Electromagnetic corrections to energy-energy correlations in high-energy e^-e^+ annihilation

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Electromagnetic corrections due to real-photon emission are computed to energy-energy correlations in high-energy e^-e^+ annihilation with γ and Z^0 exchanges.

Energy-energy correlations in e^-e^+ annihilation at high energy have been studied in Refs. 1 and 2. The new ingredient in these calculations as compared with the low-energy ones[3] is that the process proceeds through both γ and Z^0 exchanges. It has been shown[2] that the angle-integrated and normalized energy-energy correlations $\sigma_{\rm tot}^{-1}d^2\Sigma/d\cos\chi\, d\cos\vartheta$ and $\sigma_{\rm tot}^{-1}d\Sigma/d\cos\chi$ (see Fig. 1 for definitions of angles) are independent of initial-state polarizations and even more importantly do not depend on weak-interaction parameters such as the Z^0 mass and width as well as coupling constants.[4,5] Thus, they are ideal quantities to measure if the aim is to test QCD or hadron fragmentation.

The electromagnetic corrections to the energy correlation have been calculated in the low-energy region.[6] In particular, the dominant $O(\alpha\ln(W^2/m_e^2))$ correction has been determined and shown to contribute to the energy correlation. (W is the total center-of-mass energy and m_e is the electron mass.) The size of this largest contribution depends upon the experimental cuts made, but an overestimate (i.e., no cuts assumed) shows that it is generally less than 15% for the quantity $\sigma_{\rm tot}^{-1}d\Sigma/d\cos\chi$. It is an important open question how the low-energy result is modified if Z^0 exchange is also taken into account. The present paper is devoted to the study of this problem. We show that up to about $W\approx 50$ GeV Z^0 exchange does not modify the conclusions of Ref. 6, while in the Z^0 resonance region electromagnetic corrections can be disregarded. Furthermore, above Z^0 the electromagnetic corrections may be very large.

The lowest-order electromagnetic correction to the energy-energy correlation comes from the emission of a real photon by the initial electron or positron (see Fig. 2). Processes with virtual photons do not contribute, since we are interested only in angles χ different from 180°. As discussed in Ref. 6, because of collinear singularities this correction is $O(\alpha\ln(W^2/m_e^2))$. The emission of real photons by the final-state quarks should also be calculated in principle; however, it gives a much smaller correction of $O(\alpha)$.

The calculation of the contribution of Fig. 2 is in principle straightforward and may be performed by analytic methods. However, in practice it turns out to be rather complicated; therefore, in what follows we shall evaluate only the dominant [i.e. $O(\alpha\ln(W^2/m_e^2))$] corrections. Nevertheless, it is possible to start with the exact expressions and discuss the questions of Z^0 exchange in this framework.

The amplitude for the production of a $q\bar{q}$ pair with flavor f and a real photon is given by

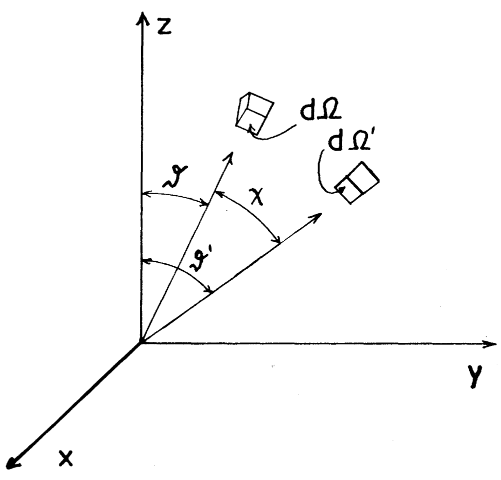

FIG. 1. Kinematics of the energy-energy correlation cross section. The e^- momentum is assumed to be parallel to the z axis, the e^+ momentum is in the opposite direction. The azimuthal angles ϕ and ϕ' are not shown in the figure.

