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B^0-$\bar{B}^0$ mixing and the weak-interaction phase

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The amount of B^0-$\bar{B}^0$ mixing as seen in the rate of same-sign leptons in $e^+e^- \to B^0\bar{B}^0 \to llX$ is a sensitive way to measure the phase (sign of $\cos\delta$) in the Kobayashi-Maskawa matrix. It is shown how a rough measurement of the same-sign dilepton rate can determine the quadrant δ is in.

One of the important unknown parameters in the standard electroweak model[1] is the phase δ in the Kobayashi-Maskawa (KM) matrix[2] describing the weak charged current of quarks. At present we have only some weak constraints[3] on combinations of θ_2, θ_3, and δ. Even knowing whether $\cos\delta$ is positive or negative would be a significant advance. Here I would like to stress one of the relatively simple ways of determining the sign of $\cos\delta$.

The experimental signature for B^0-$\bar{B}^0$ mixing in $e^+e^- \to B^0+\bar{B}^0$ is a nonzero value for the same-sign-dilepton rate in the final state characterized by the quantity[4] $r=(N^{--}+N^{++})/N^{+-}$. The main result I want to stress here is the statement that *if r is found to be large (say $r > 0.1$) then $\cos\delta$ is negative.* The assumptions that go into this and how reliable they are will be discussed below, but what is of interest is that even a fairly rough experimental limit on r can help pin down the quadrant δ is in.

To see that the above statement is true, recall that r is given by

$$r=2\Delta/(1+\Delta^2) \ , \tag{1}$$

where Δ is

$$\Delta=\frac{(\delta m/\Gamma)^2+\frac{1}{4}(\delta\Gamma/\Gamma)^2}{2+(\delta m/\Gamma)^2-\frac{1}{4}(\delta\Gamma/\Gamma)^2} \ . \tag{2}$$

δm and $\delta\Gamma$ are the mass and decay-rate differences between B_S^0 and B_L^0 and Γ is the average decay rate. In the K_L^0-K_S^0 case, both Δ and r are nearly unity. In the range of parameters of interest, $\delta\Gamma/2\Gamma$ is small[5] compared to $\delta m/\Gamma$ and can be neglected without affecting our result. Then, to a good approximation

$$\Delta \approx \frac{(\delta m/\Gamma)^2}{2+(\delta m/\Gamma)^2} \ . \tag{3}$$

In this approximation ($\delta\Gamma/2\Gamma$ small), Γ is simply the decay rate for B, which (neglecting the small contribution from $b\to u$ transition) is given by[6]

$$\Gamma=\frac{3G_F{}^2m_b{}^5}{192\pi^3}|U_{cb}|^2 \ , \tag{4}$$

including appropriate phase-space and QCD corrections. δm is evaluated in the usual way from the box diagram with vacuum insertion with the well-known result[7]:

$$|\delta m| \cong \frac{G_F{}^2\eta B_B f_B{}^2 m_B m_t{}^2}{6\pi^2}[\mathrm{Re}(U_{tb}{}^2U_{td}^{*2})] \ . \tag{5}$$

In the above we have dropped the contributions from uu, uc, ut, cc, and ct exchange[8] and kept only the tt-exchange term. For m_t in the range $25 < m_t < 60$ GeV, which we assume here, this is an excellent approximation. f_B is the analog of f_π for the B meson; one suggested[9] value for f_B is $f_B \cong 0.5$ GeV. B_B is a factor (expected to be <1) to take into account the possible deviation from vacuum saturation of the actual matrix element. The quantity $B_Bf_B{}^2$ summarizes our ignorance about the evaluation of the matrix element. We will adopt here a conservative value $B_Bf_B{}^2 \sim 0.1$ GeV^2 and allow it to vary to 0.03 GeV^2. η is the leading short-distance gluonic correction and is estimated[10] to be nearly 0.8.

