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New organization of jet calculus for colorless-cluster computations

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We write equations for a new set of leading-logarithmic parton "propagators." These differ from past "color-connecting" propagators in that they keep explicit track of the momentum of the gluons associated with the quarks in the colorless cluster. Because of this, the new functions lead in a simple and natural way to computation of the x distribution and mass distribution of colorless clusters in jets. Hence they should prove more useful for phenomenological calculations.

I. INTRODUCTION

Over the past few years, a great deal of attention has been given to the predictions possible using perturbative quantum chromodynamics (QCD). In addition to successively more complex calculations involving complete evaluation of all diagrams in some low order, there have been quite a large number of applications involving summing to all orders of the leading-logarithm terms. In this approximation, Konishi, Ukawa, and Veneziano (KUV)[1] showed how to create a "jet calculus" which allows one, given sufficient computational strength, to evaluate the inclusive distributions in x (fraction of longitudinal momentum) of any number of partons contained in the evolution of a quark or gluon jet.

Although the KUV jet calculus is useful for certain types of phenomenological work,[2] it suffers from the defect that the pair (or triplet) of partons selected may come from widely separated places in the jet spray. Because QCD has the property of "preconfinement," i.e., that colorless clusters of finite mass can be detected already in the perturbative evolution,[3] the perturbation series can be reordered so that colorless clusters (containing one quark, one antiquark, and multiple gluons) may be studied.[4]

Bassetto, Ciafaloni, and Marchesini (BCM) defined[4] "color-connected" parton propagators Γ_i^j and Γ_g^j by studying planar graphs and keeping track of the "first" quark, j, emitted from the incident line after multiple gluon emissions. They then were able to write the distribution $d\sigma/dx_1dx_2$ for the quark and antiquark in a colorless cluster emitted by a jet. This allowed an estimate of various properties of the colorless clusters, and a calculation of the upper bounds of the size of the mass of these clusters.[5]

While these estimates are enough to prove the theoretical properties of leading-logarithm QCD, they are not as useful for phenomenology as one might wish. For comparison with experiment, we need an explicit formula for the x distribution of the clusters, including the momentum carried by the gluons in the cluster. Also we would like an explicit formula for $d\sigma/dM^2$ (the mass distribution of the clusters), not just an upper bound.

In this paper we show how to write equations for some new distributions H_g^j and H_i^j. These are inspired by the BCM Γ's, but they have the property that the momentum fraction x which appears in them is the sum of the momentum carried by the first quark and that of all the gluons emitted (on the "side of the first quark" in the planar graphs) prior to the emission of the first quark. Using the H's, we can write explicit expressions for the mass of the colorless clusters and for their distribution in x.

In Sec II, we review the BCM equations for the Γ's, and then write the equations for the H's as a function of longitudinal momentum fraction x and Q^2. We show that the zeroth moment of these obeys an expected sum rule for probabilities; and that the first moment obeys the momentum-conservation condition found by BCM (i.e., as Q^2 becomes infinite, the fraction of the jet momentum going into colorless clusters approaches 1).

In Sec. III, we consider what happens when the equations are generalized to include transverse momentum. The formula for the mass distribution of colorless clusters is then given.

II. BASIC EQUATIONS

BCM equations; definition of terms

We begin by reviewing the BCM equations. The two basic points of interest here are as follows.

(i) The graphs are all written in planar form. Then one counts "clockwise" around the tree until the first quark (or antiquark) is reached. This is the parton whose momentum fraction x is being selected from the spray in Γ^j. Since only gluons are emitted on the upper half of the diagram between the incident and the measured parton, these propagators are represented as shown in Fig. 1(a) by

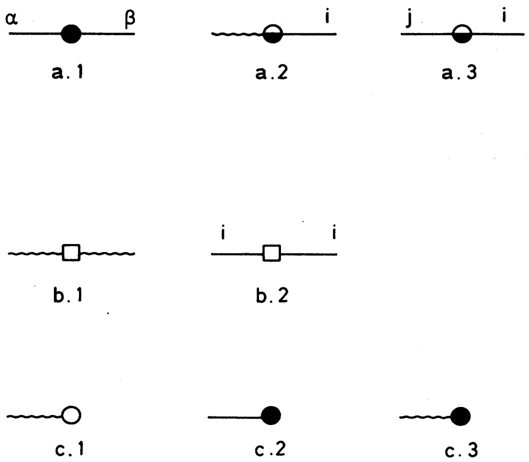

FIG. 1. Notation: (a.1) KUV symbol for $D_{\beta\alpha}(Q^2,Q_0{}^2,x)$, the probability to find parton β with momentum fraction x at $Q_0{}^2$ if one starts with parton α at Q^2. (a.2) and (a.3) BCM symbols for $\Gamma_g^i(Q^2,Q_0{}^2,x)$ and $\Gamma_j^i(Q^2,Q_0{}^2,x)$. (b.1) Representation of the virtual potential for gluons,

$$V_g(k^2)=-\int_\epsilon^{1-\epsilon} dz\frac{\alpha(z(1-z)k^2)}{2\pi}\left[\tfrac{1}{2}\hat{P}_g^{gg}(z)+\hat{P}_g^{q\bar{q}}(z)\right] .$$

