Magnetic moments of confined quarks and baryons in an independent-quark model based on Dirac equation with power-law potential

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The effect of confinement on the magnetic moment of a quark has been studied in a simple independent-quark model based on the Dirac equation with a power-law potential. The magnetic moments so obtained for the constituent quarks, which are found to be significantly different from their corresponding Dirac moments, are used in predicting the magnetic moments of baryons in the nucleon octet as well as those in the charmed and b-flavored sectors. We not only get an improved result for the proton magnetic moment, but the calculation for the rest of the nucleon octet also turns out to be in reasonable agreement with experiment. The overall predictions for the charmed and b-flavored baryons are also comparable with other model predictions.

I. INTRODUCTION

There have been a large number of papers over the years, calculating magnetic moments of old ordinary baryons in the nucleon octet[1] and also of the recent heavier baryons in the charmed and b-flavored sectors.[2,3] Nevertheless, the phenomenology of baryon magnetic moments is still not a closed chapter. Although the general pattern of magnetic moments of ordinary baryons in the nucleon octet is reproduced extremely well by various constituent-quark models, there are noticeable quantitative failures. The availability of more precise data on hyperon moments and the observation that the elementary quark models are close but not exactly adequate have stimulated many attempts[4] at improving the models with suggestions of including effects arising out of relativistic corrections, configuration mixing, anomalous quark moments, sea quarks, and various other corrections. However, in the present work we are mainly concerned with the relativistic effect only, which essentially boils down to the question of whether or not a constituent quark securely bound inside the hadron can behave as a particle with Dirac moment characteristic of the constituent mass, as is usually assumed in the simple quark-model description of baryon moments.

The effect of confinement upon the apparent magnetic moment of a quark has already been discussed by various authors[5] in the past and has been argued as an important point in favor of bag-model calculations[6,7] as against the nonreliability of the nonrelativistic quark models. Nevertheless, one of the major puzzles of the original bag-model calculations[7] is the result for the proton magnetic moment, $\mu_p \simeq 1.9\mu_N$ (μ_N = nuclear magneton), which is about 30% smaller than its corresponding experimental value. Although the predictions of the bag model for all other members of nucleon octet, when scaled by μ_p, invariably show an improvement on the naive quark model, there have been many attempts in the recent past to improve the predictions further. Donoghue and Johnson,[8] with recoil corrections, have improved the proton magnetic moment slightly, obtaining $\mu_p = 2.24\mu_N$. However, the inclusion of lowest-order pion-loop corrections in the cloudy-bag-model (CBM) calculations[9] have bettered the result even more giving $\mu_p = 2.60\mu_N$. But there is still a significant discrepancy, which may not be overlooked. It has also been observed that the spherical-bag description of hadrons containing heavy quarks may not be adequate for the mass spectrum. The discrepancy observed in the predicted mass spectrum of hadrons in the charm sector has been attributed partly to the nonsphericity of the bag[10] containing the heavy quarks. Since an analytic solution to the nonspherical bag is not quite straightforward due to mathematical complications,[11] an *ad-hoc* correction to the bag energy has been suggested[10] to get around this difficulty. Therefore, Bose and Singh,[3] in their predictions for the magnetic moments of charmed and b-flavored hadrons in the bag-model approach, had to incorporate this empirical nonsphericity correction through the effective hadronic radii obtained in Ref. 10.

In view of the above circumstances it is worthwhile to try some alternative scheme, which, while preserving the essential features of the otherwise successful bag model, can provide a simple and unified approach to the understanding of the constitutent quark dynamics particularly in the context of the magnetic-moment study of hadrons in ordinary, charmed, and b-flavored sectors. In fact, if the idea of independent constituent quarks in the hadrons and the mechanism of confinement of these quarks to hadronic dimensions are the two basic ingredients of bag models leading to their reasonable success, then one can make a simpler alternative approach based on the independent-quark Dirac equation with some average quark interaction potential of suitable Lorentz structure. Such a scheme has been followed by many authors[12] in the recent past, where the confinement of individual constituent quarks in hadrons has been achieved through some average potential with suitable Lorentz structure, without any finite boundary restrictions of the bag model. Here the confining potential replaces the effects of the external pressure on the bag. Such a scheme using a power-law potential with Lorentz structure in the form of an equal admixture of scalar and vector parts has been employed by us in connection with the study of heavy-meson spectra[13] and also in understanding the static electromagnetic properties[14] of nucleon octet.

