \ , \qquad g_A = -\tfrac{1}{2} \ . \quad (13)$$

For purposes of numerical work we shall substitute the typical value[1]

$$\sin^2\theta_W = 0.23 \ , \quad (14)$$

and obtain the total cross sections of high-energy neutrinos on electron targest in units of $\hat{\sigma}$ defined by Eq. (9),

$$\sigma(\nu_\mu e)=0.362\hat{\sigma} ,$$
$$\sigma(\bar{\nu}_\mu e)=0.309\hat{\sigma} , \quad (15)$$
$$\sigma(\nu_e e)=[2.20+(1-\xi)(1.80-0.362\xi)]\hat{\sigma} ,$$
$$\sigma(\bar{\nu}_e e)=[0.922+(1-\xi)(0.411-0.309\xi)]\hat{\sigma} .$$

These cross sections are plotted in Fig. 1 as a function of ξ. The dots on the curves at $\xi=1$ are the expectations on the basis of ν_μ-ν_e universality. The $\sigma(\bar{\nu}_e e)$ curve shows that the universality point $\xi=1$ is located inside a broad minimum (exact minimum is at $\xi=1.17$ for $\sin^2\theta_W=0.23$), and hence the existence of even significant departures from universality, $\xi\simeq(\frac{1}{2}-2)$, will not easily be noticeable in the measurements of $\sigma(\bar{\nu}_e e)$. For this reason data on $\sigma(\nu_e e)$, say from meson factories, will be very valuable. Secondly, for a given absolute value of ξ, the negative value leads to a larger cross section than the positive value—an effect due to the constructive interference between the charged-current and neutral-current interactions.

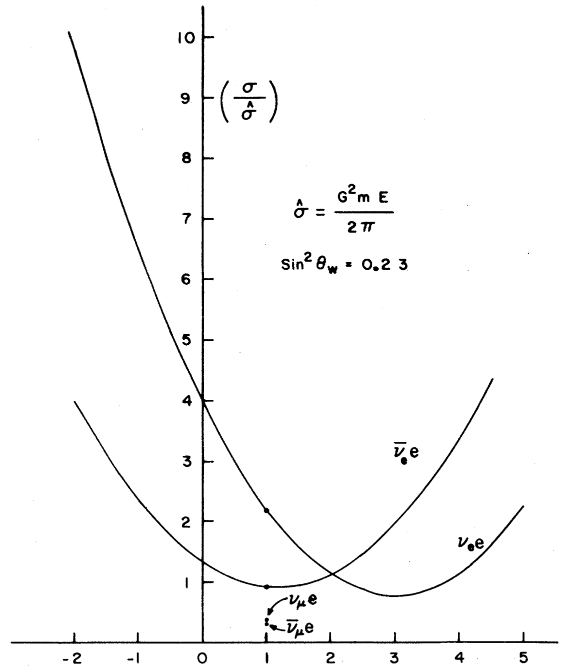

FIG. 1. The total cross sections $\sigma(\nu e)$ as a function of the nonuniversality parameter ξ. The values of $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$, and the two points marked on the curves of $\sigma(\nu_e e)$ and $\sigma(\bar{\nu}_e e)$ at $\xi=1$ are obtained from the Weinberg-Salam model when $\sin^2\theta_W=0.23$.

We now turn to an analysis of the reactor data of Reines *et al.*[7] on the difficult reaction $\bar{\nu}_e e\to\bar{\nu}_e e$ for estimating the value of ξ. The experimental results are available in terms of the charged-current cross section σ_{V-A} averaged over a calculated antineutrino spectrum and integrated over two ranges of the recoil electron's kinetic energy T_e. Avignone and Greenwood[8] have listed the numerical values of the energy integrals (called G_i's by them) which occur in the $\bar{\nu}_e e$ cross-section formula, due to the folding of the incident spectrum as a function of the electron energy cuts. (The G_i's appropriate for the recoil-electron-energy bins can also be readily extracted from Table VI of Ref. 6.) We use these integrals which incorporate the updated neutrino reactor spectrum, substitute $\sin^2\theta_W=0.23$, and obtain the following relations for ξ using the data of Reines *et al.*:

$$1.5<T_e<3.0 \text{ MeV}: \; 9.68\xi^2-14.86\xi+21.2=21.2(0.87\pm0.25) , \quad (16)$$

$$3.0<T_e<4.5 \text{ MeV}: \; 2.53\xi^2-2.28\xi+2.9=2.9(1.7\pm0.44) . \quad (17)$$

