Vector-meson mixing and hadronic decays of ψ

L. J. Clavelli
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

Gerald W. Intemann
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
and Department of Physics, University of Northern Iowa, Cedar Falls, Iowa 50614
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A phenomenological model for the hadronic decays of ψ is proposed, where the vector mesons (ρ,ω,ϕ,ψ) undergo mixing through contamination of light and charmed quarks. The decay widths of several two-body hadronic modes are predicted and compared with the current experimental data. Isospin-violating decays are included in the analysis and, in particular, the process $\psi\to\omega\pi$ is predicted to have a width large enough to be experimentally observable.

I. INTRODUCTION

A substantial amount of data[1] is now available on the exclusive decays of ψ into various ordinary two-body hadronic states (pseudoscalar + pseudoscalar, baryon + antibaryon, vector + pseudoscalar). Additional results with much higher statistics are expected soon from experiments using the Mark III detector. Current data indicates sizable deviations ($\sim 40\%$) from exact SU(3) symmetry and there is evidence of significant isospin-violating effects as well.

Motivated by the prospect of improved statistics for hadronic ψ decays, we propose in this paper to give a comprehensive description of the two-body decays of ψ in terms of a vector-meson-mixing model. In this model the vector mesons (ρ,ω,ϕ,ψ) are regarded as being admixtures of light- and charmed-quark—antiquark states. The coupling of ψ to any state of light quarks is then related to the corresponding couplings of ρ, ω, and ϕ to the same state. The idea of this type of mixing is not new and a slightly less general version of this model was proposed some time ago by one of the authors.[2] In this work we use this model to calculate all of the partial decay widths of ψ into two-body final states involving either two pseudoscalar mesons, a baryon-antibaryon pair, or a vector and pseudoscalar meson. Upon comparing our results with experiment, we find that our model gives a consistently good description for nearly all of the decays studied and predicts that certain isospin-violating decay modes, such as $\psi\to\omega\pi$, have sizable decay widths which should be accessible to measurement.

II. DESCRIPTION OF MODEL

In the vector-meson-mixing model the vector mesons ρ,ω,ϕ,ψ are written as the following quark-antiquark admixtures:

$$\begin{aligned}
\rho&=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})-\epsilon_{\omega\rho}\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})-\epsilon_{\phi\rho}s\bar{s}-\epsilon_{\psi\rho}c\bar{c}\ ,\\
\omega&=\epsilon_{\omega\rho}\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})+\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})-\epsilon_{\omega\phi}s\bar{s}-\epsilon_{\psi\omega}c\bar{c}\ ,\\
\phi&=\epsilon_{\phi\rho}\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})+\epsilon_{\omega\phi}\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})+s\bar{s}-\epsilon_{\psi\phi}c\bar{c}\ ,\\
\psi&=\epsilon_{\psi\rho}\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})+\epsilon_{\psi\omega}\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})+\epsilon_{\psi\phi}s\bar{s}+c\bar{c}\ .
\end{aligned}\tag{1}$$

The mixing of ψ with ω and ϕ which conserves isospin arises from the strong interaction through graphs of Fig. 1(a). The isospin-violating mixing of ψ and ρ arises electromagnetically through graphs of Fig. 1(b) where one gluon is replaced by a photon. For the purposes of this paper we shall consider the mixing parameters $\epsilon_{\psi\rho},\epsilon_{\psi\omega},\epsilon_{\psi\phi}$ as phenomenological constants which will be determined from the experimental data.

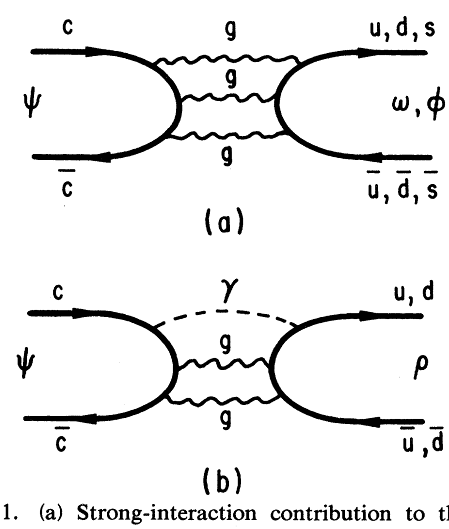

FIG. 1. (a) Strong-interaction contribution to the isospin-conserving direct transitions $\psi\to\omega$ and $\psi\to\phi$. (b) Electromagnetic contribution to the isospin-violating direct transition $\psi\to\rho$.

