Theory of neutron-antineutron oscillation using bottled ultracold neutrons

S. Marsh and K. W. McVoy
Physics Department, University of Wisconsin, Madison, Wisconsin 53706
(Received 6 June 1983)

We contrast the quantum-mechanical treatment of neutron oscillations in a "bottle" with the usual free-space formalism. The quantum-mechanical corrections appear to reduce the figure of merit for the bottled-neutron experiments, and also place constraints on the experimental arrangement.

I. INTRODUCTION

There has recently been considerable interest shown in the possibility of neutron-antineutron oscillations. These have been predicted to occur through $\Delta B=2$ transitions in some grand unified theories (GUT's) (see, for example, Refs. 1—4). At present, experimental attention has been focused on three possible methods of observation: detection of neutron oscillation in bulk matter (as in the proton-lifetime experiments[5]), detection in reactor beams,[6–11] and detection in a "bottle" of ultracold neutrons.[12,13]

The advantage of the first method is that existing proton-decay experiments may also be used to detect neutron oscillation. However, the antineutron production rate is heavily suppressed because of the large difference between neutron-nucleus and antineutron-nucleus potentials. Furthermore, uncertainties in the nuclear-structure information may make it difficult to extract accurate values for the GUT parameters.[14–17]

Experiments using the large neutron flux from reactors are already in progress or have been proposed. The difficulties with these experiments are, first, that the proximity of the reactor necessitates considerable shielding to reduce background counts and biological hazards, and second, that the long beam flight path (which is required to allow time for the antineutron amplitude to build up) must be shielded from the Earth's magnetic field. These requirements seem quite expensive to satisfy.

These problems are much reduced by storing ultracold neutrons in "bottles". This method also seems attractive since the neutrons may be contained for a long time—perhaps until they β decay. Unfortunately, the density of neutrons stored is quite low, and collisions of the neutrons with the walls modify the oscillation rate: it is therefore not clear whether such experiments are practicable, and this is the question to which this paper is addressed. In Sec. II we review the standard free-space formalism, and in Sec. III we investigate the quantum-mechanical treatment of oscillations in the bottle. We discuss the results in Sec. IV.

II. FREE-SPACE FORMALISM

Neutron oscillations in free space have been described in Ref. 18. We may apply the formalism to the case where the neutrons collide with a wall by assuming that the only effect of this collision is to introduce a time variation into the potential; that is, we neglect the effect of the collision on the spatial dependence of the wave function, and consider that dependence to be that of a local momentum eigenstate. If we allow time-varying external potentials to be applied, the Schrödinger equation may be written in matrix form as

$$i\hbar\frac{d}{dt}\begin{bmatrix}\psi\\ \bar{\psi}\end{bmatrix}=\begin{bmatrix}M-\dfrac{i\Gamma}{2}+V(t) & \delta m\\ \delta m & M-\dfrac{i\Gamma}{2}+\widetilde{V}(t)\end{bmatrix}\begin{bmatrix}\psi\\ \bar{\psi}\end{bmatrix}. \quad (1)$$

Here ψ and $\bar{\psi}$ are the neutron and antineutron components of the wave function. $V(t)$ and $\widetilde{V}(t)$ are the external neutron and antineutron potentials, M is the neutron mass, and Γ is the neutron lifetime. δm is the matrix element for the $\Delta B=2$ transition between neutrons and antineutrons.

We define

$$H_0=\begin{bmatrix}M-\dfrac{i\Gamma}{2}+V(t) & 0\\ 0 & M-\dfrac{i\Gamma}{2}+\widetilde{V}(t)\end{bmatrix}$$

and

$$U=\begin{bmatrix}0 & \delta m\\ \delta m & 0\end{bmatrix}.$$

Then writing

$$\begin{bmatrix}\psi\\ \bar{\psi}\end{bmatrix}=\exp\left[-\frac{i}{\hbar}\int^t H_0dt\right]\begin{bmatrix}\phi\\ \bar{\phi}\end{bmatrix}$$

leads to

$$i\hbar\frac{d}{dt}\begin{bmatrix}\phi\\ \bar{\phi}\end{bmatrix}=\exp\left[\frac{i}{\hbar}\int^t H_0dt\right]U\exp\left[\frac{-i}{\hbar}\int^t H_0dt\right]\begin{bmatrix}\phi\\ \bar{\phi}\end{bmatrix}. \quad (2)$$

If we neglect the β decay of the neutron and any antineutron annihilation, then $|\psi|^2=|\phi|^2$, $|\bar{\psi}|^2=|\bar{\phi}|^2$; that is, ϕ can be identified directly with the neutron component of the wave function. Although not essential, this makes the following discussion a little clearer.

