One-loop integrals in axial-type gauges

W. Konetschny
Institut für Theoretische Physik, Technische Universität Wien, Vienna, Austria
(Received 1 December 1982; revised manuscript received 24 March 1983)

A general formula for momentum-space integrals containing one noncovariant denominator is derived. These denominators arise in ghost-free gauges such as the axial gauge. The method works in dimensionally regularized Minkowski space and employs Feynman parameters as well as the Wick rotation. The pole in the noncovariant denominator is circumvented by a general $i\epsilon$ prescription. With the final formula the principal value can also be evaluated. Several explicit examples are considered.

I. INTRODUCTION

Much interest has been devoted in the past years to the quantization of Yang-Mills theories in ghost-free gauges.[1] One introduces a gauge-breaking term containing the expression $n^\mu A_\mu$, wherein the gauge field A_μ is contracted with a noncovariant "vector" n_μ. As a consequence, additional factors $k\cdot n$ appear in the momentum-space propagator of the gauge field, and loop integrals become more cumbersome than in covariant gauges, if one applies the usual techniques. Among others, continuation of the entire integrand to Euclidean space and exponentiation of denominators has been advocated[2] recently as a reliable method for computing such integrals. As an alternative it should also be legitimate to start in Minkowski space, combine denominators with Feynman parameters, and subsequently perform the Wick rotation. In this manner one expects to have a certain handle on the validity of the above continuation process. Recall that for covariant integrands the Wick rotation can be carried out below threshold, in particular in the spacelike region of the external momenta.

A notorious question is the treatment of the singularity due to the vanishing of denominators $k\cdot n$. Since these singularities are artifacts of the gauge choice and observables do not depend on n_μ one might intuitively think that one prescription is as good as any other. However, in practical life the principal-value (PV) prescription has emerged as a suitable (but by no means unique) way to implement power counting[3] and unitarity.[4] It amounts to setting[5]

$$\frac{1}{(k\cdot n)^\beta}=\lim_{\epsilon\to 0}\frac{1}{2}\left[\frac{1}{(k\cdot n+i\epsilon)^\beta}+\frac{(-1)^\beta}{(-k\cdot n+i\epsilon)^\beta}\right] . \tag{1.1}$$

Note that for even β this is *not* the Cauchy prescription which exempts a symmetrical interval around the singularity from integration. Although the bulk of the explicit calculations has been done using (1.1) the PV has come under scrutiny time and again.[6]

In this work integrals containing one denominator $(k\cdot n+i\epsilon)^\beta$ are evaluated in Minkowski space with Feynman parameters and dimensional regularization. More complicated expressions can be simplified by repeated use of the identity[2]

$$\frac{1}{k\cdot n\,(k+p)\cdot n}=\frac{1}{p\cdot n}\left[\frac{1}{k\cdot n}-\frac{1}{(k+p)\cdot n}\right] . \tag{1.2}$$

To get the PV one has to repeat the calculation with $-n_\mu$ substituted for n_μ. An alternative method (e.g., Ref. 2) immediately interprets $(k\cdot n)^{-1}$ via (1.1) as $\lim_{\epsilon\to 0}(k\cdot n)/[(k\cdot n)^2+\epsilon^2]$ prior to momentum-space integration. If it turns out that the PV has to be abandoned our results might still be useful because any prescription presumably will be formulated in terms of $k\cdot n\pm i\epsilon$.

Special cases of the general formulas derived in Sec. II have appeared before in the literature. Several examples are worked out in detail in Sec. III in order to compare with existing calculations. The peculiarities of the present method are summarized in Sec. IV.