III. ORDINARY HADRONIC ψ DECAYS

We investigate the following hadronic decay modes of ψ:

$$\begin{aligned}&\psi\to\pi^+\pi^-,\ K^+K^-,\ K_L^0K_S^0\ ,\\ &\psi\to p\bar{p},\ n\bar{n},\ \Lambda\bar{\Lambda},\ \Sigma^+\bar{\Sigma}^+,\ \Sigma^-\bar{\Sigma}^-,\ \Sigma^0\bar{\Sigma}^0,\ \Xi\bar{\Xi}\ ,\\ &\psi\to\rho\pi,\ K^{*+}K^-+\text{c.c.},\ K^{*0}\bar{K}^0+\text{c.c.}\ ,\\ &\quad\phi\eta,\ \phi\eta',\ \omega\eta,\ \omega\eta',\ \omega\pi,\ \phi\pi,\ \rho\eta,\ \rho\eta'\ .\end{aligned} \tag{2}$$

We describe these decays using the effective Lagrangian[3]

$$\mathscr{L}=g_{\psi B\bar{B}}\psi_\mu\bar{B}\gamma_\mu B+g_{\psi PP}\psi_\mu P\overleftrightarrow{\partial}_\mu P+g_{\psi VP}\epsilon_{\mu\nu\lambda\sigma}\partial_\lambda\psi_\mu\partial_\sigma V_\nu\ , \tag{3}$$

where $P=$ pseudoscalar meson, $B=$ baryon, and $V=$ vector meson. Equation (3) yields for the partial decay widths

$$\Gamma(\psi\to PP)=\frac{1}{6\pi}(g_{\psi PP})^2m_\psi\left[\frac{m_\psi{}^2-4m_P{}^2}{4m_\psi{}^2}\right]^{3/2}, \tag{4a}$$

$$\Gamma(\psi\to B\bar{B})=\frac{1}{12\pi}(g_{\psi B\bar{B}})^2m_\psi\left[1-\frac{4m_B{}^2}{m_\psi{}^2}\right]^{1/2}\left[1+\frac{2m_B{}^2}{m_\psi{}^2}\right], \tag{4b}$$

$$\Gamma(\psi\to VP)=\frac{2}{3\pi}(g_{\psi VP})^2m_\psi{}^3\left[\frac{m_\psi{}^2-(m_V+m_P)^2}{4m_\psi{}^2}\right]^{3/2}\left[\frac{m_\psi{}^2-(m_V-m_P)^2}{4m_\psi{}^2}\right]^{3/2}. \tag{4c}$$

In order to determine the strong coupling constants $g_{\psi PP}$, $g_{\psi B\bar{B}}$, $g_{\psi VP}$ we need to consider two possible contributions to hadronic ψ decays. The graphs corresponding to these contributions for the case of $\psi\to PP$ are shown in Fig. 2. The first graph represents direct $\psi\to\rho,\omega,\phi$ transitions based on our mixing model. The second graph represents a single-photon-exchange contribution. Similar graphs are possible for $\psi\to B\bar{B}$ and $\psi\to PV$. For $\psi\to PP$ decays the single-photon-exchange contribution is expected to be large due to the single-pole behavior of the electromagnetic form factor. Although such behavior has been known to occur in the spacelike region $q^2<0$, there is also some

