Boosting the bag

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The Lorentz boost transformation for the soliton bag model is constructed in the mean-field approximation. Assuming that the standard static-bag wave function is equivalent to the zero-momentum eigenstate, the Lorentz boost transformation generates quark wave functions corresponding to a moving bag. We demonstrate that the mean-field energy and momentum of such a moving bag are related to the energy of the bag at rest by the correct Lorentz-transformation formulas. The boosted quark wave functions are used to investigate relativistic recoil corrections to hadronic form factors. Electromagnetic and axial-vector form factors are discussed, as well as the pion-nucleon vertex corresponding to a hybrid chiral bag model. Various types of bags (e.g., MIT type and SLAC type) are considered. It is shown that ignoring quark binding in the boost transformation leads to an overestimate of recoil corrections to magnetic moments. Relativistic recoil generally hardens the form factors. The proton charge radius is decreased by about 15% compared to the static-approximation values. The magnetic moment is changed by less than 10%. The inclusion of recoil corrections leads to reasonable agreement with experiment for all form factors considered if the bag radius (for an MIT bag) is about 1.3 fm. A comparison is made with other works on this topic.

I. INTRODUCTION

It is a generally accepted view that quantum chromodynamics[1] (QCD) is the correct theory of strong interactions. In QCD, hadrons are made up of colored fermions in such a way that the hadron is a color singlet. Since color is not observed experimentally in isolation, it is necessary to require that the color degrees of freedom be confined. Although there is theoretical evidence that this occurs in QCD, the details of the confinement mechanism are still poorly understood.[2] In order to bypass this problem, hadronic models which postulate confinement *a priori* have been developed. The most popular such models fall under the general denomination of bag models and are characterized either by the postulation of a finite space-time region to which the quark and gluon fields are confined (MIT bag model[3]) or by the introduction of an effective confining scalar field treated in a nonperturbative fashion (soliton bag model[4,5]). In both cases, these devices are thought to approximate long-range QCD effects and it is assumed that short-range QCD effects can be treated perturbatively.

In practical applications of these models, a static approximation is usually made in the calculation of form factors and other properties. Of course, such an approximation ignores recoil and it is desirable to improve upon it. Moreover, the study of multihadron phenomena, such as nucleon-nucleon scattering and nuclear structure, in terms of quarks, requires a description of moving bags.

The description of moving bags requires (a) the knowledge of a momentum eigenstate in the theory (translational invariance) and (b) the transformation of this state from one Lorentz frame into another (covariance). Several approaches to these problems have been proposed, which address either or both of these questions. The first is a projection method[6] borrowed from nonrelativistic nuclear theory. Although it has the advantage of yielding momentum eigenstates, it takes no account of the Lorentz-transformation properties of states. Another interesting approach[7] based on the intuitive idea of relativistic center of mass has been used to calculate recoil corrections to charge radii. Yet other proposals[8] use a relativistic *free-particle* boost applied to static bag states; the corrections to calculated magnetic moments are found to be significant. However, as we shall demonstrate here, the weak-binding assumption which is implicit in this description does not seem justified for confined systems. The question of how to boost bags is addressed more carefully in Ref. 9. However, the approach used there seems limited to the calculation of electromagnetic form factors in the MIT bag model.

In this study, we construct the relativistic boost operator in the mean-field approximation (MFA) to the soliton bag model and use it to obtain recoil corrections to magnetic moments, charge radii, and axial-vector form factors. This development sheds light on the connection between the last three approaches and improves upon each of them. Also, the use of the soliton model allows us to present calculations of form factors for different types of bags. We study the effect of surface versus volume confinement and of bag-surface diffuseness. It should be emphasized that our calculations are based on the assumption that the usual static-bag wave function can be identified with the zero-momentum eigenstate. Although this provides a basis for numerical calculations, it is clearly not correct. Thus the average momentum associated with our boosted states is correctly related to their mass, but these states *are not eigenstates of the momentum operator.* The question of constructing such eigenstates is not addressed here but is left for a future publication. Therefore, no obvious comparison with the first approach mentioned above is possible at this stage.

This paper is organized as follows. In Sec. II, we introduce the boost operator and remind the reader of some of its general properties. The MFA for a soliton bag at rest is summarized in Sec. III. In Sec. IV, we construct the MFA boost and use it to obtain quark wave functions corresponding to a moving bag. This section also contains a brief discussion of some properties of moving bag states. The general form of the recoil-corrected quark-current matrix elements in MFA is discussed in Sec. V. In Sec. VI, we give explicit formulas for the electromagnetic and axial-vector form factors in terms of quark bag wave functions. The large-q^2 behavior of these form factors is compared to that obtained in the static approximation and to QCD predictions. Numerical calculations of these form factors for a variety of bag types are presented in Sec. VII; charge radii and magnetic moments are discussed. Finally, in Sec. VIII, we summarize our results and compare them to those obtained using other approaches.

II. THE BOOST TRANSFORMATION

Given any Lorentz-covariant local field theory, it is always possible to construct the boost operator,[10] i.e., the generator of pure Lorentz transformations. With applications to the soliton bag in mind, we shall restrict our considerations to spin-1/2 fermions (quarks) interacting with a scalar field. The Lagrangian has then the general form

$$\mathscr{L}=\mathscr{L}_\psi+\mathscr{L}_\sigma+\mathscr{L}_{\sigma\psi}\ , \tag{1}$$

where ψ and σ refer to the quark and boson fields, respectively. If $\mathscr{L}$ is a function of the fields and their first derivatives only, the stress-energy tensor is given by

$$T^{\mu\nu}=\frac{\partial\mathscr{L}}{\partial(\partial_\mu\sigma)}\partial^\nu\sigma+\frac{\partial\mathscr{L}}{\partial(\partial_\mu\psi)}\partial^\nu\psi-g^{\mu\nu}\mathscr{L}\ . \tag{2}$$

It follows from invariance under space-time translations that the momentum operator

$$P^i=\int d\vec{x}\, T^{0i}(x) \tag{3}$$

and the Hamiltonian

$$H=\int d\vec{x}\,\mathscr{H}(x),\quad \mathscr{H}(x)\equiv T^{00}(x)\ , \tag{4}$$

are conserved quantities. Similarly, from invariance under rotations in space-time, it follows that the generalized angular momentum tensor

$$M^{\mu\nu}=\int d\vec{x}\left[x^\nu T^{0\lambda}-x^\lambda T^{0\nu}-\frac{i}{2}\frac{\partial\mathscr{L}}{\partial(\partial_0\psi)}\sigma^{\nu\lambda}\psi\right]\ , \tag{5}$$

where

$$\sigma^{\nu\lambda}=\frac{i}{2}[\gamma^\nu,\gamma^\lambda] \tag{6}$$

is also a constant of the motion. The components of this symmetric second-rank tensor form two three-vectors, the familiar angular momentum

$$J^i=\tfrac{1}{2}\epsilon^{ijk}M^{jk} \tag{7}$$

and the Lorentz-boost operator

$$K^i=M^{i0} \tag{8}$$

which is central to our considerations. Since we are dealing with states and operators in the Heisenberg representation and since

$$\frac{d\vec{K}}{dt}=0 \tag{9}$$

we may set $t=0$ in the expression for $\vec{K}$. Then,

$$\vec{K}=\int d\vec{x}\left[\vec{x}\mathscr{H}(\vec{x})-\frac{i}{2}\psi^\dagger(\vec{x})\vec{\alpha}\psi(\vec{x})\right]\ . \tag{10}$$

Under the infinitesimal (active) Lorentz transformation characterized by a velocity $d\vec{v}$, a state transforms according to

$$|\psi'\rangle=(1+id\vec{v}\cdot\vec{K})|\psi\rangle\ . \tag{11}$$

Remembering that two successive Lorentz transformations along the same direction result in a Lorentz transformation whose rapidity ($\omega=\operatorname{artanh} v$) is the sum of the rapidities of each transformation, it is straightforward to integrate (11) to obtain

$$|\psi'\rangle=U(\vec{v})|\psi\rangle \tag{12}$$

with

$$U(\vec{v})=e^{i\omega\hat{v}\cdot\vec{K}} \tag{13}$$

for a finite Lorentz boost. Of course, U is unitary and $\vec{K}$ is Hermitian. We remind the reader that Lorentz invariance of the quantum theory demands that

$$U^\dagger HU=\cosh\omega\, H+\sinh\omega\,\hat{v}\cdot\vec{P}\ , \tag{14a}$$

$$U^\dagger\hat{v}\cdot\vec{P}U=\sinh\omega\, H+\cosh\omega\,\hat{v}\cdot\vec{P}\ , \tag{14b}$$

$$U^\dagger(\vec{P}-\hat{v}\cdot\vec{P}\,\hat{v})U=\vec{P}-\hat{v}\cdot\vec{P}\,\hat{v}\ . \tag{14c}$$

These relations simply express the fact that $(H,\vec{P})$ transforms like a Lorentz four-vector.