The numbers 21.2 and 2.9 appearing in the ξ-independent terms are the values of σ_{V-A} (in arbitrary units). Each of the quadratic equations implies a pair of alternative values of ξ,

$$\xi=\begin{cases}1.32\pm0.50 \\ 0.22\pm0.50\end{cases} \text{for } 1.5<T_e<3.0 \text{ MeV} , \quad (18)$$

$$\xi=\begin{cases}1.45\pm0.25 \\ -0.55\pm0.25\end{cases} \text{for } 3.0<T_e<4.5 \text{ MeV} . \quad (19)$$

These values as well as those of Eq. (6) coming from $\bar{\nu}_e d$ are schematically shown in Fig. 2, and compared with the ν_e-ν_μ universality value expected in the Weinberg-Salam model. Here neutrino oscillations are ignored. The dashed line denotes the alternative solution of ξ which is less than unity. The higher energy bin data of $\bar{\nu}_e e$ lead to the determination of ξ with the smallest error. It should however be noted that the value of ξ from $\bar{\nu}_e d$ data, although poorly determined, arises from the ratio of ratios and hence is less sensitive to the assumed reactor neutrino spectrum.

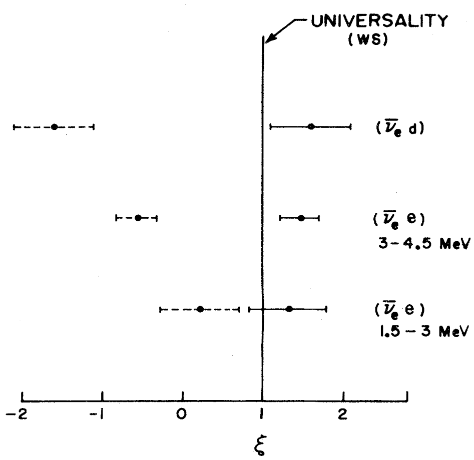

FIG. 2. Values of ξ determined from the reactor neutrino experiments $\bar{\nu}_e d$ and $\bar{\nu}_e e$. Dashed line denotes a possible value of ξ arising from the quadratic ambiguity, but from the same experimental input.

IV. INFLUENCE ON NEUTRINO OSCILLATIONS IN VACUUM

To illustrate the effect of unequal neutral-current couplings of neutrinos on the experiments which look for neutrino flavor oscillations in vacuum, we confine our attention to a simple two-level model. Let ν_1 and ν_2 denote the mass eigenstates with masses m_1 and m_2, and θ the mixing angle such that the flavor eigenstates ν_e and ν_μ are given by

$$\begin{aligned} \nu_e &= \nu_1 \cos\theta + \nu_2 \sin\theta \ , \\ \nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta \ . \end{aligned} \tag{20}$$

If we start with a specific neutrino beam (say ν_e) of energy E reaching a detector located at a distance L along the length of the beam, the "other" type of neutrino (ν_μ) will be present in the beam with relative intensity[9]

$$P(L/E) = \sin^2(2\theta)\sin^2\left[\frac{(m_1{}^2 - m_2{}^2)L}{4E}\right] . \tag{21}$$

If neutrino flavor universality ($\xi = 1$) is valid, as is generally assumed, there will be no effect of oscillations on the measured neutral-current cross sections. If, on the other hand, the neutral-current couplings of the neutrino are unequal ($\xi \neq 1$), we see that the differential cross sections of neutrino-hadron interaction by weak neutral currents are given in terms of the standard $\nu_\mu N$ cross sections as

$$d\sigma^{(L)}(\nu_\mu N \to \nu_\mu + \cdots) = [(1-P) + \xi^2 P] d\sigma^{(0)}(\nu_\mu N \to \nu_\mu + \cdots) , \tag{22}$$

$$d\sigma^{(L)}(\nu_e N \to \nu_e + \cdots) = [\xi^2(1-P) + P] d\sigma^{(0)}(\nu_\mu N \to \nu_\mu + \cdots) . \tag{23}$$

Here the probability P is given by Eq. (21) and the superscripts L and 0 denote the distances traveled by the neutrino between the points of its production and interaction; thus the $d\sigma^{(0)}$ corresponds to the case when the oscillations are altogether absent. For antineutrinos, we can simply replace ν_e by $\bar{\nu}_e$ and ν_μ by $\bar{\nu}_\mu$ in the above formulas.