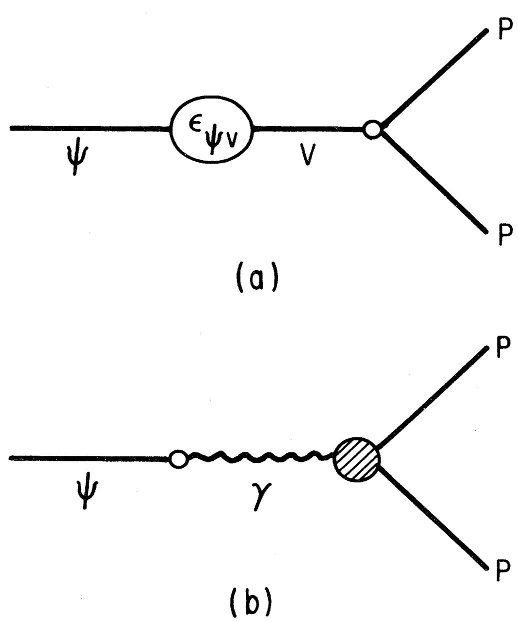

FIG. 2. (a) Contribution to $\psi\to PP$ due to a direct $\psi\to V$ transition. $V=\rho,\omega,\phi$, and $\epsilon_{\psi V}$ are mixing parameters of the model. (b) Single-photon-exchange contribution to $\psi\to PP$.

evidence[4] that it also persists for timelike $q^2\approx m_\psi{}^2$. We will assume that this is indeed the case and, by invoking vector-meson dominance to describe the pion and kaon electromagnetic form factors, we find that the single-photon (SP) exchange contributions to the ψPP couplings are given by

$$g_{\psi\pi\pi}^{\text{SP}}=\frac{4\pi\alpha g_{\rho\pi\pi}m_\rho{}^2/f_\rho}{f_\psi(m_\psi{}^2-m_\rho{}^2)}\ , \tag{5a}$$

$$g_{\psi K^-K^+}^{\text{SP}}=\frac{2\pi\alpha g_{\rho\pi\pi}}{f_\psi}\left[\frac{m_\rho{}^2/f_\rho}{(m_\psi{}^2-m_\rho{}^2)}+\frac{m_\omega{}^2/f_\omega}{(m_\psi{}^2-m_\omega{}^2)}+\frac{\sqrt{2}m_\phi{}^2/f_\phi}{(m_\psi{}^2-m_\phi{}^2)}\right], \tag{5b}$$

$$g_{\psi K^0\bar{K}^0}^{\text{SP}}=\frac{2\pi\alpha g_{\rho\pi\pi}}{f_\psi}\left[-\frac{m_\rho{}^2/f_\rho}{(m_\psi{}^2-m_\rho{}^2)}+\frac{m_\omega{}^2/f_\omega}{(m_\psi{}^2-m_\omega{}^2)}+\frac{\sqrt{2}m_\phi{}^2/f_\phi}{(m_\psi{}^2-m_\phi{}^2)}\right], \tag{5c}$$

where f_ρ, f_ω, f_ϕ, and f_ψ are related to the vector-meson—photon couplings $G_{V\gamma}=em_V{}^2/f_V$, and with our convention f_ϕ is negative.

In contrast, for the decays $\psi\to B\bar{B}$ and $\psi\to VP$ we assume that the contributions from single-photon exchange can be neglected. We base this assumption, first of all, on the evidence[5] that the electromagnetic form factors for the nucleons fall off as dipoles in the timelike region around $q^2\approx4$ GeV^2. Unfortunately, the electromagnetic form factors of other baryons have not been measured at such large q^2 but we will assume that their behavior in the timelike region is also that of a dipole.

There is very little direct information about PV electromagnetic form factors. However, if these form factors are described by a single pole one would expect such decays as $\psi\to\rho\pi$ and $\psi\to K^*\bar{K}$ to be dominated by single-photon exchange. Then U-spin invariance would require

(after correcting for phase-space differences)

$$R=\frac{\Gamma(\psi\to K^{*+}K^-)}{\Gamma(\psi\to\rho^+\pi^-)}=0.85 \ . \quad (6)$$

Experimentally this ratio is found to be[1]

$$R_{\text{expt}}=0.40\pm0.07 \ . \quad (7)$$

Thus, to the extent that U-spin is an exact symmetry, Eq. (7) implies that the decays $\psi\to K^*K$, $\rho\pi$ are not dominated by single-photon exchange and that the $\rho\pi$ and K^*K form factors have at least a dipole behavior. We will assume that this is indeed the case for all VP form factors for timelike q^2.

