RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 28, NUMBER 11 1 DECEMBER 1983

Magnetic dipole transitions in quarkonia

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(Received 5 July 1983; revised manuscript received 9 September 1983)

Amplitudes for allowed and hindered $M1$ transitions are analyzed in a theory in which relativistic effects are taken into account to order $(v/c)^2$. Relativistic effects are small for allowed transitions. For hindered transitions, on the other hand, retardation and relativistic effects and coupled-channel mixing are important. We have calculated retardation and relativistic effects, and find strong cancellation between the various contributions. Results are given for $\psi' \to \eta_c\gamma$ and for both allowed and hindered transitions in the Υ system. When coupled-channel mixing is taken into account in the $\psi' \to \eta_c\gamma$ case, we find that theory and experiment agree within error.

I. INTRODUCTION

The purpose of this paper is to report some results of a study of magnetic dipole transition-matrix elements in the charmonium and Υ systems. We use a theory in which relativistic effects are taken into account to order $(v/c)^2$ which seems to account for the observed electric dipole transitions and fine and hyperfine splitting of the levels in charmonium.[1] There are two types of magnetic dipole transitions: allowed transitions where the initial and final nonrelativistic radial wave functions are the same, and relativistic or "hindered" transitions where the initial and final nonrelativistic wave functions are orthogonal. For the allowed transition $J/\psi \to \eta_c\gamma$ there is an apparent discrepancy between the theoretical value for the rate calculated nonrelativistically and the measured value that suggests that relativistic corrections may be important.[2] Hindered transitions such as $\psi' \to \eta_c\gamma$ occur only owing to relativistic and retardation effects. We summarize the theory of leading relativistic effects for $M1$ transitions first given by Sucher,[3] verify the $(v/c)^2$ correction terms of Kang and Sucher,[4] and include that due to Grotch and Sebastian.[5] Our calculation of these effects yield small corrections to allowed transition amplitudes, smaller than previously reported.[2] Consequently there may be a discrepancy between theory and experiment for the $J/\psi \to \eta_c\gamma$ rate (see Table I). For hindered transitions, relativistic effects are relatively large. They are larger than retardation effects. The various contributions have opposite signs and tend to cancel. Consequently our results here may be model dependent. We find that, for hindered transitions, the coupling of $q\bar{q}$ states to $q\bar{q}q\bar{q}$ decay channels is important. When this coupling is taken into account, the physical charmonium states, for example, become mixtures of $1S$, $2S$, etc., $c\bar{c}$ states, and also have some probability of being $D\bar{D}$, $D^*\bar{D}^*$, and other charmed-meson states.[6] When we take this mixing into account, we find a rate for $\psi' \to \eta_c\gamma$ in rough agreement with experiment.

II. RELATIVISTIC CORRECTIONS TO $M1$ TRANSITION MATRIX ELEMENTS

We consider first $c\bar{c}$ and $b\bar{b}$ bound S states which are eigenstates (neglecting S-D mixing) of the Breit-Fermi Hamiltonian

$$\begin{aligned}H_{\rm BF} = {} & (p_1{}^2+p_2{}^2)(2m)^{-1} - (p_1{}^4+p_2{}^4)(8m^3)^{-1} + V_{\rm NR} + (V_V' - V_S')(\vec{\sigma}_1\cdot\hat{r}\times\vec{p}_1 - \vec{\sigma}_2\cdot\hat{r}\times\vec{p}_2)(4m^2)^{-1}\\ & - V_V'(\vec{\sigma}_1\cdot\hat{r}\times\vec{p}_2 - \vec{\sigma}_2\cdot\hat{r}\times\vec{p}_1)(2m^2)^{-1} + (r^{-1}V_V' - V_V'')(3\vec{\sigma}_1\cdot\hat{r}\;\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2)(12m^2)^{-1}\\ & + \vec{\sigma}_1\cdot\vec{\sigma}_2\nabla^2 V_V(6m^2)^{-1} + \nabla^2 V_{\rm NR}(4m^2)^{-1} - \vec{p}_1\cdot V_S\vec{p}_1(2m^2)^{-1}\\ & - \vec{p}_2 V_S\cdot\vec{p}_2(2m^2)^{-1} - \vec{p}_1\cdot(I - \vec{\nabla}\,\vec{\nabla}/\nabla^2)V_V\cdot\vec{p}_2(m^2)^{-1}\ ,\end{aligned} \tag{1}$$