What are the combined effects of unsymmetric neutrino couplings and neutrino oscillations in vacuum on the experimental data of the elastic reaction $\nu e \to \nu e$? This question is of great experimental interest, for in case the neutrino involved is a ν_e or $\bar{\nu}_e$, both the charged current interactions (which obey universality) and the neutral-current interactions (whose universality is under study) will be present. We shall list the y distributions for νe scattering when the neutrino flight path is L, in terms of the usual $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections obtained on the basis of no oscillations (superscript zero)

$$\frac{d\sigma^{(L)}}{dy}(\nu_\mu e) = [(1-P) + \xi^2 P] \frac{d\sigma^{(0)}}{dy}(\nu_\mu e) + P\Sigma_1 , \tag{24}$$

$$\frac{d\sigma^{(L)}}{dy}(\bar{\nu}_\mu e) = [(1-P) + \xi^2 P] \frac{d\sigma^{(0)}}{dy}(\bar{\nu}_\mu e) + P\Sigma_2 , \tag{25}$$

$$\frac{d\sigma^{(L)}}{dy}(\nu_e e) = [\xi^2(1-P) + P] \frac{d\sigma^{(0)}}{dy}(\nu_\mu e) + (1-P)\Sigma_1 , \tag{26}$$

$$\frac{d\sigma^{(L)}}{dy}(\bar{\nu}_e e) = [\xi^2(1-P) + P] \frac{d\sigma^{(0)}}{dy}(\bar{\nu}_\mu e) + (1-P)\Sigma_2 , \tag{27}$$

$$\Sigma_1 \equiv [4(1 + \xi g_V + \xi g_A) - 2\xi(g_V - g_A) my/E]\hat{\sigma} , \tag{28}$$

$$\Sigma_2 \equiv [4(1 + \xi g_V + \xi g_A)(1-y)^2 - 2\xi(g_V - g_A) my/E]\hat{\sigma} . \tag{29}$$

Here $d\sigma^{(0)}(\nu_\mu e)$, for instance, is the same as $d\sigma(\nu_\mu e)$ given in Eq. (7), while $d\sigma^{(0)}(\bar{\nu}_\mu e)$ is obtained from it

TABLE I. Coefficients a and b in the cross-section formula $\sigma(\nu e)/\hat{\sigma}=a+bP(L/E)$.

	$\xi=1$		$\xi=1.6$		$\xi=-1.6$	
$(\sigma/\hat{\sigma})$	a	b	a	b	a	b
$\nu_\mu e$	0.36	1.84	0.36	1.11	0.36	8.02
$\bar{\nu}_\mu e$	0.31	0.61	0.31	0.66	0.31	2.97
$\nu_e e$	2.20	−1.84	1.47	−1.11	8.38	−8.02
$\bar{\nu}_e e$	0.92	−0.61	0.97	−0.66	3.28	−2.97

by replacing g_A by $(-g_A)$. Note that Eqs. (26) and (27) are obtained from Eqs. (24) and (25), respectively, by replacing P by $(1-P)$, as is to be expected in a two-level oscillation scheme.

To gain some insight into the above expressions, we shall ignore the (m/E) terms, integrate over y from 0 to 1, and substitute the Weinberg-Salam-model values for g_V and g_A from Eqs. (13) and (14). The resulting total-cross-section formulas can be cast into a form which isolates the effects due to neutrino oscillations,

$$\frac{\sigma^{(L)}(\nu e)}{\hat{\sigma}}=a(\xi)+b(\xi)P(L/E)\ . \qquad (30)$$

We have listed in Table I the dimensionless constants a and b for three values of ξ [see Eq. (6)].