In this work we intend to investigate the implications of this scheme exclusively in the study of magnetic moments of ordinary, charmed, and b-flavored baryons. We present in Sec. II a brief outline of the potential model adopted and its solutions, leading to the complete description of the relativistic bound states of the independently confined constituent quarks of the hadrons. With the Dirac wave function for the ground state in hand, constituent-quark magnetic moments, taking into account the relativistic effects, are computed in the usual manner. These moments turn out to be significantly different from the corresponding Dirac moments of free quarks, illustrating thereby the importance of relativistic effects particularly for lighter constituent quarks. Then following the usual prescriptions, we express the magnetic moments of baryons in the nucleon octet and also in charmed and b-flavored baryons in terms of their corresponding constitutent-quark moments. Finally in Sec. III, we describe the phenomenology for obtaining the values of the constituent-quark magnetic moments in detailed comparison with the corresponding Dirac moments in order to have a quantitative assessment of the significance of relativistic effects on quark moments. The use of these constituent-quark moments ultimately leads to our predictions of the baryon moments. Not only do we get an improved result for the proton magnetic moment, but also the calculation for the rest of the nucleon octet turns out to be in reasonable agreement with the experiments. The overall predictions for the charmed and b-flavored baryons are also found not to be drastically different from other model predictions.

II. THE THEORETICAL FRAMEWORK

In this section we will outline briefly the adopted framework based on the potential model developed in our earlier works[13,14] and will discuss its implications with regard to the constituent-quark magnetic moments which ultimately yield the baryon moments.

A. The potential model

We start with the assumption that the constituent quarks of baryons move independently in an average flavor-independent central potential taken in the form

$$V_q(r)=(1+\beta)V(r)=(1+\beta)(a^{\nu+1}r^{\nu}+V_0)\ , \qquad (2.1)$$

where a and ν are >0 and r is the radial distance from the baryon center of mass. It is further assumed that the independent quark of rest mass m_q obeys the Dirac equation so that the four-component quark wave function $\Psi_q(\vec{r})$ satisfies the equation (with $\hbar=c=1$)

$$[\vec{\alpha}\cdot\vec{P}+\beta m_q+V_q(r)]\Psi_q(\vec{r})=E_q\Psi_q(\vec{r})\ . \qquad (2.2)$$

Following the usual approach of the bag models, if we now assume that all the three constituent quarks of the baryons are in their ground state with $J^P=\frac{1}{2}^+$, then a solution to the quark wave function $\Psi_q(\vec{r})$, written in the two-component form as

$$\Psi_q(\vec{r})=N_q\begin{bmatrix}\psi_A(\vec{r})\\ \psi_B(\vec{r})\end{bmatrix}\equiv N_q\begin{bmatrix}\varphi_q(\vec{r})x_\uparrow\\ \dfrac{\vec{\sigma}\cdot\vec{P}}{\lambda_q}\varphi_q(\vec{r})x_\uparrow\end{bmatrix}\ , \qquad (2.3)$$

can be obtained[14] with

$$\psi_A(\vec{r})=A\frac{U_q(r)}{r}Y_0^0(\theta,\varphi)x_\uparrow\equiv\varphi_q(\vec{r})x_\uparrow\ , \qquad (2.4)$$

$$\psi_B(\vec{r})=\frac{iA}{\lambda_q}\frac{d}{dr}\left[\frac{U_q}{r}\right][-(\tfrac{1}{3})^{1/2}Y_1^0x_\uparrow+(\tfrac{2}{3})^{1/2}Y_1^1x_\downarrow]$$
$$=\frac{\vec{\sigma}\cdot\vec{P}}{\lambda_q}\varphi_q(\vec{r})x_\uparrow\ . \qquad (2.5)$$

Here, $\lambda_q=(E_q+m_q)$ and A is the normalization constant of $\psi_A(\vec{r})$, whereas N_q stands for the overall normalization of $\Psi_q(\vec{r})$. This overall normalization constant N_q can be seen to appear in various dynamical expressions representing the static properties of baryons including the magnetic moments and can be easily obtained as

$$N_q{}^2=[1+(E_q-m_q-2V_0-2a^{\nu+1}\langle r^{\nu}\rangle_q)/\lambda_q]^{-1}\ . \qquad (2.6)$$