with $V_{\rm NR} = V_V + V_S$. Particle 1 is the quark and 2 the antiquark. This Hamiltonian comes from a $(v/c)^2$ expansion of the reduction to Pauli spinor form of the no-pair Dirac Hamiltonian[3]

$$H = h_1 + h_2 + \Lambda_{++}V\Lambda_{++}\ , \tag{2}$$

with $h = \vec{\alpha}\cdot\vec{p} + \beta m$ and

$$V = \beta_1\beta_2 V_S + V_V - \vec{\alpha}_1\cdot\left(I - \frac{\vec{\nabla}\,\vec{\nabla}}{\vec{\nabla}^2}\right)V_V\cdot\vec{\alpha}_2\ . \tag{3}$$

Pair-creation terms omitted in (2) are included in the wave functions perturbatively.

The transition matrix element is taken to be

$$M_{\lambda fi} = (\Phi_f, h_\lambda \Phi_i)\ , \tag{4}$$

where Φ are eigenstates of the Breit-Fermi Hamiltonian. Here h_λ has two parts; one comes from the reduction to Pauli spinors of the usual $\vec{\alpha}\cdot\vec{A}_\lambda$ coupling, and the other comes from the perturbation to the wave functions due to pair effects. The pair terms are shown diagrammatically in Fig. 1. They are $(v/c)^4$ correction terms to matrix elements which for allowed transitions are of order $(v/c)^2$, where v is the quark velocity. Therefore we include them as first-order perturbations. We separate the spin part χ and the space part ϕ of the S-state wave functions and write the matrix

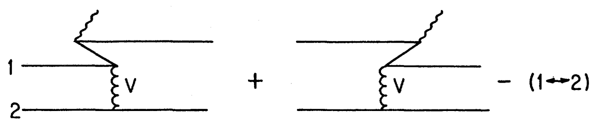

FIG. 1. The Z diagrams for photon emission which describe the pair-creation effects which have been included as perturbations on the Breit-Fermi wave functions.

element as

$$M_{\lambda fi}=ie_q\langle\chi_f|(\vec{\sigma}_1\cdot\vec{k}\times\hat{\epsilon}^*_\lambda/m)|\chi_i\rangle I \ , \tag{5}$$

with I a radial intergral. This is possible because

$$\langle\chi_f|\vec{\sigma}_1|\chi_i\rangle=-\langle\chi_f|\vec{\sigma}_2|\chi_i\rangle \tag{6}$$

for singlet-triplet transitions. The reduction to Pauli spinors of the $\vec{\alpha}\cdot\vec{A}$ terms to order $(v/c)^4$ yields what are called the "no-pair" contribution to the factor I. This contribution comes from the terms

$$-i\frac{\vec{\sigma}_1}{2m}\cdot\left[\vec{k}-\frac{\vec{p}^2\vec{k}+\vec{p}(\vec{p}\cdot\vec{k})}{2m^2}\right]\times\hat{\epsilon}^*_\lambda\exp(-i\vec{k}\cdot\vec{r}_1)+(1\leftrightarrow 2) \tag{7}$$

in h_λ. Here $\vec{p}_1=-\vec{p}_2=\vec{p}$. Including recoil, we write the final-state wave function as $e^{-i\vec{k}\cdot\vec{R}}\phi_f(r)$ with $\vec{R}=(\vec{r}_1+\vec{r}_2)/2$ and $\vec{r}=\vec{r}_1-\vec{r}_2$. Then the no-pair contribution to I is

$$I^{\text{no-pair}}=\langle\phi_f|-j_0(kr/2)+2\vec{p}^2/3m^2|\phi_i\rangle \ . \tag{8}$$