II. GENERAL INTEGRALS AND THE PRINCIPAL VALUE

We start with (metric $+---$, space-time dimension 2ω)

$$I(\alpha,\beta)=\int d^{2\omega}k\,(k^2+2p\cdot k-L+i\epsilon)^{-\alpha}(k\cdot n+i\epsilon')^{-\beta},\quad \alpha\geq 1,\ \beta\geq 1 . \tag{2.1}$$

The ϵ' associated with $k\cdot n$ need not be the same as in the covariant factor. The latter can be the result of combining several ordinary propagators with Feynman parameters. Introducing one more parameter we get

$$I(\alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int d^{2\omega}k\int_0^1 dx\,x^{\alpha-1}(1-x)^{\beta-1}[(k^2+2p\cdot k-L)x+k\cdot n\,(1-x)+i\epsilon x+i\epsilon'(1-x)]^{-\alpha-\beta} . \tag{2.2}$$

For $x\neq 0$ the k integral can be written as

$$\int d^{2\omega}k\left[\left[k+p+n\frac{1-x}{2x}\right]^2-L-\left[p+n\frac{1-x}{2x}\right]^2+i\eta\right]^{-\alpha-\beta} \tag{2.3}$$

involving the positive-definite quantity $\eta=\epsilon+\epsilon'(1-x)/x$. The outcome of the integration (2.3) will be taken to be valid also for $x=0$. This is allowed in a convergent x integral (convergence is achieved by a suitable dimension 2ω) without altering its value.

In (2.3) the Wick rotation can be performed provided

$$M(x)=4(L+p^2)x^2+4p\cdot nx(1-x)+n^2(1-x)^2 \tag{2.4}$$

is positive for $x\in[0,1]$. The terms in (2.4) do not have uniform dimension (n_μ is usually taken dimensionless) because we combined denominators of differing dimension. One may remedy this by temporarily multiplying n_μ with a positive scale factor of dimension [mass]. The conditions for positivity of $M(x)$

$$n^2>0,\ \ L+p^2>0,\ \ p\cdot n>-[(L+p^2)n^2]^{1/2} \tag{2.5}$$

are independent of such an arbitrary scale factor.

Standard dimensional integration[7] yields

$$I(\alpha,\beta)=(-1)^{\alpha+\beta}i\pi^{\omega}4^{\alpha+\beta-\omega}\frac{\Gamma(\alpha+\beta-\omega)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^1 dx\,x^{2\alpha+\beta-1-2\omega}(1-x)^{\beta-1}[M(x)]^{\omega-\alpha-\beta}\,. \tag{2.6}$$

The (positive) x integral in (2.6) exists for $2\omega-2\alpha-\beta<0$, which is just the condition for ultraviolet (UV) convergence from power counting in (2.1). It follows that $\omega-\alpha<\beta/2<\beta$. Writing

$$M(x)=n^2\left[1-\frac{x}{x_1}\right]\left[1-\frac{x}{x_2}\right] \tag{2.7}$$

the x integration gives (see Ref. 8 which will be referred to as GR)

$$I(\alpha,\beta)=(-1)^{\alpha+\beta}i\pi^{\omega}\left[\frac{4}{n^2}\right]^{\alpha+\beta-\omega}\frac{\Gamma(\alpha+\beta-\omega)}{\Gamma(\alpha)\Gamma(\beta)}B(2\alpha+\beta-2\omega,\beta)$$
$$\times F_1\left[2\alpha+\beta-2\omega,\alpha+\beta-\omega,\alpha+\beta-\omega,2\alpha+2\beta-2\omega;\frac{1}{x_1},\frac{1}{x_2}\right] \tag{2.8}$$

involving the generalized hypergeometric function F_1. Equation (2.8) is simplified with (GR 9.182.1)

$$F_1\left[2\alpha+\beta-2\omega,\alpha+\beta-\omega,\alpha+\beta-\omega,2\alpha+2\beta-2\omega;\frac{1}{x_1},\frac{1}{x_2}\right]$$
$$=\left[1-\frac{1}{x_2}\right]^{2\omega-\beta-2\alpha}F\left[2\alpha+\beta-2\omega,\alpha+\beta-\omega;2\alpha+2\beta-2\omega;\frac{1/x_1-1/x_2}{1-1/x_2}\right] \tag{2.9}$$

and (GR 9.134.1)

$$F(\alpha,\beta;2\beta;z)=\left[1-\frac{z}{2}\right]^{-\alpha}F\left[\frac{\alpha}{2},\frac{\alpha+1}{2};\beta+\frac{1}{2};\frac{z^2}{(2-z)^2}\right]\,. \tag{2.10}$$