We now define $\delta=\delta m/\hbar$ and $\Delta(t)=(V-\widetilde{V})/2\hbar$ and use the Baker-Campbell-Hausdorff expression (that is, for any operators Q,S: $e^{Q}Se^{-Q}=S+[Q,S]+(1/2!)[Q,[Q,S]]+\cdots$) to obtain the decoupled equations

$$\ddot{\phi}-2i\Delta\dot{\phi}+\delta^2\phi=0$$

and (3)

$$\ddot{\bar{\phi}}+2i\Delta\dot{\bar{\phi}}+\delta^2\bar{\phi}=0 .$$

The solution of these equations is, of course, trivial if $\Delta(t)$ is a constant. If we suppose that during a collision the neutron suddenly enters or leaves the wall, then the time dependence of Δ would just be a step function: the solution would then be obtained by matching solutions for the internal and external values of Δ at the boundary. This procedure demonstrates the main features which have been found in previous work.

If at $t=0$ we have $\phi=\rho e^{i\theta_1}$, $\bar{\phi}=\bar{\rho}e^{i\theta_2}$, and $\theta=\theta_1-\theta_2$, then after time t we have

$$|\phi|^2=\rho^2-\frac{\sin^2(\Delta^2+\delta^2)^{1/2}t}{\Delta^2+\delta^2}[\delta^2(\rho^2-\bar{\rho}^2)-2\delta\Delta\rho\bar{\rho}\cos\theta]-\frac{\rho\bar{\rho}\delta\sin\theta}{(\Delta^2+\delta^2)^{1/2}}\sin[2(\Delta^2+\delta^2)^{1/2}t] ,$$
$$|\phi|^2=1-|\bar{\phi}|^2 . \quad (4)$$

Provided $(\Delta^2+\delta^2)^{1/2}t\ll 1$, the oscillations (starting from $\rho^2=1$, $\bar{\rho}^2=0$) have unit amplitude, and proceed quadratically in time:

$$|\bar{\phi}|^2=\delta^2t^2 .$$

If $(\Delta^2+\delta^2)^{1/2}t\gg 1$, the oscillations have small amplitude $\delta/(\Delta^2+\delta^2)^{1/2}$, but high frequency $[(\Delta^2+\delta^2)^{1/2}]$ if $\Delta\gg\delta$. This is just the situation in the wall: We see that the effect of this is essentially to randomize θ (mod 2π) so that the quadratic free-space time dependence is interrupted at each collision. After N collisions the probability of finding an antineutron is

$$|\bar{\phi}|^2\approx N\delta^2\tau^2\simeq\delta^2\tau t ,$$

where τ is the time between two successive collisions (provided $\bar{\rho}$ is always small compared to ρ), i.e., increasing linearly in time. It is this well-known result, e.g., Ref. 19, which makes these experiments appear less attractive, although it might be felt that ways could be found to minimize the randomization through a suitable choice of wall material, box size, or reflection angle, for instance. We shall return to this point later.

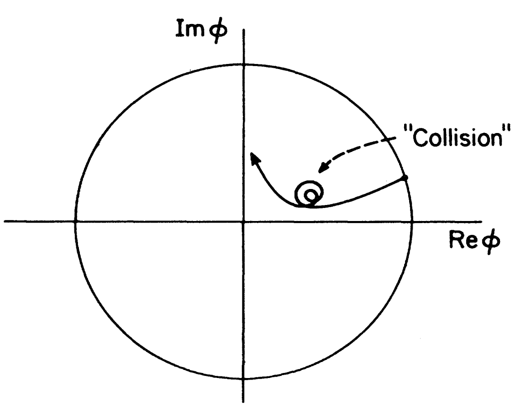

FIG. 1. Diagram showing typical time evolution of neutron wave function through a collision. The scale is, however, highly magnified.

