Brief Reports

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Neutrino and antineutrino elastic scattering and photoneutrino reactions

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(Received 31 January 1983; revised manuscript received 9 September 1983)

The processes $\nu\nu\to\nu\nu$, $\nu\bar{\nu}\to\nu\bar{\nu}$, $\nu\bar{\nu}\to 2\gamma$, and $\nu\gamma\to\nu\gamma$, which are of astrophysical interest, are calculated both in the electroweak model and in the magnetic model when the neutrino has a small magnetic moment κ.

In this work we study the elastic processes $\nu+\nu\to\nu+\nu$ and $\nu+\bar{\nu}\to\nu+\bar{\nu}$ as well as the photoneutrino processes $\nu+\bar{\nu}\to\gamma+\gamma$ and $\nu+\gamma\to\nu+\gamma$ both in the standard electroweak model (whenever applicable) and in the electromagnetic model when the neutrino has very small anomalous magnetic moment κ. All these very weak interactions are of astrophysical interest.[1]

In a recent work we have investigated[2] the neutral-current process $e+\nu\to e+\nu$ and the neutrino-antineutrino annihilation process $\nu+\bar{\nu}\to e^{+}+e^{-}$ for both models mentioned above. A small magnetic moment for the neutrino of about $\kappa\sim 10^{-9}\ \mu_B$, where μ_B is the Bohr magneton, reproduces the same total cross section as the electroweak model. The angular and energy distributions are, however, different, so that experiments on these distributions, when they become available, can provide better limits on the magnetic moment of the neutrino and provide tests between the two models.

The process $\nu+\bar{\nu}\to 2\gamma$ vanishes for local coupling for two-component neutrinos.[3] This process has recently been considered as a higher-order process in the electroweak model assuming that the neutrino has a mass,[4] and the cross section is proportional to the neutrino mass squared. In contrast, magnetic-moment interaction exists also for zero-mass, four-component neutrinos. Furthermore, whereas the electron-neutrino processes show the relation $\kappa\sim G_F$, where G_F is the Fermi coupling constant, the neutrino-neutrino processes indicate $\kappa^2\sim G_F$. All these provide interesting new tests for the models. Bandyopadhyay, Chaudhuri, and Saha[5] have assumed a new *ad hoc* $\nu\gamma$ coupling to study the photoneutrino processes, but this model seems to be excluded astrophysically.[6]

NEUTRINO-NEUTRINO SCATTERING

In the electroweak model the process $\nu+\nu\to\nu+\nu$ goes via a Z_0 exchange in the t channel (amplitude M_1) and we have to antisymmetrize it with respect to the exchange of final particles (i.e., the u-channel amplitude M_2). Thus we have

$$\sum_{\text{all spins}}(M_1-M_2)(M_1-M_2)^{\dagger}=\sum[(M_1M_1^{\dagger}+M_2M_2^{\dagger})-(M_1M_2^{\dagger}-M_2M_1^{\dagger})]$$
$$=\frac{g^4}{(16\cos^2\theta)^2}\left[\frac{1}{(t-M_Z{}^2)^2}\sum I_1I_1^{\dagger}+\frac{1}{(u-M_Z{}^2)^2}\sum I_2I_2^{\dagger}-\frac{1}{(t-M_Z{}^2)(u-M_Z{}^2)}\sum(I_1I_2^{\dagger}+I_2I_1^{\dagger})\right],$$

where

$$I_1=\bar{u}_3\gamma_\mu(1-\gamma_5)u_1\bar{u}_4\gamma^\mu(1-\gamma_5)u_2\ ,$$
$$I_2=\bar{u}_4\gamma_\mu(1-\gamma_5)u_1\bar{u}_3\gamma^\mu(1-\gamma_5)u_2\ .$$

This gives us

$$\sum I_1I_1^{\dagger}=\frac{16}{m^4}(p_1\cdot p_2)(p_4\cdot p_3)\ ,$$
$$\sum I_2I_2^{\dagger}=\frac{16}{m^4}(p_1\cdot p_2)(p_3\cdot p_4)\ ,$$
$$\sum(I_1I_2^{\dagger}+I_2I_1^{\dagger})=-\frac{32}{m^4}(p_1\cdot p_2)(p_3\cdot p_4)\ .$$