(b.2) Representation of the virtual potential for quarks,

$$V_q(k^2)=-\int_\epsilon^{1-\epsilon} dz\frac{\alpha(z(1-z)k^2)}{2\pi}\left[\tfrac{1}{2}\hat{P}_q^{qg}(z)+\tfrac{1}{2}\hat{P}_q^{gq}(z)\right] .$$

(c.1) Representation of the probability that gluons go only to gluons, $\sigma_g(k^2,Q_0{}^2)$. (c.2) The probability that a quark decays in some (any) way $=1$. (c.3) The probability that a gluon decays in any way $=1$.

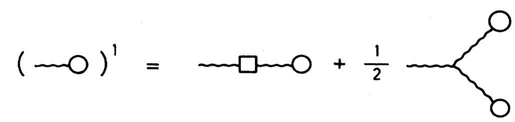

FIG. 2. Graphical depiction of Eq. (1), rate of change of σ_g with momentum.

lines with half-open circles, as contrasted with the ordinary Altarelli-Parisi or KUV propagators D_i^j, which are conventionally represented by lines with solid circles.

(2) Because of the "semiexclusive" nature of the propagators, there is extra damping at large Q^2 compared with the KUV case. This is due to the fact that infrared singularities from real and virtual emission terms do not completely cancel. This shows up in the equations as a virtual potential, represented by an open box on the line as shown in Fig 1(b).

A third point, discovered by Amati *et al.*,[6] is that the infrared singularities of the theory are handled better if the scale of the running coupling constant α_s is given by the transverse momentum k_T of the branching (instead of the mass k of the decaying parton). As shown in Ref. 5, this is necessary to get proper mass damping for the colorless clusters. In practice, this means that various factors of α_s need to be tucked inside integrals compared with BCM. We implement this throughout.

Because the gluons play a special role in this form of counting, the probability that gluons go only to gluons, σ_g, is important. This obeys the equation

$$k^2\frac{d}{dk^2}\sigma_g(k^2,Q_0{}^2)=\sigma_g(k^2,Q_0{}^2)V_g(k^2)+\tfrac{1}{2}\int dz\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2,Q_0{}^2)\sigma_g(\lambda(1-z)k^2,Q_0{}^2) , \tag{1}$$

depicted in Fig. 2. We use the notation of Altarelli and Parisi[7] for the splitting functions $P_a^{bc}(z)$; as in KUV the functions $\hat{P}$ are the P's with the δ functions at $z=1$ removed.

We now are in a position to write the equations for the BCM color-connected distributions Γ. These relations are shown in Figs. 3(a) and 3(b), which translate to

$$\begin{aligned}k^2\frac{d}{dk^2}\Gamma_g^i(k^2,Q_0{}^2,x)={}&V_g(k^2)\Gamma_g^i(k^2,Q_0{}^2,x)+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\sum_j\frac{\hat{P}_g^{q\bar{q}}(z)}{N_F}\Gamma_j^i\left[\lambda(x)k^2,Q_0{}^2,\frac{x}{z}\right]\\&+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)\Gamma_g^i\left[\lambda(z)k^2,Q_0{}^2,\frac{x}{z}\right]\\&+\int_0^{1-x}\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2,Q_0{}^2)\Gamma_g^i\left[\lambda(1-z)k^2,Q_0{}^2,\frac{x}{1-z}\right] ,\end{aligned} \tag{2a}$$

$$\begin{aligned}k^2\frac{d}{dk^2}\Gamma_j^i(k^2,Q_0{}^2,x)={}&V_q(k^2)\Gamma_j^i(k^2,Q_0{}^2,x)+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\Gamma_g^i\left[\lambda(z)k^2,Q_0{}^2,\frac{x}{z}\right]\\&+\int_0^{1-x}\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\sigma_g(\lambda(z)k^2,Q_0{}^2)\Gamma_j^i\left[\lambda(1-z)k^2,Q_0{}^2,\frac{x}{1-z}\right] .\end{aligned} \tag{2b}$$

Because of the form of the equations, they can be resolved into moments; there is no linkage between the equations for the different moments. These moment equations are given in Ref. 4.