Thus, we conclude that only for $\psi\to PP$ decays is the single-photon-exchange contribution important.

Using Eqs. (1) and (5) along with exact SU(3) symmetry among the light-meson couplings, we find[6] for the hadronic couplings of ψ

$$g_{\psi\pi\pi}=\epsilon'_{\psi\rho}g_{\rho\pi\pi} \ , \quad (8)$$

$$g_{\psi K^+K^-}=\tfrac{1}{2}(\epsilon'_{\psi\rho}+\epsilon'_{\psi\omega}-\sqrt{2}\epsilon'_{\psi\phi})g_{\rho\pi\pi} \ , \quad (9)$$

$$g_{\psi K^0\bar{K}^0}=\tfrac{1}{2}(-\epsilon'_{\psi\rho}+\epsilon'_{\psi\omega}-\sqrt{2}\epsilon'_{\psi\phi})g_{\rho\pi\pi} \ , \quad (10)$$

$$g_{\psi p\bar{p}}=\tfrac{1}{2}(3\epsilon_{\psi\omega}+\epsilon_{\psi\rho})g_{\rho\pi\pi} \ , \quad (11)$$

$$g_{\psi n\bar{n}}=\tfrac{1}{2}(3\epsilon_{\psi\omega}-\epsilon_{\psi\rho})g_{\rho\pi\pi} \ , \quad (12)$$

$$g_{\psi\Lambda\bar{\Lambda}}=\left[\epsilon_{\psi\omega}+\frac{1}{\sqrt{2}}\epsilon_{\psi\phi}\right]g_{\rho\pi\pi} \ , \quad (13)$$

$$g_{\psi\Sigma^0\bar{\Sigma}^0}=\left[\epsilon_{\psi\omega}+\frac{1}{\sqrt{2}}\epsilon_{\psi\phi}\right]g_{\rho\pi\pi} \ , \quad (14)$$

$$g_{\psi\Sigma^-\bar{\Sigma}^-}=\left[-\epsilon_{\psi\rho}+\epsilon_{\psi\omega}+\frac{1}{\sqrt{2}}\epsilon_{\psi\phi}\right]g_{\rho\pi\pi} \ , \quad (15)$$

$$g_{\psi\Sigma^+\bar{\Sigma}^+}=\left[\epsilon_{\psi\rho}+\epsilon_{\psi\omega}+\frac{1}{\sqrt{2}}\epsilon_{\psi\phi}\right]g_{\rho\pi\pi} \ , \quad (16)$$

$$g_{\psi\Xi^0\bar{\Xi}^0}=\tfrac{1}{2}(\epsilon_{\psi\rho}+\epsilon_{\psi\omega}+2\sqrt{2}\epsilon_{\psi\phi})g_{\rho\pi\pi} \ , \quad (17)$$

$$g_{\psi\Xi^-\bar{\Xi}^-}=g_{\psi\Xi^-\bar{\Xi}^-}=\tfrac{1}{2}(-\epsilon_{\psi\rho}+\epsilon_{\psi\omega}-2\sqrt{2}\epsilon_{\psi\phi})g_{\rho\pi\pi} \ , \quad (18)$$

$$g_{\psi\rho\pi}=\epsilon_{\psi\omega}g_{\omega\rho\pi} \ , \quad (19)$$

$$g_{\psi K^{*+}K^-}=\tfrac{1}{2}(\epsilon_{\psi\rho}+\epsilon_{\psi\omega}+\sqrt{2}\epsilon_{\psi\phi})g_{\omega\rho\pi} \ , \quad (20)$$