III. THE SOLITON BAG MODEL

For the purpose of discussing corrections to bag model properties that arise from relativistic recoil, it is convenient to consider the soliton bag model.[5] This model has the advantage of being derived from a standard local Lagrangian field theory, avoiding the awkward surfaces and boundary conditions that characterize the MIT bag model.[3] Contact with the latter can be made at the end of a calculation by letting the model parameters [see Eq. (16) below] assume appropriate limiting values. The Hamiltonian density of the soliton bag model is

$$\mathscr{H}(x)=\psi^\dagger(x)\left[\vec{\alpha}\cdot\frac{\vec{\nabla}}{i}+g\sigma(x)\beta\right]\psi(x)+\tfrac{1}{2}[\pi_\sigma{}^2+\tfrac{1}{2}(\vec{\nabla}\sigma)^2]+U(\sigma)\ , \tag{15}$$

where

$$U(\sigma)=\frac{a}{2}\sigma^2+\frac{b}{6}\sigma^3+\frac{c}{24}\sigma^4+p \tag{16}$$

and a, b, c, and g are model parameters.

In order to simplify the notation, flavor and color labels are suppressed in (15), as in much of what follows. The parameters a, b, c, and g are subject to the constraints $b^2 \geq 3ac$, $b < 0$. The potential (16) has two minima, at $\sigma = 0$ and

$$\sigma_{\text{vac}} = \frac{3}{2c}\left[-b + \left[b^2 - \frac{8ac}{3}\right]^{1/2}\right] . \tag{17}$$

A suitable approximation scheme is arrived at by writing

$$\sigma = \sigma_0 + \sigma_1 , \tag{18}$$

where σ_0 is a c-number field and σ_1 describes quantum fluctuations about σ_0. The quark field may be expanded as

$$\psi(\vec{x}) = \sum_k c_k \psi^k(\vec{x}) , \tag{19}$$

where $\{\psi^k\}$ is a complete, orthonormal set of Dirac spinor functions. The mean-field approximation is obtained by neglecting σ_1 altogether. Furthermore, the c-number field σ_0 corresponding to a hadron at rest is assumed to be time-independent ($\pi_0 = 0$). We shall use the phrase "rest frame" to indicate the corresponding Lorentz frame. This is somewhat inaccurate since the total momentum of the bag state in that frame is zero in expectation value only, as we shall see below. To keep the discussion simple, we shall restrict our considerations to a spherically symmetric σ_0. The extension to a nonspherical σ_0 should pose no fundamental problem. The functions ψ_0^k, σ_0 then satisfy the coupled equations[4,5]

$$[-i\vec{\alpha}\cdot\vec{\nabla} + g\sigma_0\beta]\psi_0^k = \epsilon^k \psi_0^k , \tag{20a}$$

$$-\nabla^2\sigma_0 + U'(\sigma_0) = -g \sum_{k\,\text{occ}} \bar{\psi}_0^k \psi_0^k . \tag{20b}$$

The sum in (20b) is over occupied states only; sea quark contributions are dropped. In the absence of quarks, $\sigma_0 = \sigma_{\text{vac}}$. Parameters can be chosen[5] such that for a nonvanishing quark density, the lowest-energy configuration corresponds to σ_0 near zero. The above equations therefore admit localized solutions such that the quark mass is nearly zero in the inside region and is $g\sigma_{\text{vac}}$ at infinity. In the limit of very large $g\sigma_{\text{vac}}$, confinement is achieved. For a spherically symmetric σ_0, the requirement that the quark scalar density also be spherically symmetric restricts the allowed quark single-particle states to $j = \frac{1}{2}$ ($\kappa = \pm 1$ in the standard classification of Dirac wave functions; see Appendix A). For an N-quark system built up from these states, the total MFA energy is

$$E_0 = \sum_{i=1}^{N} \epsilon^i + E_0^\sigma , \tag{21}$$

where

$$E_0^\sigma = \int d\vec{x} \left[\tfrac{1}{2} |\vec{\nabla}\sigma_0|^2 + U(\sigma_0)\right] . \tag{22}$$

Such a localized state is of course not an eigenstate of the momentum operator. However, the expectation value of the momentum vanishes since

$$-i \int d\vec{x}\, \psi_0^{i\dagger} \vec{\nabla} \psi_0^i = 0 \tag{23}$$

and the σ field carries no momentum in the rest frame.

IV. BOOSTED MFA STATES

Using (18) and (19), the boost operator (10) may be rewritten in terms of σ_0, σ_1, and $\{c_k, \psi^k\}$. Applying the exact boost transformation (13) thus obtained to the MFA states described in Sec. III would generate complicated states containing an infinite number of $q\bar{q}$ excitations and σ-field quanta. In order to reduce the problem to manageable size, we shall approximate the boost operator in a way consistent with the MFA, i.e., we shall treat the σ field as purely classical. The Lorentz transformation laws for the σ field, and the energy and momentum associated with it are then those of classical field theory. Only the quark degrees of freedom are treated quantum mechanically, and only the quark part of the boost operator (with the σ field acting as a classical source) is needed. We shall show that this MFA treatment actually maintains exact Lorentz covariance.

In this approximation, the boost operator reduces to a single-particle operator in the quark variables, and the use of a second-quantized notation becomes pedantic. Switching to first-quantized notation, the boost operator performing an infinitesimal Lorentz transformation on the rest-frame MFA state has the form

$$\vec{K}_0 = \sum_{i=1}^{N} [\vec{x}_i \mathscr{H}_0(\vec{x}_i) - \frac{i}{2}\vec{\alpha}_i] , \tag{24}$$

where

$$\mathscr{H}_0(\vec{x}) = \vec{\alpha}\cdot\frac{\vec{\nabla}}{i} + g\sigma_0(\vec{x})\beta . \tag{25}$$

The sum in (24) is over particle labels. Using (24) in (11), one obtains a state describing a system moving with velocity $d\vec{v}$. However, some care must be exercised to construct a state describing the system moving with a finite velocity $\vec{v}$. To an observer at rest the σ field corresponding to a system moving with velocity $\vec{v}$ appears Lorentz contracted. Therefore, the MFA boost generator used by this observer to boost the system from $\vec{v}$ to $\vec{v} + d\vec{v}$ is

$$\vec{K}_{\vec{v}} = \sum_{i=1}^{N} \left[\vec{x}_i \mathscr{H}_{\vec{v}}(\vec{x}_i) - \frac{i}{2}\vec{\alpha}_i\right] , \tag{26}$$

where

$$\mathscr{H}_{\vec{v}}(\vec{x}) = \vec{\alpha}\cdot\frac{\vec{\nabla}}{i} + g\sigma_{\vec{v}}(\vec{x})\beta , \tag{27}$$

$$\sigma_{\vec{v}}(\vec{x}) \equiv \sigma_0(\cosh\omega\, \vec{x}_{\parallel} + \vec{x}_{\perp}) , \quad \vec{x}_{\parallel} = \vec{x}\cdot\hat{v}\,\hat{v}, \quad \vec{x}_{\perp} = \vec{x} - \vec{x}_{\parallel} .$$

The state describing the system moving with velocity $\vec{v}$ can then be constructed using (11), which reads now (for each quark)

$$\psi_{\vec{v}+d\vec{v}}(\vec{x}) = [1 + i\,d\omega\,\hat{v}\cdot\vec{K}_{\vec{v}}]\psi_{\vec{v}}(\vec{x}) . \tag{28}$$

Because the MFA boost operator is velocity dependent, one cannot integrate (28) straightforwardly to put it in the forms (12) and (13). However, the result of performing a finite boost may in fact be written in closed form:

$$\psi_{\vec{v}}(\vec{x})=S(\vec{v})\psi_0(\cosh\omega\,\vec{x}_{\parallel}+\vec{x}_{\perp})e^{i\epsilon x_{\parallel}\sinh\omega}, \tag{29}$$

where

$$S(\vec{v})=\cosh\omega/2+(\sinh\omega/2)\hat{v}\cdot\vec{\alpha}\ . \tag{30}$$

In order to verify this, it is sufficient to show that $\psi_{\vec{v}}$ defined by (29) satisfies

$$i\hat{v}\cdot\vec{\mathbf{K}}_{\vec{v}}\psi_{\vec{v}}(\vec{x})=\frac{d}{d\omega}\psi_{\vec{v}}(\vec{x})\ . \tag{31}$$

This amounts to proving

$$S^{-1}(\vec{v})i\vec{v}\cdot\vec{\mathbf{K}}_{\vec{v}}S(\vec{v})\psi_0(\vec{x}\,')e^{i\epsilon x'_{\parallel}\tanh\omega}$$
$$=\left[\left[\tfrac{1}{2}\hat{v}\cdot\vec{\alpha}+i\epsilon x'_{\parallel}+\tanh\omega\, x'_{\parallel}\nabla'_{\parallel}\right]\psi_0(\vec{x}\,')\right]e^{i\epsilon x'_{\parallel}\tanh\omega}, \tag{32}$$

where

$$\vec{x}\,'=\cosh\omega\,\vec{x}_{\parallel}+\vec{x}_{\perp}\ .$$

From (26), (27), and (30) one finds, after some algebra,

$$S^{-1}(\vec{v})i\hat{v}\cdot\vec{\mathbf{K}}_{\vec{v}}S(\vec{v})=ix'_{\parallel}\{\mathscr{H}_0(\vec{x}\,')+\tanh\omega[\hat{v}\cdot(\vec{\sigma}\times\vec{\nabla}\,')-g\sigma_0(\vec{x}\,')\hat{v}\cdot\vec{\alpha}\beta]\}+\tfrac{1}{2}\hat{v}\cdot\vec{\alpha}\ . \tag{33}$$

It is then straightforward to verify (32) by using the equation for $\psi_0(\vec{x}\,')$,

$$\mathscr{H}_0(\vec{x}\,')\psi_0(\vec{x}\,')=\epsilon\psi_0(\vec{x}\,') \tag{34}$$

as well as

$$[-i\nabla'_{\parallel}-\hat{v}\cdot(\vec{\sigma}\times\vec{\nabla}\,')+g\sigma_0(\vec{x}\,')\hat{v}\cdot\vec{\alpha}\beta]\psi_0(\vec{x}\,')=\epsilon\hat{v}\cdot\vec{\alpha}\psi_0(\vec{x}\,')\ , \tag{35}$$

which follows from (34) after multiplication by $\hat{v}\cdot\vec{\alpha}$.