Before closing this section, we should investigate whether or not there is a qualitative change if entry to or exit from the wall is not sudden. Solving Eq. (3) with a general time-dependent Δ would be difficult, but we may determine the behavior of the solution using a simple mechanical analogy: We identify the real and imaginary parts of ϕ with the x and y coordinates of a particle moving in two dimensions. If the particle is of unit mass and is subject to a harmonic-oscillator potential $\frac{1}{2}\delta^2(x^2+y^2)$ and also to a magnetic field in the z direction of strength $eB(t)=2\Delta(t)$, then the equations of motion of the particle are identical to Eq. (3). (We assume there is no induced electric field when B varies.)

Initially we have all neutrons, which corresponds to the particle being at rest on the unit circle (see Fig. 1). It begins to accelerate towards the center if the magnetic field is zero. A collision with the wall corresponds to a rapid increase in the strength of B, followed by a decrease to zero again. During this time, the particle will move in a tight spiral which then unwinds, leaving the speed almost unchanged but the direction of motion randomized. It is just this "random walk" which reduces the quadratic time development to a linear one. This example also shows that we need not worry about "transients," and that the step-function solution is qualitatively the same as would appear if the collision were slow compared to, for example, the oscillation frequency in the wall.

III. QUANTUM-MECHANICAL TREATMENT

The free-space picture outlined in Sec. II has been used directly to estimate the antineutron annihilation rate from ultracold neutrons stored in a bottle. There is, however, an important difference between these two situations: in free space there exist neutron and antineutron states with the same momentum, which has allowed us to factor out the spatial dependence of the wave function to arrive at Eq. (1). In the bottle, however, the energy and momentum states are discrete; moreover, the splitting between neutron and antineutron levels (caused by their different interactions with the walls) may be much greater than the splitting induced by the baryon-number-violating term.

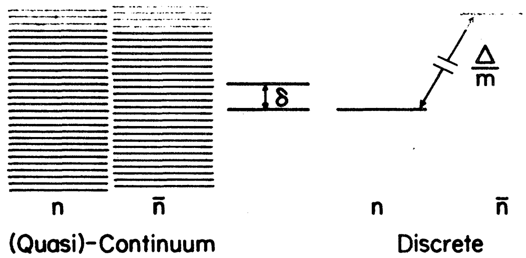

FIG. 2. Relative positions of allowed energy states in the free-space and bottle situations, compared to the estimated splitting caused by the baryon-number-violating term.

For instance, the baryon-number-violating term has been estimated to be of the order of 10^{-22} eV. The separation of consecutive neutron or antineutron levels is[19a] of the order of E/m where m is the number of nodes: E is of the same order of magnitude as the potential well of the bottle[20] ($\sim 10^{-7}$ eV) and $m=kL\sim 10^8$ for neutrons of velocity 5 m/s in a bottle of length $L\sim 1$ m ($\hbar k$ is the momentum of the neutron). The separation between adjacent neutron and antineutron levels is similarly of the order of $(V-\bar{V})/m$ where V and $\bar{V}$ are the potentials seen by the neutrons and antineutrons, respectively. The difference is probably greater than one-tenth of the magnitude of V, so that the splitting between neutron and antineutron levels $\gtrsim 10^{-16}$ eV $\gg 10^{-22}$ eV. (See Fig. 2.)

The energy levels available are, in this sense, far from being a continuum, and it is not immediately obvious that the free-space formalism is applicable. Rather than attempt a solution of the time-dependent problem on a discrete energy lattice, we instead consider the *time-independent* solutions in the bottle. $\bar{n}$ annihilation in the walls gives the energy eigenvalue an imaginary part $-i\Gamma_m/2$; since it varies only slowly with the number of nodes m, it can provide an estimate of the $\bar{n}$ annihilation rate in any realistic wave packet of states constructed to describe a neutron trapped in the bottle.