In manipulating these quantities, it is useful to remember that because of the planar nature of the graphs, in Γ_i^j i and j may both be quarks (or we may write them as $\bar{i}$ and $\bar{j}$, in which both are antiquarks), but it is impossible to have one be a quark and the other an antiquark. In Γ_g^i, we may have either a quark or an antiquark coming out; the two quantities are of course identical in size.

BCM then consider the colorless [color-singlet (CS)] clusters shown in Fig. 4, where the symbol $\sum'$ means that only the vertices $qgq, \bar{q}g\bar{q}$ and $\frac{1}{2}ggg$ are included in the summation. (The vertices $gq\bar{q}$ do not lead to colorless clusters with this construction.) The differential cross section

$$\left.\frac{k^2 d\sigma_\alpha}{\sigma_\alpha dk^2 dx_1 dx_2}\right|_{\text{CS}} = \sum_{\beta\gamma\delta}{}' \int_{(x_1+x_2)}^{1} \frac{dx}{x^2} D_\alpha^\beta(Q^2,k^2,x) \times \int_{x_1/x}^{1-x_2/x} \frac{dz}{z(1-z)} \frac{\alpha(z(1-z)k^2)}{2\pi} \hat{P}_\beta^{\gamma\delta}(z) \Gamma_\gamma^{q_l}\left[\lambda(z)k^2, Q_0^2, \frac{x_1}{xz}\right] \Gamma_\delta^{\bar{q}_m}\left[\lambda(1-z)k^2, Q_0^2, \frac{x_2}{x(1-z)}\right] \quad (3)$$

then contains information about the x_i of the quark and antiquark.

New equations

The problem with Eq. (3) is that it does not contain explicit information about the x of the entire cluster, including the gluons. Clearly, therefore, one would like to substitute for the "half circle" propagators similar ones in which the x variable includes not only the momentum of the quark but also the momentum of all the gluons emitted on the open side of the circle.

We depict such propagators H by half-open squares. Following the lead of Fig. 3, we write the equations in Fig. 5. To convert these to a form for computation, it is only necessary to keep track of the momentum. We then arrive at

$$k^2 \frac{d}{dk^2} H_g^i(k^2,x) = V_g(k^2) H_g^i(k^2,x) + \int_x^1 \frac{dz}{z} \frac{\alpha(z(1-z)k^2)}{2\pi} \sum_j \frac{\hat{P}_g^{q\bar{q}}(z)}{N_F} H_j^i\left[\lambda(z)k^2, \frac{x}{z}\right] + \int_x^1 \frac{dz}{z} \frac{\alpha(z(1-z)k^2)}{4\pi} \hat{P}_g^{gg}(z) H_g^i\left[\lambda(z)k^2, \frac{x}{z}\right] + \int_0^x \frac{dz}{1-z} \frac{\alpha(z(1-z)k^2)}{4\pi} \hat{P}_g^{gg}(z) \sigma_g(\lambda(z)k^2, Q_0^2) H_g^i\left[\lambda(1-z)k^2, \frac{x-z}{1-z}\right], \quad (4a)$$

$$k^2 \frac{d}{dk^2} H_j^i(k^2,x) = V_q(k^2) H_j^i(k^2,x) + \int_x^1 \frac{dz}{z} \frac{\alpha(z(1-z)k^2)}{2\pi} \hat{P}_q^{gq}(z) H_g^i\left[\lambda(z)k^2, \frac{x}{z}\right] + \int_0^x \frac{dz}{1-z} \frac{\alpha(z(1-z)k^2)}{2\pi} \hat{P}_q^{gq}(z) \sigma_g(\lambda(z)k^2, Q_0^2) H_j^i\left[\lambda(1-z)k^2, \frac{x-z}{1-z}\right]. \quad (4b)$$

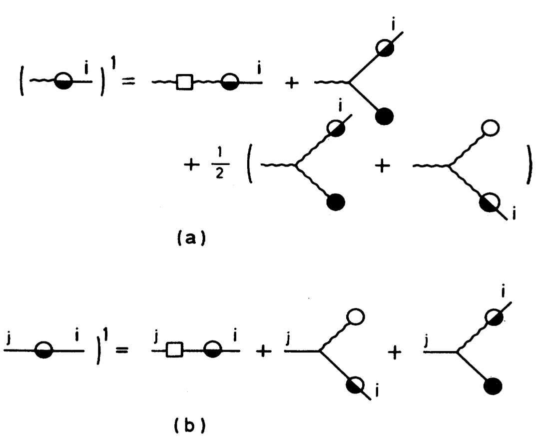

FIG. 3. Graphical depiction of Eqs. (2a) and (2b).