$$g_{\psi K^{*0}\bar{K}^0}=\tfrac{1}{2}(-\epsilon_{\psi\rho}+\epsilon_{\psi\omega}+\sqrt{2}\epsilon_{\psi\phi})g_{\omega\rho\pi} \ , \quad (21)$$

$$g_{\psi\phi\eta}=-\sqrt{2}\cos(\theta_c-\theta_P)\epsilon_{\psi\phi}g_{\omega\rho\pi} \ , \quad (22)$$

$$g_{\psi\phi\eta'}=-\sqrt{2}\sin(\theta_c-\theta_P)\epsilon_{\psi\phi}g_{\omega\rho\pi} \ , \quad (23)$$

$$g_{\psi\omega\eta}=-\sin(\theta_c-\theta_P)\epsilon_{\psi\omega}g_{\omega\rho\pi} \ , \quad (24)$$

$$g_{\psi\omega\eta'}=\cos(\theta_c-\theta_P)\epsilon_{\psi\omega}g_{\omega\rho\pi} \ , \quad (25)$$

$$g_{\psi\omega\pi}=\epsilon_{\psi\rho}g_{\omega\rho\pi} \ , \quad (26)$$

$$g_{\psi\phi\pi}=\epsilon_{\psi\rho}\epsilon_{\omega\phi}g_{\omega\rho\pi} \ , \quad (27)$$

$$g_{\psi\rho\eta}=-\sin(\theta_c-\theta_P)\epsilon_{\psi\rho}g_{\omega\rho\pi} \ , \quad (28)$$

$$g_{\psi\rho\eta'}=\cos(\theta_c-\theta_P)\epsilon_{\psi\rho}g_{\omega\rho\pi} \ , \quad (29)$$

where θ_P is the pseudoscalar mixing angle, $\theta_c=35.3°$, and

$$\epsilon'_{\psi\rho}=\epsilon_{\psi\rho}+\frac{4\pi\alpha m_\rho{}^2/f_\rho}{f_\psi(m_\psi{}^2-m_\rho{}^2)} \ , \quad (30)$$

$$\epsilon'_{\psi\omega}=\epsilon_{\psi\omega}+\frac{4\pi\alpha m_\omega{}^2/f_\omega}{f_\psi(m_\psi{}^2-m_\omega{}^2)} \ , \quad (31)$$

$$\epsilon'_{\psi\phi}=\epsilon_{\psi\phi}+\frac{4\pi\alpha m_\phi{}^2/f_\phi}{f_\psi(m_\psi{}^2-m_\phi{}^2)} \ . \quad (32)$$

In deriving the couplings of ψ to $B\bar{B}$ states we have invoked vector-meson universality[7] which allows us to relate the $VB\bar{B}$ couplings to the VPP couplings. The second terms in Eqs. (30)—(32) are the single-photon contributions which are assumed negligible except for the case of decays to two pseudoscalars.

IV. NUMERICAL RESULTS AND DISCUSSION

Using as input the experimental vector-meson leptonic widths[1] as well as hadronic widths $\Gamma(\rho\to\pi\pi)=160.0$ MeV and $\Gamma(\omega\to3\pi)=8.91$ MeV, we obtain[8] $(g_{\rho\pi\pi})^2/4\pi=3.11$, $(g_{\omega\rho\pi})^2/4\pi=12.4m_\omega{}^{-2}$, $f_\rho{}^2/4\pi=2.42$, $f_\omega{}^2/4\pi=18.4$, $f_\phi{}^2/4\pi=12.2$, and $f_\psi{}^2/4\pi=11.8$. Combining these values with Eqs. (4) and (8)—(32) we calculate the various two-body hadronic decay rates for ψ as a function of the pseudoscalar mixing angle. The vector-meson mixing parameters, determined from the data, are found to have the best-fit values

$$\epsilon_{\psi\rho}=9.5\times10^{-6}, \ \epsilon_{\psi\omega}=1.2\times10^{-4} \ , \quad \epsilon_{\psi\phi}=2.9\times10^{-5}, \ \epsilon_{\omega\phi}=4.5\times10^{-2} \ . \quad (33)$$

Shown in Table I are our predicted branching ratios. We have expressed those ratios which involve η or η' in the final state in terms of a general pseudoscalar mixing angle θ_P because of the considerable uncertainty in the correct value for this angle.