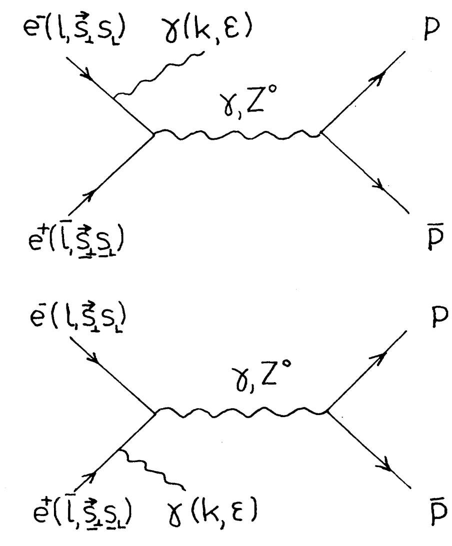

FIG. 2. Graphs representing the dominant electromagnetic corrections to the energy-energy correlation cross section.

$$T\propto e^2\bar{v}(\bar{l})\left[\gamma^\mu\frac{\gamma\cdot(l-k)}{2l\cdot k}\gamma\cdot\epsilon+\gamma\cdot\epsilon\frac{\gamma\cdot(k-\bar{l})}{2\bar{l}\cdot k}\gamma^\mu\right]u(l)\frac{g_{\mu\nu}}{q^2}\bar{u}(p)eQ_f\gamma^\nu v(\bar{p})$$
$$\times e\bar{v}(\bar{l})\left[\gamma^\mu(g_V+\gamma_5 g_A)\frac{\gamma\cdot(l-k)}{2l\cdot k}\gamma\cdot\epsilon+\gamma\cdot\epsilon\frac{\gamma\cdot(k-\bar{l})}{2\bar{l}\cdot k}\gamma^\mu(g_V+\gamma_5 g_A)\right]u(l)$$
$$\times\frac{g_{\mu\nu}}{q^2-M_Z{}^2+iM_Z\Gamma_Z}\bar{u}(p)\gamma^\nu(G_{Vf}+\gamma_5 G_{Af})v(\bar{p})\ , \quad (1)$$

with the notation of momenta explained in Fig. 2. M_Z and Γ_Z mean the Z^0 mass and width, respectively. The couplings are those of the standard model with the same notation as in Ref. 2. It is convenient to rewrite Eq. (1) as

$$a_{1f}v_\mu V^\mu+a_{2f}v_\mu A^\mu+a_{3f}a_\mu V^\mu+a_{4f}a_\mu A^\mu\ , \quad (2)$$

with

$$a_{1f}=\frac{e^2}{q^2}Q_f+\frac{g_V G_{Vf}}{q^2-M_Z{}^2+iM_Z\Gamma_Z}\ ,\quad a_{2f}=\frac{g_V G_{Af}}{q^2-M_Z{}^2+iM_Z\Gamma_Z}\ ,\quad a_{3f}=\frac{g_A G_{Vf}}{q^2-M_Z{}^2+iM_Z\Gamma_Z}\ ,\quad a_{4f}=\frac{g_A G_{Af}}{q^2-M_Z{}^2+iM_Z\Gamma_Z}\ , \quad (3)$$

and

$$v_\mu=e\bar{v}(\bar{l})\left[\gamma_\mu\frac{\gamma\cdot(l-k)}{2l\cdot k}\gamma\cdot\epsilon+\gamma\cdot\epsilon\frac{\gamma\cdot(k-\bar{l})}{2\bar{l}\cdot k}\gamma_\mu\right]u(l)\ ,\quad a_\mu=e\bar{v}(\bar{l})\left[\gamma_\mu\gamma_5\frac{\gamma\cdot(l-k)}{2l\cdot k}\gamma\cdot\epsilon+\gamma\cdot\epsilon\frac{\gamma\cdot(k-\bar{l})}{2\bar{l}\cdot k}\gamma_\mu\gamma_5\right]u(l)\ ,$$
$$V^\mu=\bar{u}(p)\gamma^\mu v(\bar{p})\ ,\quad A^\mu=\bar{u}(p)\gamma^\mu\gamma_5 v(\bar{p})\ . \quad (4)$$

There are two final states contributing to the energy-energy correlation; namely, it is possible that the quark (antiquark) moves in the direction $d\Omega$ ($d\Omega'$) and vice versa. Summing over the two final states as well as quark and antiquark spins we get

$$\sum V^\mu V^{\nu *}=\sum A^\mu A^{\nu *}\propto H^{\mu\nu}\ ,\quad \sum V^\mu A^{\nu *}=\sum A^\mu V^{\nu *}=0\ , \quad (5)$$

where

$$H^{\mu\nu}=p^\mu\bar{p}^\nu+p^\nu\bar{p}^\mu-p\cdot\bar{p}g^{\mu\nu}\ . \quad (6)$$

