Partial screening of classical color

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The static classical gauge potentials generated by a shell source of SU(2) color are examined. This is done within an ansatz for the gauge potentials which incorporates not only the Abelian, or Coulomb, solution but also spans the space of static fluctuations which have been shown by Magg to lower the energy from its Coulomb value. We find that, for supercritical coupling $\alpha>\alpha_c=\frac{3}{2}$, configurations corresponding to the minimum of the ansatz energy functional only partially screen the external source. Qualifications of this result are discussed.

I. INTRODUCTION

It has long been known[1] that the classical gauge potentials generated by c-number external sources do not exhibit confinement in the conventional sense; there is no infinite energy gap between states of a given physical system which carry zero and nonzero charge, or color. This is evident, for example, in the case of the SU(2) external source

$$g\rho^a(r)=\delta_3^a g\rho(r)=\delta_3^a\alpha\frac{\delta(r-r_0)}{{r_0}^2}\ ,\qquad \alpha=\frac{gQ}{4\pi}\ , \tag{1}$$

which describes a charge of magnitude Q uniformly distributed over a spherical shell of radius r_0. Choosing all color components of the gauge potential A^a to lie parallel to ρ^a gives the Abelian, or Coulomb, solution

$$A_0^a(\vec{x})=\int d^3x'\frac{1}{4\pi|\vec{x}-\vec{x}'|}\rho^a(\vec{x}')\ ,\qquad \vec{A}^a(\vec{x})=0\ , \tag{2}$$

possessing the finite energy

$$\mathscr{E}_{\rm COUL}=\frac{Q^2}{8\pi r_0}\ .$$

Moreover, the external source in this case is unscreened, so that the total color of the system is simply equal to that of the source:

$$\alpha_{\rm tot}\equiv\frac{gQ_{\rm tot}}{4\pi}=\alpha\ .$$

Thus, even if colorless states of lower energy exist, they must be separated by a finite gap from the Coulomb energy $\mathscr{E}_{\rm COUL}$ since the Hamiltonian is bounded from below by zero.

The problem, of course, resides in the severity of the classical approximation, wherein both the gauge potentials A_μ^a and the quark color densities ρ^a are regarded as commuting c-number fields. By the correspondence principle, this implies that we are making two assumptions: the first is that the number of gluons is large and exist in coherent states with fixed phase relationships, the second is that the representation of the quarks in the gauge group is large.[2] The first assumption may indeed be true; the usual description of confined quark sources maintains that as the sources are separated, regions of intense gluon energy density appear between the sources (flux tubes). This could correspond to a significant population of many gluonic modes with correlated phases in a manner susceptible to a classical treatment. However, the fact that quantum chromodynamics (QCD) describes quarks in the fundamental representation is in direct contradiction with the second assumption.

Nevertheless, classical Yang-Mills theories with external sources, or classical chromodynamics (CCD),[1] possess a remarkable property which may be pertinent to confinement in the quantum theory and should not be ignored. Studies of SU(2) CCD with source (1) have shown that once the external source exceeds a certain critical value

$$\alpha>\alpha_c=\tfrac{3}{2} \tag{3}$$

the Coulomb solution becomes unstable.[3] Components of the gauge potential, which were zero in the Coulomb configuration, through their self-interactions produce color which tends to screen the source and lower the energy.

This is reminiscent of the critical behavior observed in classical massless scalar electrodynamics in the presence of an external electric charge Q. Below α_c (here equal to $\frac{1}{2}$), the charged scalar field is zero and the gauge potential is simply the long-range Coulomb potential of the unscreened external charge. Once α_c is exceeded, however, the scalar field becomes nonzero, reducing the range of the gauge potential. In other words, a charged scalar field builds up around the external source so as to totally screen its charge. That this scenario is consistent with a solution of the static equations of motion has been demonstrated by Mandula.[4] Furthermore, this total-screening solution corresponds to the global minimum of the Hamiltonian restricted to spherically symmetric fields. Thus we conclude that the lowest-energy configuration of massless scalar electrodynamics has zero total charge.

Therefore it is natural to ask whether or not the same phenomenon occurs in CCD: Does the global minimum of the CCD energy functional correspond to a configuration of gauge potentials which totally screen external color? While absolute confinement of quark sources is unattainable in a classical treatment for the reasons mentioned above, the question of whether this remnant of confinement, that the energetically favored state is a color-singlet, is addressable in the formalism of CCD. We shall call this phenomenon, if it occurs, *color* confinement. In this paper we shall attempt to determine if color confinement is a property of SU(2) CCD with external source (1).