Then we have for $|\delta m/\Gamma|$

$$|\delta m/\Gamma|=\frac{32\pi\eta B_Bf_B{}^2m_t{}^2\mathrm{Re}(U_{tb}{}^2U_{td}^{*2})}{3m_b{}^4|U_{cb}|^2} \tag{6}$$

$$=6.863(B_Bf_B{}^2/0.1\ \mathrm{GeV}^2)(m_t/40\ \mathrm{GeV})^2[\mathrm{Re}(U_{tb}{}^2U_{td}^{*2})/|U_{cb}|^2] \ . \tag{7}$$

Now consider $\cos\delta > 0$; then the angular factor in Eq. (7) satisfies

$$\frac{\mathrm{Re}(U_{tb}{}^2U_{td}^{*2})}{|U_{cb}|^2} < \frac{c_2{}^2c_3{}^2s_1{}^2}{1+s_3{}^2/s_2{}^3} \ . \tag{8}$$

The right-hand side of Eq. (8) is smaller than $s_1{}^2$; in fact for almost the entire range[3] of the allowed values of θ_2 and θ_3 it is smaller than $(0.9s_1)^2$. Hence for $m_t=40$ GeV, $B_Bf_B{}^2=0.1$ GeV^2, and $\cos\delta>0$, we have $\delta m/\Gamma<0.29$ and $r<0.082$. The results for other values of m_t are shown as a solid curve in Fig. 1, and those for $f_B{}^2B_B=0.03$ GeV^2 are shown as the dashed curve in Fig. 1. For $\cos\delta<0$ the largest $|U_{td}/U_{cb}|$ can be is[11]

$$|U_{td}/U_{bc}|_{\max}=\frac{0.13}{0.05}=2.6 \ .$$

From $m_t=40$ GeV and $f_B{}^2B_B=0.1$ GeV^2, $\delta m/\Gamma$ is 41.9 and $r_{\max}=1$. In fact $r_{\max}$ for $\cos\delta<0$ remains at 1 for all $m_t>25$ GeV and $f_B{}^2B_B$ as low as 0.03 GeV^2. Hence the region bounded by the line $r=1$ and the solid curve corre-

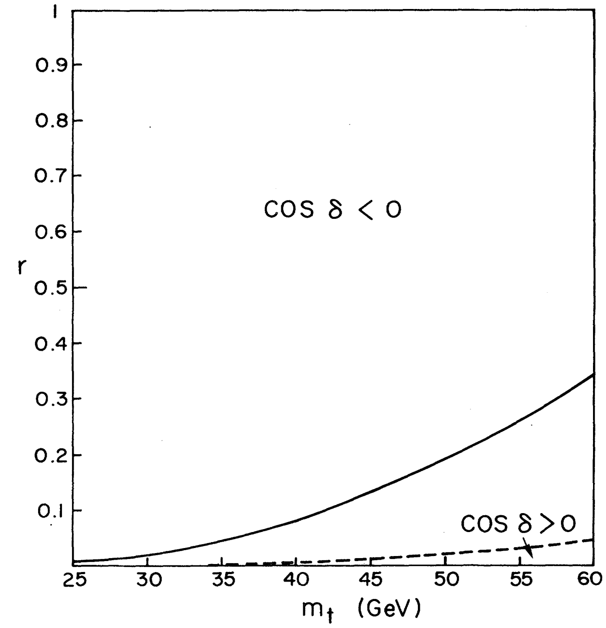

FIG. 1. The solid curve shows the maximum value of r (in B^0-$\bar{B}^0$) for $\cos\delta > 0$ for $(f_B{}^2 B_B) = 0.1$ GeV^2, the dashed curve for $(f_B{}^2 B_B) = 0.03$ GeV^2.