Putting the pieces together we end up with

$$I(\alpha,\beta)=(-1)^{\alpha+\beta}2i\pi^{\omega+1/2}\frac{\Gamma(2a)}{\Gamma(\alpha)\Gamma(c)}(n^2)^{\alpha-\omega}(2p\cdot n)^{-2a}F\left[a,b;c;1-\frac{L+p^2}{p_L{}^2}\right]\,, \tag{2.11}$$

where $a=\alpha+\beta/2-\omega$, $b=\alpha+(\beta+1)/2-\omega$, $c=\alpha+\beta+\frac{1}{2}-\omega$,

$$p_L{}^2=\frac{(p\cdot n)^2}{n^2}\,.$$

It is easily checked using $F(a,b;b;z)=(1-z)^{-a}$ that in the limit $\beta\to 0$ the familiar formula[7] from dimensional regularization is recovered.

The hypergeometric function in (2.11) is taken below its cut, but for noninteger 2ω there is an apparent cut for $p\cdot n<0$ due to $(p\cdot n)^{-2a}$. In order to obtain a formula which is useful around $p\cdot n=0$ one continues the hypergeometric function in (2.11) with the help of GR 9.132.1:

$$I(\alpha,\beta)=(-1)^{\alpha+\beta}i\pi^{\omega+1/2}\frac{(n^2)^{\alpha-\omega}}{\Gamma(\alpha)}\left\{\frac{\Gamma(a)}{\Gamma((\beta+1)/2)}[(L+p^2)n^2]^{-a}F\left[a,\frac{\beta}{2};\frac{1}{2};\frac{p_L{}^2}{L+p^2}\right]\right.$$

$$\left.-2p\cdot n\frac{\Gamma(a+\frac{1}{2})}{\Gamma(\beta/2)}[(L+p^2)n^2]^{-a-1/2}F\left[a+\frac{1}{2},\frac{1+\beta}{2};\frac{3}{2};\frac{p_L{}^2}{L+p^2}\right]\right\}. \tag{2.12}$$

Equation (2.12) displays the cut in $p\cdot n$ as expected from condition (2.5). At $p\cdot n=0$, (2.12) yields immediately[9]

$$I(\alpha,\beta;p\cdot n=0)=(-1)^{\alpha+\beta}i\pi^{\omega+1/2}(n^2)^{-\beta/2}(L+p^2)^{\omega-\alpha-\beta/2}\frac{\Gamma(\alpha+\beta/2-\omega)}{\Gamma(\alpha)\Gamma((\beta+1)/2)}. \tag{2.13}$$

The case $L+p^2=0$ is conveniently treated with (2.11). One uses

$$F(a,b;c;1)=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)},$$

which is valid for $c-a-b=\omega-\alpha>0$, to get

$$I(\alpha;\beta;L+p^2=0)=(-1)^{\alpha+\beta}i\pi^{\omega}\left[\frac{n^2}{4}\right]^{\alpha-\omega}(p\cdot n)^{2\omega-\beta-2\alpha}\frac{\Gamma(\omega-\alpha)\Gamma(2\alpha+\beta-2\omega)}{\Gamma(\alpha)\Gamma(\beta)}. \tag{2.14}$$

For the integral

$$I_\mu(\alpha,\beta)=\int d^{2\omega}k\, k_\mu(k^2+2p\cdot k-L+i\epsilon)^{-\alpha}(k\cdot n+i\epsilon')^{-\beta} \tag{2.15}$$

we go through the same steps as before (2.11) leading to

$$I_\mu(\alpha,\beta)=(-1)^{\alpha+\beta+1}2i\pi^{\omega+1/2}\frac{\Gamma(2a)}{\Gamma(\alpha)\Gamma(c)}(n^2)^{\alpha-\omega}(2p\cdot n)^{-2a}$$

$$\times\left[p_\mu F\left[a,b;c;1-\frac{L+p^2}{p_L{}^2}\right]+n_\mu\frac{p\cdot n}{n^2}\frac{\beta}{2a-1}F\left[a-\tfrac{1}{2},a;c;1-\frac{L+p^2}{p_L{}^2}\right]\right]. \tag{2.16}$$