Consequently, the differential cross section in the limit $m\to 0$ is given by

$$\frac{d\sigma^{\nu\nu}}{dt}=\left[\frac{g}{4\cos\theta}\right]^4\frac{2}{\pi}\left[\frac{s+2M_Z{}^2}{(t-M_Z{}^2)(s+t+M_Z{}^2)}\right]^2\ ,\qquad(1)$$

where $s=(p_1+p_2)^2$, $t=(p_3-p_1)^2$, $u=(p_4-p_1)^2$, and $g/4\cos\theta$ is the $Z_0\nu\bar{\nu}$-vertex coupling constant with θ being the Weinberg angle.

In the model with a neutrino magnetic moment we have first, if the two neutrinos are different, i.e., for $\nu_1+\nu_2\to\nu_1+\nu_2$, a single t-channel diagram so that the amplitude is given by

$$M=\frac{\kappa_1\kappa_2}{t}\bar{u}_3\sigma_{\mu\nu}(1-\gamma_5)u_1\bar{u}_4\sigma^{\mu\rho}(1-\gamma_5)u_2k^\nu k_\rho \ . \tag{2}$$

We then evaluate

$$\sum_{\text{all spins}} MM^\dagger=\frac{\kappa_1^2\kappa_2^2}{t^2}\frac{1}{4m_1^2m_2^2}\mathrm{Tr}[\not{p}_3\sigma_{\mu\nu}\not{p}_1\sigma_{\mu'\nu'}(1+\gamma_5)]\,\mathrm{Tr}[\not{p}_4\sigma^{\mu\rho}\not{p}_2\sigma^{\mu'\rho'}(1+\gamma_5)]k^\nu k^{\nu'}k_\rho k_{\rho'} \ ,$$

which, in the limit of very small neutrino masses $m_1\rightarrow 0$, $m_2\rightarrow 0$, becomes

$$\sum MM^\dagger=\frac{\kappa_1^2\kappa_2^2}{m_1^2m_2^2}(2s+t)^2 \ ,$$

so that the differential cross section is

$$\frac{d\sigma^{\nu_1\nu_2}}{dt}=\frac{\kappa_1^2\kappa_2^2}{8\pi}\frac{1}{s^2}(2s+t)^2 \ , \tag{3}$$

and the total cross section becomes

$$\sigma^{\nu_1\nu_2}=\frac{7\kappa_1^2\kappa_2^2}{24\pi}s \ . \tag{4}$$

In the case of two identical neutrinos $\nu+\nu\rightarrow\nu+\nu$ we have to subtract the u-channel amplitude M' so that

$$\sum(M-M')(M-M')^\dagger=\frac{\kappa^4}{k^4}\sum II^\dagger+\frac{\kappa^4}{l^4}\sum I'I'^\dagger-\frac{\kappa^4}{k^2l^2}\sum(II'^\dagger+I'I^\dagger)$$
$$=\frac{\kappa^4}{m^4}[(2s+t)^2+(s-t)^2+2(2s+t)(s-t)]=\frac{\kappa^4}{m^4}(3s)^2 \ .$$

The differential and the total cross sections are then

$$\frac{d\sigma^{\nu\nu}}{dt}=\frac{9\kappa^4}{8\pi} \ , \tag{5}$$

$$\sigma^{\nu\nu}=\frac{9\kappa^4}{8\pi}s \ . \tag{6}$$

It is interesting that both the energy dependence of $d\sigma/dt$ at fixed t and the t dependence at fixed s are qualitatively the same in both models: The asymptotic values of $d\sigma/dt$ for large s, at fixed t, are $g^4/[(16\cos^2\theta)^2 2\pi t^2]$ and $g\kappa^4/8\pi$, respectively. For fixed s, $d\sigma/dt$ varies slightly as a function of t, and has essentially the same behavior in both models.