Since the b's for $\nu_\mu e$ and $\bar{\nu}_\mu e$ are positive, the measured cross sections of these reactions will be larger than what they would be in a "no-oscillation" theory; the cross sections for $\nu_e e$ and $\bar{\nu}_e e$ are correspondingly depressed.[10] The ratio $|b/a|$ which controls the relative importance of the oscillation term, is largest for $\nu_\mu e$ and smallest for the reactor neutrino experiment $\bar{\nu}_e e$. As a function of ξ, we observe that even relatively modest deviations from ν_e-ν_μ universality can lead to rather large differences between the measured cross sections and those estimated in the standard model—especially for negative values of ξ. The actual values of cross sections, however, depend on the assumed values of the neutrino-mixing parameter and the masses which enter the expression for P in Eq. (21); moreover, a comparison of data with the theoretical expectations in any case requires the folding in of the incident neutrino energy spectrum and the ubiquitous y cuts. We therefore defer such calculations for the present, as we only want to illustrate here the effects of $\xi\neq 1$ on νe scattering when the incident neutrinos undergo flavor oscillations.

V. NONUNIVERSALITY AND MATTER EFFECTS ON NEUTRINO OSCILLATIONS

When a neutrino beam passes through a piece of matter the emerging forward beam includes the neutrinos that had been scattered coherently in the forward direction. As a result the oscillation pattern gets affected, as was first emphasized in a fascinating paper by Wolfenstein.[11] This coherent forward scattering of the neutrinos arises in two ways: (i) by the charged-current interaction of the neutrino with the atomic electrons, when the neutrino involved is a ν_e or $\bar{\nu}_e$,and (ii) by the neutral-current interaction of the neutrino with the e, p, and n of the medium. Thus while the charged-current interaction selectively affects a ν_e or $\bar{\nu}_e$ involved in the oscillation and modifies the oscillation pattern, the neutral current will not affect the oscillations when the neutrino types involved have equal vector coupling. What we wish to point out is that a violation of the ν_e-ν_μ universality results only in a minor but important change in the oscillation length, which may lead to interesting experimental consequences.

Focusing our attention once again on the ν_e-ν_μ oscillations, the time evolution of the two states ν_1 and ν_2 will be governed by the Schrödinger equation $i\partial_t\psi=H\psi$, where the $H=H_0+H_W$ now includes the interaction of neutrinos with the matter through which they are passing; H is a 2×2 matrix with elements

$$H_{12}=\langle \nu_1\,|\,H_0+H_W\,|\,\nu_2\rangle\ ,$$

etc. If we start initially with a beam of definite type, say ν_e, which is a mixture of mass eigenstates with mixing angle θ according to Eq. (20), then at a subsequent time the eigenstates will be given by

$$\begin{aligned}\nu_{1m}&=\nu_e\cos\theta_m-\nu_\mu\sin\theta_m\ ,\\ \nu_{2m}&=\nu_e\sin\theta_m+\nu_\mu\cos\theta_m\ ,\end{aligned} \qquad (31)$$

$$\tan(2\theta_m)=\frac{\tan(2\theta)}{1-\beta\sec(2\theta)}\ , \qquad (32)$$

$$\beta\equiv\frac{\langle \nu_\mu\,|\,H_W\,|\,\nu_\mu\rangle-\langle \nu_e\,|\,H_W\,|\,\nu_e\rangle}{({m_1}^2-{m_2}^2)/2E}\ , \qquad (33)$$

where the matrix elements in the last equation refer to elastic scattering of neutrinos on matter. The parameter β does not vanish because the coherent-forward-scattering amplitudes of ν_μ and ν_e on the intervening matter are not equal irrespective of the universality question.[12] The relative intensity of the ν_μ component in matter at a distance L along a ν_e beam is now given by[11]

$$P_m(L/E)=(l_m/l_v)^2\sin^2(2\theta)\sin^2(\pi L/l_m)\ , \qquad (34)$$

where l_v is the conventional oscillation length in vacuum,

$$l_v=\frac{4\pi E}{{m_1}^2-{m_2}^2}\ , \qquad (35)$$

and l_m is the oscillation length in matter given by

$$l_m = \frac{l_v}{[1+\beta^2-2\beta\cos(2\theta)]^{1/2}} \ . \tag{36}$$

It is important to realize[13] that the "lengths" l_v and l_m can be positive or negative. Although a change of sign of l_v does not affect the intensity formula Eq. (21), it does affect the corresponding expression Eq. (34) in matter because β depends on l_v.