In order to demonstrate that our boosted wave functions are sensible we consider the energy and momentum of the moving system in the MFA. They are given by

$$E_{\vec{v}}=\int d\vec{x}\left[\sum_{i=1}^{N}\psi^{i\dagger}_{\vec{v}}\mathscr{H}_{\vec{v}}\psi^{i}_{\vec{v}}+\tfrac{1}{2}\pi_{\vec{v}}{}^{2}+\tfrac{1}{2}\,|\,\vec{\nabla}\sigma_{\vec{v}}\,|^{2}+U(\sigma_{\vec{v}})\right], \tag{36}$$

$$\vec{\mathbf{P}}_{\vec{v}}=\int d\vec{x}\left[-i\sum_{i=1}^{N}\psi^{i\dagger}_{\vec{v}}\vec{\nabla}\psi^{i}_{\vec{v}}-\pi_{\vec{v}}\vec{\nabla}\sigma_{\vec{v}}\right], \tag{37}$$

where $\pi_{\vec{v}}$ is canonically conjugate to $\sigma_{\vec{v}}$. The time dependence of the σ field associated with the moving bag is dictated by the assumption that the σ field is time independent for a bag at rest:

$$\sigma_{\vec{v}}(\vec{x},t)=\sigma_0(\cosh\omega\,\vec{x}_{\parallel}-\sinh\omega\, t\hat{v}+\vec{x}_{\perp}) \tag{38}$$

so that

$$\pi_{\vec{v}}(\vec{x})\equiv[\partial_t\sigma_{\vec{v}}(\vec{x},t)]_{t=0}=-\tanh\omega\,\hat{v}\cdot\vec{\nabla}\sigma_{\vec{v}}(\vec{x})\ . \tag{39}$$

By using (21)–(23), (27), (29), and (39), expressions (36) and (37) may be rewritten in terms of the static solutions ψ_0^i and σ_0. After some algebra, one finds

$$E_{\vec{v}}=\cosh\omega\, E_0+\sinh\omega\,\hat{v}\cdot(\vec{\mathscr{P}}_0+\tanh\omega\,\vec{\mathscr{E}}_0)\ , \tag{40}$$

$$\hat{v}\cdot\vec{\mathbf{P}}_{\vec{v}}=\sinh\omega\, E_0+\sinh\omega\,\hat{v}\cdot(\vec{\mathscr{E}}_0+\tanh\omega\,\vec{\mathscr{P}}_0)\ , \tag{41}$$

$$\vec{\mathbf{P}}_{\vec{v}\perp}=\tanh\omega\,\vec{\mathscr{E}}_{0\perp}\ , \tag{42}$$

where

$$\vec{\mathscr{P}}_0=\int d\vec{x}\sum_{i=1}^{N}\epsilon^i\psi_0^{i\dagger}\vec{\alpha}\psi_0^i\ , \tag{43}$$

$$\vec{\mathscr{E}}_0=\int d\vec{x}\left[\sum_{i=1}^{N}\psi_0^{i\dagger}\hat{v}\cdot\vec{\alpha}\frac{\vec{\nabla}}{i}\psi_0^i+\hat{v}\cdot\vec{\nabla}\sigma_0\vec{\nabla}\sigma_0\right]-\hat{v}E_0^{\sigma}\ . \tag{44}$$

The energy and momentum of the boosted system will be correctly related to those of the system at rest [see (14)] provided the solutions of (20) satisfy

$$\vec{\mathscr{P}}_0=\vec{\mathscr{E}}_0=0\ . \tag{45}$$

That this is indeed the case is proved in Appendix A. Thus, even though they are not momentum eigenstates, the hadronic states constructed from the boosted quark wave functions (29) correspond to an average momentum and a mean-field energy correctly related to the hadron mass, if by the latter one means the mean-field energy calculated in the frame in which the σ field is time independent.

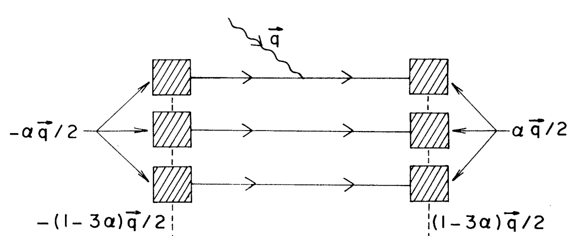

FIG. 1. Nucleon-current interaction in the soliton mean-field model. The solid lines represent quarks, the dashed line represents the soliton mean field. The boxes stand for the MFA quark wave functions. In the average, each quark carries a fraction α of the nucleon momentum ($\alpha=\epsilon/m$); the soliton field carries the rest. Note that even though the soliton field carries momentum and energy, it is not treated here as a quantum-mechanical degree of freedom and there is no probability amplitude associated with it in Eq. (53). Its role is akin to that of a potential in nonrelativistic theory.

We note that the conclusions of this section could have been reached by considering the Lorentz-transformation properties of the time-dependent mean-field equations and of the associated energy-momentum stress tensor. This alternate and equivalent viewpoint is briefly discussed in Appendix B.

V. RECOIL CORRECTIONS TO CURRENT MATRIX ELEMENTS

The calculation of hadronic form factors requires the consideration of current matrix elements of the form[10]

$$\langle p' \,|\, J_\mu(x) \,|\, p \rangle = \langle p' \,|\, J_\mu(0) \,|\, p \rangle e^{-iq\cdot x} , \tag{46}$$

where spin, isospin, and other labels are suppressed for simplicity and $|p\rangle$ denotes a four-momentum eigenstate of the composite hadron. The conventional normalization is

$$\langle p' \,|\, p \rangle = (2\pi)^3 \delta^3(\vec{p}\,' - \vec{p}\,)\rho(\vec{p}\,) \tag{47}$$

with $\rho(\vec{p}\,) = E_p/m$ for fermions and $\rho(\vec{p}\,) = 1$ for bosons. Equation (46), where $q = p' - p$, follows in general from space-time translation invariance. The transition from plane-wave states to localized states can be made formally as follows.[11] Set $t=0$ in (46) and integrate both sides over space to get

$$\langle p' \,|\, J_\mu(0) \,|\, p \rangle = \frac{1}{(2\pi)^3\delta^3(0)}[\rho(\vec{p}\,')\rho(\vec{p}\,)]^{1/2} \int d\vec{x}\, e^{i\vec{q}\cdot\vec{x}} \langle m \,|\, U^\dagger(\vec{v}\,') J_\mu(\vec{x}\,) U(\vec{v}\,) \,|\, m \rangle , \tag{48}$$

where $|m\rangle$ denotes a state with four-momentum $(m,\vec{0}\,)$. The unitary transformation $U(\vec{v}\,)$ was introduced in Sec. II ($\vec{v} = \vec{p}/E_p$). The factor $[\rho(\vec{p}\,')\rho(\vec{p}\,)]^{1/2}$ is necessary to maintain the conventional normalization (47).

The standard expression used to calculate form factors in the static approximation can be obtained by (i) letting $\rho(\vec{p}\,)^{1/2}U(\vec{v}\,) \to 1$ and (ii) making the following identification between the zero-momentum eigenstate $|m\rangle$ and the static localized bag state $|B_0\rangle$:

$$|m\rangle \to [(2\pi)^3\delta^3(0)]^{1/2} \,|\, B_0 \rangle , \tag{49}$$

where the proportionality factor guarantees the normalization (47) if $|B_0\rangle$ is normalized to unity. In this work, we shall keep the identification (49), but shall improve upon the static approximation by taking into account the transformation $\rho(\vec{p}\,)^{1/2}U(\vec{v}\,)$. This amounts to identifying (up to an appropriate normalization) the momentum eigenstate $|p\rangle$ with a moving bag state for which the expectation value of the momentum is correctly given by $\vec{p}$, rather than with a static bag. We note that identification (49) only guarantees that the states used in the calculation of current matrix elements are correctly normalized. Moving MFA states corresponding to different momenta are not in general orthogonal to each other, as required in principle by (47). This shortcoming could be remedied by projecting these states onto states of good momentum. Thus, boosting is not an alternative for projecting. The situation here is analogous to that encountered in nonrelativistic nuclear physics, where simple projection[12] leads to states which are momentum eigenstates but have unphysical properties (e.g., the intrinsic state is not independent of the total momentum). These problems can be cured by taking into account the transformation properties of states under Galilean transformation,[13] the nonrelativistic analog of the Lorentz boost. This suggests that projection approaches to bag recoil, as discussed so far in the literature,[6] could be significantly improved by combining projection and boost. This technically complex program is left for future work.