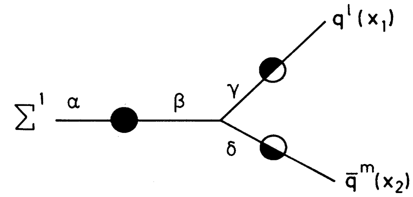

FIG. 4. We can select colorless clusters in the α jet by using the Γ's together with those vertices which can produce disconnected $q\bar{q}$ (multigluon) clusters.

These equations no longer decouple moment by moment. However, the equations for the nth moment involve only the nth and lower moments, so solution using moments can be used if necessary.

In the equation corresponding to Eq. (3),

$$\frac{d\sigma_\alpha}{\sigma_\alpha dx_1 dx_2}\Big|_{CS}=\sum_{\beta\gamma\delta}{}' \int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\int_{x_1+x_2}^{1}\frac{dx}{x^2}D_\alpha^\beta(Q^2,k^2,x) \times\int_{x_1/x}^{1-x_2/x}\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_\beta^{\gamma\delta}(z)H_\gamma^{q_l}\left[\lambda(z)k^2,\frac{x_1}{xz}\right] \times H_\delta^{\bar{q}_m}\left[\lambda(1-z)k^2,\frac{x_2}{x(1-z)}\right], \tag{5}$$

the momentum distribution of the colorless clusters can now be computed exactly as

$$\frac{d\sigma_\alpha}{\sigma_\alpha dx}\Big|_{CS}=\int dx_1 dx_2 \frac{d\sigma_\alpha}{\sigma_\alpha dx_1 dx_2}\Big|_{CS}\delta(x-x_1-x_2)\ . \tag{6}$$

Sum rules

By taking the integral of Eq. (4) over all x, we arrive at the sum rules

$$\int_0^1 dx\sum_i H_j^i(Q^2,x)=1\ ,$$
$$\int_0^1 dx\sum_i H_g^i(Q^2,x)=1-\sigma_g(Q^2,Q_0^2)\ , \tag{7}$$

the same as the sum rules for the Γ's.

The interesting question related to momentum (the first moment) then is to ascertain that the fraction of momentum in the jet going into colorless clusters approaches 1 as Q^2 becomes infinite. There are two ways to accomplish this. The first, and most straightforward, is simply to multiply Eq. (6) by x, and integrate over x. By use of the equations for H and σ_g, this can be cast into the form of a total derivative in k^2; integration over k^2 then leads us to

$$S_q(Q^2,1)=\int_0^1\frac{d\sigma_q}{\sigma_q dx}\Big|_{CS}x\,dx$$
$$=1-\sum_i H_j^i(Q^2,1)\ , \tag{8a}$$

$$S_g(Q^2,1)=\int_0^1\frac{d\sigma_g}{\sigma_g dx}\Big|_{CS}x\,dx$$
$$=1-\sigma_g(Q^2,Q_0^2)-2\sum_i H_g^i(Q^2,1)\ . \tag{8b}$$

To see the variation of this explicitly, we compute the first moment of Eq. (4) and solve these simultaneously with Eq. (1). The solution is shown in Fig. 6.

Another approach to this momentum question is to tackle the gluon momentum explicitly. This is basically the approach taken by BCM in their original paper. One begins with the double distributions Γ_a^{bg} and the auxiliary function ϕ_g^g defined by the equations

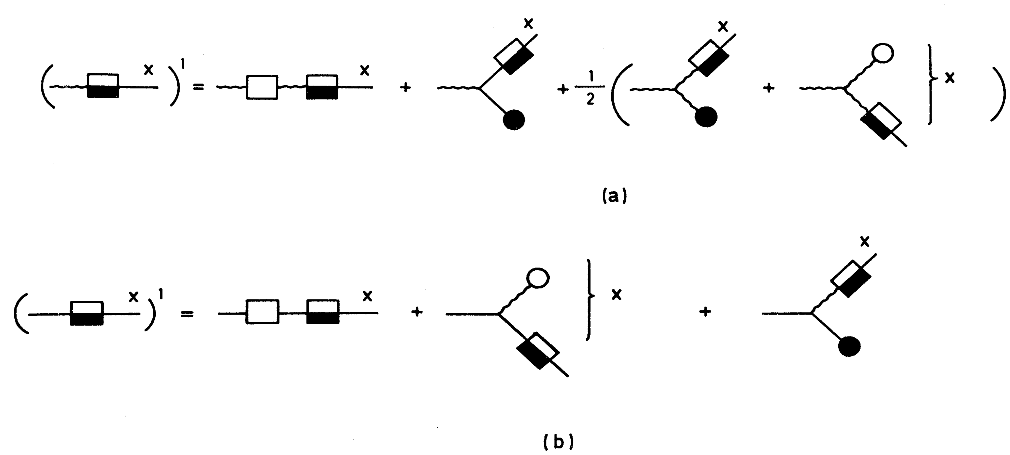

FIG. 5. Graphical depiction of Eqs. (4a) and (4b).