Total-screening solutions of other CCD systems may be found in the literature, but to date these have only considered source distributions which are extended over finite spatial volumes[5] or do not model single fixed quark sources.[6] Hughes[7] has shown, however, that the former solutions may be gauge artifacts. This is due to the possibility that, by nonsingular gauge transformations, solutions with extended sources may be gauge equivalent to finite-energy configurations with colorless sources; these are clearly uninteresting. Thus, by working with a source of unextended color, we avoid this inherent ambiguity of CCD. In this regard, a point-charge distribution would do as well were it not for the difficulty of applying boundary conditions at the source or the linear divergence of the classical Coulomb energy. Both problems are ameliorated by distributing the charge over a spherical shell.

Below we search for energy minima by solving the static SU(2) Yang-Mills equations with source (1) within a certain ansatz for the color and spatial dependences of the gauge potential. This ansatz designates a sector of function space including the Coulomb solution (2) as well as fluctuations about (2) which have been shown by Magg[8] to destabilize the Coulomb solution in the supercritical region of coupling (3). After preliminaries and a discussion of the merits of the ansatz in Sec. II, we present our solution of the resulting equations of motion in Sec. III. While for all supercritical values of the external charge probed a minimum of the ansatz energy functional is found with energy $\mathscr{E} < \mathscr{E}_{\rm COUL}$, we discover that the total color of the system is always nonzero, though of subcritical value. This suggests that color confinement is not a property of classical Yang-Mills systems. Some further qualifications of this result and our conclusions are given in Sec. IV.

II. THE ANSATZ

We shall assume the following ansatz for the color dependence of the gauge potentials:

$$gA_0^a=\delta_3^a A_0\ ,\qquad g\vec{A}^a=\delta_2^a\vec{A}\ . \tag{4}$$

From the definition of the field strength tensor

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+g\epsilon^{abc}A_\mu^b A_\nu^c\ ,$$

we may extract the chromoelectric and chromomagnetic fields via the definitions

$$E^{ia}\equiv F_{0i}^a\ ,\qquad B^{ia}\equiv-\tfrac{1}{2}\epsilon^{ijk}F_{jk}^a\ .$$

This gives

$$g\vec{E}^a=-\delta_3^a\vec{\nabla}A_0+\delta_1^a A_0\vec{A}\ ,\qquad g\vec{B}^a=\delta_2^a\vec{\nabla}\times\vec{A}\ .$$

The total energy of the system is

$$\mathscr{E}=\int d^3x\,(\tfrac{1}{2}\vec{E}^a\cdot\vec{E}^a+\tfrac{1}{2}\vec{B}^a\cdot\vec{B}^a)=\frac{1}{2g^2}\int d^3x\,(|\vec{\nabla}A_0|^2+|A_0\vec{A}|^2+|\vec{\nabla}\times\vec{A}|^2)\ , \tag{5}$$

which we shall express in terms of the reduced energy

$$\overline{\mathscr{E}}\equiv\frac{\mathscr{E}-\mathscr{E}_{\rm COUL}}{\mathscr{E}_{\rm COUL}}\ .$$

Finally, the Lagrangian

$$L=\int d^3x\,(\tfrac{1}{2}\vec{E}^a\cdot\vec{E}^a-\tfrac{1}{2}\vec{B}^a\cdot\vec{B}^a-\rho^a A_0^a)=\frac{1}{g^2}\int d^3x\,[\tfrac{1}{2}|\vec{\nabla}A_0|^2+\tfrac{1}{2}|A_0\vec{A}|^2-\tfrac{1}{2}|\vec{\nabla}\times\vec{A}|^2-(g\rho)A_0]$$

may be varied to determine the static equations of motion:

$$-\nabla^2A_0+(\vec{A}\cdot\vec{A})A_0=g\rho\ , \tag{6a}$$

$$\nabla^2\vec{A}+(A_0)^2\vec{A}=\vec{\nabla}(\vec{\nabla}\cdot\vec{A})\ . \tag{6b}$$

Thus we see that the gauge coupling g and the source strength Q enter the problem as a single parameter, namely, α.

In a moment we shall further restrict ansatz (4) with a specification of the spatial dependences of $A_0(\vec{x})$ and $\vec{A}(\vec{x})$. However, already at this stage, the ansatz possesses a number of desirable features which we now consider.