The angular brackets $\langle r^{\nu}\rangle_q$ mean the expectation value with respect to $\varphi_q(\vec{r})$, the normalized radial-angular part of $\psi_A(\vec{r})$. Finally, the reduced radial part $u_q(r)$ corresponding to the ground-state wave function of the confined quark in Eqs. (2.4) and (2.5) can be found to satisfy the equation

$$\frac{d^2U_q(r)}{dr^2}+\lambda_q(E_q-m_q-2V_0-2a^{\nu+1}r^{\nu})U_q(r)=0\ . \qquad (2.7)$$

Now if we define a dimensionless variable $\rho=(r/r_0)$ with the scalar factor r_0 chosen as

$$r_0=(2\lambda_q a^{\nu+1})^{-1/(\nu+2)}\ , \qquad (2.8)$$

then Eq. (2.7) can be transformed to a convenient form:

$$\frac{d^2U_q(\rho)}{d\rho^2}+(\epsilon_q-\rho^{\nu})U_q(\rho)=0 \qquad (2.9)$$

when

$$\epsilon_q=\left[\frac{2a^{\nu+1}}{\lambda_q{}^{\nu/2}}\right]^{-2/(\nu+2)}(E_q-m_q-2V_0)\ . \qquad (2.10)$$

Equation (2.9) provides the basic eigenvalue equation whose solution by any standard numerical method or WKB approximation method would give ϵ_q and the normalized function $u_q(r)$ for a particular choice of $\nu > 0$ and independent of any other parameters such as V_0, a, and m_q. As for example, the WKB solution gives[15]

$$\epsilon_q^{\mathrm{WKB}} = \left[\frac{3\sqrt{\lambda}}{2} \frac{\Gamma(\frac{3}{2}+1/\nu)}{\Gamma(1+1/\nu)} \right]^{2\nu/(\nu+2)} \tag{2.11}$$

and

$$U_q^{\mathrm{WKB}}(r) \simeq \frac{\mathrm{const}}{(\epsilon_q - \rho^\nu)^{1/4}} \times \cos\left[\int_0^\rho d\rho' (\epsilon_q - \rho'^\nu)^{-1/2} - \pi/4 \right] . \tag{2.12}$$

Once ϵ_q is known, relation (2.10) can be inverted to obtain the individual quark binding energy E_q, which shall now depend on the parameters V_0, m_q, and a through the relation

$$E_q = (m_q + 2V_0 + a x_q) , \tag{2.13}$$

where x_q is the solution of the root equation obtained through substitutions from (2.10) in the form

$$x_q^{(\nu+2)/\nu} \left[x_q + \frac{2}{a}(m_q + V_0) \right] = 2^{2/\nu}(\epsilon_q)^{(\nu+2)/\nu} . \tag{2.14}$$

Thus, the simple model under discussion provides a complete description of the relativistic bound states of the confined constituent quarks of the baryons with the quark wave function $\Psi_q(\vec{r})$ given as in (2.4) and (2.5) and the corresponding binding energy E_q given by (2.13). It is rather trivial and interesting to note that an ultrarelativistic limit to these solutions also exists when $m_q \to 0$, implying thereby the fact that massless quarks can also be confined in such potential models as it happens in bag models. It can further be shown that the nonrelativistic limit to the solutions can also be realized when $m_q \to \infty$ ($m_q \gg |2V_0|$). In that case we can neglect x_q and V_0 inside the square brackets of Eq. (2.14) as compared to m_q, so as to obtain the value of x_q and hence E_q in a reasonably good approximation as

$$E_q \simeq m_q + 2V_0 + a \left[2^{(2-\nu)/(2+\nu)} \left[\frac{a}{m_q} \right]^{\nu/(\nu+2)} \epsilon_q \right] . \tag{2.15}$$

This result is well in accord with the expectation that the confined particle energy must approach free-particle mass in this limit. Now approximating $\lambda_q = (E_q + m_q) \simeq 2m_q$ and putting $E_q' = (E_q - m_q)$ and $V'(r) = 2(V_0 + a^{\nu+1} r^\nu)$ in Eq. (2.7), we can obtain a Schrödinger-type equation,

$$\frac{d^2 U_q(r)}{dr^2} + 2m_q[E_q' - V'(r)]U_q(r) = 0 . \tag{2.16}$$

The solution of this equation would ultimately give the normalized upper component $\psi_A(\vec{r})$ of $\Psi_q(\vec{r})$ which obviously in this limit must be appreciably larger compared to $\psi_B(\vec{r})$. This fact can be made transparent by inspecting the expression for N_q^2 in Eq. (2.6). If we take the WKB value of $\langle r^\nu \rangle_q$ in the form[15]