We make the following observations.

(1) The agreement with experiment is generally quite good and for the most part better than earlier phenomenological analyses.[9]

(2) The inclusion of the isopin-violating parameter $\epsilon_{\psi\rho}$ in our model predicts a small difference in the $p\bar{p}$ and $n\bar{n}$ decay widths and a more substantial difference in the $K^{*\mp}K^\pm$ and $K^{*0}\bar{K}^0$ decay widths. Both of these predictions are consistent with experiment. A small difference in decay widths also predicted for the case of $\Sigma^+\bar{\Sigma}^+$ and $\Sigma^-\bar{\Sigma}^-$ cannot be confirmed until better data are obtained on $\Sigma\bar{\Sigma}$ decay modes.

(3) The values obtained for the mixing parameters $\epsilon_{\psi\rho},\epsilon_{\psi\omega},\epsilon_{\psi\phi}$ indicate that the degree of mixing of ψ with ω is of the order of 0.01% whereas the mixing of ψ with ρ or ϕ states only amount to about 0.001%. As a consequence, $\psi\to\pi\pi$ decay is dominated by single-photon exchange due to the small value of $\epsilon_{\psi\rho}$, whereas for the process $\psi\to K\bar{K}$ contributions from ψ-ω mixing are comparable to those for single-photon exchange.

(4) The large suppression of the decay $\psi\to K_S^0K_L^0$ rela-

TABLE I. Comparison of predicted and experimental branching ratios for various modes of $\psi \to$ ordinary hadrons. Input parameters are $\Gamma(\rho \to \pi\pi) = 160.9$ MeV and $\Gamma(\omega \to \pi\pi\pi) = 8.91$ MeV. Results are expressed in terms of the pseudoscalar mixing angle θ_P and $\theta_c = 35.3°$.

Decay mode	Predicted branching ratio (%)	Experiment[a]
$\pi^+\pi^-$	0.012	0.011 ± 0.005
K^+K^-	0.027	0.022 ± 0.008
$K_S^0 K_L^0$	0.004	< 0.009
$p\bar{p}$	0.16	0.22 ± 0.02
$n\bar{n}$	0.15	0.18 ± 0.09
$\Lambda\bar{\Lambda}$	0.09	0.11 ± 0.02
$\Sigma^0\bar{\Sigma}^0$	0.08	0.13 ± 0.04
$\Sigma^-\bar{\Sigma}^-$	0.07	0.24 ± 0.26
$\Sigma^+\bar{\Sigma}^+$	0.09	...
$\Xi\bar{\Xi}$	0.07	0.32 ± 0.08
$\rho\pi$	1.40	1.22 ± 0.12
$K^{*\mp}K^{\pm}$	0.40	0.67 ± 0.26
$\bar{K}^{*0}K^0$ + c.c.	0.31	0.27 ± 0.06
$\phi\eta$	$0.04\cos^2(\theta_c - \theta_P)$	0.10 ± 0.06
$\phi\eta'$	$0.03\sin^2(\theta_c - \theta_P)$	< 0.13
$\omega\eta$	$0.42\sin^2(\theta_c - \theta_P)$	...
$\omega\eta'$	$0.33\cos^2(\theta_c - \theta_P)$	...
$\rho\eta$	$0.0026\sin^2(\theta_c - \theta_P)$	...
$\rho\eta'$	$0.0021\cos^2(\theta_c - \theta_P)$	...
$\omega\pi$	0.003	...
$\phi\pi$	5×10^{-6}	...

[a]Values are from Ref. 1.

tive to $\psi \to K^+K^-$, which is predicted by the model and confirmed by experiment, is due primarily to electromagnetic effects arising from the single-photon-exchange contributions to $\psi \to K\bar{K}$.