First, we note that by specifying the color dependences of the gauge potential, we have already fixed the gauge in which we work. It is possible to ascertain to what extent a gauge has been specified by considering all (necessarily time independent) gauge transformations which leave the ansatz (4) and the external source (1) in the same general form. If the operators T^a form a representation of the SU(2) Lie algebra.

$$[T^a,T^b]=i\epsilon^{abc}T^c\ ,$$

then these gauge transformations may be written as

$$\rho^aT^a\to U(\rho^aT^a)U^{-1}\ ,\qquad A_0^aT^a\to U(A_0^aT^a)U^{-1}\ ,\qquad \vec{A}^aT^a\to U\left[\vec{A}^aT^a-\frac{i}{g}\vec{\nabla}\right]U^{-1}\ , \tag{7}$$

where $U\in$ SU(2). Using (7), it is easy to show that all gauge degrees of freedom have been eliminated save for the transformations

$$\vec{A} \to -\vec{A} .$$

We leave this remaining discrete symmetry intact, as it will pose no difficulty toward the numerical solution we undertake later.

Second, we have specified a static external source, with $\dot{\rho}^a=0$. However, due to the antisymmetry of $F^a_{\mu\nu}$ with respect to its Lorentz indices, ρ^a must also satisfy the consistency condition

$$gA_0^a \rho^b \epsilon^{abc}=0 .$$

Clearly we must have A_0^a color parallel to ρ^a.

Next we notice that the ansatz accommodates the Coulomb solution, given by

$$A_0(\vec{x})=\tilde{\phi}(\vec{x})=\frac{\alpha}{r_0}\phi\left[\alpha\frac{|\vec{x}|}{r_0}\right], \tag{8}$$

where

$$-\nabla^2\tilde{\phi}=g\rho , \tag{9}$$

or

$$-\frac{1}{z}\frac{d^2}{dz^2}[z\phi(z)]=\frac{\delta(z-\alpha)}{\alpha} .$$

The solution

$$\phi(z)=\begin{cases}1, & z\le\alpha\\ \dfrac{\alpha}{z}, & z\ge\alpha\end{cases}$$

is regular at the origin and possesses the finite energy $\mathscr{E}_{\text{COUL}}$, i.e., $\mathscr{E}=0$.

Perhaps most important, the ansatz incorporates, in the simplest possible manner, a nontrivial color structure for the gauge potential. We have allowed for a vector component $\vec{A}$ which admits screening solutions, as seen by computing the total color vector Q^a_{tot}:

$$gQ^a_{\text{tot}}=\delta^a_3\int d^3x(g\rho-\vec{A}\cdot\vec{A}A_0) +\delta^a_1\int d^3x(-\vec{A}\cdot\vec{\nabla}A_0) . \tag{10}$$

Thus, as expected, the gauge potential itself carries color contingent upon the existence of a nonzero $\vec{A}$. Later we shall further specify A_0 and $\vec{A}$ so as to eliminate the second term in this expression. In that case, all color, source and field, lies color parallel and the occurrence of a total-screening solution arises as a distinct possibility.

Finally we may observe that if nontrivial solutions exist, then it is possible to show that they must possess energy lower than the Coulomb value, that is, $\mathscr{E}<0$:

Theorem. Given the ansatz (4), the Coulomb solution is a local maximum of the energy.

This theorem, due to Frenkel[9] and Ball[10] who utilized ansatz (4) in a different context, follows simply from Eqs. (5) and (6). If we define the field

$$f(\vec{x})=A_0(\vec{x})-\tilde{\phi}(\vec{x}) , \tag{11}$$

then the proof amounts to showing that the reduced energy is negative semidefinite:

$$\frac{\alpha}{r_0}\mathscr{E}=-\int d^3x\,|\vec{\nabla}f|^2 . \tag{12}$$

Since, in view of Eq. (6a), f is necessarily nonzero whenever $\vec{A}$ is, the theorem follows.

Magg has shown[8] that the axial-vector modes $\vec{A}$, satisfying the conditions

$$\vec{\nabla}\cdot\vec{A}=0$$

and

$$\hat{r}\cdot(\vec{\nabla}\times\vec{A})=0 , \tag{13}$$

are responsible for the instability of the Coulomb solution at α_c. Thus we supplement the specification of the ansatz with conditions (13), implying $\vec{A}$ is of the form

$$\vec{A}(\vec{x})=\frac{1}{r}\sum_{j=1}^{\infty}\sum_{m=-j}^{j}h_{jm}\left[\alpha\frac{r}{r_0}\right]i\vec{T}_{jm}(\theta,\varphi) , \tag{14}$$