In practice, we shall perform calculations in the Breit frame, for which $\vec{p}\,' = -\vec{p} = \vec{q}/2$. Then, using (49) in (48), one gets

$$\left\langle \frac{\vec{q}}{2} \left| J_\mu(0) \right| -\frac{\vec{q}}{2} \right\rangle = \rho\left[\frac{\vec{q}}{2}\right] \int d\vec{x}\, e^{i\vec{q}\cdot\vec{x}} \langle B_{\vec{v}} \,|\, J_\mu(\vec{x}\,) \,|\, B_{-\vec{v}} \rangle , \tag{50}$$

where the moving bag state

$$|B_{\vec{v}}\rangle \equiv U(\vec{v}\,) \,|\, B_0 \rangle \tag{51}$$

is constructed using the boosted quark wave functions (29). In a first quantized notation, a quark current operator may be written

$$J_\mu(\vec{x}\,) = \sum_{i=1}^{N} \Gamma_\mu^i \delta(\vec{x} - \vec{x}_i) , \tag{52}$$

where Γ_μ^i is a Dirac matrix and an operator in flavor space. For simplicity, we shall assume[14] from now on that all quarks are in the $1s$ ($\kappa=-1$) orbit with energy ϵ_0. Then, using (29), (51), (52), and (A2), it is straightforward to derive

$$\left\langle \frac{\vec{q}}{2} \left| J_\mu(0) \right| -\frac{\vec{q}}{2} \right\rangle = \left\langle \sum_{i=1}^{N} \int d\vec{x} \exp[i(q - 2\epsilon_0 \sinh\omega) x_{\parallel}] \phi_i^\dagger(\vec{x}\,') S_i(\omega) \Gamma_\mu^i S_i(-\omega) \phi_i(\vec{x}\,') \right\rangle \times \left[\int d\vec{x} \exp(-2i\epsilon_0 \sinh\omega\, x_{\parallel}) \phi^\dagger(\vec{x}\,')\phi(\vec{x}\,') \right]^{N-1} , \tag{53}$$

where

$$\phi_i(\vec{x}\,') = \frac{1}{\sqrt{4\pi}} \begin{bmatrix} u(x') \\ i\vec{\sigma}_i\cdot\hat{x}\,' l(x') \end{bmatrix}, \quad \vec{x}\,' = \cosh\omega\, \vec{x}_{\parallel} + \vec{x}_{\perp}, \quad \omega = \operatorname{artanh}\frac{q}{2E_{q/2}} \tag{54}$$

and the bracket indicates that the spin-flavor matrix element is to be taken using the appropriate SU(6) wave function. Formula (53) is our main result; it is used in the next two sections to calculate the electromagnetic and axial-vector form factors of the nucleon. The physical content of (53) is easily understood and is illustrated in Fig. 1. The first term represents the amplitude for the quark which absorbs the current to make the appropriate transition from a bag state moving with velocity $-\vec{v}$ to one moving with velocity $\vec{v}$. The second term represents the probability amplitude for the $N-1$ spectator quarks to follow along. The assumption underlying this picture is that the soliton mean field provides the interaction required for this to happen, since initial- and final-state wave functions are generated from that interaction only.

VI. RECOIL-CORRECTED NUCLEON FORM FACTORS

The proton electromagnetic and axial-vector form factors can be calculated using formula (53) with

$$\Gamma_\mu = e(\tfrac{1}{6}+\tau^3)\gamma_0\gamma_\mu \equiv \lambda\gamma_0\gamma_\mu \quad \text{(electromagnetic)} \tag{55}$$

and

$$\Gamma_\mu = \underline{\tau}\gamma_0\gamma_\mu\gamma_5 \quad \text{(axial vector)} . \tag{56}$$

Here τ^i $(i=1,2,3)$ stands for a quark isospin Pauli matrix and λ is the quark electric charge. The form factors G_E, G_M, G_A, and G_P are conventionally defined by[10]

$$\langle p's' | J_\mu^{elm}(0) | ps \rangle = \bar{u}_{s'}(p') \left[\gamma_\mu F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu}q^\nu F_2(q^2) \right] u_s(p) , \tag{57}$$

$$G_E(q^2) \equiv F_1(q^2) + \frac{q^2}{4m^2}F_2(q^2) , \tag{58}$$

$$G_M(q^2) \equiv F_1(q^2) + F_2(q^2) , \tag{59}$$

$$\langle p's' | \underline{A}_\mu^5(0) | ps \rangle = \langle \tfrac{1}{2} | \underline{\tau}_N | \tfrac{1}{2} \rangle \bar{u}_{s'}(p')[\gamma_\mu G_A(q^2) + q_\mu G_P(q^2)]\gamma_5 u_s(p) , \tag{60}$$

where $u_s(p)$ denotes a nucleon Dirac spinor. Specializing to the Breit frame, one gets

$$\left\langle \frac{\vec{q}}{2}s' \left| J_0^{elm}(0) \right| -\frac{\vec{q}}{2}s \right\rangle = \chi_{s'}^\dagger \chi_s G_E(q^2) , \tag{61}$$

$$\left\langle \frac{\vec{q}}{2}s' \left| \vec{J}^{elm}(0) \right| -\frac{\vec{q}}{2}s \right\rangle = \chi_{s'}^\dagger \frac{i\vec{\sigma}_N \times \vec{q}}{2m} \chi_s G_M(q^2) , \tag{62}$$

$$\left\langle \frac{\vec{q}}{2}s' \left| \underline{\vec{A}}^5(0) \right| -\frac{\vec{q}}{2}s \right\rangle = \langle \tfrac{1}{2} | \underline{\tau}_N | \tfrac{1}{2} \rangle \frac{E}{m} \chi_{s'}^\dagger \left\{ \vec{\sigma}_N G_A(q^2) - \frac{\vec{q}\,\vec{\sigma}_N\cdot\vec{q}}{2E} \left[G_P(q^2) + \frac{G_A(q^2)}{2(E+m)} \right] \right\} \chi_s . \tag{63}$$

Using (55) and (56) in (53), and referring to (61)—(63), it is straightforward to derive expressions for the recoil-corrected form factors. We simply quote the final formulas:

$$G_E(q^2) = e\left[\frac{m}{E}\right]^2 Q(v)\mathcal{O}^2(v) , \tag{64}$$

$$G_M(q^2) = 2e\left[\frac{m}{E}\right]^2 \left[\tfrac{1}{2}R(v) + P(v)\right]\mathcal{O}^2(v) , \tag{65}$$

$$G_A(q^2) = \frac{5}{3}\left[\frac{m}{E}\right]^2 \left[R(v) + \frac{q^2}{2E^2}P(v)\right]\mathcal{O}^2(v) , \tag{66}$$

$$G_P(q^2) = \frac{5}{3m}\left[\frac{m}{E}\right]^3 [P(v) - \mathcal{F}(v)]\mathcal{O}^2(v) - \frac{G_A(q^2)}{2(E+m)} , \tag{67}$$

with

$$\mathcal{O}(v) = \int_0^\infty dx\, x^2 j_0(2\epsilon_0 vx)[u^2(x)+l^2(x)] , \tag{68}$$

$$Q(v) = \int_0^\infty dx\, x^2 j_0(2(m-\epsilon_0)vx)[u^2(x)+l^2(x)] , \tag{69}$$

$$P(v) = \int_0^\infty dx\, x^2 \frac{1}{v} j_1(2(m-\epsilon_0)vx) u(x) l(x) , \tag{70}$$

$$R(v)=\int_0^\infty dx\,x^2\left\{j_0(2(m-\epsilon_0)vx)[u^2(x)-l^2(x)]+2\frac{j_1(2(m-\epsilon_0)vx)}{2(m-\epsilon_0)vx}l^2(x)\right\}, \tag{71}$$

$$\mathscr{F}(v)=\int_0^\infty dx\,x^2\frac{1}{2v^2}[j_0(2(m-\epsilon_0)vx)(\sqrt{1-v^2}-1)u^2(x)]$$
$$+\left[j_0(2(m-\epsilon_0)vx)(\sqrt{1-v^2}+1)-2(2\sqrt{1-v^2}+1)\frac{j_1(2(m-\epsilon_0)vx)}{2(m-\epsilon_0)vx}\right]l^2(x), \tag{72}$$

where

$$E=\left[m^2+\frac{q^2}{4}\right]^{1/2},\quad v=\frac{q}{2E},$$

and j_0 and j_1 are the usual spherical Bessel functions. For comparison, we also quote the corresponding static approximation results, obtained by disregarding the unitary transformation $U(\vec{v})$ [see (48)] and neglecting $q^2/4$ compared to m:

$$G_E(q^2)=e\int_0^\infty dx\,x^2 j_0(qx)[u^2(x)+l^2(x)], \tag{73}$$

$$G_M(q^2)=4m\int_0^\infty dx\,x^3\frac{j_1(qx)}{qx}u(x)l(x), \tag{74}$$

$$G_A(q^2)=\tfrac{5}{3}\int_0^\infty dx\,x^2\left\{j_0(qx)[u^2(x)-l^2(x)]+2\frac{j_1(qx)}{qx}l^2(x)\right\}, \tag{75}$$

$$G_p(q^2)=-\frac{4m}{q^2}\tfrac{5}{3}\int_0^\infty dx\,x^2\left[j_0(qx)-3\frac{j_1(qx)}{qx}\right]l^2(x). \tag{76}$$