As before, we write the wave function of the system with two components, the upper one corresponding to neutrons, and the lower one to antineutrons. If the external potential in the volume of the bottle is zero, then we have, in the cavity,

$$\begin{bmatrix} -\frac{\hbar^2}{2M}\frac{d^2}{dx^2}-E & \delta \\ \delta & -\frac{\hbar^2}{2M}\frac{d^2}{dx^2}-E \end{bmatrix}\begin{bmatrix} f \\ \bar{f} \end{bmatrix}=0 . \tag{5}$$

Then writing $F\propto e^{ik_\pm x}$ we have the two solutions

$$\psi_+=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}e^{ik_+x}, \quad \text{where } \frac{\hbar^2}{2M}k_+{}^2+\delta\equiv E ,$$

$$\psi_-=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}e^{ik_-x}, \quad \text{where } \frac{\hbar^2}{2M}k_-^2-\delta\equiv E .$$

Similarly, if the potentials acting on neutrons and antineutrons in the walls are V and $\bar{V}$, then we have, in the walls,

$$\begin{bmatrix} -\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+V-E & \delta \\ \delta & -\frac{\hbar^2}{2M}\frac{d^2}{dx^2}+\bar{V}-E \end{bmatrix}\begin{bmatrix} f \\ \bar{f} \end{bmatrix}=0 . \tag{6}$$

We write $f\propto e^{-K_\pm|x|}$ and introduce

$$\Delta=(V-\bar{V})/2, \quad D=\Delta-(\Delta^2+\delta^2)^{1/2}$$

to obtain the solutions

$$\psi_+=\frac{1}{(\delta^2+D^2)^{1/2}}\begin{bmatrix}\delta\\-D\end{bmatrix}e^{-K_+|x|} ,$$

where

$$\frac{\hbar^2}{2M}K_+{}^2+D\equiv V-E ,$$

and

$$\psi_-=\frac{1}{(\delta^2+D^2)^{1/2}}\begin{bmatrix}D\\\delta\end{bmatrix}e^{-K_-|x|} ,$$

where

$$\frac{\hbar^2}{2M}K_-{}^2-D\equiv \bar{V}-E .$$

We note, in passing, that $D\sim -\delta^2/2\Delta$ if $\Delta\gg\delta$, so that in the walls these components are predominantly neutron or antineutron.

The complete wave function is then formed from linear combinations of the $+$ and $-$ solutions in the cavity and in the wall, and these, as usual, must be continuous and have a continuous derivative at the boundary. We take the bottle to be of length L, centered at $x=0$, and consider only even-parity states, the odd-parity states being obtainable in a similar fashion. Matching the wave function at $x=L/2$ gives

$$\frac{A}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\cos(k_+L/2)+\frac{A'}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}\cos(k_-L/2)=\frac{B}{(\delta^2+D^2)^{1/2}}\begin{bmatrix}\delta\\-D\end{bmatrix}e^{-K_+L/2}+\frac{B'}{(\delta^2+D^2)^{1/2}}\begin{bmatrix}D\\\delta\end{bmatrix}e^{-K_-L/2} , \tag{7}$$

and for the derivative

$$\frac{A}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}k_+\sin(k_+L/2)+\frac{A'}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}k_-\sin(k_-L/2)=\frac{B}{(\delta^2+D^2)^{1/2}}\begin{bmatrix}\delta\\-D\end{bmatrix}K_+e^{-K_+L/2}+\frac{B'}{(\delta^2+\Delta^2)^{1/2}}\begin{bmatrix}D\\\delta\end{bmatrix}K_-e^{-K_-L/2} \tag{8}$$

These four equations allow us to obtain B, B', and A' in terms of A, and also provide a constraint on the energy eigenvalue:

$$2\left[k_+\tan\left[\frac{k_+L}{2}\right]k_-\tan\left[\frac{k_-L}{2}\right]+K_+K_-\right](\delta^2+D^2)$$
$$=k_+\tan\left[\frac{k_+L}{2}\right][K_+(\delta+D)^2+K_-(\delta-D)^2]+k_-\tan\left[\frac{k_-L}{2}\right][K_+(\delta-D)^2+K_-(\delta+D)^2]\ . \quad (9)$$

