Quark anomalous magnetic moments and constraints on the composite structure of quarks

S. Fajfer*
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60201

R. J. Oakes
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
and Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60201
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The angular distribution of lepton pairs produced in hadronic interactions and the angular distribution of two-jet events in electron-positron annhihilation are shown to provide a possible means to measure the anomalous magnetic moment $\kappa=(g-2)/2$ of a confined quark. Present data imply that $\kappa \lesssim 10^{-4}$ for u and d quarks, compared to 10^{-8} for muons and 10^{-10} for electrons, leaving open the interesting possibility of a genre of models in which leptons behave as elementary Dirac particles while quarks may exhibit substructure throughout some intermediate energy regime. The magnitude of the anomalous magnetic moment one expects due to quantum-chromodynamic vertex corrections to a *confined* quark's electromagnetic interaction is estimated and found to be not much smaller than the present experimental upper limits on κ, suggesting that the first observation of $g-2$ for quarks may be feasible in the not too distant future.

A great deal of interest and speculation has been focused on the possibility that the quarks and leptons may be composite,[1] perhaps revealing some structure at distances smaller than have so far been probed, which correspond to an energy scale $\Lambda \sim 100$ to 200 GeV. The possibilities for speculation are very intriguing, indeed, particularly in the case of quarks, which themselves seem to exist only as confined constituents of the hadrons—the question of structure of a permanently confined object is a new concept in elementary-particle physics.

Precision measurements at low energies of the electron and muon anomalous magnetic moments provide strong contraints on composite models of these leptons. Although the analogous direct measurements of the magnetic moments of free quarks are not possible, even in principle, the structure of the electromagnetic interaction of confined quarks can be explored in the asymptotically free region.

In the following, we point out that the production of lepton pairs in hadronic interactions, in the regime where the Drell-Yan process is applicable, provides a particularly sensitive test for the presence of an anomalous Pauli term in the electromagnetic interactions of quarks. Specifically, the angular distribution of high-mass Drell-Yan lepton pairs produced at zero transverse momentum provides a direct probe of such an anomalous magnetic moment, at least for the u and d quarks. At zero transverse momentum the lepton-pair angular distribution is uncomplicated by either the intrinsic transverse-momentum distribution of the quarks or the quantum-chromodynamic radiative corrections, which are proportional to $Q_\perp^2$, making the test particularly clean.

We also show that the angular distribution of two-jet events in electron-positron annihilation at high energies, due to the process $e^+ + e^- \rightarrow q+\bar{q}$, is similarly affected by a quark anomalous magnetic moment. Present data, limiting the deviation from the usual $1+\cos^2\theta$ angular distribution, already impose a quite small bound on the quarks' anomalous magnetic moments.

Finally, we estimate the quark anomalous magnetic moment arising from quantum-electrodynamic (QED) and quantum-chromodynamic (QCD) radiative corrections. Comparing these estimates with the limits imposed by the present data indicates that current experiments are not far from observing the effect of a quark anomalous magnetic moment of the magnitude expected from the QCD vertex correction. Of course, only a disagreement between the observed quark anomalous magnetic moment and the theoretical prediction can be interpreted as an indication of quark compositeness. And, unfortunately, this could prove to be a far more delicate issue for confined quarks than for free electrons and muons, even when better data show evidence for the quark anomalous magnetic moment, since there is some inherent uncertainty in the theoretical calculations of the QCD (and QED) radiative corrections arising from our imperfect understanding of quark confinement.

The Drell-Yan mechanism[2] for the hadronic production of lepton pairs $(A+B \rightarrow l+\bar{l}+X_A+X_B)$ is shown in Fig. 1, where the particles and their momenta are labeled. The differential cross section in the center of mass for the subprocess $q(p)+\bar{q}(\bar{p}) \rightarrow l(k)+\bar{l}(\bar{k})$ is

$$\frac{d\sigma}{d\Omega}=\frac{e_q{}^2\alpha^2}{12Q^2}\left[G_M{}^2(Q^2)(1+\cos^2\theta)+\frac{4M^2}{Q^2}G_E{}^2(Q^2)\sin^2\theta\right], \tag{1}$$