Assuming ν_μ-ν_e universality ($\xi=1$), the effect of matter arises only through the charged-current interaction of ν_e with the e's. In this case it can be shown that[11,13,14]

$$\beta = l_v / l_W \ , \tag{37}$$

where we define the Wolfenstein length l_W by

$$l_W \equiv \frac{\sqrt{2}\pi}{GN_e} \simeq \left[\frac{N_A}{N_e}\right] 1.63\times 10^9 \text{ cm} \ . \tag{38}$$

Here N_e denotes the number of electrons per unit volume of the target medium and N_A is the Avogadro number. Note that l_W does not depend on the energy E, in contrast to l_v. At small enough energies, when $l_v \ll l_W$, we have $l_m \simeq l_v$ and the matter effects can be ignored. In the opposite extreme when $l_v \gg l_W$, we have $l_m \simeq l_W$, but the intensity becomes small due to Eq. (34); in fact, l_W then serves as a "cutoff" on l_m (since l_m no longer increases with E as l_v does) and the oscillations through matter would not be characterized by the difference $(m_1{}^2 - m_2{}^2)$.

With the nonuniversal neutral-current couplings the modification to be introduced is quite simple —instead of l_W being given by Eq. (38), it will be modified to l'_W, which is given by

$$\frac{\sqrt{2}\pi}{l'_W} = GN_e + (G_eN_e + G_pN_p + G_nN_n) - (G'_eN_e + G'_pN_p + G'_nN_n) \ , \tag{39}$$

where G_x is the neutral-current vector coupling of ν_e to the target particle x $(=p,n,e)$, G'_x refers to the corresponding vector couplings of the ν_μ, and N_x denotes the target particles of type x per unit volume in the medium. From Eq. (1) we see that $G_e = G\xi g_V$, etc. For a medium (such as the terrestrial medium) in which $N_e = N_p = N_n$, we obtain

$$\frac{\sqrt{2}\pi}{l'_W} = GN_e[1+(\xi-1)(g_V+g_V^p+g_V^n)] \ . \tag{40}$$

In the Weinberg-Salam model the sum of the vector-current couplings of ν_μ with e, p, and n is $g_V+g_V^p+g_V^n = -\frac{1}{2}$, and this gives

$$l'_W = \frac{2\sqrt{2}\pi}{GN_e(3-\xi)} \ . \tag{41}$$

Thus the deviation from strict ν_e-ν_μ universality changes the Wolfenstein length l_W of Eq. (38) by the simple factor $2/(3-\xi)$.[15]

