Creation of $q\bar{q}$ pairs in a chromoelectric flux tube

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Schwinger's result for the rate of pair creation in a uniform external field has been transcribed previously into QCD. We reexamine this problem. The mutual interaction of the pair and strict energy conservation (which were implicitly neglected by Schwinger), while generally negligible in QED, are of major importance in QCD. In the picture of strict color confinement in a tube we derive a new result which is free of these defects.

In a classic paper on quantum electrodynamics, Schwinger derived, among other things, the persistence of the vacuum against e^+e^- creation in a uniform external electric field.[1] It is implicit in his derivation that the mutual interaction of the pair is neglected and that the energy stored in the external field is large compared to $2m_e$. His explicit formula for the rate per unit volume that a e^+e^- pair will be created in the uniform field of strength ϵ is

$$p=\frac{(e\epsilon)^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-\pi m_e^2 n/e\epsilon) \quad , \tag{1}$$

where e is the electron charge and m_e its mass. Recently a number of authors have applied this result to hadronic production in high-energy e^+e^- annihilation.[2-4] In the annihilation event, a tube of color flux is supposed to be formed connecting a rapidly receding quark and antiquark. In this uniform color field a virtual $q\bar{q}$ pair may tunnel to a real state causing the tube to fission and thus creating mesons. The mass of the created quark appears explicitly in (1), thus giving different production rates for K and π.

However, there are essential differences between the QED and QCD processes of pair creation. While it is true in QED that Schwinger's approximations can be rendered as innocuous as one pleases because the external field is at the disposition of the experimenter, this is not true for QCD. The lines of color flux in QCD are believed to be confined to a narrow tube connecting opposite color charges. We adopt this picture quite literally. A virtual pair in such a field will generate a color field of the *same* strength as that in which they were born. Thus the mutual interaction is not negligible. Indeed, if it were negligible, as has been tacitly assumed in earlier work,[2] it would be inconsistent to assume that pair creation causes the fission of a color tube. It is precisely because the field is equal and oppositely directed to the original field that the region between the created pair is devoid of color flux and causes fission. Moreover, the energy of created pairs in a jet need not be negligible in comparison with the energy of the jet. Fortunately, in contrast with QED, the confinement of the color field allows us to account precisely for the mutual interaction and energy conservation (in the idealization of a uniform field in the tube). This we now do.

Suppose that a virtual pair appears spontaneously at some point in the flux tube. The quark and antiquark of this pair can have any combination of momenta, spins, flavors, and colors if they carry as a whole the same quantum numbers as the vacuum from which they emerge. This means that the spin, flavor, color charge, and momenta of each component of the pair should be opposite. Let the magnitude of the transverse momentum be denoted by p_T. (We shall use longitudinal to denote the orientation of the tube and transverse to denote an orthogonal direction.) First we calculate the probability that each component will tunnel from the virtual state to a real state having the same energy as the original. The logitudinal momentum of each component at the point where the virtual pair first appears must satisfy

$$p_L{}^2+p_T{}^2+m^2=0 \tag{2}$$

or

$$p_L=iE_T\equiv i(p_T{}^2+m^2)^{1/2} \quad . \tag{3}$$

As they move apart in the field of the tube their mutual interaction produces a field equal in magnitude but opposite in direction to the field in the tube, thus destroying the field between. After they have each moved a distance r in opposite directions from the point of first appearance, the energy balance reads

$$2[p_L{}^2(r)+p_T{}^2+m^2]^{1/2}=2\sigma r \quad . \tag{4}$$

The right-hand side is the energy of the field of the tube that is destroyed when the quark and antiquark each moves a distance r. The string constant σ is the energy per unit length stored in the field, and its value, deduced from Regge trajectories, is about 1 GeV/fm. The longitudinal momentum is therefore

$$p_L(r)=i[E_T{}^2-(\sigma r)^2]^{1/2}, \quad E_T=(p_T{}^2+m^2)^{1/2} \quad . \tag{5}$$

The action of both quarks integrated from the initial point to the point where they materialize given by $p_L(r)=0$ is

$$J=2\int_0^{E_T/\sigma}|p_L|dr=\frac{\pi E_T{}^2}{2\sigma} \quad . \tag{6}$$

The probability that a virtual pair can tunnel to a real state in the field of the tube, with each component having transverse momentum p_T is therefore

$$P(p_T)=|e^{-J}|^2=\exp(-\pi E_T{}^2/\sigma) \quad . \tag{7}$$

Knowing the correct tunneling probability from a virtual to a real state we can now calculate the probability that a pair will actually be created. Following Casher, Neuberger, and Nussinov, we compute the vacuum persistence probability, which is the probability that no such tunneling event for any spin, flavor, or transverse momentum has occurred at any point r in the tube at any time t during the existence of the tube,

$$\langle 0_+|0_-\rangle^2 = \prod_{\text{flavor}}\prod_{\text{spin}}\prod_{(\vec{p}_T)}\prod_r\prod_t [1-P(p_T)]$$
$$= \exp\left\{\sum\sum\sum\sum\sum \ln[1-P(p_T)]\right\} . \quad (8)$$