$$\langle r^\nu \rangle_q = \frac{2 r_0^{\ \nu}}{(\nu+2)} \epsilon_q , \tag{2.17}$$

then with $\xi_q = (m_q + V_0)$, the expression (2.6) for N_q^2 simplifies to

$$N_q^2 = \frac{(\nu+2)}{2(\nu+1) - 2\nu(\xi_q/\lambda_q)} . \tag{2.18}$$

But in the limit $m_q \gg |2V_0|$, the ratio (ξ_q/λ_q) approximating to $\frac{1}{2}$ would lead to the limiting value of $N_q^2 = 1$. This would imply that $|\psi_B(\vec{r})|^2 \ll |\psi_A(\vec{r})|^2$. Hence, a reasonable description of the confined-quark wave function by the normalized two-component function $\psi_A(\vec{r})$ is possible in this nonrelativistic limit.

B. Magnetic moment of confined quarks

We would now investigate here the effect of confinement on the apparent magnetic moment of the constituent quarks. With the ground-state wave function $\Psi_q(\vec{r})$ of the quark known in the form given by Eqs. (2.3)—(2.5), it is straightforward to compute the magnetic moment $\vec{\mu}_q = \mu_q \vec{\sigma}$ of the quarks. In doing this one introduces into the original Dirac equation a minimal coupling of an external electromagnetic field with the vector potential

$$\vec{A} \equiv \tfrac{1}{2}(-yB, xB, 0) , \tag{2.19}$$

so that

$$\vec{B} = \vec{\nabla} \times \vec{A} = \hat{k} B . \tag{2.20}$$

Then the change in the quark binding energy (ΔE_q) can be given by

$$\Delta E_q = \int d^3\vec{r} [\vec{A}(\vec{r}) \cdot \vec{J}_q(\vec{r})] \equiv \langle -\vec{\mu}_q \cdot \vec{B} \rangle . \tag{2.21}$$

Hence, the confined-quark magnetic moment can be obtained from the relation

$$\mu_q \langle \sigma_z \rangle = |\Delta E_q| / B . \tag{2.22}$$

Using $\vec{J}_q(\vec{r}) = e_q \bar{\Psi}_q(\vec{r}) \vec{\gamma} \Psi_q(\vec{r})$ in (2.21), expression (2.22) can be simplified to

$$\mu_q \langle \sigma_z \rangle = \frac{e_q}{2} \int d^3\vec{r} \Psi_q^\dagger(\vec{r}) [(\vec{r} \times \vec{\alpha})_z] \Psi_q(\vec{r}) . \tag{2.23}$$

Now with the explicit form of $\Psi_q(\vec{r})$ as given in Eqs. (2.3)—(2.5) and after some algebra, it is straightforward to show that in units of the nuclear magneton μ_N, the confined-quark magnetic moment

$$\mu_q = \frac{2M_p e_q}{\lambda_q} N_q^2 \mu_N . \tag{2.24}$$

Here M_p is the proton mass and e_q is the electric charge of the quark in the unit of the proton charge.

For a comparative study it would be worthwhile to remind ourselves of the corresponding expression obtained in the bag model. The magnetic moment of a confined quark in the bag model can be given by

$$\mu_q^{\text{bag}} = \frac{e_q R}{6m_q} f(\omega R) , \tag{2.25}$$

with

$$f(\omega R) = \frac{4\omega R + 2m_q R - 3}{2(\omega R)^2 - 2\omega R + m_q R} , \tag{2.26}$$

where R is the hadronic bag radius and ω is the binding energy of the quark in the lowest mode given by

$$\omega = [x^2 + (mR)^2]^{1/2}/R . \tag{2.27}$$

Here x is the root of the transcendental equation

$$\tan x = x/[1 - mR - (x^2 + m^2R^2)^{1/2}] . \tag{2.28}$$

If one recognizes the fact that the energy ω here plays the role of an effective mass for the confined quarks and then compares the magnetic moment of the confined quark with the Dirac moment $\mu_q^{D'}$ of a free quark with mass ω, then one arrives at the useful ratio

$$R_2 = (\mu_q^{\text{bag}}/\mu_q^{D'}) = \frac{\omega R}{3} f(\omega R) . \tag{2.29}$$

Quigg[16] points out that although obviously at the nonrelativistic limit, this ratio is equal to unity; the two moments do not differ by more than 20% even for the extreme case of a confined massless quark. Therefore, he concludes that it may not be nonsensical for the confined quarks to display Dirac-moments characteristic of their constituent masses.