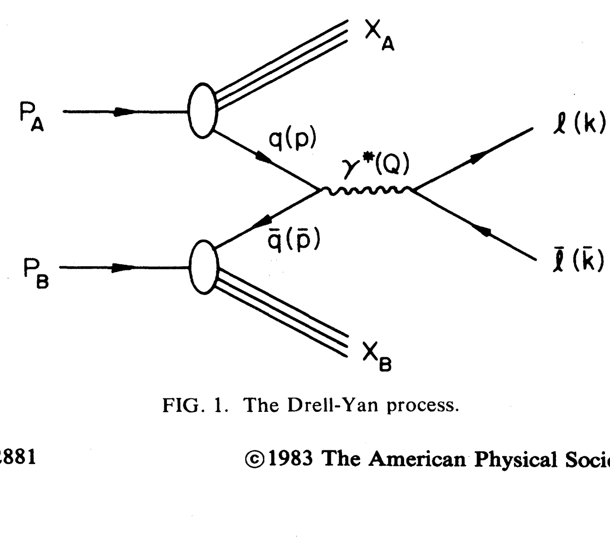

FIG. 1. The Drell-Yan process.

where e_q is the quark charge in units of e and M is the quark mass, presumably the running quark mass $M(Q^2)$. The quark electric and magnetic form factors are defined in terms of the Dirac and Pauli form factors as usual:

$$G_E(Q^2)=F_1(Q^2)+(Q^2/4M^2)\kappa F_2(Q^2) \ , \quad (2)$$

$$G_M(Q^2)=F_1(Q^2)+\kappa F_2(Q^2) \ . \quad (3)$$

In Eq. (1) the color factor $(\frac{1}{3})$ has been included for completeness even though we shall only be concerned with the *shape* of the lepton-pair angular distribution, which can be expressed in the form

$$\frac{dN}{d\Omega}=1+\alpha\cos^2\theta \ , \quad (4)$$

where

$$\alpha=\frac{G_M{}^2(Q^2)-(4M^2/Q^2)G_E{}^2(Q^2)}{G_M{}^2(Q^2)+(4M^2/Q^2)G_E{}^2(Q^2)} \ . \quad (5)$$

We emphasize that this angular distribution is independent of the normalization of the Drell-Yan cross section, which is not yet completely understood; and, in addition, we shall consider only zero transverse momentum, thereby avoiding the corrections proportional to $Q_\perp{}^2$ coming either from the quarks' intrinsic transverse-momentum distributions or QCD corrections.

In the absence of an anomalous Pauli term κF_2 one finds the familiar $1+\cos^2\theta$ angular distribution; i.e., $\alpha=1$, at $Q_\perp{}^2=0$ [provided, of course, that one is well above the threshold at $Q^2=4M^2$, where the kinematical constraint $G_E(4M^2)=G_M(4M^2)$ requires the angular distribution to be isotropic]. The present experimetnal data[3] on lepton-pair production at $Q_\perp{}^2=0$ are indeed consistent with a pure $1+\cos^2\theta$ angular distribution and it is the rather small experimental uncertainty in α that limits the magnitude of κF_2. For convenience in extracting limits on the anomalous Pauli interaction from the various experimental data available, we write Eq. (5) in the form

$$\alpha=\frac{1-\beta}{1+\beta} \ , \quad (6)$$

where we have defined

$$\beta=\frac{4M^2}{Q^2}\frac{\{1+(Q^2/4M^2)[\kappa F_2(Q^2)/F_1(Q^2)]\}^2}{1+\kappa F_2(Q^2)/F_1(Q^2)} \ . \quad (7)$$

Note that because $\alpha\leqslant 1$ [c.f. Eq. (5)] it necessarily follows that $\beta\geqslant 0$, which is also explicitly evident in Eq. (7); and since the data at $Q_\perp{}^2=0$ are consistent with $\alpha=1$, β cannot be very large.

For convenient, quantitative comparison with the data we have computed the ratio $\kappa F_2(Q^2)/F_1(Q^2)$ as a function of the dimensionless variable $Q^2/4M^2$ for various values of α. The numerical results for the representative values[3] $\alpha=0.6$ and $\alpha=0.8$ are shown in Fig. 2 for the range $10^2 < Q^2/4M^2 < 10^5$. Asymptotically, for very large $Q^2/4M^2$,

$$|\kappa F_2(Q^2)/F_1(Q^2)| < \left[\frac{4M^2}{Q^2}\beta\right]^{1/2} \ . \quad (8)$$