where

$$\vec{T}_{jm}(\Omega)=\frac{-i(\vec{r}\times\vec{\nabla})}{[j(j+1)]^{1/2}}Y_{jm}(\Omega)$$

is a vector spherical harmonic and $h_{jm}(r)$ is a radial function to be determined. Note that, with conditions (13), we have broken the manifest spherical symmetry of the gauge potentials. If, as a final assumption, we take symmetry to be broken only along a particular axis ($\hat{z}$), that is, if we take

$$f(\vec{x})=f(r,\theta) ,$$
$$\vec{A}(\vec{x})=\vec{A}(r,\theta) , \tag{15}$$

then we may restrict the sum (14) over m to $m=0$, and drop the m index on the functions h_{jm}. This further assumption also permits the expansion of $f(\vec{x})$ in terms of Legendre polynomials:

$$f(\vec{x})=\frac{1}{r}\sum_{l=0}^{\infty}f_l\left[\alpha\frac{r}{r_0}\right]Y_{l0}(\theta) . \tag{16}$$

Together conditions (13) and (15) imply

$$\vec{A}\cdot\vec{\nabla}A_0=0 .$$

As promised, this assures that the total color density (10) points solely along the third direction in color space.

III. THE SOLUTION

Inserting (11) into Eqs. (6a) and (6b), eliminating $g\rho$ via (9), and using (13) we obtain the equations of motion for f and $\vec{A}$:

$$(-\nabla^2+\vec{A}\cdot\vec{A})f=-(\vec{A}\cdot\vec{A})\tilde{\phi} , \tag{17a}$$

$$[\nabla^2+(\tilde{\phi}+f)^2]\vec{A}=0 . \tag{17b}$$

Reexpressed in terms of the functions $f_l(z)$ and $h_j(z)$ and using the relation (8) between $\tilde{\phi}(\vec{x})$ and $\phi(z)$, we obtain the coupled set of nonlinear differential equations:

$$\left[z^2\frac{d^2}{dz^2}-l(l+1)\right]f_l-\sum_{j,j'=1}^{\infty}\sum_{l'=0}^{\infty}I_{jj'}^{l'}h_jh_{j'}\left[\delta_{ll'}z\phi+\sum_{k=0}^{\infty}J_{ll'k}f_k\right]=0,\quad l\geq 0 \tag{18a}$$

and

$$\left[z^2\frac{d^2}{dz^2}-j(j+1)+z^2\phi^2\right]h_j+\sum_{l,l'=0}^{\infty}\sum_{j=1}^{\infty}f_lI_{jj'}^{l}h_{j'}\left[\delta_{ll'}2z\phi+\sum_{k=0}^{\infty}J_{ll'k}f_k\right]=0,\quad j\geq 1\ , \tag{18b}$$

where

$$I_{jj'}^{l'}=\int d\Omega Y_{l0}^{*}\vec{T}_{j0}^{*}\cdot\vec{T}_{j'0}\ ,\quad l'\geq 0,\quad j,j'\geq 1\ ,$$

and

$$J_{ll'k}=\int d\Omega Y_{l0}Y_{l'0}Y_{k0},\quad l,l',k\geq 0\ .$$

Our task is now to solve this coupled set of equations, with appropriate boundary conditions, in a series of approximations beginning with a solution of the truncated set of functions (f_0,h_1) and including additional functions f_l and h_j as the approximation is improved. Convergence in the reduced energy $\overline{\mathscr{E}}$ and total color

$$\alpha_{\rm tot}=\alpha-\int d^3x(\tilde{\phi}+f)\vec{A}\cdot\vec{A}$$

$$=\alpha+\left.\frac{f_0(z)}{\sqrt{4\pi}}\right|_{z\to\infty}\ , \tag{19}$$

establishes the validity of this approximation scheme. [The second equality in (19) follows from Gauss's law (17a) and the definition of $f(\vec{x})$ in terms of the functions $f_l(z)$ (16).]