However, if we now return to our present model to make a similar investigation, we find

$$R_2 = (\mu_q/\mu_q^{D'}) = \frac{2E_q}{\lambda_q} N_q^{\,2} . \tag{2.30}$$

In the nonrelativistic limit, since $\lambda_q \simeq 2E_q \simeq 2m_q$ and $N_q^{\,2} \Longrightarrow 1$, the constitutent-quark moment approaches the Dirac moment characteristic of the constituent mass. However, in the ultrarelativistic limit when $m_q \to 0$, using (2.18), we find that

$$R_2 = (\nu+2)/[(\nu+1) - \nu V_0/E_q] . \tag{2.31}$$

For a simple estimate, if we take the potential constant $V_0 = 0$, then for a class of potentials with $\nu = 0$, 1, and 2 (logarithm, linear, and harmonic, respectively), the ratio $R_2 = 2$, $\frac{3}{2}$, and $\frac{4}{3}$, respectively, suggesting the fact that the two moments would differ significantly. Therefore, we expect that the situation, in a realistic potential model of such type describing hadrons, may be quite different from what one encounters in the bag models, where the relativistic effects seem to be apparently suppressed. Hence, we believe that the relativistic expression (2.24) for the magnetic moments of the light constituent quarks in particular may bring forth some significant improvement in the proton magnetic moment as well as in the moments for the rest of the baryons in the nucleon octet.

C. Baryon magnetic moments

If we make the usual assumption that the baryon moments arise solely from the constituent-quark moments,[17] then following Johnson and Shah-Jahan[3] and also the earlier work of Franklin,[18] we can obtain expressions for the magnetic moments of ordinary, charmed, and b-flavored baryons in terms of the magnetic moments of the corresponding constituent quarks in the following manner:

$$\mu_B = \sum_q \langle B\uparrow | \mu_q \sigma_z^q | B\uparrow \rangle , \tag{2.32}$$

where $|B\uparrow\rangle$ stands for the state vectors of the baryons. In the case of octet nucleons $|B\uparrow\rangle$ represents the regular SU(6) state vectors. For the charmed or b-flavored baryons, the corresponding state vectors are the straightforward extensions as given by Singh.[2] We denote the magnetic moments of quarks u, d, s, c, and b by μ_u, μ_d, μ_s, μ_c, and μ_b, respectively, whereas for a baryon we use the baryon symbol itself to stand for its magnetic moment. The symbols used for charmed and b-flavored baryons are according to Pandit *et al.*[3] Then for ready reference, we list the well known relations between the baryon magnetic moments and the corresponding constitutent-quark moments:

(i) Nucleon octet:

$$\begin{aligned}
&p = \tfrac{1}{3}(4\mu_u - \mu_d), \quad n = \tfrac{1}{3}(4\mu_d - \mu_u), \quad \Lambda = \mu_s ,\\
&\Sigma^+ = \tfrac{1}{3}(4\mu_u - \mu_s), \quad \Sigma^- = \tfrac{1}{3}(4\mu_d - \mu_s) ,\\
&\Sigma^0 = \tfrac{1}{3}(2\mu_u + 2\mu_d - \mu_s), \quad (\Sigma^0,\Lambda) = \frac{1}{\sqrt{3}}(\mu_d - \mu_u) ,\\
&\Xi^0 = \tfrac{1}{3}(4\mu_s - \mu_u), \quad \Xi^- = \tfrac{1}{3}(4\mu_s - \mu_d) .
\end{aligned} \tag{2.33}$$

(ii) Charmed baryons:

$$\begin{aligned}
&\Sigma_c^{++} = \tfrac{1}{3}(4\mu_u - \mu_c), \quad \Sigma_c^{+} = \tfrac{1}{3}(2\mu_u + 2\mu_d - \mu_c) ,\\
&\Sigma_c^0 = \tfrac{1}{3}(4\mu_d - \mu_c), \quad \Omega_c^0 = \tfrac{1}{3}(4\mu_s - \mu_c) ,\\
&\Xi_c^+ = \tfrac{1}{3}(2\mu_u + 2\mu_s - \mu_c), \quad \Xi_c^0 = \tfrac{1}{3}(2\mu_d + 2\mu_s - \mu_c) ,\\
&\Xi_{cc}^{++} = \tfrac{1}{3}(4\mu_c - \mu_u), \quad \Xi_{cc}^{+} = \tfrac{1}{3}(4\mu_c - \mu_d) ,\\
&\Omega_{cc}^{+} = \tfrac{1}{3}(4\mu_c - \mu_s), \quad \Lambda_c^+ = \mu_c = \Xi_c'^{\,+} = \Xi_c'^{\,0} .
\end{aligned} \tag{2.34}$$