Another quantity of interest is the πNN vertex which corresponds to a hybrid chiral bag version[15] of the soliton bag. To lowest order in the pion field, the πNN vertex is generated by a pseudovector quark-pion coupling:

$$H_{\rm int}=\frac{1}{2f}\int d\vec{x}\,\bar{\psi}(\vec{r})\vec{\gamma}\gamma_5\underline{\tau}\psi(\vec{x})\cdot\vec{\nabla}\underline{\phi}(\vec{x}), \tag{77}$$

where ψ and $\underline{\phi}$ denote the quark and pion field, respectively; f is the pion decay constant. For a $\pi N\to N$ transition, the relevant operator is therefore, in the Breit frame,

$$\left\langle\frac{q^2}{2}s't'\left|H_{\rm int}(\vec{q})\right|-\frac{q^2}{2}st\right\rangle=\frac{i}{2f(2\pi)^{3/2}\sqrt{2\omega_q}}\underline{a}(\vec{q})\cdot\left\langle\frac{q^2}{2}s't'\left|\vec{q}\cdot\vec{\underline{A}}^5(0)\right|-\frac{q^2}{2}st\right\rangle$$
$$=\frac{i}{2f(2\pi)^{3/2}\sqrt{2\omega_q}}\underline{a}(\vec{q})\cdot\langle t'\,|\,\underline{\tau}_N\,|\,t\rangle\langle s'\,|\,\vec{\sigma}_N\cdot\vec{q}\,|\,s\rangle G_{\pi NN}(q^2), \tag{78}$$

where $\vec{q}$ is the pion momentum, ω_q the pion energy, $\underline{a}(\vec{q})$ the pion destruction operator, t the nucleon isospin, and

$$G_{\pi NN}(q^2)=G_A(q^2)-\frac{q^2}{2m}G_p(q^2)=\frac{5}{3}\left[\frac{m}{E}\right]^2\mathscr{V}(v)\mathscr{O}^2(v), \tag{79}$$

with

$$\mathscr{V}(v)=\int_0^\infty dx\,x^2\left\{j_0(2(m-\epsilon_0)vx)[u^2(x)+l^2(x)]-4\frac{j_1(2(m-\epsilon_0)vx)}{2(m-\epsilon_0)vx}l^2(x)\right\}. \tag{80}$$

The static-approximation result is

$$G_{\pi NN}(q^2)=\tfrac{5}{3}\int_0^\infty dx\,x^2\left\{j_0(qx)[u^2(x)+l^2(x)]-4\frac{j_1(qx)}{qx}l^2(x)\right\}. \tag{81}$$

Before turning to numerical results in the next section, we briefly comment on some general features of these form factors. First we consider their analytic structure. Mathematical proofs of the statements made below parallel those of Ref. 16 and are left to the reader. Noting that all expressions above are even functions of q, we consider them as functions of the complex variable $t=-\vec{q}^{\,2}$. As pointed out in Ref. 16, form factors for systems of absolutely confined quarks, calculated in the static approximation, generally possess an essential singularity at infinity in the t plane. This is readily checked on expressions (73)—(76) for the MIT bag limit ($u,l=0$ for $x>R$). In the soliton model, the quark mass at an infinite distance from the bag center is $m_q=g\sigma_{\rm vac}$, and the quark wave functions fall off at large distance as $e^{-m_q x}$. Then, it is easy to see that all form factors possess an isolated singularity for $t=4m_q^2$, corresponding to the possibility of ionization. Of course, for reasonable values of the parameters, m_q is very large and this singularity is very far from the physical region. If relativistic recoil is taken into account, it is easy to see, by comparing (64)—(72) to (73)—(76), that the singularity structure of the form factors[17] is now as follows. First, there is a purely kinematical singularity at $t=4m^2$, due to the factors m/E. In the absolutely confining case, the essential singularity at infinity is in the variable v^2. In the variable t, this also translates into a singularity at $t=4m^2$. In the finite binding case, singularities occur for[18] $v^2=-m_q^2/\epsilon_0^2$ and $v^2=-m_q^2/(m-\epsilon_0)^2$, which translates into

$$t=4m^2\left[1-\frac{\epsilon_0^2}{m_q^2}\right]$$

and

$$t=4m^2\left[1-\frac{(m-\epsilon_0)^2}{m_q^2}\right]$$

if $m_q \gg m$. In the limit $m_q\to\infty$, the singularity structure of the absolute confinement case is smoothly recovered. The presence of an essential singularity at infinity in the form factors has been invoked[16] to criticize quark models with absolute confinement and/or sharp surfaces. The above discussion shows that, even though the singularity remains if recoil is taken into account, its location becomes more compatible with dispersion theory arguments.[10]

It is also of some interest to compare the large-q^2 behavior of the static and recoil-corrected form factors. Static form factors typically behave as $F(q^2)q^{-n}$ for $q^2\to\infty$, with $F(q^2)$ a bounded oscillating function of q^2. They therefore possess an infinite number of zeros. In contrast, because the integrands of (68)—(72) are functions only of v^2, which tends to the finite value 1 as $q\to\infty$, the recoil-corrected form factors behave as Cq^{-n}, with C a constant.[19] These form factors have only a finite (and in fact small) number of zeros before falling off monotonously to zero. Of course these considerations are somewhat academic since the bag model is not expected to be a good description of reality for very large momentum transfers. In particular, the above properties must be contrasted with the predictions of perturbative QCD, which yields an $O(q^{-4})$ behavior for the electromagnetic nucleon form factors.[20]

VII. NUMERICAL RESULTS

In this section, we present numerical results for recoil-corrected nucleon form factors. We consider first the magnetic moment and the charge radius. They can be derived by expanding expressions (64) and (65) and (73) and (74) in powers of q^2. Alternatively, one may go back to the formulas of Sec. IV and notice that, since recoil corrections are $O(q)$ at least, the Lorentz-contraction effect in the boost operator is $O(q^3)$ and can be neglected in an expansion to $O(q^2)$. Then one may use (24), (25), and (20a) to write (for each quark):

$$\vec{\mathbf{K}}_0\psi_0(\vec{\mathbf{x}})=\left[\vec{\mathbf{x}}\epsilon_0-\frac{i}{2}\vec{\alpha}\right]\psi_0(\vec{\mathbf{x}})\ , \tag{82a}$$

$$\psi_0^\dagger(\vec{\mathbf{x}})\vec{\mathbf{K}}_0=\psi_0^\dagger(\vec{\mathbf{x}})\left[\epsilon_0\vec{\mathbf{x}}+\frac{i}{2}\vec{\alpha}\right]\ . \tag{82b}$$

Noting that

$$\langle\psi_0\,|\,\vec{\alpha}\,|\,\psi_0\rangle=\langle\psi_0\,|\,\vec{\mathbf{x}}\,|\,\psi_0\rangle=0 \tag{83}$$

and using (50)—(52), (28), and (55), one gets to $O(q)$:

$$\left\langle\frac{\vec{\mathbf{q}}}{2}s'\left|\vec{\mathbf{J}}^{\,elm}(0)\right|-\frac{\vec{\mathbf{q}}}{2}s\right\rangle=i\left\langle B_0(s')\left|\sum_{i=1}^{3}\lambda_i\left[\left[1-\frac{\epsilon_0}{m}\right]\vec{\alpha}_i\vec{\mathbf{q}}\cdot\vec{\mathbf{x}}_i+\frac{1}{2m}\vec{\sigma}_i\times\vec{\mathbf{q}}\right]\right|B_0(s)\right\rangle+O(q^3)\ , \tag{84}$$

where the matrix elements are to be taken using static bag wave functions. The recoil-corrected magnetic moment is therefore

$$\mu_p^{(R)}=\left[1-\frac{\epsilon_0}{m}\right]\mu_p^{(S)}+\Delta\mu_p\ , \tag{85}$$

where $\mu_p^{(S)}$ is the static-approximation value

$$\mu_p^{(S)}=e\tfrac{2}{3}\int_0^\infty dx\, x^3 u(x) l(x) \tag{86}$$

and

$$\Delta\mu_p=\frac{e}{2m}\int_0^\infty dx\, x^2[u^2(x)-\tfrac{1}{3}l^2(x)]\ . \tag{87}$$

