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Chiral-symmetry restoration in large-N quantum chromodynamics at finite temperature

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It is shown that, at large N, $\langle\bar{\psi}\psi\rangle$ is independent of the temperature in the confining phase. This implies that the temperature at which chiral symmetry is restored is larger than or equal to the critical temperature for deconfinement. If $T_{\text{chiral}}=T_{\text{deconfining}}$, then the chiral transition must be first order.

It is commonly believed that the temperature at which chiral symmetry is restored (T_{ch}) is greater than or equal to the temperature at which a gauge theory switches from a confining phase to a gas of free quarks and gluons (the deconfining temperature T_b). Coleman and Witten[1] showed that, at large N, chiral symmetry is necessarily broken at zero temperature. The same is generally believed to be true also at finite N. Monte Carlo studies seem to support this claim.[2] At finite temperature, however, it is not at all clear that chiral symmetry *must* be broken in the confining phase, because the chiral-anomaly argument (see Ref. 1) cannot easily be extended to finite temperature (see Ref. 3). In this Rapid Communication we will argue that, at least in the limit of an infinite number of colors, $T_{\text{ch}}\geqslant T_b$, i.e., confinement implies chiral-symmetry breaking. Note that at $N=\infty$ both temperatures are uniquely defined since Polyakov loops are a good order parameter for confinement (dynamic quark loops are suppressed) and $\langle\bar{\psi}\psi\rangle$ can be used as an order parameter for chiral symmetry. Our proof relies on the observation that, at large N, $\langle\bar{\psi}\psi\rangle$ must be independent of the temperature in the confining phase to leading order in $1/N$ (N for $\langle\bar{\psi}\psi\rangle$). This of course proves our statement since if $\langle\bar{\psi}\psi\rangle\neq 0$ at $T=0$ it will also be different from zero up to T_b. The easiest way to convince oneself that $\langle\bar{\psi}\psi\rangle$ is indeed independent of the temperature is as follows: In the confining phase the theory behaves effectively as a gas of noninteracting glueballs and mesons. Then, because of the large-N counting rules, the shift in expectation value of $\langle\bar{\psi}\psi\rangle$ in a state where a finite number of glueballs and mesons is present is only of order 1, while the vacuum expectation value is of order N. To leading order the expectation value of $\bar{\psi}\psi$, in a state of n glueballs (and m mesons), is equal to the vacuum value,

$$\langle nm|\bar{\psi}\psi|nm\rangle=\langle 0|\bar{\psi}\psi|0\rangle+O(1)\ . \tag{1}$$

Therefore,

$$\langle\bar{\psi}\psi(0)\rangle_\beta\equiv\frac{\sum_{n,m}e^{-\beta E_{nm}}\langle 0|\bar{\psi}\psi|0\rangle}{\sum_{n,m}e^{-\beta E_{nm}}}=\langle 0|\bar{\psi}\psi(0)|0\rangle\ . \tag{2}$$

Equation (2) of course expresses the fact that, in the noninteracting theory (1), $\langle\bar{\psi}\psi(0)\rangle$ is independent of the temperature. The importance of the effective free glueball theory for the behavior of large-N QCD at finite temperature was first observed by Thorn, Ref. 4 (see also Ref. 5). The same conclusion can also be reached in a more formal way. Following Ref. 6 we can derive the large-N version of the Schwinger-Dyson equations for quark bilinears at finite temperature. We will write them using the "naive" action

$$S=\sum_n\sum_{\mu<\nu=1}^{d}\frac{N}{\lambda}[\text{tr}(U_{n,\mu}U_{n+\mu,\nu}U^\dagger_{n+\nu,\mu}U^\dagger_{n,\nu})+\text{H.c})]$$
$$+\sum_n\left\{k\sum_\mu[\bar{\psi}(n)\gamma_\mu U_{n,\mu}\psi(n+\mu)-\bar{\psi}(n+\mu)\gamma_\mu U_{n+\mu,-\mu}\psi(n)]+\bar{\psi}(n)\psi(n)\right\}\ . \tag{3}$$