To find an approximate solution we expand D to first order in δ/D, $D\simeq-\delta^2/2\Delta$, and keep lowest-order terms. The details of the algebra are contained in the Appendix. We write $k^2=2ME/\hbar^2$ and apply the conditions $kL\gg1$ and (order of magnitude) $k\sim K$, $E\sim V\sim\Delta$ to retain dominant terms. This leads to

$$\left[k\tan\frac{kL}{2}-K_+\right]\left[k\tan\frac{kL}{2}-K_-\right]=+k^2\frac{\delta^2}{4E^2}\frac{k^2L^2}{4}\left[\frac{V}{E}\right]^2,$$

where we have used

$$\frac{1}{\cos^2(kL/2)}\simeq\frac{V}{E}\ . \quad (10)$$

We choose the solution corresponding to a large neutron component and write $k=k_0+\epsilon$ where $k_0\tan k_0L/2=K_+$ to obtain

$$\epsilon=-\frac{\delta^2}{4}\frac{L}{2k}\left[\frac{V}{E}\right]\frac{1}{(K_+-K_-)}\left[\frac{2M}{\hbar^2}\right]^2\ . \quad (11)$$

Ignoring the β decay of the neutron, V is real, as is k_0 and K_+, so we may write

$$\frac{\Gamma}{2}\equiv\mathrm{Im}(E)=2\,\mathrm{Re}k\,\mathrm{Im}k\frac{\hbar^2}{2M}\simeq2\frac{\hbar^2k_0}{2M}\mathrm{Im}\epsilon\ . \quad (12)$$

Now the time between successive collisions with the walls is

$$L\Big/\left[\frac{\hbar k}{M}\right]\equiv\tau\ ,$$

say. So we finally have

$$\frac{\Gamma}{2}=+\delta^2\tau\frac{1}{2\hbar}\left[\frac{V}{E}\right]\mathrm{Im}\left[\frac{k}{K_+-K_-}\right]\ . \quad (13)$$

(This is for a one-dimensional bottle; in three dimensions, Γ is 3 times as large.) Since the energy eigenvalue is complex, the wave function decays with times as $e^{-\Gamma t/\hbar}$ so the number of annihilations is approximately $\Gamma t/\hbar$ for short times. If we start at $t=0$ with n neutrons in the bottle, then the number of them which will have converted to antineutrons and annihilated by time t is

Number of annihilations

$$=\left[n\left[\frac{\delta}{\hbar}\right]^2\tau t\right]\left[\left[\frac{V}{E}\right]\mathrm{Im}\frac{k}{(K_+-K_-)}\right]\ . \quad (14)$$

IV. DISCUSSION

We see that the quantity in curly brackets in the final result [Eq. (14)] is of order unity. (The singularity if $K_+-K_-\to0$ is spurious since the derivation has assumed that $V-\bar{V}\gg\delta$.) Thus, the annihilation rate predicted by the quantum-mechanical calculation is very similar to that predicted by the free-space calculation in the limit that *the phase is completely randomized* at each collision.

While real experiments will not be able to ensure that the potentials in the cavity are precisely zero, we believe that the result will be unchanged if we satisfy the condition derived from the continuum calculation, namely that

$$\frac{1}{\delta^2+\Delta^2}\sin^2[(\delta^2+\Delta^2)^{1/2}\tau]\simeq\tau^2\ ,$$

i.e.,

$$(\delta^2+\Delta^2)^{1/2}\tau\ll1\ ,$$

where Δ is here the potential difference in the cavity.

Although the annihilation rates predicted by the two calculations are similar, there is an important difference to note. In the continuum case, if there were no loss of flux at all (either through antineutron annihilation or β decay), an equilibrium would eventually be reached in which the bottle contained equal numbers of neutrons and antineutrons. In the quantum case the steady-state ratio of neutrons to antineutrons (for the mainly neutron component) is

$$1:\left[\frac{\delta}{\Delta}\frac{kL}{2}\right]^2$$

independent of time (see Appendix). We may view this either as approximately the number of antineutrons appearing after one crossing of the cavity, or else as the mixing of neutron and antineutron levels expected from simple first-order perturbation theory. The significance of this result is that it makes the practical detection of the n-$\bar{n}$ oscillations more difficult; for example, nothing would be gained by trying to accumulate antineutrons in a bottle with highly reflecting walls (i.e., a low annihilation cross section) over a long period, and only occasionally exposing a detector. Because the n-$\bar{n}$ ratio is fixed, this method could not be used to increase the signal to noise ratio.