The charge radius can be obtained in a similar fashion. Noting that the boost operator commutes with the electric charge, one gets

$$\left\langle \frac{q^2}{2}s' \left| J_0^{elm}(0) \right| -\frac{q^2}{2}s \right\rangle = e\left[1+\frac{q^2}{8m^2}\right],$$

$$-\frac{1}{2}\left\langle B_0(s') \left| \sum_{i=1}^{3} \lambda_i \left[\left[1-\frac{2\epsilon_0}{m}+\frac{3\epsilon_0^2}{m^2}\right](\vec{q}\cdot\vec{x}_i)^2+\frac{3q^2}{4m^2}\right] \right| B_0(s)\right\rangle + O(q^4) \equiv e\left[1-\frac{q^2\langle r_c\rangle^2}{6}+O(q^4)\right]. \quad (88)$$

The first term in (88) arises from the expansion of the normalization factor $\rho(\vec{q}/2)$. The recoil-corrected charge radius is

$$\langle r_c^2\rangle^{(R)} = \left[1-\frac{2\epsilon_0}{m}+\frac{3\epsilon_0^2}{m^2}\right]\langle r_c^2\rangle^{(S)}+\frac{3}{2m^2}, \quad (89)$$

where $\langle r_c^2\rangle^{(S)}$ is the static-approximation value

$$\langle r_c^2\rangle^{(S)} = \int_0^\infty dx\, x^4[u^2(x)+l^2(x)]. \quad (90)$$

Before discussing numerical results, we briefly compare the above formulas to those derived by other authors. Result (89) for the charge radius is identical to that obtained by Dethier *et al.*[7] using the concept of relativistic center-of-mass operator. This operator is related to our boost by $\vec{R}=\vec{K}/m$. Here and in what follows, we identify the hadron mass m with the MFA energy in the rest frame. As discussed in Sec. IV, this is consistent with the Lorentz-transformation properties of the mean-field theory. It follows that, to $O(q^2)$, formula (5) may be written

$$\left\langle \frac{\vec{q}}{2} \left| J_\mu(0) \right| -\frac{\vec{q}}{2} \right\rangle = \rho\left(\frac{\vec{q}}{2}\right)\int d\vec{x}\,\langle B_0 | e^{i\vec{q}\cdot(\vec{x}-\vec{R})/2} J_\mu(\vec{x}) e^{i\vec{q}\cdot(\vec{x}-\vec{R})/2} | B_0\rangle \quad (91)$$

from which one gets

$$\langle r_c^2\rangle^{(R)}+\frac{3}{4m^2} = \left\langle B_0 \left| \sum_{i=1}^{N} \lambda_i(\vec{x}_i-\vec{R})^2 \right| B_0\right\rangle \quad (92)$$

which is precisely the expression used by Dethier *et al.* We note in passing that the commutation properties of the different components of $\vec{R}$, which, as discussed in Ref. 7, do not quite befit a particle localization operator, are exactly those required of a boost operator, as a consequence of the structure of the Poincaré group. The close connection between relativistic center-of-mass and boost comes as no surprise if one remembers that the boost transformation reduces in the nonrelativistic limit to the Galilean transformation, which is generated by the usual center-of-mass operator.

Relativistic recoil corrections to magnetic moments have been calculated by some authors[8] using a free Dirac particle boost for each quark. As a result of this approximation, the factor $1-\epsilon_0/m$ multiplying the first term of (85) does not appear in their work. As we shall see shortly, a large amount of cancellation occurs between this correction and the second term of (85). Therefore, the recoil corrections of Ref. 8 are overestimated and the nice agreement with measured magnetic moments quoted by Ref. 8 appears spurious.

Despite rather large conceptual differences,[21] our formulas for electromagnetic form factors are identical to those of Barnhill[9] in the MIT bag limit. In this limit

$$u(x)=N_0 j_0(\epsilon_0 x),\quad l(x)=N_0 j_1(\epsilon_0 x),$$

$$\epsilon_0=\omega_0/R,\quad \omega_0=2.04, \quad (93)$$

$$N_0=\left[\frac{\omega_0}{2(\omega_0-1)j_0^2(\omega_0)R^3}\right]^{1/2},\quad m=4\epsilon_0,$$

TABLE I. Recoil corrections to magnetic moment and charge radius in the soliton bag model. The static ($\mu^{(S)}$) and recoil-corrected ($\mu^{(R)}$) magnetic moments are given in Bohr magnetons ($e/2m_p$). The parameter $b=-\sqrt{3ac}$. In the soliton bag, the large-c entries correspond to a sharp surface transition in $\sigma(r)$ while the small-c entries correspond to a very diffuse bag ($0.2R$). Entries with $G_A(0)\sim 1$ correspond to MIT-type bags while entries with smaller $G_A(0)$ correspond to SLAC-type bags. The last line gives the exact MIT-bag results using the formulas in the text.

$\frac{a}{10^2}$ (fm^{-2})	$\frac{c}{10^4}$ (fm^0)	$g\sigma_v$ (GeV)	$(\langle r_c^2\rangle^{(S)})^{1/2}$ (fm)	$\mu^{(S)}$	$\mu^{(R)}$	$G_A(0)$	m (GeV)	ϵ (GeV)
26.2	320	3.88	0.953	2.92	2.68	0.780	1.081	0.238
28.0	320	2.41	0.958	2.82	2.63	0.906	1.182	0.259
29.5	320	1.65	0.963	2.73	2.57	0.993	1.251	0.277
32.3	320	0.86	0.967	2.62	2.49	1.078	1.301	0.297
0.426	0.625	4.49	0.944	2.99	2.71	0.641	1.081	0.220
0.419	0.625	2.21	0.949	2.93	2.70	0.743	1.134	0.233
0.529	0.625	0.63	0.967	2.61	2.48	1.095	1.325	0.300
MIT			0.968	2.57	2.44	1.09	1.210	0.303

where R is the bag radius. Formulas (85) and (89) then give

$$\mu_p^{(R)}=0.192R,\ \ \langle r_c{}^2\rangle^{(R)}=0.39R^2\ , \tag{94}$$

compared to the static-approximation values

$$\mu_p^{(S)}=0.202R,\ \ \langle r_c{}^2\rangle^{(S)}=0.53R^2\ . \tag{95}$$

Recoil effects significantly lower the charge radius but affect the magnetic moment only slightly (5% decrease). In deriving formulas (94), we have used the simple MIT bag mass, $m=4\epsilon_0$. If one uses instead the physical nucleon mass or the centroid mass of the $N\Delta$ isomultiplet (as done in Ref. 9), one obtains a slight increase in the magnetic moment due to recoil. However, the effect is still small (less than 10%).

The recoil-corrected charge radii and magnetic moments for a variety of soliton bag parameters are compared to static approximation values in Table I. In all cases, the parameters are adjusted to give the experimental charge radium (with recoil). The mass used in the calculations is the total soliton bag energy. The trends described above for the MIT bag are present in all cases. The recoil effect on $\langle r_c{}^2\rangle$ is roughly independent of bag type. The effect on μ_p is a decrease varying with bag type from negligible (for MIT type) to roughly 15% (for SLAC type). The results are quite insensitive to surface diffuseness. This is a consequence of the fact that the quark wave functions are very similar for entries in Table I with similar values of $G_A(0)$. We note that the soliton bags considered here are rather far from absolutely confining (see the values of $g\sigma_{\rm vac}$, the quark mass at infinity). Our motivation for considering models with approximate confinement is purely technical; it is difficult to obtain numerical solutions to Eqs. (20) for nearly discontinuous quark wave functions. We emphasize that the parameters are kept unchanged for each type in the comparison of Table I. Of course, as discussed in Ref. 7, fitting the recoil corrected charge radius requires a larger bag size than would be inferred by fitting the static-approximation $\langle r_c{}^2\rangle^{(S)}$. This led to a scaling up of all magnetic moments compared to the usually quoted bag model values.

The electromagnetic form factors calculated using formulas (64)—(72) are compared to the static-approximation results in Figs. 2 and 3. Again, parameters are constrained to give $\langle r_c{}^2\rangle^{(R)}=0.69\ \text{fm}^2$. The form factors for the MIT and SLAC bags are quite similar. In both cases, recoil corrections push the first zero to larger values of q^2. This effect was already pointed out in Ref. 9. For the magnetic form factor, a rather good agreement with experiment[22] is achieved, once recoil corrections are included. In contrast, bag-model electric form factors fall off too fast as q^2 increases.

The axial-vector form factor $G_A(q^2)$ is also hardened by

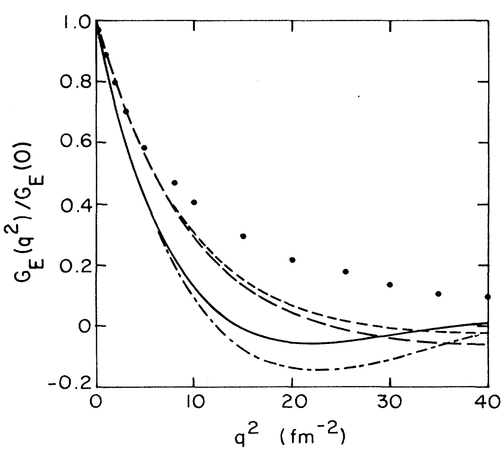

FIG. 2. Static and recoil-corrected electric form factors $G_E(q^2)$ and comparison with experiment. The different curves are solid = MIT bag without recoil; long stroke = MIT bag with recoil; dash-dot = SLAC-type soliton bag without recoil; dash = SLAC-type soliton bag with recoil. The experimental data are denoted by the solid circles. The experimental errors quoted are smaller or equal to the size of these circles. The form factor is normalized to one at $q^2=0$.