The counterparts of (2.12) and (2.13) are straightforward. The analog of (2.14) becomes

$$I_\mu(\alpha,\beta;L+p^2=0)=(-1)^{\alpha+\beta+1}i\pi^{\omega}\frac{\Gamma(\omega-\alpha)\Gamma(2\alpha+\beta-2\omega)}{\Gamma(\alpha)\Gamma(\beta)}$$

$$\times\left[\frac{n^2}{4}\right]^{\alpha-\omega}(p\cdot n)^{2\omega-\beta-2\alpha}\left[p_\mu+2n_\mu\frac{\omega-\alpha}{2\alpha+\beta-1-2\omega}\frac{p\cdot n}{n^2}\right]. \tag{2.17}$$

We note [compare Ref. 10, (A.10)]

$$I_\mu(\alpha,1;L+p^2=0)=\left[\frac{p\cdot n}{n^2}n_\mu-p_\mu\right]I(\alpha,1;L+p^2=0) \tag{2.18}$$

consistent with dimensional regularization since

$$n^\mu I_\mu(\alpha,1;L+p^2=0)=\int d^{2\omega}k\,[(k+p)^2]^{-\alpha}=\int d^{2\omega}k'(k'^2)^{-\alpha}=0\,.$$

The case $p=0$ is very simple because $I_\mu(\alpha,\beta)$ is proportional to n_μ:

$$I_\mu(\alpha,\beta;p=0)=\frac{n_\mu}{n^2}I(\alpha,\beta-1;p=0)\,. \tag{2.19}$$

In order to implement the prescription (1.1) we have to do the above calculation for n_μ and $-n_\mu$. The simultaneous positivity of $M(x)$ imposes the condition $(p\cdot n)^2<(L+p^2)n^2$ [see Eq. (2.5)]. In this region (2.12) is to be applied yielding the principal value (denoted by an overbar)

$$\bar{I}(\alpha,\beta)=(-1)^{\alpha+\beta}i\pi^{\omega+1/2}\frac{(n^2)^{\alpha-\omega}}{\Gamma(\alpha)}\left\{\tfrac{1}{2}[1+(-1)^\beta]\frac{\Gamma(a)}{\Gamma((\beta+1)/2)}[(L+p^2)n^2]^{-a}F\left[a,\frac{\beta}{2};\frac{1}{2};\frac{p_L{}^2}{L+p^2}\right]\right.$$

$$\left.-[1-(-1)^\beta]p\cdot n\frac{\Gamma(a+\frac{1}{2})}{\Gamma(\beta/2)}[(L+p^2)n^2]^{-a-1/2}F\left[a+\frac{1}{2},\frac{\beta+1}{2};\frac{3}{2};\frac{p_L{}^2}{L+p^2}\right]\right\}. \tag{2.20}$$

If $p\cdot n=0$, $\bar{I}(\alpha,\beta)$ vanishes for β odd consistent with symmetric integration.

For $L+p^2=0$, positivity of $M(x)$ for n_μ and $-n_\mu$ requires $p\cdot n\gtrless 0$, and we have to define the PV by analytic continuation in $p\cdot n$ to a common domain.[11] Thus $\bar{I}(\alpha,\beta;L+p^2=0)$ is obtained if $(p\cdot n)^{-2\alpha}$ is (2.14) is substituted by

$$\tfrac{1}{2}[(p\cdot n)^{-2\alpha}+(-1)^{\beta}(-p\cdot n)^{-2\alpha}]=(p\cdot n)^{-2\alpha}\tfrac{1}{2}[1+(-1)^{2\omega}]\ . \tag{2.21}$$

For general ω

$$(-1)^{2\omega}=e^{2\omega i\pi(1+2l)}\ , \tag{2.22}$$

where the integer l indicates the choice of continuation path of the function $z^{-2\alpha}$ to negative z in the cut plane (for $l=0$ and -1 one stays in the first Riemann sheet). In physical quantities, $\omega=2$ and all values of l are equivalent. However, a problem arises in the context of dimensional renormalization[7] because expanding (2.22) around $\omega=2$ one encounters an l-dependent imaginary part in (ultraviolet) divergent integrals [which contain $\Gamma(2-\omega)$]. One has to drop this contribution in order to obtain agreement with other methods of calculation (see the example in Sec. III). Finally we note that for 2ω odd the zero in (2.21) can be compensated by a pole in $\Gamma(2\alpha+\beta-2\omega)$.