V. SUMMARY

We have shown that neutron oscillations in bottles (as in nuclei[15]) produce an $\bar{n}$ annihilation rate which is only linear in time. Current sizes of bottles (~ 1 m^3) and filling densities (~ 100 neutrons per cm^3 at velocities ~ 5 m s^{-1}) might produce a figure of merit [annihilation rate/$(\delta/\hbar)^2$] $\simeq 2\times 10^7$. This is considerably better than some current reactor experiments,[13] while worse than proposed reactor experiments [e.g., 8×10^9 at Grenoble III (Ref. 12)]. However, there seems to be considerable scope for improvement, even allowing for the linear time progression, through larger containment volumes, higher densities, and clever choice of wall material.

ACKNOWLEDGMENTS

The authors wish to thank Professor C. Goebel for helpful discussions. This work was supported in part by the National Science Foundation.

APPENDIX

Expanding Eq. (9) to first order in δ/Δ we find

$$\left[k_+\tan\left[k_+\frac{L}{2}\right]k_-\tan\left[k_-\frac{L}{2}\right]+K_+K_-\right]$$
$$=\frac{(K_++K_-)}{2}\left[k_+\tan\left[k_+\frac{L}{2}\right]+k_-\tan\left[k_-\frac{L}{2}\right]\right]-\frac{\delta}{\Delta}\frac{(K_+-K_-)}{2}\left[k_+\tan\left[k_+\frac{L}{2}\right]-k_-\tan\left[k_-\frac{L}{2}\right]\right]. \tag{A1}$$

We write $k^2=2ME/\hbar^2$; then $k_+{}^2=k^2-2M\delta/\hbar^2$,

$$k_+\simeq k\left[1-\frac{\delta}{2E}\right],\quad k_-\simeq k\left[1+\frac{\delta}{2E}\right]. \tag{A2}$$

Also

$$\tan\left[k_+\frac{L}{2}\right]=\tan\left[\frac{kL}{2}\right]-\frac{kL}{2}\frac{\delta}{2E}\frac{1}{\cos^2(kL/2)},\quad \tan\left[k_-\frac{L}{2}\right]\simeq\tan\left[\frac{kL}{2}\right]+\frac{kL}{2}\frac{\delta}{2E}\frac{1}{\cos^2(kL/2)}. \tag{A3}$$

So we have from Eq. (A1)

$$k^2\left[1-\frac{\delta^2}{4E^2}\right]\left[\tan^2\frac{kL}{2}-\frac{k^2L^2}{4}\frac{\delta^2}{4E^2}\frac{1}{\cos^4(kL/2)}\right]+K_+K_-$$
$$=\frac{(K_++K_-)}{2}\left[2k\tan\frac{kL}{2}+k\left[\frac{kL}{2}\right]\left[\frac{\delta}{2E}\right]^2\frac{1}{\cos^2(kL/2)}\right]+\frac{\delta}{\Delta}\frac{(K_+-K_-)}{2}\frac{k\delta}{2E}\left[2\tan\frac{kL}{2}+\frac{kL}{\cos^2(kL/2)}\right]. \tag{A4}$$

Rearranging terms, we have

$$\left[k\tan\frac{kL}{2}-K_+\right]\left[k\tan\frac{kL}{2}-K_-\right]\cong\frac{k^2\delta^2}{4E^2}\left[\left[\tan^2\frac{kL}{2}+\frac{k^2L^2}{4}\frac{1}{\cos^4(kL/2)}\right]+\frac{(K_++K_-)}{2k}\left[\frac{kL}{2}\right]\frac{1}{\cos^2(kL/2)}\right.$$
$$\left.+\frac{E}{\Delta}\frac{(K_+-K_-)}{2k}\left[2\tan\frac{kL}{2}+\frac{kL}{\cos^2(kL/2)}\right]\right]. \tag{A5}$$