Then it follows that

$$|T|^2\propto[v_\mu v_\nu^*(|a_{1f}|^2+|a_{2f}|^2)+a_\mu a_\nu^*(|a_{3f}|^2+|a_{4f}|^2)+a_\mu v_\nu^*(a_{3f}a_{1f}^*+a_{4f}a_{2f}^*)+v_\mu a_\nu^*(a_{1f}a_{3f}^*+a_{2f}a_{4f}^*)]H^{\mu\nu} \quad (7)$$

and actually only the symmetric parts of the leptonic tensors $v_\mu v_\nu^*$, ... contribute. A sum over the photon polarizations ϵ_λ is performed; however, we keep the initial e^- and e^+ polarized. Throughout the calculation we put the lepton masses equal to zero except for the denominators ($l\cdot k$ and $\bar{l}\cdot k$) of the lepton tensors. The description of lepton polarization is taken over from Ref. 1. The e^- [e^+] polarizations are characterized by ($\vec{s}_\perp,s_L$) [$\vec{\underline{s}}_\perp,\underline{s}_L$)]. In order to get the energy correlation Eq. (7) should be integrated as

$$\int\frac{p^2dp}{(2\pi)^3}\frac{\bar{p}^2d\bar{p}}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\frac{1}{2k^0}|T|^2(2\pi)^4\delta^4(p+\bar{p}+k-l-\bar{l})\ . \quad (8)$$

We obtain

$$\frac{d\Sigma^{\rm EM}}{d\Omega\, d\Omega'}\propto\int_0^{W/2}\frac{\bar{p}^2d\bar{p}}{(2\pi)^5}\frac{p^2}{2[W+\bar{p}(\cos\chi-1)]}|T|^2\ , \quad (9)$$

where

$$p=\frac{W^2-2\bar{p}W}{2[W+\bar{p}(\cos\chi-1)]} \quad (10)$$

and $\vec{k}=-\vec{p}-\vec{\bar{p}}$ must be set everywhere.[7]

$|T|^2$ is a rather complicated expression; therefore, the analytic integration of Eq. (9) is difficult. In order to obtain simpler results let us define

$$\frac{d^3\Sigma}{d\cos\bar{\chi}\, d\cos\vartheta\, d\cos\vartheta'}=\int\frac{d\Sigma}{d\Omega\, d\Omega'}\delta(\cos\bar{\chi}-\cos\chi)d\phi\, d\phi'\ , \quad (11)$$

where

$$\cos\chi=\cos\vartheta\cos\vartheta'+\sin\vartheta\sin\vartheta'\cos(\phi-\phi')\ . \quad (12)$$

$d^3\Sigma^{\rm EM}/d\cos\bar{\chi}\, d\cos\vartheta\, d\cos\vartheta'$ may be written down in the form of an integral over the variable $\bar{p}$. Namely, substituting Eqs. (7) and (9) into Eq. (11), the ϕ,ϕ' integrations may be easily carried out. ϕ and ϕ' dependences occur only in $\delta(\cos\bar{\chi}-\cos\chi)$ and in $|T|^2$ in the scalar products $\vec{s}_\perp\cdot\vec{p}$, $\vec{s}_\perp\cdot\vec{\bar{p}}$, $\vec{\underline{s}}_\perp\cdot\vec{p}$, and $\vec{\underline{s}}_\perp\cdot\vec{\bar{p}}$ ($\vec{k}=-\vec{p}-\vec{\bar{p}}$). The denominators $l\cdot k$ and $\bar{l}\cdot k$ do not depend on ϕ and ϕ'. Calculating the traces as well as the ϕ,ϕ' integrals the transverse polarizations drop out and the longitudinal polarization dependence is simple. Putting in all the factors we get

$$\frac{d^3\Sigma^{\rm EM}}{d\cos\chi\, d\cos\vartheta\, d\cos\vartheta'}=\frac{\alpha}{W^4}\int_0^{W/2}\bar{p}^2d\bar{p}\frac{p^2}{[W+\bar{p}(\cos\chi-1)]}P(q^2)[F(l,\bar{l},p,\bar{p},k)+F(\bar{l},l,p,\bar{p},k)+F(l,\bar{l},\bar{p},p,k)+F(\bar{l},l,\bar{p},p,k)]\ . \quad (13)$$