NEUTRINO-ANTINEUTRINO SCATTERING

Again we begin with the electroweak model. There are two diagrams, Z_0 exchange in the t channel (amplitude M_1) and Z_0 exchange in the s channel (amplitude M_2), with

$$M_1=\frac{ig^2}{16\cos^2\theta}\frac{1}{k^2-M^2}[\bar{u}_3\gamma_\mu(1-\gamma_5)u_1\bar{v}_2\gamma^\mu(1-\gamma^5)v_4-\frac{1}{m^2}\bar{u}_3\not{k}(1-\gamma_5)u_1\bar{v}_2\not{k}(1-\gamma_5)v_4] \ ,$$
$$M_2=\frac{ig^2}{16\cos^2\theta}\frac{1}{q^2-M^2}[\bar{u}_3\gamma_\mu(1-\gamma_5)v_4\bar{v}_2\gamma^\mu(1-\gamma_5)u_1-\frac{1}{M^2}\bar{u}_3\not{q}(1-\gamma_5)v_4\bar{v}_2\not{q}(1-\gamma_5)u_1] \ .$$

We then evaluate

$$\sum_{\text{all spins}}(M_1-M_2)(M_1-M_2)^\dagger$$
$$=\frac{g^4}{64\cos^4\theta}\frac{16}{m^4}(s+t)^2\left[\frac{s+t-2M^2}{(t-M^2)(s-M^2)}\right]^2 \ ,$$

where we have neglected small terms involving $1/M_Z{}^2$, and obtain

$$\frac{d\sigma^{\nu\bar{\nu}}}{dt}=\left[\frac{g}{4\cos\theta}\right]^4\frac{2}{\pi}\left[\frac{(s+t)(s+t-2M^2)}{(t-M^2)s(s-M^2)}\right]^2 \ . \tag{7}$$

In the magnetic-moment model we have also two diagrams, photon exchange in the t channel (amplitude M_1') and in the s channel (amplitude M_2'), where

$$M_1'=\frac{-i\kappa^2}{k^2}[\bar{u}_3\sigma_{\mu\nu}(1-\gamma_5)u_1]$$
$$\times[\bar{v}_2\sigma^{\mu\rho}(1-\gamma_5)v_4]k^\nu k_\rho \ ,$$
$$M_2'=\frac{-i\kappa^2}{q^2}[\bar{v}_2\sigma_{\mu\nu}(1-\gamma_5)u_1]$$
$$\times[\bar{u}_3\sigma^{\mu\rho}(1-\gamma_5)v_4]q^\nu q_\rho \ .$$

After trace calculations we obtain

$$\sum_{\text{all spins}}(M_1-M_2)(M_1-M_2)^\dagger=\frac{9\kappa^4}{m^4}(s+t)^2 \ ;$$

hence the differential cross section becomes ($m \to 0$)

$$\frac{d\sigma^{\nu\bar{\nu}}}{dt}=\frac{9\kappa^4}{8\pi}\frac{1}{s^2}(s+t)^2 \ . \tag{8}$$

Then the total cross section is

$$\sigma^{\nu\bar{\nu}}=\frac{3\kappa^4}{8\pi}s \ . \tag{9}$$

Again the behavior of $d\sigma/dt$ in both t and s is surprisingly similar in both models.

NEUTRINO-PHOTON PROCESSES

The neutrino-antineutrino annihilation process $\nu_L+\nu_R \to 2\gamma$ is allowed for a four-component neutrino with an anomalous magnetic moment κ. There is a t-channel amplitude M_1 plus a u-channel amplitude M_2 given by

$$M_1=\frac{2i\kappa^2}{4k^2}\bar{v}_2\sigma_{\nu\rho}\not{k}\,\sigma_{\mu\lambda}(1-\gamma_5)u_1 p_4^{\rho}p_3^{\lambda}\epsilon_{(4)}^{\nu}\epsilon_{(3)}^{\mu} \ ,$$