$$\left[\frac{d}{d\ln k^2}-V_q(k^2)\right]\Gamma_q^{qg}(k^2,Q_0^2,x_q,x_g)$$
$$=\int\frac{dz}{z^2}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_q^{qg}(z)\sigma_g(\lambda(1-z)k^2,Q_0^2)\Gamma_q^{qg}\left[\lambda(z)k^2,Q_0^2,\frac{x_q}{z},\frac{x_g}{z}\right]$$
$$+\int\frac{dz}{(1-z)^2}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_q^{qg}(z)\Gamma_g^{qg}\left[\lambda(1-z)k^2,Q_0^2,\frac{x_q}{1-z},\frac{x_g}{1-z}\right]$$
$$+\int\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_q^{qg}(z)\Phi_g^g\left[\lambda(1-z)k^2,Q_0^2,\frac{x_g}{1-z}\right]\Gamma_q^q\left[\lambda(z)k^2,Q_0^2,\frac{x_q}{z}\right], \quad (9a)$$

$$\left[\frac{d}{d\ln k^2}-V_g(k^2)\right]\Gamma_g^{qg}(k^2,Q_0^2,x_q,x_g)$$
$$=\int\frac{dz}{z^2}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_g^{q\bar q}(z)\Gamma_q^{qg}\left[\lambda(z)k^2,Q_0^2,\frac{x_q}{z},\frac{x_g}{z}\right]$$
$$+\int\frac{dz}{z^2}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat P_g^{gg}(z)[1+\sigma_g(\lambda(1-z)k^2,Q_0^2)]\Gamma_g^{qg}\left[\lambda(z)k^2,Q_0^2,\frac{x_q}{z},\frac{x_g}{z}\right]$$
$$+\int\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat P_g^{gg}(z)\Phi_g^g\left[\lambda(1-z)k^2,Q_0^2,\frac{x_g}{1-z}\right]\Gamma_g^q\left[\lambda(z)k^2,Q_0^2,\frac{x_q}{z}\right], \quad (9b)$$

$$\left[\frac{d}{d\ln k^2}-V_g(k^2)\right]\Phi_g^g(k^2,Q_0^2,x_g)=\int_{x_g}^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_g^{gg}(z)\Phi_g^g\left[\lambda(z)k^2,Q_0^2,\frac{x_g}{z}\right]\sigma_g(\lambda(1-z)k^2,Q_0^2), \quad (9c)$$

shown in Fig. 7. Note that $\Phi_g^g(k^2,1)=\sigma_g(k^2,Q_0^2)$.

The first moment of Eq. (6), the total momentum in the colorless clusters, of course can be written in the form

$$S_a(Q^2,1)=\langle x^q\rangle_a\,|_{\rm CS}+\langle x^{\bar q}\rangle_a\,|_{\rm CS}+\langle x^g\rangle_a\,|_{\rm CS}\,.$$

Following BCM [Eqs. (4.16)–(4.18) of the Nucl. Phys. paper in Ref. 4], we can write

$$\langle x^q\rangle_a\,|_{\rm CS}=D_a^q(Q^2,Q_0^2,1)-\Gamma_a^q(Q^2,Q_0^2,1), \quad (10a)$$
$$\langle x^g\rangle_q\,|_{\rm CS}=D_q^g(Q^2,Q_0^2,1)-\Gamma_q^{qg}(Q^2,Q_0^2,0,1), \quad (10b)$$
$$\langle x^g\rangle_g\,|_{\rm CS}=D_g^g(Q^2,Q_0^2,1)-\Gamma_g^{qg}(Q^2,Q_0^2,0,1)-\Gamma_g^{\bar q g}(Q^2,Q_0^2,0,1)-\Phi_g^g(Q^2,Q_0^2,1), \quad (10c)$$

so the momentum in the colorless clusters assumes the form

$$S_q(Q^2,1)=1-\Gamma_q^q(Q^2,Q_0^2,0,1)-\Gamma_q^{qg}(Q^2,Q_0^2,0,1), \quad (11a)$$