The solution of Eqs. (18) seemingly poses a formidable problem. Fortunately our results indicate that, for the region of coupling $\frac{3}{2}<\alpha<8$, the set of functions (f_0,f_2,h_1) determine $\overline{\mathscr{E}}$ and $\alpha_{\rm tot}$ to a relative accuracy of better than 1%. Thus the entire complexity in solving (18) lies in the nonlinearity of these equations. The technique we employ to surmount this difficulty, first developed by Henyey and Wilets,[11] is a functional generalization of Newton's method for finding the roots of a, in general nonlinear, function. We outline the method below. Let $H(F;z)$ be a nonlinear (differential) operator on a space of functions $F(z)$. We wish to solve

$$H(F;z)=0 \tag{20}$$

subject to a prescribed set of boundary conditions on F. Let

$$F^{(i+1)}(z)=F^{(i)}(z)+\delta^{(i)}(z)\ ,$$

where $F^{(i)}$ and $F^{(i+1)}$ satisfy the boundary conditions and $\delta^{(i)}$ satisfies the appropriate difference boundary conditions. Then if $F^{(i)}$ is an approximate solution of (20), we may expand to first order in $\delta^{(i)}$:

$$H(F^{(i+1)};z)\simeq H(F^{(i)};z)+\left.\frac{\delta H}{\delta F}\right|_{F=F^{(i)}}\delta^{(i)}(z)\ . \tag{21}$$

This linear, inhomogeneous differential equation for $\delta^{(i)}$ can be solved by any of several standard techniques. An iterative algorithm is thus defined; provided convergence is achieved, the fixed-point function $F^{(\infty)}$ will be the desired solution of the full nonlinear equation (20). In practice, the method has proved to be quite stable provided the initial guess $F^{(0)}(z)$ is "reasonable." Stability is related to the imposition of the boundary conditions at each iteration of the algorithm.

To apply this method to Eqs. (18), we must know the asymptotic behavior of the functions f_l and h_j. This will supply us with the required boundary conditions. At the origin, it is easy to see that the conditions

$$f_l\sim z^{l+1}\ ,\qquad h_j\sim z^{j+1}\ , \tag{22}$$

as $z\to 0$, are consistent with Eqs. (18). Series expansion analysis about the point $z=0$ shows that relative corrections to (22) are of the order z^2, and so may be neglected.

Determination of the asymptotic behavior as z approaches infinity is a more difficult proposition. We could begin with the supposition that f_l falls off faster than $f_{l'}$, and similarly for h_l and $h_{l'}$, for $l>l'$. In this case, one could naively drop all but coupling to h_1 in (18a) and f_0 in (18b). This yields f_l and h_j approximately satisfying the equations

$$\left[z^2\frac{d^2}{dz^2}-l(l+1)\right]f_l-\sum_{l'=0,2}I_{11}^{l'}(h_1)^2\left[\delta_{ll'}z\phi+\sum_{k=0}^{\infty}J_{ll'k}f_k\right]\approx 0,\quad l\geq 0 \tag{23a}$$

and

$$\left[z^2\frac{d^2}{dz^2}-j(j+1)+\left[z\phi+\frac{f_0}{\sqrt{4\pi}}\right]^2\right]h_j\approx 0,\quad j\geq 1\ . \tag{23b}$$

A consistent asymptotic behavior for $f_{l=0,1}$ and h_j in this case is

$$f_l\sim z^{-l},\quad l=0,1\ ,$$

$$h_j\sim z^{-\nu_j+1/2},\quad j\geq 1\ ,$$

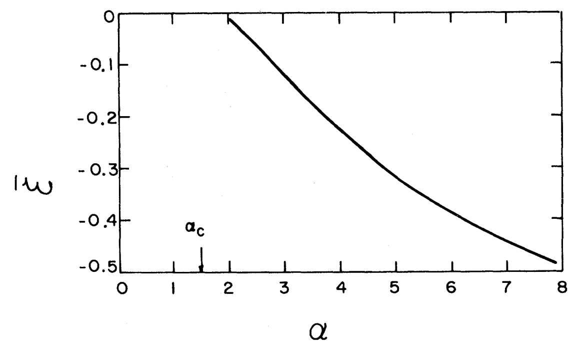

FIG. 1. The reduced energy $\mathscr{E}$ as a function of the external color α.

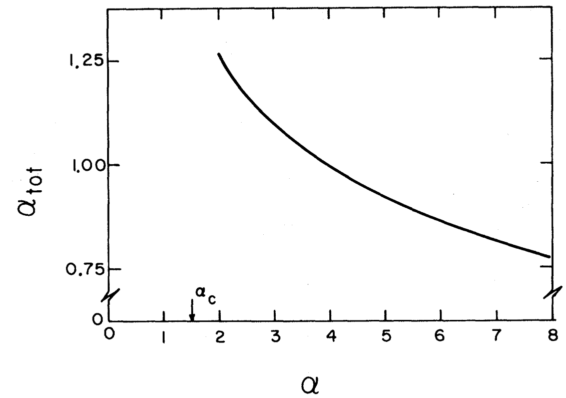

FIG. 2. Total color $\alpha_{\rm tot}$ as a function of the external color α.

where $\nu_j^2=(j+\frac{1}{2})^2-\alpha_{\rm tot}^2$. Note that the functions h_j fall off faster the better the external source is screened.