The present data, which come mostly from the region $Q^2>10$ GeV^2, indicate that $\alpha\gtrsim 0.8$. Assuming a (current) quark mass of $M=10$ MeV/:c^2 one then finds

$$|\kappa F_2(Q^2)/F_1(Q^2)|\lesssim 2\times 10^{-3} \quad (9)$$

which is presumably a limit on the anomalous magnetic moment $\kappa F_2(0)$ itself, since these values of Q^2 are certainly small compared to the scale $\Lambda\sim 100$ to 200 GeV of any finite-size effects.[4] As future experiments provide more precise data, Eqs. (7) and (8), as well as Fig. 2, will be very useful for extracting improved limits on an anomalous Pauli

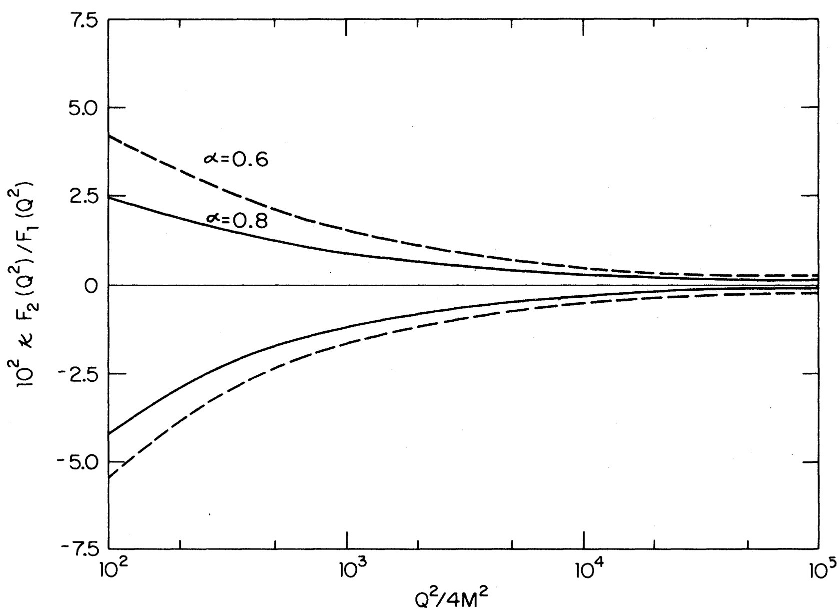

FIG. 2. The anomalous Pauli interaction strength $\kappa F_2(Q^2)/F_1(Q^2)$ as a function of $Q^2/4M^2$. The curves shown are the bounds for $\alpha=0.8$ (solid curves) and $\alpha=0.6$ (dashed curves), where α characterizes the angular distribution $1+\alpha\cos^2\theta$.

interaction. Note also that the u- and d-quark anomalous magnetic moments can be separately determined from experiments using π^- and π^+ beams, respectively.

In electron-positron annihilation the two-jet events, whose origin is the process $e^+ + e^- \to q + \bar{q}$, have the same angular distribution as given by Eqs. (4) and (5). Of course, the form factors in these equations now refer to whichever quark pair is responsible for the jets. Precisely as in the Drell-Yan process, discussed above, Eqs. (7) and (8), and/or Fig. 2, can be used to extract a limit on the quark anomalous magnetic moments from the experimental limits on deviations of the two-jet angular distribution from simply $1+\cos^2\theta$. At the highest PETRA energies, $Q^2 = (37\ \text{GeV})^2$ and the data[4] look completely consistent with pure $1+\cos^2\theta$, within the experimental uncertainties of about 15%. It is, no doubt, conservative to take $\alpha > 0.6$, or $\beta < 0.25$, for purposes of an illustrative numerical estimate. Moreover, if the two-jet events are dominated by the light u and d quarks, which seems reasonable, the appropriate running mass $M(Q^2)$ at such a large Q^2 should be close to the current mass $M_u = M_d = 10\ \text{MeV}/c^2$. Consequently, since even these values of Q^2 are well below the scale of any finite-size effects, one finds for the upper limit on κ for the u and d quarks

$$\kappa \lesssim 3\times 10^{-4} \ . \tag{10}$$

This bound is considerably stronger than was found above from the Drell-Yan data [c.f., Eq. (9)]. It is also comparable to the results of Silverman and Shaw,[5] who found limits of 3×10^{-4} and 6×10^{-4} for u and d quarks, respectively, from an analysis of R_{hadron} in electron-positron annihilation.

Instead of taking the numerical values of these limits too seriously at this point, we rather wish to emphasize the usefulness for the future of using the angular distribution of (i) lepton pairs in Drell-Yan production and (ii) two-jet events in electron-positron annihilation to investigate the anomalous magnetic moments of the quarks. However, our numerical estimates given above do indicate that present experiments are not too far from observing a quark magnetic moment at the level one expects from the vertex radiative corrections, which we discuss next.