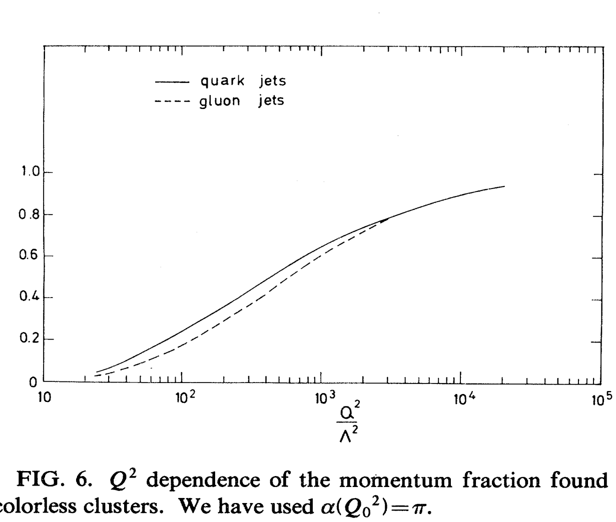

FIG. 6. Q^2 dependence of the momentum fraction found in colorless clusters. We have used $\alpha(Q_0^2)=\pi$.

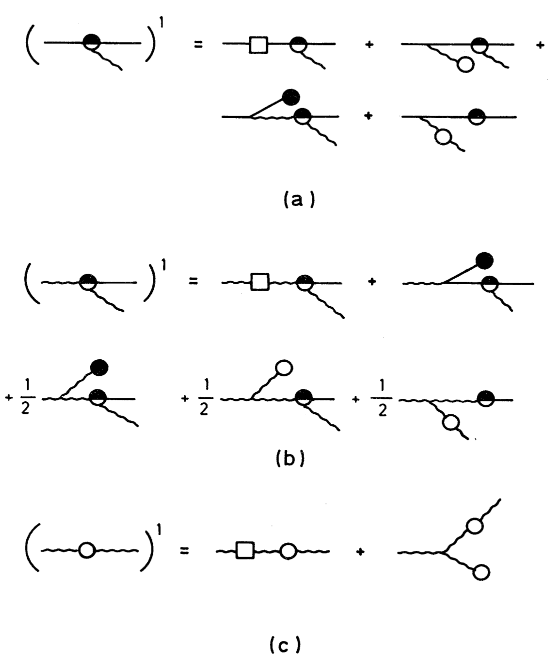

FIG. 7. Equations for the additional gluon-related distributions of BCM. (a) Γ_q^{qg}. (b) Γ_g^{qg}. (c) Φ_g^g. This function is related to σ (solution of Fig. 2) by $\Phi_g^g(k^2,1)=\sigma_g(k^2,Q_0^2)$.

$$S_g(Q^2,1)=1-2\Gamma_g^q(Q^2,Q_0^2,1)-2\Gamma_g^{qg}(Q^2,Q_0^2,0,1)$$
$$-\Phi_g^g(Q^2,Q_0^2,1)\ . \tag{11b}$$

The quantities in Eq. (11) now obey differential equations in k^2 which can be compared with the differential equation obeyed by the first moment of Eq. (6). We find that the differential equations agree if the first moments of the double distributions are related to the H functions as follows:

$$H_q^q(k^2,1)=\sum_i H_J^i(k^2,1)$$
$$=\Gamma_q^q(k^2,Q_0^2,1)+\Gamma_q^{qg}(k^2,Q_0^2,0,1)\ , \tag{12a}$$

$$H_g^q(k^2,1)=\sum_i H_g^i(k^2,1)$$
$$=\Gamma_g^q(k^2,Q_0^2,1)+\Gamma_g^{qg}(k^2,Q_0^2,0,1)\ . \tag{12b}$$

Looking back, we check that these substitutions in fact make Eqs. (11) and (8) identical.

III. TRANSVERSE MOMENTUM

In order to be able to compute the mass of the colorless cluster, we must know the complete four-momentum vector coming from each leg. The equations in the previous section determine only the longitudinal-momentum fraction. We need three other degrees of freedom, which we choose to be the mass p^2 of the system whose longitudinal momentum is specified in H, and its traverse momentum p_T.