The oscillation length in matter is now given by

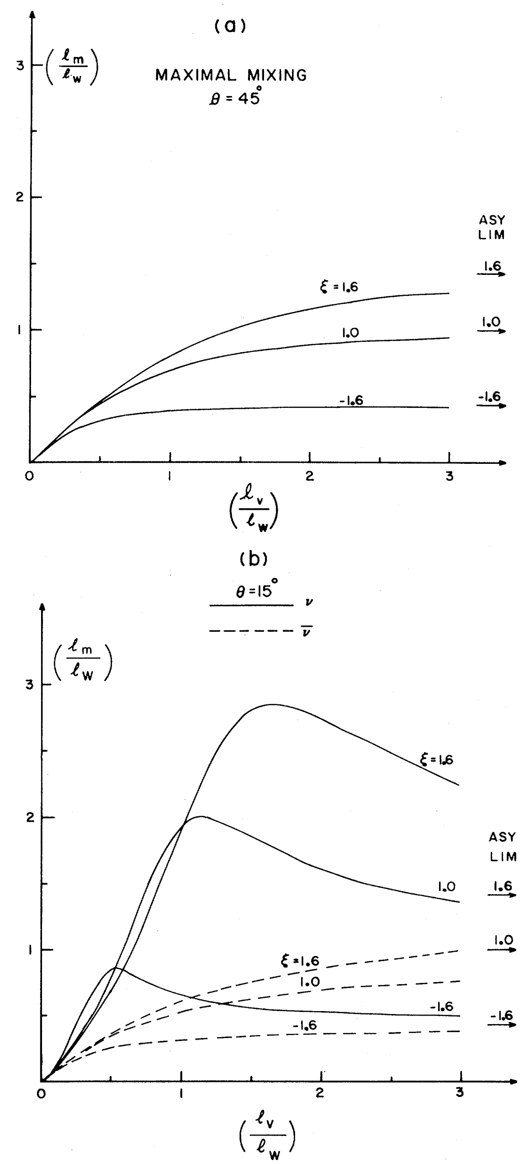

FIG. 3. Variation of the matter oscillation length l_m with respect to the vacuum oscillation length l_v (both measured in units of l_W) for some values of the nonuniversality parameter ξ. The arrows denote the asymptotic limits of the curves at infinite neutrino energy. The neutrino mixing angle in vacuum θ is taken to be (a) $\theta=45°$, for which case the curves are identical for ν and $\bar{\nu}$; (b) $\theta=15°$. In the latter case if the solid curve refers to ν the dashed curve would refer to $\bar{\nu}$ (or vice versa); the curves have a common asymptotic limit.

$$\frac{l_m}{l_W}=\frac{l_v}{l_W}\left[1+\frac{1}{4}\left[\frac{l_v}{l_W}\right]^2(3-\xi)^2 - \frac{l_v}{l_W}(3-\xi)\cos(2\theta)\right]^{-1/2} . \quad (42)$$

Instead of considering l_m as a function of the incident neutrino energy E, it is convenient to look at it as a function of $E/(m_1{}^2-m_2{}^2)$ which is essentially l_v. In Fig. 3 we plotted (l_m/l_W) as a function of (l_v/l_W) for some typical values of $\xi(=-1.6,\ 1.0,$ and $1.6)$. Since the only difference between the cases of ν and $\bar{\nu}$ amounts to changing the sign in front of the $\cos(2\theta)$ term in Eq. (42), the curves for ν and $\bar{\nu}$ are the same for maximal mixing, $\theta=\pi/4$, shown in Fig. 3(a). For arbitrary mixing angles there will be a difference between the matter oscillation lengths of ν and $\bar{\nu}$. This difference is depicted as a function of l_v in Fig. 3(b) for the case of $\theta=15°$. In this case we see that the lengths $l_m(\nu)$ and $l_m(\bar{\nu})$ could be different by more than a factor 3, but it would not be possible to decide which is the larger of the two since the intrinsic sign of $[(3-\xi)l_v\cos(2\theta)]$ is *a priori* unknown.

Negative values of ξ imply constructive interference between the two types of weak interactions and lead to smaller matter oscillation lengths than positive values of ξ, at sufficiently large energies of the ν or $\bar{\nu}$. In this context, let us consider the following interesting, but somewhat extreme possibility. Suppose ν_e's undergo flavor oscillations but $l_v \gg D_e$, where $D_e=1.3\times10^9$ cm is the earth's diameter. Then vacuum oscillations cannot be observed in a terrestrial experiment of the type proposed by Mann and Primakoff.[16] However, can one detect the effects due to matter in such a case? Assuming a uniformly dense medium for the earth, we substitute $N_e \simeq 2N_A$ in Eq. (38) and obtain $l_W \simeq 0.6D_e$. From Eq. (42), since $l_m \simeq 2l_W/(3-\xi)$, negative values of ξ would be favorable for the observation of matter effects. For instance, $\xi \simeq -2$ (allowed by Fig. 2) leads to $l_m \simeq 0.25D_e$ and the effects due to matter may be barely observable.