III. SOME EXAMPLES

For $\beta=1$, $L+p^2=m^2$, and using GR 9.132.2 one can cast (2.20) into the form

$$\bar{I}(\alpha,1)=-(-1)^{\alpha}\frac{i\pi^{\omega}}{p\cdot n\,\Gamma(\alpha)}\left[-(p_L{}^2)^{\omega-\alpha}\frac{\Gamma(\frac{1}{2}+\alpha)\Gamma(\frac{1}{2}-\alpha)\sqrt{\pi}}{\Gamma(1-\alpha)}(-1)^{-\alpha-1/2}\left[1-\frac{m^2}{p_L{}^2}\right]^{-\alpha}\right.$$
$$\left.+(m^2)^{\omega-\alpha}\Gamma(\alpha-\omega)F\left[1,\tfrac{1}{2};\tfrac{3}{2}-\alpha;\frac{m^2}{p_L{}^2}\right]\right]\ . \tag{3.1}$$

The multivalued functions occurring in this formula may be taken at any side of their cut such that the entire quantity in square brackets in (3.1) does not contain an imaginary part, as dictated by the underlying Eq. (2.20) (remember $p_L{}^2<m^2$). Choosing for instance $-1=e^{i\pi}$ we get

$$-\frac{\Gamma(\frac{1}{2}+\alpha)\Gamma(\frac{1}{2}-\alpha)\sqrt{\pi}}{\Gamma(1-\alpha)}(-1)^{-\alpha-1/2}=\Gamma(\alpha+\tfrac{1}{2})B(\alpha,\tfrac{1}{2}-\alpha)\tfrac{1}{2}[1+(-1)^{2\omega-2\alpha}]\ .$$

With this relation Eq. (3.1) coincides with a formula derived by Kazama and Yao.[12]

Let us now compute the integral

$$\bar{I}(1,1)=\mathrm{PV}\int\frac{d^{2\omega}k}{(q-k)^2k\cdot n}\ . \tag{3.2}$$

Equations (2.14), (2.21), and (2.22) yield

$$\bar{I}(1,1)=i\pi^{\omega}\Gamma(3-2\omega)\Gamma(\omega-1)\left[\frac{n^2}{4}\right]^{1-\omega}(-q\cdot n)^{2\omega-3}\tfrac{1}{2}[1+(-1)^{2\omega}]$$
$$=-i\pi^{\omega}\Gamma(2-\omega)B(\omega-1,\tfrac{1}{2})(n^2)^{1-\omega}(-q\cdot n)^{2\omega-3}(-1)^{\omega}\frac{\cos\omega\pi(1+2l)}{\cos\omega\pi}\ . \tag{3.3a}$$

The method of Ref. 2 gives (in Euclidean space)

$$\bar{I}(1,1)=\pi^{\omega}\Gamma(2-\omega)B(\omega-1,\tfrac{1}{2})(n^2)^{1-\omega}(q\cdot n)^{2\omega-3}\ . \tag{3.3b}$$

Near $\omega=2$, putting $(-1)^{2\omega}=1$ either expression leads to [compare Ref. 13, (A5), $n^2<0$ there]

$$\bar{I}(1,1)=(i)\pi^{\omega}\frac{2q\cdot n}{n^2}\left[\Gamma(2-\omega)-\ln\frac{4(q\cdot n)^2}{n^2}+2+O(\omega-2)\right]\ . \tag{3.4}$$

Notice that $n^2>0$ both in (3.3a) and (3.3b). The intrinsic factor i in (3.4) is of course only there in Minkowski space.