We therefore define distributions

$$\mathscr{H}_a^{\,i}(k^2,p^2,x,\bar p_\perp - x\bar k_\perp)\ ,$$

which, when integrated over the variables p^2 and p_T, give our previous functions

$$H_g^i(k^2,x)\ ;\ \ H_J^i(k^2,x)\ .$$

In terms of these functions, the formula for the color-singlet mass distribution (see Fig. 8) becomes

$$S_a(Q^2,m^2)=\sum_{cc_1c_2}{}' \int \frac{dk^2}{k^2}\frac{dx}{x^2}d\bar k_\perp D_a^c(Q^2,k^2,x,\bar k_\perp)$$
$$\times\int\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat P_c^{c_1c_2}(z)$$
$$\times\int\frac{d\bar q_\perp}{\pi}\delta(z(1-z)k^2-\bar q_\perp{}^2)$$
$$\times\left[\int dp_1{}^2dp_2{}^2dx_1dx_2d\bar p_{1\perp}d\bar p_{2\perp}\delta\left[m^2-\frac{x_1+x_2}{x_1}p_1{}^2-\frac{x_1+x_2}{x_2}p_2{}^2-\left[\frac{x_2}{x_1}\bar p_{1\perp}-\frac{x_1}{x_2}\bar p_{2\perp}\right]^2\right]\right.$$
$$\times\mathscr{H}_{c_1}^{\,q}\left[\lambda(z)k^2,p_1{}^2,\frac{x_1}{xz},\bar p_{1\perp}-\frac{x_1}{xz}\bar q_\perp-\frac{x_1}{x}\bar k_\perp\right]$$
$$\left.\times\mathscr{H}_{c_2}^{\,\bar q}\left[\lambda(1-z)k^2,p_2{}^2,\frac{x_2}{x(1-z)},\bar p_{2\perp}+\frac{x_2}{x(1-z)}\bar q_\perp-\frac{x_2}{x}\bar k_\perp\right]\right]\ , \tag{13}$$

where q_T is given by

$$\bar q_\perp=\bar k_{1\perp}-z\bar k_\perp$$

(k_{1T} is defined in Fig. 8).

We are now ready to write the generalized equations corresponding to Fig. 5. This is chiefly an exercise in careful kinematics. Consider, for instance, contributions of the form shown in Fig. 9. The appropriate H function will have the form

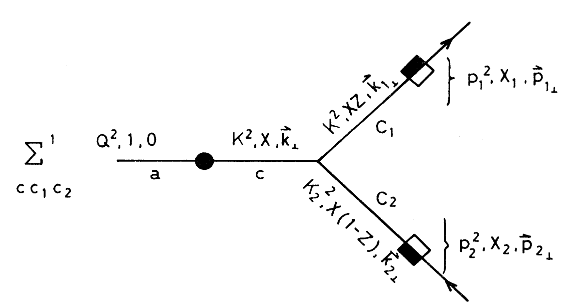

FIG. 8. Labeling of kinematics for calculation of the mass of the colorless clusters.

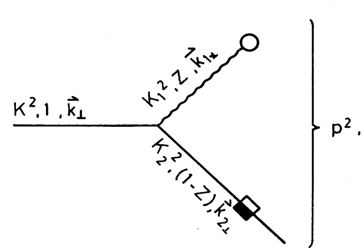

FIG. 9. Labeling of kinematics for sample graph in the equation for the generalized distributions.

$$\mathscr{H}_q^q\left[k_2{}^2,(p-k_1)^2,\frac{x-z}{1-z},\bar{p}_\perp-\bar{k}_{1\perp}-\frac{x-z}{1-z}k_{2\perp}\right].$$

Since we wish to use at the splitting vertex the transverse momentum relative to the incoming k line, $q_T=k_{1T}-zk_T$, the arguments become

$$(p-k_1)^2=\frac{x-z}{x}p^2+\frac{z-x}{z}k_1{}^2-[\sqrt{x/z}\,\bar{q}_\perp+\sqrt{xz}\,\bar{k}_\perp-\sqrt{z/x}\,\bar{p}_\perp]^2$$

and

$$\bar{p}_\perp-\bar{k}_{1\perp}-\frac{x-z}{1-z}\bar{k}_{2\perp}=\bar{p}_\perp-x\bar{k}_\perp-\frac{1-x}{1-z}\bar{q}_\perp .$$

We see that there is a residual dependence on $k_1{}^2$. Since we are trying to determine the variation of H with $(p-k_1)^2$, we must be a little careful and use the basic equation for discontinuities[4,6] rather than the "integrated" version used above. Hence the graph of Fig. 9 is written as

$$\int\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\int dk_1{}^2dk_2{}^2\frac{d\bar{q}_\perp}{\pi}\left[\frac{x-z}{x}\right]\times\delta(z(1-z)k^2-(1-z)k_1{}^2-zk_2{}^2-\bar{q}_\perp{}^2)\frac{d}{dk_1{}^2}\sigma_g(k_1{}^2,Q_0{}^2)\times\frac{d}{dk_2{}^2}\mathscr{H}_q^q\left[k_2{}^2,(p-k_1)^2,\frac{x-z}{1-z},\bar{p}_\perp-x\bar{k}_\perp-\frac{1-x}{1-z}\bar{q}_\perp\right].\quad(14)$$