VI. COMMENTS AND SUMMARY

While there is considerable experimental evidence for the validity of the e-μ universality in the charged-current sector, it is still an open question whether the neutral-current couplings of ν_μ and ν_e are in fact identical. It is possible that the neutral-current interaction of the neutrinos with matter is not universal, and if such a possibility is realized experimentally it would provide an interesting clue to the vexing problem of generations. As a first step, therefore, it is worthwhile examining, purely from a phenomenological angle, how a simple deviation from the ν_e-ν_μ universality, as for instance the one characterized by the parameter ξ in Eq. (1), could affect the various experimental measurements.

As an example of a rather remote but interesting ramification due to the unequal neutral-current strengths of ν_e and ν_μ, we shall briefly digress to consider the cosmic neutrino background. A suggestion originally due to Opher,[17] and subsequently elucidated by Lewis,[14] to detect the cosmic neutrino background ($E\sim10^{-2}$ eV) is to make use of the coherent scattering of neutrinos from the entire target of the detector system. According to this proposal, one measures the force exerted on a collector plate when neutrinos get totally reflected from its surface. If the collector plate has nuclei with atomic and mass numbers Z and A, the refractive indices will be given by

$$n(\nu_e)=1+\frac{GN_e}{\sqrt{2}E}[2Z-(A-Z)\xi] , \quad (43)$$

$$n(\nu_\mu)=1-\frac{GN_e}{\sqrt{2}E}[A-Z] , \quad (44)$$

the expressions for the corresponding antineutrinos being obtained by changing the sign in front of G. The sign of $(n-1)$ has important implications for the design of the Opher-Lewis detector. For $\xi \gtrsim 2$, the refractive index of ν_e scattering on most materials of interest (Cu,Fe,. . .,) is less than unity and thus cannot easily be discriminated from the case of ν_μ; for $\xi \lesssim 2$ (which includes the case of universality), however, the ν_e exhibits the phenomena of total internal reflection ($n>1$), as the $\bar{\nu}_\mu$.

As for the current experimental information on ξ, the only available data are from the reactor experiments of the Irvine group. The information extracted from these data is displayed in Fig. 2, which may be summarized by noting the two *alternative* values of ξ (ensuing mainly from the $\bar{\nu}_e e$ data): $\xi=1.45\pm0.20$, the weighted average of the three values of ξ which are greater than unity, and $\xi=-0.6\pm0.2$, the weighted average of the three values less than unity. The first is about two standard deviations from the universality value $\xi=1$. The errors on these values however should be much larger than what are quoted, to account for the many uncertainties, including that due to the assumed spectrum of reactor antineutrinos.

It should be mentioned that we have not considered here the possible violations of e-μ universality in neutral currents. From a phenomenological viewpoint, the e-μ universality need not be related to the ν_e-ν_μ universality discussed in this paper. For this purpose one may have to consider the reactions

ν_μ and ν_e scattering on μ, and $e^+e^- \rightarrow (e^+e^-, \mu^+\mu^-)$.

In summary, starting with the reasonable assumption that the structure of the neutral-current interaction of ν_e is the same as that established from the ν_μ reactions with matter, we have assumed the ν_e-ν_μ nonuniversality to be of a factorizable type characterized by the strength parameter ξ, and studied the consequences of it. Future scattering data using the ν_e or $\bar{\nu}_e$ beams of known fluxes and purity (as for example at the meson factories) on nuclear targets will be valuable in checking whether $\xi = 1$. Secondly, even significant departures from universality, when ξ lies in the range 0.5—2, will only cause minor changes in $\sigma(\bar{\nu}_e e)$ as in Fig. 1; in contrast, the corresponding changes in $\sigma(\nu_e e)$ are appreciable. Thirdly, the influence of (possible) ν_e-ν_μ oscillations on the data obtained on $\sigma(\nu e)$ could be considerably modified by nonuniversality, as exemplified in Table I; here negative values of ξ could lead to spectacular increases in the measured $\sigma(\nu_\mu e)$, and to a less extent in $\sigma(\bar{\nu}_\mu e)$, as a result of oscillations. Lastly, the elusive matter effects on oscillations, in the context of experiments which seek to detect GeV neutrinos passing through the earth, may become observable provided sizable departures from universality ($\xi \lesssim -3$) exist.

ACKNOWLEDGMENTS

We would like to thank J. S. Bell, S. Pakvasa, and L. M. Sehgal for useful conservations.

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