The photon one-loop QED vertex correction gives the well-known result

$$\kappa_{\text{QED}} = (e_q)^2 \frac{\alpha}{2\pi} \ , \tag{11}$$

where e_q is the electric quark charge in units of e. The analogous gluon one-loop QCD vertex correction is

$$\kappa_{\text{QCD}} = \frac{4}{3}\frac{\alpha_s(Q^2)}{2\pi} \ , \tag{12}$$

where $\frac{4}{3}$ is the color factor. Since $\alpha_s(Q^2)$ is not so small, even at $Q^2 = (37\ \text{GeV})^2$, κ_{QCD} is quite large, exceeding the experimental upper limits by at least one, probably two, orders of magnitude. Even κ_{QED} is comparable to the present experimental upper bound. But, recall, these one-loop radiative corrections have been calculated for *free quarks*—and consequently are grossly misleading. For a *confined quark*, arbitrarily large wavelengths are not possible and therefore the loop integration should not extend to arbitrarily small momentum. Physically, one expects the effect of cutting off the loop integration to reduce the anomalous magnetic moments by a factor of the ratio of the quark mass to the hadron mass scale appropriate to the process being considered.[6] Thus the leading one-loop correction becomes

$$\kappa = \frac{4}{3}\frac{\alpha_s(Q^2)}{2\pi}\frac{M_{\text{quark}}}{M_{\text{hadron}}} \ . \tag{13}$$

It should be emphasized that κ is not entirely independent of the process in which it is observed, since the nature of the confinement of the quark enters in an essential way. Compare, for example, the Drell-Yan annihilation of quarks initially confined to pions or nucleons with quark hadronization into jets in electron-positron annihilation.

For a typical estimate of κ we might take $\alpha_s \simeq 0.2$, $M_{\text{quark}} \simeq 10\ \text{MeV}/c^2$, and $M_{\text{hadron}} \simeq 1\ \text{GeV}/c^2$, in which case

$$\kappa \simeq 4\times 10^{-4} \ , \tag{14}$$

which is comparable to the present experimental upper limits discussed above. While our estimates are somewhat crude, their order of magnitude is certainly correct, and one can reasonably expect to see the effects of the gluon one-loop QCD vertex correction to the quark anomalous magnetic moment as the experimental data improve in the near future.

Of course, to infer a composite structure of the quarks the observed anomalous magnetic moment must differ from the value given by the radiative vertex corrections due to the quarks' interactions. And, unfortunately, the one-loop corrections are complicated by quark confinement and will therefore be dependent somewhat on the quark-confinement model used to compute the vertex correction of the confined quark. Nevertheless, it is probably safe to conclude, at present, that there is no contribution to the quark anomalous magnetic moment due to composite structure at the level of 10^{-3} to 10^{-4} compared to 10^{-8} for muons and 10^{-10} for electrons. Therefore, one cannot rule out the interesting possiblity of composite quarks but elementary leptons throughout some intermediate energy regime.

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*On leave from the University of Sarajevo, Sarajevo, Yugoslavia.

[1]See, for example, the recent reviews of M. E. Peskin, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Bonn, 1981,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981), p. 880, and L. Lyons, Oxford University Report No. 52/82, 1982 (unpublished).

[2]S. D. Drell and T. -M. Yan, Phys. Rev. Lett. 25, 316 (1970); Ann. Phys. (N.Y.) 66, 578 (1971). For a recent review see the talk of

E. L. Berger at the Fermilab Workshop on Drell-Yan Processes, Fermilab, 1982 [Argonne Report No. ANL-HEP-CP-82-72 (unpublished)].

[3]For summaries see J. E. Pilcher, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, 1979,* edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1979) and the talk of A. Michelini at the European Physical Society International Conference on High Energy Physics, Lisbon, 1981 [CERN Report No. CERN-EP/81-128, 1981 (unpublished)].

[4]For a recent review of the experimental data see K. H. Mess and B. H. Wiik, DESY Report No. DESY 82-011 (unpublished).

[5]D. J. Silverman and G. L. Shaw, Phys. Rev. D 27, 1196 (1983).

[6]This argument is based on the very similar calculations of the QCD vertex corrections to the weak interactions of confined quarks due to A. Halprin, B. W. Lee, and P. Sorba, Phys. Rev. D 14, 2343 (1976).