We thus can write the generalized equations for Fig. 5 as

$$\left[\frac{d}{d\ln k^2}-V_g(k^2)\right]\mathscr{H}_g^q(k^2,p^2,x,\bar{p}_\perp-x\bar{k}_\perp)$$
$$=\int\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_g^{q\bar{q}}(z)\int\frac{d\bar{q}_\perp}{\pi}\delta(z(1-z)k^2-\bar{q}_\perp{}^2)\mathscr{H}_q^q\left[\lambda(z)k^2,p^2,\frac{x}{z},\bar{p}_\perp-x\bar{k}_\perp-\frac{x}{z}\bar{q}_\perp\right]$$
$$+\tfrac{1}{2}\int\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_g^{gg}(z)\int\frac{d\bar{q}_\perp}{\pi}\delta(z(1-z)k^2-\bar{q}_\perp{}^2)\mathscr{H}_g^q\left[\lambda(z)k^2,p^2,\frac{x}{z},\bar{p}_\perp-x\bar{k}_\perp-\frac{x}{z}\bar{q}_\perp\right]$$
$$+\tfrac{1}{2}\int\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_g^{gg}(z)\int dk_1{}^2dk_2{}^2\frac{d\bar{q}_\perp}{\pi}\left[\frac{x-z}{x}\right]\times\delta(z(1-z)k^2-zk_2{}^2-(1-z)k_1{}^2-\bar{q}_\perp{}^2)\frac{d}{dk_1{}^2}\sigma_g(k_1{}^2,Q_0{}^2)\times\frac{d}{dk_2{}^2}\mathscr{H}_g^q\left[k_2{}^2,(p-k_1)^2,\frac{x-z}{1-z},\bar{p}_\perp-x\bar{k}_\perp-\frac{1-x}{1-z}\bar{q}_\perp\right],\quad(15a)$$

$$\left[\frac{d}{d\ln k^2}-V_q(k^2)\right]\mathscr{H}_q^q(k^2,p^2,x,\bar{p}_\perp-x\bar{k}_\perp)$$
$$=\int\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\int\frac{d\bar{q}_\perp}{\pi}\delta(z(1-z)k^2-\bar{q}_\perp{}^2)\mathscr{H}_g^q\left[\lambda(z)k^2,p^2,\frac{x}{z},\bar{p}_\perp-x\bar{k}_\perp-\frac{x}{z}\bar{q}_\perp\right]$$
$$+\int\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}\int dk_1{}^2dk_2{}^2\frac{dq_\perp}{\pi}\left[\frac{x-z}{z}\right]\times\delta(z(1-z)k^2-zk_2{}^2-(1-z)k_1{}^2-\bar{q}_\perp{}^2)\frac{d}{dk_1{}^2}\sigma_g(k_1{}^2,Q_0{}^2)\times\frac{d}{dk_2{}^2}\mathscr{H}_q^q\left[k_2{}^2,(p-k_1),\frac{x-z}{1-z},\bar{p}_\perp-x\bar{k}_\perp-\frac{1-x}{1-z}\bar{q}_\perp\right].\quad(15b)$$

with the boundary conditions

$$\mathscr{H}_a^q(Q_0^2,p^2,x,\bar{p}_\perp - x\bar{k}_\perp) = \delta_a^q \delta(p^2-Q_0^2)\delta(1-x)\delta(\bar{p}_\perp - \bar{k}_\perp) \ .$$

If Eqs. (15) are integrated over the variables $d\bar{p}_T dp^2$, one recovers Eqs. (4).

IV. CONCLUSIONS AND OUTLOOK

As we mentioned in Sec. II, when moments of Eq. (4) are calculated, the nth moment is coupled to all the lower moments. However, in practice there are only three independent functions for each n (H_g^i, H_i^i, and H_i^j for i,j different). Thus, the coupled equations involving σ_g and the moments can be solved numerically by Runge-Kutta techniques if one is not overly ambitious in the number of moments requested. Moments of Eq. (6) can also be computed. To be specific, Eq. (5) is written in the form of a derivative in k^2, and the moments of this in $X=x_1+x_2$ are then solved simultaneously with the moments of Eq. (4). These moments may then be inverted using Yndurain's technique to obtain an x distribution.

Results of these calculations have been reported in Refs. 8 and 9, where comparison with other authors and other approaches is made. Solution of the more complicated Eq. (15) is currently under study; results will be reported later.

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