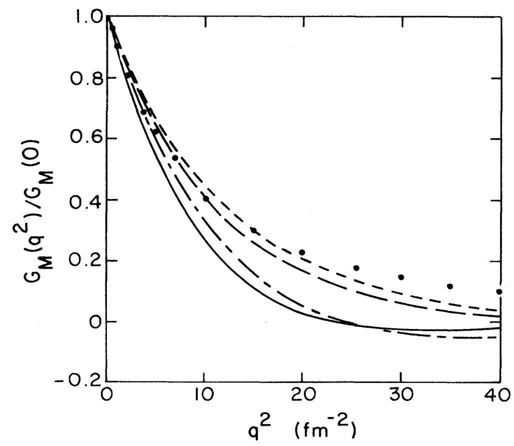

FIG. 3. The same as Fig. 2 for the magnetic form factor $G_M(q^2)$.

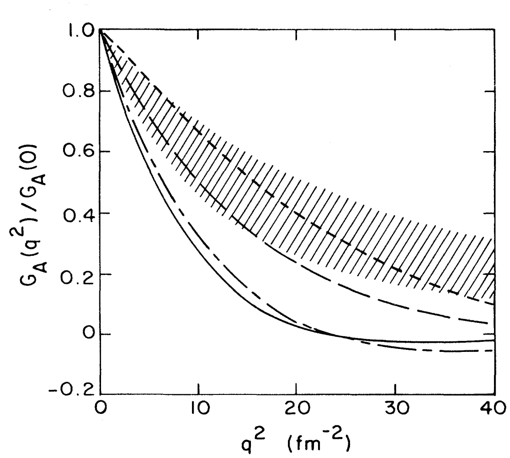

FIG. 4. The same as Fig. 2 for the axial-vector form factor $G_A(q^2)$. The experimental results (dipole fit) are denoted by the diagonal-line band. The length of the lines is approximately the uncertainty of the fit.

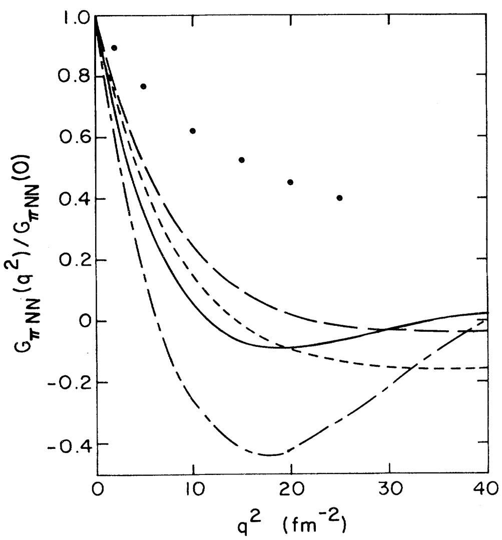

FIG. 5. The same as Fig. 2 for the pion-nucleon form factor $G_{\pi NN}(q^2)$. The phenomenological monopole form factor with $m = 0.8$ GeV cutoff is indicated by the solid circles.

recoil corrections, as shown in Fig. 4. Phenomenological analyses[23] of neutrino reactions and pion photoproduction and electroproduction yield dipole fits $G_A(q^2) \propto (q^2 - m_A{}^2)^{-1}$ with m_A in the range (0.9—1.4) GeV. The recoil-corrected bag-model form factors are quite compatible with these values. Rather than showing the induced pseudoscalar form factor $G_P(q^2)$, about which no direct experimental information is available, we show in Fig. 5 the πNN form factor, which is a combination of G_A and G_P [see Eq. (79)]. Again recoil hardens the form factor and pushes the first zero to larger q^2. We note that this form factor is quite different for the SLAC and MIT bags. This is because $G_A(0)$ is strongly suppressed for the SLAC bag (see Table I), so that the cancellation between the two terms of (79) occurs at much smaller q^2 in that case. Chiral bag model πNN form factors are typically much softer than standard semiphenomenological monopole fits.[24] Recoil corrections are not so drastic as to change this state of affairs. An attempt at determining the bag radius by fitting peripheral nucleon-nucleon phase shifts in a pion-exchange model with the static form factor[25] (81) was made in Ref. 26. The result is $R = 0.8 \pm 0.1$ fm, in agreement with the value obtained from pion-nucleon scattering.[27] If these analyses were redone using our recoil-corrected form factor, one may estimate from Fig. 5 and the scaling properties of the form factors that $R = 1.1 \pm 0.1$ fm would obtain.

It is important to realize that the present results will be somewhat modified by the inclusion of pionic and gluonic corrections. Pionic corrections can be included in a chiral-bag-model approach. Assuming that recoil and pionic corrections are independent in first approximation, an estimate of $\langle r_c{}^2 \rangle$ and μ_p can be made using results tabulated in Ref. 28. There, charge radii and magnetic moments were corrected for recoil using the approach of Donoghue and Johnson.[6] If instead,[29] we use formulas (94) to obtain recoil-corrected quark core contributions, Tables 1 and 2 of Ref. 28 are replaced by Tables II and III. Although pionic corrections do not have the desirable effect of increasing the charge radius and the magnetic moment for a given bag radius, the effects are insufficient to bring a chiral bag model with relatively small radius (1 fm or less) into agreement with experiment.

Gluonic corrections have been considered in Ref. 30, where they were found to produce a 10% decrease in μ_p. However, the calculation was performed in a severely truncated space of quark orbitals ($1s_{1/2}$ only). An alternative calculation has recently been performed,[31] with vastly different results. In the latter work, free quark and gluon propagators with cutoff masses, treated as parameters, are used in place of bag-model propagators. It is found that a large increase in μ_p is possible, though the results are quite sensitive to the values of the cutoff masses. Since it is apparent that the values for gluonic corrections to the magnetic moment and the charge radius are not established, an analysis of their effects on our results cannot be carried out at this time. A more careful study of these corrections, in conjunction with those discussed above, is clearly needed.

VIII. CONCLUSIONS

We have developed a method for boosting soliton-bag-model states in the mean-field approximation which, within the assumption that the zero-momentum eigenstate is well approximated by the static-bag-model wave functions, provides a prescription for calculating bag-model form factors. The effect of the boost on the electromagnetic and axial-vector form factors is quite significant, affecting both the slope at $q^2 = 0$ and the large-q^2 behavior of the form factors. If the bag radius is adjusted to reproduce the experimental proton charge radius, the agreement with experiment for the directly measurable form factors

TABLE II. Pionic and recoil corrections to proton charge radius. Experimentally $\langle r^2 \rangle^{1/2} = 0.83$ fm. Here R is the bag radius, $\langle r_c{}^2 \rangle_B^{(R,S)}$ are the quark contributions with and without recoil to the chiral bag charge radius, $\langle r_c{}^2 \rangle = \langle r_c{}^2 \rangle_\pi$ is the pionic correction, and $\langle r_c{}^2 \rangle = \langle r_c{}^2 \rangle_B^{(R)} + \langle r_c{}^2 \rangle_\pi$. The last column is the MIT recoil-corrected charge radius for the same R.

R (fm)	$\langle r_c{}^2 \rangle_B^{(S)}$ (fm^2)	$\langle r_c{}^2 \rangle_B^{(R)}$ (fm^2)	$\langle r_c{}^2 \rangle_\pi$ (fm^2)	$\langle r_c{}^2 \rangle^{1/2}$ (fm)	$\langle r_c{}^2 \rangle_{\rm MIT}{}^{1/2}$ (fm)
0.8	0.273	0.201	0.228	0.655	0.499
0.9	0.365	0.269	0.207	0.690	0.562
1.0	0.468	0.344	0.190	0.731	0.624
1.1	0.582	0.428	0.173	0.775	0.687

TABLE III. Pionic and recoil corrections to proton magnetic moment (in Bohr magnetons). Experiment $=2.79$. R is the bag radius, $\mu_B^{(S)}$, $\mu_B^{(R)}$, and μ_π are the quark static, recoil-corrected, and pionic contributions to the magnetic moment. The calculated magnetic moment is $\mu=\mu_\pi+\mu_B^{(R)}$; the MIT value is given in the last column.

R (fm)	$\mu_B^{(S)}$	$\mu_B^{(R)}$	μ_π	μ	μ_{MIT}
0.8	1.36	1.28	0.83	2.11	1.46
0.9	1.56	1.48	0.70	2.18	1.65
1.0	1.76	1.67	0.57	2.24	1.83
1.1	1.96	1.86	0.46	2.32	2.01

$G_E(q^2)$, $G_M(q^2)$, and $G_A(q^2)$ is food in the low-q^2 region, even though only the simple MIT bag model (with $R=1.3$ fm) is used. The magnetic moment comes within 10% of the experimental value.