The situation for $f_{l>1}$ is less clear. Based on the supposed approximate equations (23), involved analysis shows that

$$f_{2k}\sim z^{-k(2\nu_1-1)},$$
$$f_{2k+1}\sim z^{-k(2\nu_1-1)-1},\quad k\geq 1\;,$$

as $z\rightarrow\infty$. These expressions may indeed be invalid, however, since we have neglected couplings in (23) of h_j to f_l of the form

$$h_jh_{j'}f_k\;,$$

where $k=|j-j'|$. The angular momentum k, in such instances, could be small, indicating that this coupling, as opposed to the $(h_1)^2$ coupling supposed above, may be a dominant factor at spatial infinity.

To extricate ourselves from this difficulty, we shall simply adopt the following boundary conditions:

$$f_l\sim z^{-l}\;,\qquad h_j\sim z^{-\nu_j+1/2}\;, \tag{24}$$

as $z\rightarrow\infty$. It is possible to show that if we integrate the equations out to sufficiently large z, the particular boundary conditions we apply are unimportant.[12]

Our final results for $\mathscr{E}$ and $\alpha_{\rm tot}$ for $2<\alpha<8$ are presented in Figs. 1 and 2, respectively. As mentioned above, these results, involving a truncated calculation including only the functions f_0, f_2, and h_1, are in relative error by less than a percent. Numerical difficulties associated with the slow falloff of $f_{l=0,2}$ preclude consideration of the range $\alpha_c=\frac{3}{2}<\alpha<2$. Nevertheless extrapolation from $\alpha=2$ to α_c in these figures is self-evident.

From Fig. 1 we see that for all α probed, $\mathscr{E}<0$. In accordance with the theorem above, we have therefore determined, within the context of the ansatz, energy minima corresponding to gauge potentials of truly nontrivial, non-Abelian nature.

However, while these configurations screen the external source, they do not color confine (Fig. 2): for all values of α examined, no $\alpha_{\rm tot}$ is identically zero. Instead, $\alpha_{\rm tot}$ is reduced to subcritical ($<\alpha_c$) but finite values. This partial screening of the external source is the primary result of our work.[13] In the next section we qualify this result, which appears to infer that color confinement, that is, total screening of an external source, is not a property of the CCD ground state.

IV. CONCLUDING REMARKS

The ansatz [Eqs. (4), (13), and (15)] for the gauge potentials, while not obviously restrictive *vis-à-vis* a classical description of color confinement, is still of limited color and spatial structure. While we have determined energy minima (one for each value of α) within the function space of the ansatz, these do not, of course, necessarily correspond to the minima of the full CCD energy functional. Indeed, the true minimum may lie connected in function space to the solution we have determined here by paths which monotonically decrease in energy. This possibility may be tested by performing energy stability analysis[14] about our solutions, an involved task we have not performed.

Because of the spatial dependence of the ansatz, the gauge field contribution to the total color (19) is spherically asymmetric. One may argue that total screening of a spherically symmetric source would be difficult to achieve with this restriction. We point out, however, that the gauge color density is a gauge-variant object [cf. Eq. (7)], so that spherical symmetry is not *a priori* requisite.

A more likely possibility is that our solution corresponds to a true local minimum of the full energy functional, but nevertheless is not the global minimum. By construction we have determined the closest minimum to the Coulomb solution in function space. We cannot preclude the existence of another minimum of lower energy.

In this regard it may be interesting to consider the alternative ansatz due to Witten.[15] It describes, within gauge transformations, a spherically symmetric gauge potential which couples, in a nontrivial fashion, Lorentz and color indices. It is possible to show that the sector of function space delineated by this ansatz excludes the Coulomb solution. Furthermore it incorporates quartic interactions of the spatial components of the gauge potential $\vec{\mathrm{A}}$, a feature which appears to be associated with the total screening observed in classical Yang-Mills systems with an infinite plane of color.[16] A study of this alternative, with source (1), is currently underway.

While this manuscript was in preparation, we learned of similar work by J. Mandula, D. Meiron, and S. Orszag,[17] who determined the static axially symmetric Yang-Mills potentials generated by an external point source. In spite of the different source distribution, our values of α_{tot} are in accord with their results for the total effective charge at spatial infinity.

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