Multiquark interactions in the strong-coupling expansion of lattice gauge theories

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We show that the strong-coupling expansion of lattice gauge theories leads to static multiquark interactions which are in contrast to the additive-potential model and avoid the difficulties of this model.

The nonrelativistic quark model, enriched by ingredients from quantum chromodynamics, has been remarkably successful in explaining many features of hadron spectroscopy. The application to low-energy hadron-hadron scattering, however, meets serious difficulties for several reasons. There are kinematical difficulties due to the composite nature of hadrons, and there is our lack of knowledge about long-range quark-quark interactions for multiquark states. Though at present it is certainly unavoidable to try different *Ansätze* for this interaction, one should always be aware which of these are compatible with or even supported by more fundamental theoretical concepts. A particular popular model for multiquark interactions in the investigation of low-energy hadron-hadron scattering is the confining *additive* two-body potential.[1–6] It leads, however, to two serious difficulties:

(1) It implies strong van der Waals forces[7,8] which fall off only as inverse powers of the distance. They would lead to large effects at large distances, which are not observed.

(2) The *local* gauge invariance of QCD makes a strong long-range potential based on exchange of objects carrying color for more than three quarks in general ill defined.[9]

In this Brief Report we show that the strong-coupling expansion of lattice gauge theories[10–12] leads to a supersaturation in multiquark interactions and avoids the difficulties with van der Waals forces between color singlets.

Since we are interested in large-distance effects, we may use a rather coarse lattice and hence apply the strong-coupling expansion. In the strong-coupling limit the Abelian U(1) gauge theory is confining,[10] and for simplicity we first restrict ourselves to this model.

We consider two quark-antiquark pairs propagating in parallel rectangular paths from the points A and A' to D and D', respectively (see Fig. 1). The spatial separation between the two consitiuents of a pair is $l=AB=CD=A'B'=C'D'$. They travel over the Euclidean time period $t=AC=A'C'=BD=B'D'$. The distance between the two planes $ABCD$ and $A'B'C'D'$ is d. We evaluate the statistical weight W of these paths in the four-point function of the two-quark pairs $T(AA';DD')$ with the measure given by the lattice action of U(1) gauge theory.[10] This weight W receives its dominant contribution from two distinct gauge-field configurations. One is obtained by filling the planes in the rectangular loops $ABDC$ and $A'B'D'C'$ with plaquettes, the other one by filling the walls of the brick-shaped solid, i.e., the planes $AA'B'B$, $BB'D'D$, $CC'D'D$, and $AA'C'C$. Each plaquette gives a factor $1/g^2$,[10,11,13] where g is the coupling constant on the lattice. The statistical weight of this path configuration is hence given by

$$W(l,d,t)=[1/g^2(a)]^{t2l/a^2}[1+O(1/g^2(a))] + [1/g^2(a)]^{(2td+2ld)/a^2}[1+O(1/g^2(a))] \;, \quad (1)$$

where a is the lattice spacing. The equivalent static potential can be obtained from W by comparing it with the corresponding weight in the Feynman path integral[14] of nonrelativistic quantum mechanics. This yields (see, e.g., Refs. 15 and 16)

$$V(l,d)=-\lim_{t\to\infty}\frac{1}{t}\ln W(l,d,t) \;. \quad (2)$$

If $d>l$ (not necessarily $d\gg l$), one obtains

$$V=(2l/a^2)\ln g^2(a)+O(1/g^2(a)) \;,$$

i.e., just the sum of the two linearly rising potentials inside the two quark-antiquark pairs plus terms of order $1/g^2(a)$. For large spacings a the lattice coupling $g^2(a)$ scales like $\exp(+\Lambda a^2)$, where Λ is the slope of the linearly rising potential (string tension). Hence the terms of order $1/g^2(a)$ are suppressed exponentially for large distances and can only accommodate the exponentially decreasing potentials from hadron exchange.