(iii) b-flavored baryons:

$$\begin{aligned}
&\Sigma_b^+ = \tfrac{1}{3}(4\mu_u - \mu_b), \quad \Sigma_b^0 = \tfrac{1}{3}(2\mu_u + 2\mu_d - \mu_b) ,\\
&\Sigma_b^- = \tfrac{1}{3}(4\mu_d - \mu_b), \quad \Omega_b^- = \tfrac{1}{3}(4\mu_s - \mu_b) ,\\
&\Xi_{cb}^+ = \tfrac{1}{3}(2\mu_d + 2\mu_c - \mu_b), \quad \Xi_{cb}^0 = \tfrac{1}{3}(2\mu_d + 2\mu_c - \mu_b) ,\\
&\Omega_{ccb}^+ = \tfrac{1}{3}(4\mu_c - \mu_b), \quad \Xi_{bb}^0 = \tfrac{1}{3}(-\mu_u + 4\mu_b) ,\\
&\Xi_{bb}^- = \tfrac{1}{3}(-\mu_d + 4\mu_b), \quad \Omega_{bb}^- = \tfrac{1}{3}(-\mu_s + 4\mu_b) ,\\
&\Omega_{cbb}^0 = \tfrac{1}{3}(-\mu_c + 4\mu_b), \quad \Xi_b^0 = \tfrac{1}{3}(2\mu_u + 2\mu_s - \mu_b) ,\\
&\Xi_b^- = \tfrac{1}{3}(2\mu_d + 2\mu_s - \mu_b), \quad \Omega_{cb}^0 = \tfrac{1}{3}(2\mu_s + 2\mu_c - \mu_b) .
\end{aligned} \tag{2.35}$$

III. RESULTS AND CONCLUSION

In this section we would describe the procedure adopted to compute the constituent-quark magnetic moments as

TABLE I. Bound-state solutions of the constituent quarks given in terms of $\lambda_q=(E_q+m_q)$ in MeV and N_q^2, along with the quark magnetic moments μ_q in nuclear magnetons and the magnetic-moment ratios $R_1=(\mu_q/\mu_q^D)$ and $R_2=(\mu_q/\mu_q^{D'})$ obtained by (i) the WKB method and (ii) the numerical method.

q	u	d	s	c	b
(i) λ_q	498.36	498.36	976.1	3514.63	9823.55
N_q^2	0.728	0.728	0.844	0.954	0.983
μ_q	1.8539	−0.926 97	−0.5453	0.340 38	−0.0627
R_1	0.54	0.54	0.84	1.014	1.022
R_2	0.91	0.91	0.85	0.895	0.944
(ii) λ_q	459.71	459.71	948.02	3492.08	9802.8
N_q^2	0.7038	0.7038	0.8354	0.952	0.983
μ_q	1.9155	−0.9577	−0.5512	0.3411	−0.0627
R_1	0.45	0.45	0.81	1.006	1.02
R_2	0.96	0.96	0.87	0.898	0.946

given by the expression (2.24) which ultimately leads to the calculation of magnetic moments of baryons in ordinary, charmed, and b-flavored sectors.

First of all we make the usual assumption that the average potential taken in this model for the confined independent quarks inside the hadrons is flavor independent. Then in encompassing the charm and b-flavor sectors as well, we can still use the same set of values for the potential parameters a, V_0, and ν in Eq. (2.1) as obtained in Ref. 14, in explaining successfully the static properties of baryons in the nucleon octet. So we fix these parameters accordingly as

$$(\nu,a,V_0)\equiv(0.1,1.5562\text{ GeV},-1.89\text{ GeV})\ . \tag{3.1}$$

The quark masses m_q for u, d, s, c, and b quarks are obtained by making appropriate references to some hadronic ground-state masses, which in this independent-quark-model approach would be given by the sum total of the constituent-quark binding energies in the form