Let $L_xL_yL_zT$ be the space-time region of the tube in which no such event is supposed to have occurred. Let z be the longitudinal direction. Divide it into cells of length equal to that required for the materialization of a pair, which is, according to (5),

$$\Delta z = 2E_T/\sigma \ . \quad (9)$$

The time interval T is divided into cells according to the frequency with which such tunneling attempts can occur in accordance with the uncertainty principle,

$$\Delta t = \frac{2\pi}{\omega} = \frac{2\pi}{2E_T} = \frac{\pi}{E_T} \ . \quad (10)$$

Since $P(p_T)$ is independent of r, t, and spin, we obtain

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{\gamma \frac{L_z}{\Delta z}\frac{T}{\Delta t}\sum_f \int \frac{p_T dp_T d\phi}{(2\pi/L_x)(2\pi/L_y)} \ln[1-P(p_T)]\right\} = \exp(-L_xL_yL_zTp) \ , \quad (11)$$

where

$$p = -\gamma\frac{\sigma}{8\pi^2}\sum_{\text{flavor}}\int_{m_f^2}^{\infty} dE_T^2 \ln[1-P(p_T)]$$
$$= \frac{\sigma^2}{4\pi^3}\sum_{\text{flavor}}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-\pi m_f^2 n/\sigma) \ . \quad (12)$$

In the above $\gamma = 2$ is the spin degeneracy.

The above p may be interpreted as the rate at which a $q\bar{q}$ pair having opposite color charges $g/2$ are created per unit volume inside the tube containing the color field. Of course, it is independent of position in the tube and of time.

It is important to note that Casher *et al.* have demonstrated that Schwinger's exact QED treatment of the approximate problem (noninteracting e^+e^- pairs) is precisely obtained by a WKB calculation of the tunneling probability from a virtual to real state and the subsequent calculation of the persistence probability. This is reassuring because otherwise one would not have been sure that the enumeration of tunneling events employed above, involving the equality in the uncertainty relation (10) and the materialization distance (9), would have yielded the exact numerical factor in (12).

Now we wish to compare our new result (12), which does include the mutual interaction of the created pair, with the problem solved by Schwinger and by Casher *et al.* To do this, we adopt the Abelian version of Gauss's law used by Casher *et al.* to relate the color field strength ϵ, the cross section A of the flux tube, and the quark charge $g/2$,

$$\epsilon A = g/2 \ . \quad (13)$$

They also assume that the string tension is related to the energy density $\frac{1}{2}\epsilon^2$ by

$$\sigma = \tfrac{1}{2}\epsilon^2 A \ . \quad (14)$$

(This would be modified by the additon of the term BA on the right if one assumed that the flux is confined by an external pressure B.) These relations imply

$$\sigma = g\epsilon/4 \ . \quad (15)$$

Substituting into (12) our result reads

$$p = \frac{(g\epsilon)^2}{64\pi^3}\sum_f\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-4\pi m_f^2 n/g\epsilon) \ . \quad (16)$$

This has precisely the same form as Schwinger's formula (1) but differs in the numerical constants. This can be understood easily, because if the mutual interaction of the pair is neglected in QED the geometry of the constant external field is the same as that of the flux tube *whether or not* the mutual interaction is included in QCD.

As an additional note, if one conjectures that the color field is confined by an external pressure B, then

$$\sigma = \tfrac{1}{2}\epsilon^2 A + BA = \frac{1}{8}\frac{g^2}{A} + BA \rightarrow (g/2)\epsilon \ , \quad (17)$$

where the final result corresponds to the equilibrium value of A.

In view of our derivation of the probability for creation of interacting pairs in a flux tube, it becomes clear how the substitution procedure $e\epsilon \rightarrow \sigma$ in Schwinger's formula, employed in Refs. 3 and 4, which neglects the mutual interaction, nonetheless yields our result (12).

In summary, we have derived the probability per unit four-volume for creation of a quark-antiquark pair in a chromoelectric flux tube. Our result differs from Schwinger's because we are able to take into account the mutual interaction of the pair and strict energy conservation. While neglect of these in QED are justified for macroscopic electric fields, they are not justified in QCD. It is remarkable that with the assumption of confinement of the color field it is possible to solve a problem in QCD that has remained intractable in QED. It is, of course, precisely the confinement of color that allows an easy solution.

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