If we denote the gauge-invariant quantity

$$\frac{1}{N}\left\langle\bar{\psi}(x')\prod_{l\in\Gamma}U(l)\psi(x)\right\rangle$$

by $G(x,x',\Gamma)$, where Γ is a path from x to x', the Schwinger-Dyson equations read

$$\frac{1}{\lambda} d_{l\in\Gamma} G(x,x',\Gamma) = \sum_{l'\in\Gamma} \sum_{k=-\infty}^{+\infty} \delta(l|l'+k\beta)\, W(C_{ll'})\, G(x,x',\Gamma_{l'l}) \ , \tag{4}$$

$$\Delta_{x'} G(x,x',\Gamma) = \sum_{k=-\infty}^{\infty} (-)^k \delta(x'|x+k\beta)\, W(\Gamma_{xx'}) - G(x,x',\Gamma) \ . \tag{5}$$

Here $d_{l\in\Gamma}$ is the "link derivative" corresponding to the replacement

$$U_{n,\nu} \to \sum_{\substack{\mu\neq\nu \\ \mu\neq-\nu}} (U_{n,\mu} U_{n+\mu,\nu} U_{n+\mu+\nu,-\mu} - U_{n,\nu} U_{n+\nu,\mu} \times U_{n+\nu+\mu,-\nu} U_{n+\mu,-\mu} U_{n,\nu}) \ . \tag{6}$$

W denotes a Wilson loop:

$$W(C) \equiv \left\langle \frac{1}{N} \operatorname{tr} \prod_{l\in C} U(l) \right\rangle \ . \tag{7}$$

The "link delta" $\delta(l|l')$ is equal to 1 if l and l' are traversed in the same direction and -1 if they are traversed in the opposite direction. In Eq. (5) $\Delta_{x'}$ is the variation with respect to the end point of the path Γ and consists of the replacement

$$\bar{\psi}(x') \to k \sum_{\mu} [\bar{\psi}(x'-\mu)\gamma_\mu U_{x'-\mu,\mu} - \bar{\psi}(x'+\mu)\gamma_\mu U_{x'+\mu,-\mu}] \ . \tag{8}$$

Equation (5) follows from the simple identity

$$\int d\psi \frac{\partial}{\partial\psi} (U\psi e^{-S}) = 0 \ . \tag{9}$$

Note that on the right-hand side (RHS) of Eq. (4) we have used large-N factorization. Furthermore, note the absence of "string splitting" terms on the RHS of (4). These correspond to the creation of quark-antiquark pairs and are absent at large N. The effect of finite physical temperature enters in Eqs. (4) and (5) through the periodicity of the link delta: The notation $\delta(l|l'+k\beta)$ means that one obtains a contribution whenever links l and l' are separated by a multiple of the "time" extent of our lattice related to the temperature by

$$n_T = \frac{\beta}{a} \ . \tag{10}$$

$C_{ll'}$ in (4) is a closed path in Γ going from l to l' and $\Gamma_{l'l}$ is the remaining part of Γ after $C_{ll'}$ has been cut out.[6] Another source of temperature dependence, of course, is the possible explicit dependence of the Wilson loops in Eqs. (4) and (5). In the confining phase, however, these are, to leading order in $1/N$, independent of the temperature.[7] Furthermore, in the confining phase all terms with $k\neq 0$ on the RHS of Eqs. (4) and (5) vanish since they are proportional to Polyakov loops which are identically zero in the confining phase. Note that this is true only at large N, where the creation of additional quark-antiquark pairs is suppressed. Hence only the $k=0$ terms survive and the equations in the confining phase are identical to the zero-temperature equations derived in Ref. 6. Assuming that (4) and (5) uniquely specify G we conclude that it must be independent of the temperature in the confining phase. This implies also that $\langle\bar{\psi}\psi(0)\rangle$ is independent of T, since it can be obtained from G by some limiting procedure. Note that the antiperiodic boundary conditions on ψ enter through the factor $(-)^k$ on the right-hand side of Eq. (5). To summarize, we have shown that, to leading order in $1/N$, the order parameter for chiral-symmetry breaking is constant in the confining phase. Hence the chiral-symmetry-restoring temperature T_{ch} must be greater than or equal to the deconfinement temperature T_b. If $T_{\mathrm{ch}} = T_b$, our argument shows that the chiral transition is a first-order phase transition, since $\langle\bar{\psi}\psi\rangle$ can only go to zero discontinuously at T_b if it is constant for $T < T_b$. Since we do not know of any proof of $T_{\mathrm{ch}} = T_b$, we cannot exclude a second-order chirality transition, but in such a case we must have $T_{\mathrm{ch}} > T_b$, that is, an intermediate phase exists in which quarks are not confined, but chiral symmetry is broken.

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