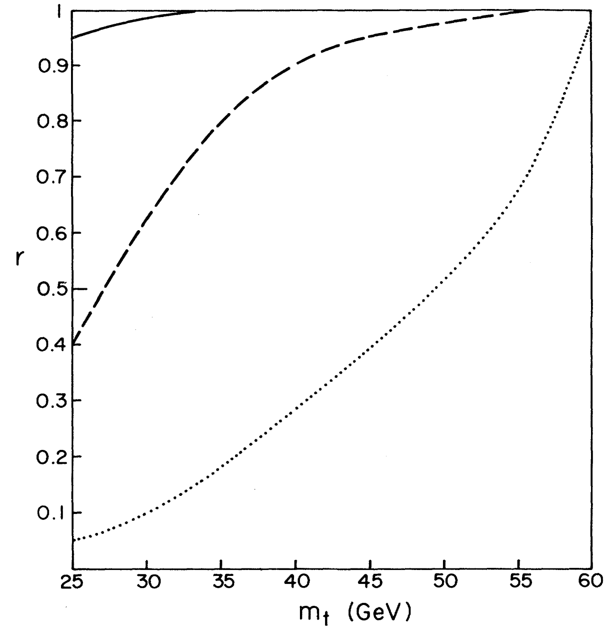

FIG. 2. The solid curve shows r (in B_s^0-$\bar{B}_s^0$) for $(f_B{}^2 B_B) = 0.1$ GeV^2, the dashed curve for $(f_B{}^2 B_B) = 0.03$ GeV^2, and the dotted curve for $(f_B{}^2 B_B) = 0.01$ GeV^2.

sponds to $\cos\delta < 0$. (If $f_B{}^2 B_B$ is as small as 0.03 GeV^2 the $\cos\delta < 0$ region expands to the dashed curve.) *An experimental determination that puts r anywhere in this area fixes* $\cos\delta$ *to be negative,* proving our assertion. The region below the dashed curve can be safely taken to correspond to $\cos\delta > 0$.

In principle, the slight remaining ambiguity, the dependence on $f_B{}^2 B_B$, can be removed. However, it involves the measurement of r for the B_s^0-$\bar{B}_s^0$ system, admittedly a difficult task. For the B_s^0-$\bar{B}_s^0$ system, $\delta m/\Gamma$ is given by (in the same approximation as before)

$$\delta m/\Gamma = 6.863(m_t/40\text{ GeV})^2(f_B{}^2 B_B/0.1\text{ GeV}^2) \times \mathrm{Re}(U_{tb}{}^2 U_{ts}^{*2})/|U_{cb}|^2 \quad (9)$$

Now, for any δ, $U_{ts}{}^2/|U_{bc}|^2 \approx 1$ and so $(\delta m/\Gamma)_{B_s} \approx 5.81$ for $m_t = 40$ GeV, $f_B{}^2 B_B = 0.1$ GeV^2, and $U_{tb} \simeq 0.95$, and hence $r = 1$. For $f_B{}^2 B_B = 0.1$ GeV^2, r is very near 1 for all m_t (Fig. 2, solid curve), whereas when $f_B{}^2 B_B$ is changed to 0.03 GeV^2, r drops to 0.4 for $m_t = 25$ GeV (Fig. 2, dashed curve). Assuming $f_B{}^2 B_B$ is not too different for the B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ systems, a measurement of r in the B_s^0-$\bar{B}_s^0$ system determines $f_B{}^2 B_B$ (especially if m_t is also known).

The results agree with those of detailed calculations[7] with more specific assumptions. However, it is clear that they are quite general and a detailed knowledge of the KM angles is unnecessary. The method proposed here to fix $\cos\delta$ is complementary to the one proposed[12] recently; the connection being that, for a given m_t, longer τ_B corresponds to higher r and to $\cos\delta < 0$. The actual observed value of r at the $\Upsilon(4s)$ would be diluted by the "contamination" of B^+B^- production which contributes to N^{+-} but not to N^{--} or N^{++}.

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[8]The *ct* term which contributes to the square bracket a term

$$(m_c^2/m_t^2)\ln(m_t^2/m_c^2)\mathrm{Re}(U_{tb}U_{td}^*U_{cb}U_{cd}^*)$$

has an opposite (same) sign to the *tt* term for $\cos\delta > 0$ (< 0). However, it is numerically unimportant. I have also dropped terms of order m_B^2/m_t^2.

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[11]$|U_{td}|_{\max}$ and $|U_{cb}|_{\min}$ are from unitarity of U_{KM} and $\tau_B < 1.4\times10^{-12}$ sec; see Ref. 3.

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