As a second example we consider

$$\bar{I}(1,2;p=0)=\mathrm{PV}\int\frac{d^{2\omega}k}{(k^2-L)(k\cdot n)^2}\ . \tag{3.5}$$

One may either reduce (3.5) to an integral without[14] $k\cdot n$,

$$\bar{I}(1,2;p=0)=-\frac{2\omega-2}{n^2}\int\frac{d^{2\omega}k}{k^2(k^2-L)}\ ,$$

or evaluate it directly with (2.13). In both ways one obtains

$$\bar{I}(1,2;p=0)=-i\pi^{\omega}\frac{2}{n^2}L^{\omega-2}\Gamma(2-\omega)\ . \tag{3.6}$$

Finally we want to calculate the more complicated integral

$$I(\beta)=\int\frac{d^{2\omega}k}{k^2(q-k)^2(k\cdot n+i\epsilon')^{\beta}}\ . \tag{3.7}$$

Equation (2.11) gives

$$I(\beta)=(-1)^{\beta}2i\pi^{\omega+1/2}(n^2)^{2-\omega}\frac{\Gamma(4+\beta-2\omega)}{\Gamma(\frac{5}{2}+\beta-\omega)}$$
$$\times\int_0^1 dx\,(-2q\cdot nx)^{2\omega-\beta-4}F\left[2+\frac{\beta}{2}-\omega,\frac{5+\beta}{2}-\omega;\tfrac{5}{2}+\beta-\omega;1+\frac{q^2n^2}{(q\cdot n)^2}\frac{1-x}{x}\right]\ . \tag{3.8}$$

The condition $L+p^2>0$ requires $q^2<0$ (remember $n^2>0$). The UV divergence of $I(\beta)$ as determined from power counting is contained in $\Gamma(4+\beta-2\omega)$. Naive "infrared (IR)" power counting (behavior of the integrand for small k with q arbitrary) reveals another divergence for $\omega\le 1+\beta/2$. In fact, using GR 9.132.2 to investigate the vicinity of $x=0$ it is seen that the x integral (3.8) only exists for $\omega>1+\beta/2$ (a similar analysis with GR 9.131.2 shows convergence at $x=1$ for $\omega>1$). Performing a Laurent expansion of the integrand around $x=0$ the IR pole can be isolated. In this manner one derives for $\beta=2$ the IR pole[2]: $[i\pi^2/(2-\omega)]2/q^2n^2$. This is also the IR pole if the PV is taken in (3.7).

For $\omega=2$ only $I(1)$ is both UV and IR convergent. If we set $\omega=2$ in (3.8), the PV$\bar{I}(1)$ according to (1.1) is also given by (3.8). Defining $y=(q\cdot n)^2/q^2n^2<0$, Eq. (3.8) may be transformed into

$$\bar{I}(1)=\frac{2i\pi^2}{q\cdot n}y\left[\int_0^1 du\,(1-y+yu^2)^{-1}\ln\frac{1-u}{1+u}-2\int_0^{\infty}du\,(1-y-yu^2)^{-1}\arctan u\right]\ . \tag{3.9}$$

In terms of the "Spence function"[15]

$$\mathrm{Sp}(x)=-\int_0^x\frac{\ln|1-t|}{t}dt\ , \tag{3.10}$$

it is straightforward to evaluate the first integral of (3.9):

$$\int_0^1 du\,(u^2-\xi^2)^{-1}\ln\frac{1-u}{1+u}=\frac{1}{2\xi}\left[\ln\left|\frac{1-\xi^2}{\xi^2}\right|\ln\left|\frac{1-\xi}{1+\xi}\right|\right.$$
$$\left.+\mathrm{Sp}\left[\frac{\xi-1}{\xi+1}\right]-\mathrm{Sp}\left[\frac{\xi+1}{\xi-1}\right]-2\mathrm{Sp}\left[\frac{\xi}{\xi+1}\right]+2\mathrm{Sp}\left[\frac{\xi}{\xi-1}\right]\right]\ , \tag{3.11}$$

where $\xi^2=|(1-y)/y|>1$. Partial integration converts the second term of (3.9) into

$$\int_0^{\infty}\frac{\arctan u}{u^2+\xi^2}du=\frac{\pi^2}{4\xi}-\frac{1}{\xi}\int_0^{\infty}\frac{\arctan u/\xi}{1+u^2}du\ .$$

After differentiation with respect to ξ the u integration is performed with GR 3.264.2; the subsequent ξ integration leads again to Spence functions:

$$\int_0^{\infty}\frac{\arctan u}{u^2+\xi^2}du=\frac{1}{2\xi}\ln\xi\ln\left|\frac{1-\xi}{1+\xi}\right|+\frac{1}{2\xi}[\mathrm{Sp}(\xi)-\mathrm{Sp}(-\xi)]\ . \tag{3.12}$$

Combining the Spence functions[15] in (3.11) and (3.12) we find

$$\bar{I}(1)=\frac{i\pi^2}{q\cdot n\xi}\left[\tfrac{1}{2}\ln^2\left[\frac{\xi-1}{\xi+1}\right]+2\mathrm{Sp}\left[\frac{\xi-1}{\xi+1}\right]-\frac{\pi^2}{3}\right]\ . \tag{3.13}$$

This agrees with Ref. 10, (A9).