Now

$$\tan\frac{kL}{2}\sim\frac{K}{k},\quad \frac{1}{\cos^2 kL/2}\sim 1+\frac{K^2}{k^2}\sim\frac{V}{E},$$

and $kL\gg 1$, whereas $K\sim k$, $E\sim V\sim\Delta$ (order of magnitude). So retaining the largest term we have

$$\left[k\tan\frac{kL}{2}-K_+\right]\left[k\tan\frac{kL}{2}-K_-\right]=\frac{k^2\delta^2}{4E^2}\frac{k^2L^2}{4}\left[\frac{V}{E}\right]^2. \tag{A6}$$

Write

$$k=k_0+\epsilon,$$

where

$$k_0\tan\frac{k_0L}{2}=K_+;$$

then

$$\tan\frac{kL}{2}\simeq\tan\frac{k_0L}{2}+\frac{\epsilon L}{2}\left[\frac{V}{E}\right]$$

and

$$\left[k\tan\frac{kL}{2}-K_+\right]\simeq\epsilon\left[\frac{K_+}{k_0}+\frac{k_0L}{2}\left[\frac{V}{E}\right]\right],\quad\left[k\tan\frac{kL}{2}-K_-\right]\simeq(K_+-K_-)\,. \tag{A7}$$

So again using

$$\frac{k_0L}{2}\left[\frac{V}{E}\right]\gg\frac{K_+}{k_0}\,,$$

we have

$$\epsilon\simeq\frac{k^2\delta^2}{4E^2}\times\frac{kL}{2}\left[\frac{V}{E}\right]\times\frac{1}{(K_+-K_-)}=\frac{\delta^2}{4}\times\frac{L}{2k}\left[\frac{V}{E}\right]\left[\frac{2M}{\hbar^2}\right]^2\times\frac{1}{(K_+-K_-)}\,. \tag{A8}$$

Now

$$\mathrm{Im}E\simeq\frac{\hbar^2}{2M}2\,\mathrm{Re}k\;\mathrm{Im}k\simeq\frac{\hbar^2}{2M}2k_0\mathrm{Im}\epsilon\,.$$

Writing

$$\tau=\frac{L}{\hbar k/M}\,,$$

we arrive at

$$\frac{\Gamma}{2}\equiv\mathrm{Im}E=\delta^2\tau\frac{1}{2\hbar}\left[\frac{V}{E}\right]\mathrm{Im}\left[\frac{k}{K_+-K_-}\right]\,. \tag{A9}$$

We now use this result to calculate the steady-state ratio of the amplitude of antineutrons to neutrons. The relation between A and A' of Eq. (7) is

$$\frac{A'}{A}=-\frac{\cos(k_+L/2)\left[k_+\tan k_+(L/2)+K_+\delta/2\Delta-K_-\left[1-\delta/2\Delta\right]\right]}{\cos(k_-L/2)\left[k_-\tan k_-L/2+K_+\delta/2\Delta-K_-\left[1-\delta/2\Delta\right]\right]}\,. \tag{A10}$$

Using the previous approximations to k_+ and k_- gives

$$\frac{A'}{A}=-\frac{\cos k_+\dfrac{L}{2}\left[k\left[1+\dfrac{\delta}{2E}\right]\left[\tan\dfrac{kL}{2}+\dfrac{kL}{2}\dfrac{\delta}{2E}\dfrac{V}{E}\right]+K_+\dfrac{\delta}{2\Delta}-K_-\left[1-\dfrac{\delta}{2\Delta}\right]\right]}{\cos k_-\dfrac{L}{2}\left[k\left[1-\dfrac{\delta}{2E}\right]\left[\tan\dfrac{kL}{2}-\dfrac{kL}{2}\dfrac{\delta}{2E}\dfrac{V}{E}\right]+K_+\dfrac{\delta}{2\Delta}-K_-\left[1+\dfrac{\delta}{2\Delta}\right]\right]} \tag{A11}$$

$$=-\frac{\cos k_+\dfrac{L}{2}\left[\left[k\tan\dfrac{kL}{2}-K_-\right]+\left[k\tan\dfrac{kL}{2}+k\dfrac{kL}{2}\dfrac{V}{E}\right]\dfrac{\delta}{2E}+(K_++K_-)\dfrac{\delta}{2\Delta}\right]}{\cos k_-\dfrac{L}{2}\left[\left[k\tan\dfrac{kL}{2}-K_-\right]-\left[k\tan\dfrac{kL}{2}+k\dfrac{kL}{2}\dfrac{V}{E}\right]\dfrac{\delta}{2E}+(K_+-K_-)\dfrac{\delta}{2\Delta}\right]} \tag{A12}$$