The πNN vertex corresponding to the hybrid chiral bag model is hardened by recoil corrections, but is still much softer than standard phenomenological monopole fits. If our recoil corrections are applied to the quark core contributions, the charge radii and magnetic moment predicted by the chiral bag model are smaller than the experimental values for bag radii consistent with analyses of πN and NN scattering ($R\approx 1.1$ fm). The large bag size favored by our analysis is somewhat contrary to the beliefs which led to the development of chiral bags and their application to nuclear phenomena. This may be indicative of the need for developing better models for bag-bag interactions, bag nuclear physics, and related problems.

Our results are nevertheless quite encouraging and suggest that a global investigation combining all known corrections to bag models is in order. Applications of our boost procedure to problems of practical interest, such as the $\pi\pi N$ system[32] and $N\bar{N}$ annihilation[33] are in progress and will be reported in future publications. Our treatment of the Lorentz boost improves upon earlier work regarding covariances[7,8] in bag models. However, the question of constructing translationally invariant (i.e., momentum) eigenstates *consistent* with the boost transformation still remains unanswered.

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APPENDIX A

In this appendix, we sketch the proof of

$$\vec{\mathscr{P}}_0=\vec{\mathscr{E}}_0=0\ , \tag{A1}$$

where $\vec{\mathscr{P}}_0$ and $\vec{\mathscr{E}}_0$ are defined by (43) and (44). For simplicity, we consider the case of a spherically symmetric σ_0 field only. Then the solutions[14] of (20a) take the form

$$\psi_0^i(\vec{x})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} u_i(x) \\ i\,\vec{\sigma}\cdot\hat{x}l_i(x)\end{bmatrix}\Gamma_{\kappa_i}^{\mu_i}\ , \tag{A2}$$

where κ and μ stand for the eigenvalues of $\beta(\vec{\sigma}\cdot\vec{1}+1)$ and J_z, respectively. For $j=\frac{1}{2}$, $\kappa=\pm 1$, and $\Gamma_{-1}^{\mu}=\chi_\mu$ (Pauli spinor); $\Gamma_1^{\mu}=-\vec{\sigma}\cdot\hat{x}\chi_\mu$. The functions u_i, l_i, and σ_0 are solutions of the coupled equations

$$(\epsilon^i-g\sigma_0)u_i=l_i'+\frac{l_i}{x}-\frac{\kappa_i l_i}{x}\ , \tag{A3}$$

$$(\epsilon^i+g\sigma_0)l_i=-u_i'-\frac{u_i}{x}-\frac{\kappa_i u_i}{x}\ , \tag{A4}$$

$$-\nabla^2\sigma_0+U'(\sigma_0)=-\frac{g}{4\pi}\sum_{i=1}^{N}(u_i^2-l_i^2)\ . \tag{A5}$$

Using the explicit form of the quark wave functions (A2), it is straightforward to verify that $\vec{\mathscr{P}}_0=0$ and to derive

$$\int d\vec{x}\sum_{i=1}^{N}\psi_0^{i\dagger}\hat{v}\cdot\vec{\alpha}\frac{\vec{\nabla}}{i}\psi_0^i=\tfrac{2}{3}\hat{v}\left[\sum_{\substack{i=1\\(\kappa=1)}}^{N^\dagger}\int_0^\infty dx\,x^2 l_i'u_i-\sum_{\substack{i=N^++1\\(\kappa=-1)}}^{N^\dagger+N^-}\int_0^\infty dx\,x^2 l_i u_i'\right]\ , \tag{A6}$$

where $N^\pm$ stands for the number of filled orbitals with $\kappa=\pm 1$ ($N^++N^-=N$). The right-hand side of (A6) can be rewritten in terms of the quark single-particle energies and the σ_0 field as follows. Multiply (A3) by u_i and (A4) by l_i; add the resulting equations and sum over i. Integrate over space and use (A5) to eliminate the scalar fermion density. The result is

$$\int d\vec{x}\sum_{i=1}^{N}\psi_0^{i\dagger}\hat{v}\cdot\hat{\alpha}\frac{\vec{\nabla}}{i}\psi_0^i=\hat{v}\left\{\tfrac{1}{3}\sum_{i=1}^{N}\epsilon^i+\frac{4\pi}{3}\int_0^\infty dx\,x^2\sigma_0[-\nabla^2\sigma_0+U'(\sigma_0)]\right\}\ . \tag{A7}$$

The quark single-particle energies can be eliminated from this expression with the use of a virial theorem first derived by Rafelski[34]:

$$\sum_{i=1}^{N} \epsilon^i = 4\pi \int_0^\infty dx\, x^2 (x\sigma_0' + \sigma_0)[\nabla^2\sigma_0 - U'(\sigma_0)] \ . \quad \text{(A8)}$$

Substituting this in (A7), and using the resulting expression together with expression (22) for E_0^σ, it is easy to verify that $\mathscr{E}_0$, defined by (44), vanishes identically.

APPENDIX B

In this appendix, we discuss briefly how the results of Sec. IV can be obtained by considering the Lorentz-transformation properties of the time-independent equations of motion. The time-dependent MFA equations read

$$(i\gamma^\mu\partial_\mu - g\sigma)\Psi^k = 0 \ , \quad \text{(B1)}$$

$$\partial^\mu\partial_\mu\sigma + U'(\sigma) = -g \sum_{k\,\text{occ}} \overline{\Psi}^k\Psi^k \ . \quad \text{(B2)}$$

In the "rest frame," the solutions are $\sigma_0(\vec{x}\,')$ (time independent) and

$$\Psi_0^k(\vec{x}\,',t') = \psi_0^k(\vec{x}\,')e^{-i\epsilon^k t'} \ . \quad \text{(B3)}$$

Rest-frame variables are primed for consistency with the notation of Sec. IV. The Lorentz-transformation properties of the solutions follow from the requirement that the equations of motion take the same form in any frame. This demands[10]

$$\Psi_{\vec{v}}^k(\vec{x},t) = S(\vec{v})\Psi_0^k[L^{-1}(\vec{x},t)] \ , \quad \text{(B4)}$$

$$\sigma_{\vec{v}}(\vec{x},t) = \sigma_0[L^{-1}(\vec{x},t)] \ , \quad \text{(B5)}$$

where $S(\vec{v})$ is given by (30) and the active Lorentz transformation $(\vec{x},t) = L(\vec{x},t')$ reads explicitly

$$x_{\|} = \cosh\omega\, x_{\|}' + \sinh\omega\, t' \ ,$$
$$t = \cosh\omega\, t' + \sinh\omega\, x_{\|}' \ , \quad \text{(B6)}$$
$$\vec{x}_\perp = \vec{x}_\perp' \ .$$

For a system moving with velocity $\vec{v}$, a quark wave function at $t=0$ is therefore

$$\Psi_{\vec{v}}^k(\vec{x},0) = S(\vec{v})\psi_0^k(\cosh\omega\,\vec{x}_{\|} + \vec{x}_\perp)\exp(i\epsilon^k \sinh\omega\, x_{\|}) \quad \text{(B7)}$$

which is precisely (29).

The MFA energy-momentum stress tensor is

$$T_{\text{MFA}}^{\mu\nu}(x) = i \sum_{k\,\text{occ}} \overline{\Psi}_\gamma^{k\mu}\partial^\nu\Psi^k + \partial^\mu\sigma\partial^\nu\sigma - g^{\mu\nu}\left[\tfrac{1}{2}\partial^\gamma\sigma_\partial\lambda\sigma - U(\sigma) + \sum_{k\,\text{occ}} \overline{\Psi}^k(i\gamma\mu\partial_\mu) - g\sigma)\Psi^k\right] \quad \text{(B8)}$$

equations of motion (B1) and (B2), it follows that

$$\partial_\mu T_{\text{MFA}}^{\mu\nu}(x) = 0 \ . \quad \text{(B9)}$$

Therefore the MFA energy and momentum,

$$E_{\text{MFA}}(t) = \int d\vec{x}\, T_{\text{MFA}}^{00}(x) = \int d\vec{x}\left[\sum_k \Psi^{k\dagger}\left[\vec{\alpha}\cdot\frac{\vec{\nabla}}{i} + g\sigma\beta\right]\Psi^k + \tfrac{1}{2}\pi_\sigma{}^2 + \tfrac{1}{2}|\vec{\nabla}\sigma|^2 + U(\sigma)\right] \ , \quad \text{(B10)}$$

$$P_{\text{MFA}}^i(t) = \int d\vec{x}\, T_{\text{MFA}}^{0i}(x) = \int d\vec{x}\left[\sum_k \Psi^{k\dagger}\frac{\nabla^i}{i}\Psi^k - \pi_\sigma\nabla_\sigma^i\right] \ , \quad \text{(B11)}$$

satisfy

$$\frac{\partial}{\partial t}E_{\text{MFA}}(t) = \frac{\partial}{\partial t}P_{\text{MFA}}^i(t) = 0 \ . \quad \text{(B12)}$$

The Lorentz-transformation properties (B4) and (B5) ensure that $T_{\text{MFA}}^{\mu\nu}$ transforms like a second-rank tensor. This, together with (B9) guarantees that $(E_{\text{MFA}},\vec{\text{P}}_{\text{MFA}})$ form a Lorentz four-vector. The proof, based on the use of Green's theorem in four-dimensional space-time, is standard.[35] The explicit verification of this result in the case of a spherically symmetric σ_0 was presented in Appendix A.

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