The contribution of the pair effects is of particular interest because it is sensitive to the Lorentz-transformation properties of the interaction potential V. Following Sucher[3] we separate the potential into even and odd parts,

$$V=\underbrace{(\beta_1\beta_2V_S+V_V)}_{V_{\text{even}}}+\underbrace{(\vec{\alpha}_1\cdot\vec{\alpha}_2Y+\vec{\alpha}_1\cdot\hat{r}\,\vec{\alpha}_2\cdot\hat{r}Z)}_{V_{\text{odd}}} \ , \tag{9}$$

and allow the transverse-gluon-exchange part of V to have a more general form than in (3). We take it to be given by some $f(r)$, viz.,

$$-\vec{\alpha}_1\cdot\left[I-\frac{\vec{\nabla}\vec{\nabla}}{\nabla^2}\right]f(r)\cdot\vec{\alpha}_2 \ . \tag{10}$$

If one evaluates (10) in the usual way, one finds (9) with

$$Y=-(2f+Z)/3 \tag{11}$$

and

$$Z=r^{-3}\int_0^r s^3f'(s)\,ds \ . \tag{12}$$

The four terms in (9) give to I the contribution

$$I^{\text{pair}}=\langle\phi_f|(V_S/m)j_0(kr/2) -[(rV_V'+rY'-Z)/mkr]j_1(kr/2)|\phi_f\rangle \ . \tag{13}$$

With $f(r)=V_V$, the transverse-gluon-exchange piece cancels the V_V part of V_{even}, and only the V_S term in (13) survives.

Combining pair with no-pair terms, we find that the radial integral is given by

$$I=I_1+I_2+I_4 \ , \tag{14}$$

where

$$I_1=\langle\phi_f|-j_0(kr/2)|\phi_i\rangle \ , \tag{15}$$

$$I_2=\langle\phi_f|2\vec{p}^2/3m^2|\phi_i\rangle \ , \tag{16}$$

$$I_4=\langle\phi_f|(V_S/m)j_0(kr/2)|\phi_i\rangle \ . \tag{17}$$

If the wave functions in (15)–(17) are taken to be eigenfunctions of a nonrelativistic Hamiltonian, these expressions agree with those of Kang and Sucher.[4] In their paper there is an additional contribution I_3 which is nonvanishing only for hindered transitions. It is the first-order perturbative effect of the hyperfine interaction on the wave functions. With nonrelativistic wave functions ϕ_i and ϕ_f, it is given by

$$I_3=-4\langle\phi_f|\nabla^2V_V|\phi_i\rangle/6m^2(E_i-E_f) \ . \tag{18}$$

If instead one uses Breit-Fermi wave functions, the effect of the hyperfine interaction is included in I_1, and I_3 is redundant. The Hamiltonian we use is Hermitian, so effects of the spin-independent $(v/c)^2$ correction terms in the Hamiltonian do not contribute to I_1 to order $(v/c)^2$. Because $(v/c)^2$ correction terms are relatively small, results obtained with the use of Breit-Fermi wave functions are the same, to the accuracy of our calculations, as those obtained treating the hyperfine and other $(v/c)^2$ effects perturbatively. Consequently all retardation and relativistic effects to order $(v/c)^2$ except recoil corrections are included in I if we evaluate I_1 through I_4 using nonrelativistic wave functions and include I_3 for hindered transitions.