$$M(\text{hadron})=\sum_q E_q\ . \tag{3.2}$$

To start with we determine the up- and down-quark masses in conformity with the nucleon mass. For this, we set as usual the apparently small mass difference (m_d-m_u) to be zero, since it does not upset the magnetic-moment calculations significantly. Then $m_u=m_d$ is chosen suitably so that when confined within the nucleon by an average potential given in (2.1) and (3.1), the up and down quark would have, from the solutions of Eq. (2.7), the binding energy $E_u=E_d=\frac{1}{3}M_p$. Similarly for fixing m_s and m_c, we take the masses of Λ and Λ_c, respectively, as inputs so as to obtain $E_s=(M_\Lambda-2E_u)$ and $E_c=(M_{\Lambda_c}-2E_u)$. But in the absence of any knowledge of the b-flavored baryons, we simply refer to the Υ mass in fixing m_b so as to obtain $E_b=\frac{1}{2}M_\Upsilon$. In other words, the constituent-quark binding energies E_q's corresponding to the ground states of the baryons with $J^P=\frac{1}{2}^+$, calculated from the appropriate hadronic masses as stated above, were used effectively as inputs in order to determine the quark masses m_q's and hence, the solutions for their bound states. These inputs are roughly,

$$\begin{aligned}(E_u=E_d,E_s)&\equiv(312.76,490.08)\text{ MeV}\ ,\\ (E_c,E_b)&\equiv(1647.38,4716.8)\text{ MeV}\ .\end{aligned} \tag{3.3}$$

For a quick estimate of the quantities of interest, we adopt first the WKB method in solving Eq. (2.8). The quark masses m_q leading to the appropriate binding energies as given in (3.3) are then obtained as

TABLE II. Magnetic moments of the nucleon octet calculated by the present model in (i) the WKB method and (ii) the numerical method as compared with the results of the cloudy bag model (CBM) (Ref. 9) and the experimental data (Ref. 19) (all numbers in nuclear magneton).

Baryons	Present calculation (i)	Present calculation (ii)	CBM calculation	Experimental
p	2.7809	2.8732	2.60	2.793
n	−1.8539	−1.9154	−2.01	−1.913
Λ	−0.5453	−0.5512	−0.58	−0.614±0.005
Σ^+	2.6536	2.7377	2.34	2.33 ±0.13
Σ^-	−1.0542	−1.0932	−1.08	−1.41 ±0.25 −0.89 ±0.14
Σ^0	0.7997	0.8222		0.46 ±0.28
Ξ^-	−0.4181	−0.4157	−0.51	−0.69 ±0.04
Ξ^0	−1.3450	−1.3734	−1.27	−1.25 ±0.014
(Λ,Σ)	−1.6056	−1.6588		$-1.82^{+0.18}_{-0.25}$

TABLE III. (i) WKB and (ii) numerical results for the magnetic moments of charmed baryons obtained in the present model as compared to other calculations (all numbers in nuclear magnetons).

Baryon symbol (Ref. 3)	Quark content	Present calculation (i)	(ii)	DGG model (Ref. 20)	Bag model (Ref. 21)
Σ_c^{++}	cuu	2.358	2.44	2.36	1.955
Σ_c^{+}	cud	0.505	0.525	0.43	0.363
Σ_c^{0}	cdd	−1.349	−1.391	−1.43	−1.23
Ξ_c^{+}	cus	0.759	0.796	0.73	0.475
Ξ_c^{0}	cds	−1.095	−1.12	−1.16	−1.09
Ω_c^{0}	ssc	−0.841	−0.85	−0.89	−0.98
Ξ_{cc}^{++}	ccu	−0.164	−0.184	−0.12	−0.167
Ξ_{cc}^{+}	ccd	0.763	0.774	0.82	0.865
Ω_{cc}^{+}	ccs	0.636	0.639	0.69	0.838
Λ_c^{+}	$c(ud)_a$	0.34	0.341	0.37	0.503
$\Xi_c'^{+}$	$c(us)_a$	0.34	0.341	0.37	0.503
$\Xi_c'^{0}$	$c(ds)_a$	0.34	0.341	0.37	0.503

$$(m_u=m_d,m_s)\equiv(185.6,486.02)\text{ MeV},\quad (m_c,m_b)\equiv(1867.25,5106.75)\text{ MeV}. \tag{3.4}$$