IV. SUMMARY

We have calculated general momentum-space integrals occurring in axial-type gauges with an arbitrary power of one noncovariant factor $k \cdot n$. The calculation proceeds in dimensionally regularized Minkowski space by combining denominators with Feynman parameters and subsequently performing the Wick rotation. The $i\epsilon$ needed for circumvention of the "gauge pole" $k \cdot n = 0$ is kept distinct from the one used with the ordinary propagators. A closed expression in terms of hypergeometric functions emerges, which (depending on the number of covariant denominators) still have to be integrated over Feynman parameters. The principal-value prescription for the $k \cdot n = 0$ singularity is implemented by adding two integrals, with n_μ and $-n_\mu$, respectively, after the k integration has been carried out. We have considered several explicit examples and found agreement with published literature. In a special case, evaluation of the principal value entails a sign factor $(-1)^{2\omega}$ (2ω = dimension), which is peculiar to the present method. It does not appear if the two denominators corresponding to the principal value are combined before momentum-space integration.

ACKNOWLEDGMENTS

I am much indebted to G. Wirthumer for his incisive criticism and aid in calculations. I also thank P. Schaller for assistance.

[1]R. Delbourgo, A. Salam, and J. Strathdee, Nuovo Cimento 23A, 237 (1974); W. Kummer, Acta Phys. Austriaca 41, 315 (1975); W. Konetschny and W. Kummer, Nucl. Phys. B100, 106 (1975); J. Frenkel, Phys. Rev. D 13, 2325 (1976); W. Konetschny, Phys. Lett. 90B, 263 (1980); Yu. L. Dokshitser, D. I. D'yakonov, and S. I. Troyan, Phys. Rep. 58, 269 (1980).

[2]D. M. Capper and G. Leibbrandt, Phys. Rev. D 25, 1009 (1982).

[3]W. Kummer, Ref. 1.

[4]W. Konetschny and W. Kummer, Nucl. Phys. B108, 397 (1976).

[5]I. M. Gel'fand and G. E. Shilov, *Generalized Functions* (Academic, New York, 1964), Vol. I.

[6]Y. Frishman, C. T. Sachrajda, H. Abarbanel, and R. Blankenbecler, Phys. Rev. D 15, 2275 (1977); T. T. Wu, Phys. Lett. 71B, 142 (1977); V. F. Müller and W. Rühl, Ann. Phys. (N.Y.) 133, 240 (1981); S. Caracciolo, G. Curci, and P. Menotti, Phys. Lett. 113B, 311 (1982); H. D. Dahmen, B. Scholz, and F. Steiner, *ibid.* 117B, 339 (1982).

[7]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).

[8]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1965).

[9]This case was handled by a different method in W. Konetschny, Nucl. Phys. B152, 153 (1979), Appendix.

[10]W. L. van Neerven, Z. Phys. C 14, 241 (1982).

[11]For a similar idea in a different context see P. Cvitanović, Nucl. Phys. B130, 114 (1977); W. Konetschny, Nuovo Cimento 47A, 546 (1978).

[12]Y. Kazama and Y. P. Yao, Phys. Rev. D 19, 3121 (1979).

[13]B. Humpert and W. L van Neerven, Phys. Rev. D 24, 2245 (1981).

[14]W. Kainz, W. Kummer, and M. Schweda, Nucl. Phys. B79, 484 (1974).

[15]K. Mitchell, Philos. Mag. 40, 351 (1949); R. Barbieri, J. Mignaco, and E. Remiddi, Nuovo Cimento 11A, 824 (1972); W. Gröbner and N. Hofreiter, *Integraltafeln* (Springer, Wien, 1949).