In the above analysis recoil effects in the final-state wave function were taken into account only by allowing the center of mass to recoil (Galilean invariance). Lorentz invariance gives rise to additional effects on the wave function. This induces a spin-dependent difference between the constituent and center-of-mass coordinates; Grotch and Sebastian[5] have pointed out that this generates a contribution to $M1$ transitions from the convection current. Their result is that there is an additional $(v/c)^2$ correction to I given by

$$I_{\text{GS}}=\langle\phi_f|\vec{p}^2/6m^2|\phi_i\rangle \ . \tag{19}$$

Since this combines simply with the $(v/c)^2$ correction to the

TABLE I. Values for the overlap integral I for some allowed transitions. The contributions I_1, $I_2'=I_2+I_{\text{GS}}$, and I_4 are explained in text; see Eqs. (15)–(17). We obtained the experimental values for the magnitude of I from the data of Ref. 8 using our Eq. (21).

	$-I_1$	$-I_2'$	$-I_4$	$-I$	$\lvert I\rvert_{\text{expt}}$
$c\bar{c}$					
$1\,^3S_1\rightarrow1\,^1S_0$	1.00	−0.17	0.25	1.08	0.7 ±0.2
$2\,^3S_1\rightarrow2\,^1S_0$	1.00	−0.20	0.07	0.87	1.1–1.7
$b\bar{b}$					
$1\,^3S_1\rightarrow1\,^1S_0$	1.00	−0.07	0.11	1.04	· · ·
$2\,^3S_1\rightarrow2\,^1S_0$	1.00	−0.06	0.07	1.01	· · ·
$3\,^3S_1\rightarrow3\,^1S_0$	1.00	−0.07	0.03	0.96	· · ·

TABLE II. Calculated values for the overlap integral I for some hindered transitions. When coupled-channel mixing is taken into account for the $\psi' \rightarrow \eta_c \gamma$ decay, there are additional contributions to I of the same order of magnitude as those shown here. See text. (We include the photon momenta in this table because I_1 is proportional to k^2. The photon momenta are taken from Ref. 8 for $\psi' \rightarrow \eta_c$ and from Ref. 1 for the $b\bar{b}$ transitions.)

	k (MeV)	$-I_1$	$-I_2'$	$-I_3$	$-I_4$	$-I$
$c\bar{c}$						
$2\,^3S_1 \rightarrow 1\,^1S_0$	634	0.052	−0.136	0.127	0.073	0.116
$b\bar{b}$						
$2\,^3S_1 - 1\,^1S_0$	639	0.019	−0.052	0.098	0.016	0.080
$3\,^3S_1 \rightarrow 2\,^1S_0$	368	0.021	−0.048	0.095	0.027	0.095
$3\,^3S_1 \rightarrow 1\,^1S_0$	948	0.010	−0.033	0.056	0.008	0.041

$\vec{\sigma}\cdot\vec{B}$ interaction term I_2, we quote results for I_2', where

$$I_2' = I_2 + I_{GS} = \langle \phi_f | 5\vec{p}^2/6m^2 | \phi_i \rangle \quad . \tag{20}$$

To exhibit the relative sizes of the various effects, we give in Tables I and II the values of I_1, I_2', I_3, and I_4, all calculated with nonrelativistic wave functions. Here I_1 simply exhibits retardation effects, I_2' is given by (20), I_3 is the hyperfine interaction effect, and I_4 gives the effect of the Z diagrams.

So far we have neglected the coupling of $q\bar{q}$ states to multiparticle channels such as $q\bar{q}q\bar{q}$. Taking coupling to closed $q\bar{q}q\bar{q}$ decay channels into account, Eichten *et al.*[6] showed that the physical J/ψ and ψ' states are mixtures of $1S$, $2S$, and other $c\bar{c}$ states, and also have some probability of being virtual $D\bar{D}$, $D^*\bar{D}$, etc., charmed-meson states. If the ψ' has some probability of being a $1S$ state, since the η_c is primarily a $1S$ state, this mixing can be important in a calculation of the $\psi' \rightarrow \eta_c\gamma$ amplitude. To estimate the effect of such mixing, we used the results of Eichten *et al.* for the amplitudes for the ψ' to be $1S$ and the η_c to be $2S$ $c\bar{c}$ states (see below) and found that this mixing contributes significantly to the $\psi' \rightarrow \eta_c\gamma$ amplitude.