Then it is straightforward to calculate the constituent-quark magnetic moments from expressions (2.24) and (2.18). In order to realize the significance of the relativistic effects on the constituent-quark magnetic moments, we compare first our results with the corresponding Dirac moment μ_q^D of the free quarks with mass m_q, in terms of the ratio

$$R_1=(\mu_q/\mu_q^D)=\frac{2m_q}{\lambda_q}N_q^{\,2}. \tag{3.5}$$

However, if we argue that the binding energy E_q here plays the role of an effective mass for the confined quarks, then we can consider the Dirac moments of the confined quarks with masses E_q and hence calculate the ratio as

$$R_2=(\mu_q/\mu_q^{D'})=\frac{2E_q}{\lambda_q}N_q^{\,2}. \tag{3.6}$$

The results of such calculations are presented in Table I. We observe that in terms of the ratio R_1, the two moments differ quite significantly for the lighter quarks (u,d,s) with the difference varying between 20% to 60%, where as is almost negligible for heavier quarks (c,b), in accordance with one's usual expectation. However, in terms of the ratio R_2, we find that over the entire range of the required constituent-quark masses, the difference between the two moments are still of the order of 10% to 20%, which is unlike the observation suggested by Quigg[16] for bag-model results.

To be more realistic, we repeat all the above calculations by following the exact numerical methods. We find that for the same quark-binding energies [Eq. (3.3)] taken as the inputs, the quark masses come out to be

$$(m_u=m_d,m_s)\equiv(146.95,457.94)\text{ MeV},\quad (m_c,m_b)\equiv(1844.7,5086.0)\text{ MeV}, \tag{3.7}$$

which are slightly different from the ones obtained in WKB method. However, the magnetic moments of the

TABLE IV. (i) WKB and (ii) numerical results for the magnetic moments of b-flavored baryons obtained in the present model as compared to other calculations (all numbers in nuclear magnetons).

Baryon symbol	Quark content	Present calculation (i)	(ii)	DGG model (Ref. 20)	Bag model (Ref. 21)
Σ_b^{+}	uub	2.493	2.575	2.5	2.318
Σ_b^{0}	udb	0.639	0.659	0.61	0.587
Σ_b^{-}	ddb	−1.215	−1.256	−1.28	−1.117
Ω_b^{-}	ssb	−0.706	−0.714	−0.55	−0.838
Ξ_{cb}^{+}	ucb	1.484	1.525	1.5	2.04
Ξ_{cb}^{0}	dcb	−0.37	−0.39	−0.38	−0.39
Ω_{ccb}^{+}	ccb	0.475	0.476	0.51	0.894
Ξ_{bb}^{0}	ubb	−0.702	−0.722	−0.7	−0.614
Ξ_{bb}^{-}	dbb	0.225	0.236	0.23	0.14
Ω_{bb}^{-}	sbb	0.098	0.10	0.105	0.084
Ω_{cbb}^{0}	cbb	−0.197	−0.197	−0.21	0.31
Ξ_b^{0}	sub	0.893	0.93	0.87	0.73
Ξ_b^{-}	sdb	−0.961	−0.985	−1.05	−0.977
Ω_{cb}^{0}	scb	−0.116	−0.119	−0.11	−0.223

constituent quarks and the so-called ratios R_1 and R_2, which are also presented in Table I, are not drastically different from the corresponding WKB values.

Hence, we can come to the conclusion that unlike bag-model calculations, the relativistic effects in the present model appear to have quite significant bearing on the constituent-quark magnetic moment. Therefore, we expect that making use of the individual quark magnetic moments so obtained, the situation for predicting the baryon moments may be improved. Once we have the right constituent-quark moments, it is straightforward to compute the magnetic moments of baryons using the expressions (2.33)—(2.35). The magnetic moments calculated in this manner for ordinary, charmed, and b-flavored baryons are presented in Tables II, III, and IV, respectively. The results obtained for ordinary baryons in the nucleon octet compare reasonably well with the available experimental data. The proton magnetic moment in particular comes out in agreement with the experiment within 2% as compared to 7% in the cloudy bag model.[9] On the other hand, in the absence of any experimental data for the magnetic moments of charmed and b-flavored baryons, we just compare our results with the predictions of some other model.[3] It may be a long time before one can actually measure the static moments of charmed and b-flavored baryons. Nevertheless, the predictions for these heavy-hadron magnetic moments may be useful in some other calculations more easily accessible to experimental tests. We observe that our results are not very different from the prediction of other models as shown in Tables III and IV.

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