$$M_2=\frac{2i\kappa^2}{4l^2}\bar{v}_2\sigma_{\nu\rho}\not{l}\,\sigma_{\mu\lambda}(1-\gamma_5)u_2 p_3^{\rho}p_4^{\lambda}\epsilon_{(3)}^{\nu}\epsilon_{(4)}^{\mu} \ ,$$

where p_3 and p_4 are the momenta of the neutrinos, k and l those of intermediate neutrinos. The trace calculation gives

$$\sum_{\text{all spins}}(M_1+M_2)(M_1+M_2)^{\dagger}=\frac{2\kappa^4}{m_\nu^{\,2}}t(t+s) \ ,$$

hence the differential and total cross sections are

$$\frac{d\sigma}{dt}=\frac{\kappa^4}{16\pi}\frac{1}{s^2}t(t+s) \tag{10}$$

and

$$\sigma^{\nu\bar{\nu}\to 2\gamma}=\frac{\kappa^4}{96\pi}s \ . \tag{11}$$

If we take $\kappa=10^{-9}\mu_B$, the numerical value of the total cross section is

$$\sigma^{\nu\bar{\nu}\to 2\gamma}=(5\times 10^{-87}\text{cm}^4)s \ ,$$

or

$$\sigma \cong 10^{-65}\ \text{cm}^2 \text{ at } s=1\ \text{MeV}^2,$$

$$\sigma \cong 10^{-59}\ \text{cm}^2 \text{ at } s=1\ \text{GeV}^2 \ .$$

For the neutrino Compton scattering $\nu+\gamma\to\nu+\gamma$ we evaluate the two usual Compton diagrams in the s and u channels, but now with the Pauli coupling. The amplitudes are

$$M_1=\frac{2i\kappa^2}{4q^2}\bar{u}_3\sigma_{\nu\rho}\not{q}\,\sigma_{\mu\lambda}(1-\gamma_5)u_1 p_4^{\rho}p_2^{\lambda}\epsilon_{(4)}^{\nu}\epsilon_{(2)}^{\nu} \ ,$$

$$M_2=\frac{2i\kappa^2}{4l^2}\bar{u}_3\sigma_{\nu\rho}\not{l}\,\sigma_{\mu\lambda}(1-\gamma_5)u_1 p_2^{\rho}p_4^{\lambda}\epsilon_{(2)}^{\nu}\epsilon_{(4)}^{\mu} \ .$$

Consequently,

$$\sum_{\text{all spins}}(M_1+M_2)(M_1+M_2)^{\dagger}=\frac{2\kappa^4}{m_\nu^{\,2}}s(s+t) \ ,$$

so that we obtain the following differential and total cross sections:

$$\frac{d\sigma}{dt}=\frac{\kappa^4}{16\pi}\frac{1}{s}(s+t) \tag{12}$$

and

$$\sigma^{\nu\gamma\to\nu\gamma}=\frac{\kappa^4}{32\pi}s \ , \tag{13}$$

which is again of the order of 10^{-65} cm^2 at $s=1$ MeV^2, and about 10^{-59} cm^2 at $s=1$ GeV^2. These are all reasonable numbers from the astrophysical point of view.

This work was supported in part by the Scientific and Technical Council for Research of Turkey, and by a NATO Research Grant. We would like to thank M. Abak and S. A. Baran for valuable discussions.

[1]See, for example, M. Abak, Lett. Nuovo Cimento 13, 49 (1975), and references therein.
[2]A. O. Barut, Z. Z. Aydin, and I. H. Duru, Phys. Rev. D 26, 1794 (1982).
[3]M. Gell-Mann, Phys. Rev. Lett. 6, 70 (1961).
[4]R. J. Crewter, J. Finjord, and P. Minkowski, Nucl. Phys. B207, 269 (1982).
[5]P. Bandyopadhyay, P. R. Chaudhuri, and S. K. Saha, Phys. Rev. D 1, 377 (1970); P. R. Chaudhuri, Can. J. Phys. 48, 944 (1970).
[6]R. B. Stothers, Phys. Rev. D 2, 1417 (1970).