Ignoring the difficulties coming from the fact that the lattice is not rotationally invariant, one may infer from the leading strong-coupling expansion the following model for a four-body interaction:

$$\begin{aligned} V(\vec{x}_1,\vec{x}_2,\vec{x}_3,\vec{x}_4)=\ln g^2\{&\theta(|\vec{x}_1-\vec{x}_4|+|\vec{x}_2-\vec{x}_3|-|\vec{x}_1-\vec{x}_3|-|\vec{x}_2-\vec{x}_4|)\\ &\times[|\vec{x}_1-\vec{x}_3|a^\dagger(\vec{x}_1)a(\vec{x}_1)b^\dagger(\vec{x}_3)b(\vec{x}_3)+|\vec{x}_2-\vec{x}_4|a^\dagger(\vec{x}_2)a(\vec{x}_2)b^\dagger(\vec{x}_4)b(\vec{x}_4)]\\ &+\theta(|\vec{x}_1-\vec{x}_3|+|\vec{x}_2-\vec{x}_4|-|\vec{x}_1-\vec{x}_4|-|\vec{x}_2-\vec{x}_3|)\\ &\times[|\vec{x}_1-\vec{x}_4|a^\dagger(\vec{x}_1)a(\vec{x}_1)b^\dagger(\vec{x}_4)b(\vec{x}_4)|\vec{x}_2-\vec{x}_3|a^\dagger(\vec{x}_2)a(\vec{x}_2)b^\dagger(\vec{x}_3)b(\vec{x}_3)]\} \;, \end{aligned} \quad (3)$$

where $a,a^\dagger$, and $b,b^\dagger$ are annihilation and creation operators of quarks and antiquarks, respectively.

For non-Abelian gauge groups the situation for comparable distances between all quarks is more complex. In Fig. 2(b), e.g., a connection scheme for $\pi^+\pi^+$ in color SU(3) is shown, which is not present for the Abelian case. Thus we will have, in general, more complicated contributions to the statistical weight, since there are more field configurations possible; in the static limit, however, only the configuration with the largest weight will contribute. If two hadrons are

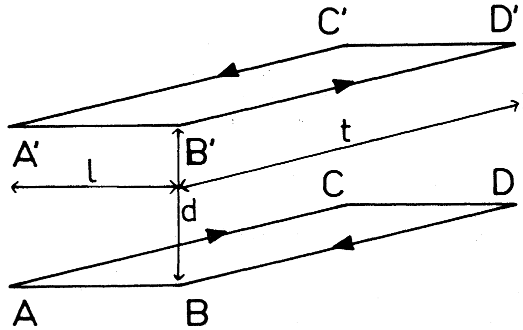

FIG. 1. Paths for two quark-antiquark pairs propagating from A and A' to D and D'. Arrows indicate charge flow.

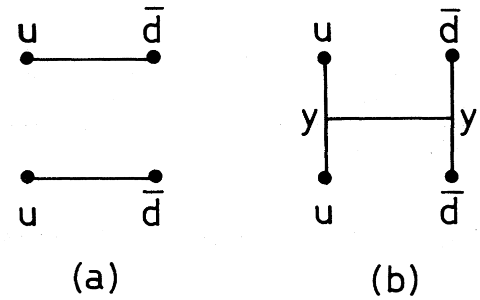

FIG. 2. Different possibilities to connect 2 u quarks (u) and 2 anti-d-quarks ($\bar{d}$) by plaquettes in color SU(3). The figure gives the configuration at a fixed time. The solid lines denote edges of plaquettes. At the lines going through y and y' in the "time" direction (i.e., perpendicular to the paper plane) the edges of three plaquettes are coupled to color singlets.

separated so far from each other that the sum of the distances betweeen the quarks in each hadron is smaller than the smallest distance between two quarks of different clusters, then up to $1/g^2$ terms the potential will be just the sum of the internal potentials of the two clusters. Thus, as in the Abelian case, there are no power-behaved potentials between the clusters. It should, however, be noted that the explicit form of the multibody potential will in general be much more complicated than for the Abelian case [see, e.g., the $3q$ potential in color SU(3) of Ref. 15].

Concluding we may remark that the strong-coupling expansion of lattice gauge theory does not only lead to realistic confining potentials for quark-antiquark pairs and three-quark states, but also avoids the difficulties of the additive-potential model for multiquark states.

Note added. After the submission of this paper we learned of a paper by M. W. Gross[17] which comes to the same conclusions.

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