III. RESULTS OF THE MODEL CALCULATION

We have calculated the various contributions to I using the one-gluon-exchange-plus-linear-confinement model of Ref. 1. The model includes transverse-gluon exchange (in Coulomb gauge) and treats the linearly confining potential as a Lorentz scalar. The confining potential is given by $V_S = r/a^2 + C$ with $a = 0.46$ fm. The vector part of the potential is given as $V_V = -\kappa\,\mathrm{erf}(\sqrt{2}mr)/r$, which is the Coulomb potential of a Gaussian-distributed color charge whose root-mean-square radius is given by the Compton wavelength m^{-1} of the quark. The parameters of the potential and quark masses are given in Table III. The constant C in V_S is necessary in order to get the observed masses for the charmonium and Υ states. McClary[7] found it necessary to consider C as part of V_S rather than V_V in order to fit the observed charmonium level structure taking

TABLE III. Parameter values for quark masses and V (see text).

	$c\bar{c}$	$b\bar{b}$
m (GeV)	1.84	5.17
κ	0.65	0.53
C (MeV)	−761	−745

$(v/c)^2$ corrections to the nonrelativistic Hamiltonian into account. He smeared the color charge in order to calculate relativistic corrections to $E1$ transition rates. The $E1$ rates are insensitive to this smearing, and it was simply a convenient way to get results. On the other hand, the hyperfine splitting of the levels in this model is sensitive to the short-distance behavior of V_V. The model correctly accounts for the hyperfine splitting of the ground states of charmonium and is in rough agreement with the ψ'-η_c' mass difference. Note that the diagonal matrix elements of the operator in I_3 just give the fine-structure splitting of the corresponding S states. Since I_3 only contributes to hindered transition amplitudes, our results for the allowed transitions are insensitive to the smearing of the color charge. In the case of the hindered transitions, however, I_3 plays an important role. Variation of the color charge radius from a point charge to the quark Compton wavelength can change the values of I_3 by as much as 30%.

The rates are related to I by ($e_q = \frac{2}{3}$ for $c\bar{c}$ and $-\frac{1}{3}$ for $b\bar{b}$, and k is the photon momentum)

$$\Gamma(^3S_1 \rightarrow {}^1S_0) = e_q{}^2\alpha(4k^3/3m^2)|I|^2(M_i{}^2 + M_f{}^2)/2M_i{}^2 \quad . \tag{21}$$

Rates are very sensitive to the hyperfine splitting of the levels owing to the k^3 factor in (21). The experimental value for k should be used. For allowed transitions, I is insensitive to k though I_1 has some k dependence because it contains the retardation correction. For hindered transitions, on the other hand, I_1 is proportional to k^2.

When coupled-channel mixing effects are included, the physical ψ' has some probability of being a $1\,^3S_1$ state and the η_c a $2\,^1S_0$ state. Consequently allowed transitions occur in the $\psi' \rightarrow \eta_c\gamma$ amplitude. One should use the physical value for k to calculate the retardation effects; they are relatively large because the photon frequency is relatively large.

IV. COUPLED-CHANNEL MIXING EFFECTS

To estimate the effect of coupled-channel mixing, we calculate I for $\psi' \rightarrow \eta_c\gamma$ using the results of Eichten *et al.*[6] They give probability amplitudes that the physical J/ψ and ψ' states are $1S$, $2S$, etc., $c\bar{c}$ states. For the η_c, we assume weak spin dependence of coupled-channel mixing in the ground states, and approximate the probability amplitudes for the physical η_c to be $1S$ and $2S$ $c\bar{c}$ states by the values given by Eichten *et al.* for the J/ψ. These are shown in Table IV. When we include the $1\,^3S_1 \rightarrow 1\,^1S_0$ and $2\,^3S_1 \rightarrow 2\,^1S_0$ transitions in what before was taken to be a $2\,^3S_1 \rightarrow 1\,^1S_0$ transition, the effect is striking. Our value of

TABLE IV. Modification of $c\bar{c}$ states due to decay. The probability amplitude for a physical particle (J/ψ or ψ') to be in a charmonium state is given by the number under that state. $Z_{(c\bar{c})}$ gives the norm of the physical particle in the $c\bar{c}$ sector. This table is a portion of Table VIII in Ref. 6.