Here $q^2=(p+\bar{p})^2$,

$$P(q^2)=\sum_f [(1-s_L\underline{s}_L)(|a_{1f}|^2+|a_{2f}|^2+|a_{3f}|^2+|a_{4f}|^2)+2(\underline{s}_L-s_L)\,\mathrm{Re}(a_{3f}a_{1f}^*+a_{4f}a_{2f}^*)] \quad (14)$$

and

$$F(l,\bar{l},p,\bar{p},k)=\{p\cdot k\bar{p}\cdot\bar{l}k\cdot\bar{l}+p\cdot l[l\cdot\bar{l}\bar{p}\cdot(\bar{l}-k)+\bar{p}\cdot l\bar{l}\cdot k-\bar{p}\cdot\bar{l}l\cdot k]\}/(k\cdot lk\cdot\bar{l}) \ . \quad (15)$$

The total cross section σ_{tot} has a similar dependence on the longitudinal polarizations:

$$\sigma_{tot}=\frac{W^2}{4\pi}P(W^2)A(W) \ , \quad (16)$$

where $A(W)=1+\alpha_s(W)/\pi$ to low orders in QCD. (α_s is the running strong coupling.) In contrast to the purely hadronic case, the propagator factor $P(q^2)$ containing also the polarization dependence does not drop out of the normalized quantity.

Using the approximation scheme of Barr and Brown,[6] we may get an estimate for the size of the electromagnetic corrections. In this dominant approximation the events are coplanar, i.e.,

$$\frac{d\Sigma^{EM}}{d\Omega\, d\Omega'}\propto\delta(\phi'-\phi-\pi) \ . \quad (17)$$

Going over to the triple-differential distribution we get

$$\frac{1}{\sigma_{tot}}\frac{d^3\Sigma^{EM}}{d\cos\chi\, d\cos\vartheta\, d\cos\vartheta'}=\frac{3\alpha}{64\pi^2}\ln\left[\frac{W^2}{m_e^2}\right]2\pi\delta(\cos\chi-\cos(\vartheta+\vartheta'))\times\frac{1}{\cos\frac{\chi}{2}}\frac{1}{\sin^6\frac{\chi}{2}}\frac{\left[\sin^2\frac{\chi}{2}+\sin^2\frac{\vartheta'-\vartheta}{2}\right]\left[\cos^2\frac{\chi}{2}+\cos^2\frac{\vartheta'-\vartheta}{2}\right]}{\left[\cos\frac{\chi}{2}+\cos\frac{\vartheta'-\vartheta}{2}\right]^3}\frac{(q^2)^2P(q^2)}{W^4P(W^2)} \ . \quad (18)$$

Under Eq. (17) the value of q^2 is completely fixed by the angles ϑ,ϑ', and W:

$$q^2=\frac{2W^2\sin\vartheta\sin\vartheta'(1-\cos\chi)}{(\sin\vartheta+\sin\vartheta'+\sin\chi)^2} \ . \quad (19)$$

Assuming no cuts on the final-state energies (yielding an overestimate of a true experimental situation), we may integrate Eq. (18) to determine

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma^{EM}}{d\cos\chi}=\int d\cos\vartheta\, d\cos\vartheta'\frac{1}{\sigma_{tot}}\frac{d^3\Sigma^{EM}}{d\cos\chi\, d\cos\vartheta\, d\cos\vartheta'} \ . \quad (20)$$

Note that it is $\sigma_{tot}^{-1}d\Sigma/d\cos\chi$ which is measured in experiments.[8] The resulting one-dimensional integral may be performed analytically. However, considering its intricacy, we choose a numerical integration for the unpolarized case only.

Putting in $\sin^2\theta_W=0.23$ and three families, the function $P(q^2)$ is completely determined (in the standard model). The only modification as compared with the case of the pure γ exchange is the factor $(q^2)^2P(q^2)/W^4P(W^2)$ in Eq. (18). For a fixed χ the allowed values of q^2 are in the range

$$\left[0,W^2\frac{\sin^2\frac{\chi}{2}}{\left[1+\cos\frac{\chi}{2}\right]^2}\right] \ , \quad (21)$$

which means that even the maximum value q_{max}^2 is usually much smaller than W^2.