$$=-\frac{\cos k_+\dfrac{L}{2}\left[1+\dfrac{\delta}{2E}\dfrac{kL}{2}\dfrac{V}{E}\dfrac{k}{K_+-K_-}\right]}{\cos k_-\dfrac{L}{2}\left[1-\dfrac{\delta}{2E}\dfrac{kL}{2}\dfrac{V}{E}\dfrac{k}{K_+-K_-}\right]}\,. \tag{A13}$$

Now

$$\frac{|\bar{n}\rangle}{|n\rangle}=\frac{A\cos k_+\frac{L}{2}-A'\cos k_-\frac{L}{2}}{A\cos k_+\frac{L}{2}+A'\cos k_-\frac{L}{2}}\simeq\frac{1-\left[1+\frac{\delta}{2E}\frac{kL}{2}\frac{V}{E}\frac{k}{K_+-K_-}\right]^2}{2} \tag{A14}$$

$$=\frac{\delta}{2E}\frac{kL}{2}\frac{V}{E}\frac{k}{K_+-K_-}\ . \tag{A15}$$

Again $V\sim E\sim\Delta$ and $k\sim K_+\sim K_-$ (order of magnitude). So

$$\frac{|\bar{n}\rangle}{|n\rangle}\simeq\frac{\delta}{\Delta}\frac{kL}{2}\ . \tag{A16}$$

[1]R. E. Marshak and R. N. Mohapatra, Phys. Rev. Lett. 44, 1316 (1980).

[2]R. N. Mohapatra and R. E. Marshak, Phys. Lett. 94B, 183 (1980).

[3]L. N. Chang and N. P. Chang, Phys. Lett. 92B, 103 (1980).

[4]G. Costa and A. H. Zimerman, Nuovo Cimento 64B, 285 (1981).

[5]R. J. Ellis, CERN Report No. CERN-EP/83-26, 1983 (unpublished); D. C. Peaslee, report (unpublished).

[6]S. Ratti, in *Proceedings of the International Conference on Baryon Nonconservation, Bombay, 1982* (Tata Institute, Bombay report).

[7]R. Wilson, in *Proceedings of the International Conference on Baryon Nonconservation, Bombay, 1982* (Ref. 6).

[8]V. A. Kuz'min, in *Proceedings of the International Conference on Baryon Nonconservation, Bombay, 1982* (Ref. 6).

[9]H. L. Anderson, in *Proceedings of the Informal Workshop on n-$\bar{n}$ Oscillations, Cambridge, Massachusetts, 1983,* edited by M. S. Goodman, M. Machacek, and P. D. Miller, (Harvard University report).

[10]R. J. Ellis *et al.*, LAMPF Proposal No. 647 (unpublished).

[11]M. S. Goodman *et al.*, Proposal No. ORNL/PHYS-82/1, 1982 (unpublished).

[12]M. Baldo-Coelin, in Proceedings of the International Conference on Unified Theories and their Experimental Tests, Venice, 1982 (unpublished).

[13]A. G. Petschek, LASL Internal Report No. P/BR-83.192, 1983 (unpublished).

[14]K. G. Chetyrkin *et al.*, Phys. Lett. 99B, 358 (1981).

[15]P. G. H. Sandars, J. Phys. G 6, L161 (1980).

[16]W. M. Alberico, A. Bottino, and A. Molinari, Phys. Lett. 114B, 266 (1982).

[17]C. B. Dover, A. Gal, and J. M. Richard, Phys. Rev. D 27, 1090 (1982).

[18]O. Sawada, M. Fukugita, and J. Arafume, Astrophys. J. 248, 1162 (1981).

[19]H. Yoshiki, in Proceedings of the IVth International Conference on Applied Nuclear Structure, KEK, Japan, 1980 (unpublished).

[19a]For definiteness, we consider a rectangular "bottle," for which the x, y, and z motions are independent; consequently we only need to work out the properties of a one-dimensional bottle.

[20]R. Golub and J. M. Pendlebury, Rep. Prog. Phys. 42, 439 (1979).