Particle	$1S$	$2S$	$3S$	$4S$	$1D$	$2D$	$Z_{(c\bar{c})}$
J/ψ	0.982	0.040	−0.010	0.003	-2×10^{-4}	-7×10^{-6}	0.966
ψ'	−0.090	0.883	0.046	−0.015	−0.031	0.006	0.791

$-I$ is reduced from 0.116 to 0.032.

Coupled-channel mixing does not affect the rate for $J/\psi\rightarrow\eta_c\gamma$ significantly; the amplitude changes by about 4%. The effect is probably larger in the $\psi'\rightarrow\eta_c'\gamma$ case. Note in Table IV that $Z_{(c\bar{c})}=0.79$ for ψ'. We hesitate to use the same weak-spin-dependence approximation for η_c' as we used for the η_c because the η_c' is closer to open-charm threshold. We are calculating these amplitudes directly.

It seems likely that coupled-channel mixing will also be significant in hindered $b\bar{b}$ transitions. We expect this will be true in the lowest $\Upsilon'\rightarrow\eta_b\gamma$ case as well as in transitions in higher states. Though open-b channels are relatively farther away, their effect is expected to be significant because the relativistic and retardation effects that account for the $2\,^3S_1\rightarrow1\,^1S_0$ transition are also smaller in the $b\bar{b}$ system. Study of coupled-channel effects in both charmonium and Υ systems is underway.

V. DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

Our result for the relativistic correction to the $J/\psi\rightarrow\eta_c\gamma$ rate is smaller than previously reported results.[2] This is due to our inclusion of the constant term in the scalar potential. Had we omitted this constant we would have gotten 0.16 instead of −0.25 for I_4, and our final result for I would have been −0.63. Such a decrease in magnitude seems to be indicated by Crystal Ball measurements; Gaiser[8] gives values for the branching fraction and photon energy which, when combined with the width given in the 1982 Particle Data Group tables[9] and used in (21), give $|I|=0.7\pm0.2$. On the other hand, no such decrease is indicated by the experimental observations in the $\psi'\rightarrow\eta_c'\gamma$ case. Gaiser's values give $|I|$ between 1.1 and 1.7 (90% C.L.). Neglecting coupled-channel mixing, we find 0.86 for the magnitude of I. If we omit the contribution I_4 from C, the magnitude of I would decrease to 0.46. This would give a rate about four times smaller than the reported experimental result. From a theoretical point of view it is inconsistent to omit the constant C because, as McClary showed,[7] it must be considered part of the scalar potential if a model such as ours is to fit the charmonium spectrum. If it were part of the vector potential it would not contribute to I_4, but then the relativistic corrections to the energy-level spectrum would make it impossible for the model to fit the experimental data. This discussion of our results for these allowed transitions ignores coupled-channel mixing effects. Using the results of Eichten *et al.*,[6] we estimate that these effects decrease the $J/\psi\rightarrow\eta_c\gamma$ amplitude by about 4%. We do not have an estimate of these effects for the $\psi'\rightarrow\eta_c'\gamma$ decay.

Our results for some hindered transition matrix elements are shown in Table II. The value of $|I|=0.116$ for the $2\,^3S_1\rightarrow1\,^1S_0$ $c\bar{c}$ transition is considerably larger than the experimental value of 0.047 ± 0.008 obtained from the measured rate for $\psi'\rightarrow\eta_c\gamma$.[8] Taking coupled-channel mixing into account as described above, we find that the magnitude of I is decreased to 0.032 which agrees reasonably well with the experimental value considering the crudeness of our estimation.

We thank J. Sucher and E. Eichten for very useful conversations. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AT03-81ER40024-Task B.

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