For $W=30$ GeV the factor $(q^2)^2P(q^2)/W^4P(W^2)$ is essentially constant and equals to unity. Therefore, the results of Ref. 6—in particular Eq. (1.9) and Fig. 4—remain unchanged; weak-interaction effects are completely negligible.

The situation changes dramatically for $W\approx M_Z$. Here $W^4P(W^2)$ takes on its maximum value, while $(q^2)^2P(q^2)$ is much smaller. The result is reduction by about a factor of 10^{-3} of the electromagnetic correction at $W=30$ GeV, so that in the Z^0 resonance region the $O(\alpha\ln(W^2/m_e^2))$ electromagnetic correction is totally negligible. The nondominant corrections are also reduced by the same mechanism; therefore, in the Z^0 region the total correction should be negligible.

For $W>M_Z$ a reversed situation is possible, i.e., $(q^2)^2P(q^2)/W^4P(W^2)\gg 1$. This results in a peak in the W dependence of $\sigma_{tot}^{-1}d\Sigma^{EM}/d\cos\chi$. The position of this

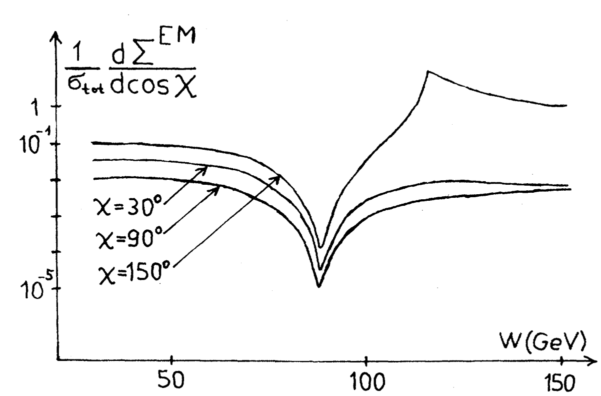

FIG. 3. Energy dependence of the electromagnetic correction to the normalized energy-energy correlation at various χ values.

peak (W_{peak}) is χ dependent. A crude estimate gives

$$W_{\text{peak}} = (\sqrt{q_{\max}{}^2}/W_{\text{peak}})^{-1} M_Z = \left(1 + \cos\frac{\chi}{2}\right) M_Z \Big/ \left(\sin\frac{\chi}{2}\right) , \tag{22}$$

i.e., we are at the energy W_{peak}, when $q_{\max}{}^2$ equals $\sim M_Z{}^2$. It is clear that for larger χ W_{peak} is smaller. Figure 3 shows the energy dependence of $\sigma_{\text{tot}}{}^{-1} d\Sigma^{\text{EM}}/d\cos\chi$ for three characteristic angles. At W_{peak} the electromagnetic correction turns out to be very large. For comparison we note that $\sigma_{\text{tot}}{}^{-1} d\Sigma^{\text{EM}}/d\cos\chi$ calculated with only γ exchange is a logarithmically growing smooth function of W.

In conclusion, electromagnetic corrections to high-energy hadronic energy-energy correlations have been determined. Numerical results are obtained in the dominant approximation of Ref. 6. At energies $W < 50$ GeV Z^0 exchange does not change the results calculated with pure γ exchange in Ref. 6. In the Z^0 region the correction is negligible. For higher energies it may be very large, especially at the χ-dependent W_{peak} energies.

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[2]F. Csikor, G. Pócsik, and A. Tóth, Phys. Rev. D 28, 1206 (1983).

[3]C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. Lett. 41, 1585 (1978); Phys. Rev. D 19, 2018 (1979); L. S. Brown and S. D. Ellis, *ibid.* 24, 2382 (1981).

[4]For the definition of kinematics see Fig. 1.

[5]By the methods of Ref. 2 it is easy to show that already $\sigma_{\text{tot}}{}^{-1} d^3\Sigma/d\cos\chi\, d\cos\theta\, d\cos\theta'$ is independent of initial-state polarizations as well as weak-interaction parameters.

[6]S. M. Barr and L. S. Brown, Phys. Rev. D 25, 1229 (1982).

[7]If experimental cuts on the energies are allowed for, the integration region in Eq. (10) should be modified accordingly.

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