(5) The results shown in Table I indicate that for hadronic ψ decays there are several deviations from exact-SU(3) predictions. According to SU(3) one expects[10] $\Gamma(\pi\rho):\Gamma(KK^*):\Gamma(\phi\eta)$ to be 1.0:0.84:0.36 after correcting for phase space. However, our model predicts these ratios to be 1.0:0.42:0.06 whereas experiment yields $1.0 \pm 0.1 : 0.41 \pm 0.06 : 0.24 \pm 0.14$. The overall suppression of $K\bar{K}$ modes compared to $K^*\bar{K}$ modes, although in accord with SU(3), is somewhat lessened by the sizable deviation of $\epsilon_{\psi\omega}$ and $\epsilon_{\psi\phi}$ from the SU(3) relation

$$\epsilon_{\psi\omega} = \sqrt{2}\epsilon_{\psi\phi} , \tag{34}$$

which follows[11] from assuming that ψ couples to the SU(3)-singlet part of ω and ϕ. In the case of $B\bar{B}$ decays SU(3) predicts that the relative branching ratios should be given by just the ratio of phase-space factors. Thus, for example, in the decays $\psi \to p\bar{p}$, $\Lambda\bar{\Lambda}$ one expects from SU(3) symmetry that $\Gamma(\psi \to \Lambda\bar{\Lambda})/\Gamma(\psi \to p\bar{p}) = 0.93$. However, our model predicts this ratio to equal 0.55 which is in excellent agreement with the experimental value[1] 0.50 ± 0.14. SU(3) breaking for other $B\bar{B}$ decays of ψ is also predicted and experimentally observed.

(6) The branching ratio for the isospin-violating decay $\psi \to \omega\pi$ is predicted to be

$$\frac{\Gamma(\psi \to \omega\pi)}{\Gamma(\psi \to \text{all})} = 0.003\% , \tag{35}$$

which should be large enough to be accessible to measurement by the Mark III detector experiments.

(7) The predicted width for the decay $\psi \to \Xi\bar{\Xi}$ agrees with the prediction of Okubo's model,[9] but is about a factor of 4 smaller than the experimental value and no reasonable adjustment of the mixing parameters will change this prediction.

(8) Our model correctly predicts the relative branching ratios of $\psi \to B\bar{B}$ and $\psi \to VP$ which are *a priori* unrelated by any symmetry considerations.

V. CONCLUDING REMARKS

It would also be useful if estimates could be obtained for the decay widths of the ordinary hadronic modes of the radial excitation ψ'. Since the production of ordinary hadrons and e^+e^- both involve annihilation of $c\bar{c}$ at the origin, it has been suggested[9,12] that the following relation should be satisfied for each hadronic mode:

$$\frac{\Gamma(\psi' \to \text{hadrons})}{\Gamma(\psi \to \text{hadrons})} \simeq \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(\psi \to e^+e^-)} , \tag{36}$$

which then enables one to estimate the partial width of ψ' for each hadron decay channel. Although Eq. (36) is approximately valid for such channels as $p\bar{p}$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-$, $p\bar{p}\pi^+\pi^-$, and $2\pi^+2\pi^-$, it is poorly satisfied for the $\pi^+\pi^-$ and K^+K^- channels and thus its general validity is suspect.

The vector-meson-mixing model that we have used in this paper can also be extended to study the hadronic decays of ψ'. However, since there is only a limited amount of data on hadronic decay modes of ψ' it would be premature at this time to attempt the kind of comprehensive treatment of two-body final states that we have carried out for the case of ψ. However, it is interesting to consider the isospin-violating decay $\psi' \to \pi^+\pi^-$. If $\psi', \psi \to \pi^+\pi^-$ decays are both dominated by single-photon exchange, then one expects

$$\frac{\Gamma(\psi' \to \pi^+\pi^-)}{\Gamma(\psi \to \pi^+\pi^-)} = \frac{\Gamma(\psi' \to e^+e^-)}{\Gamma(\psi \to e^+e^-)} \frac{F_{\pi\pi}{}^2(m_{\psi'}{}^2)}{F_{\pi\pi}{}^2(m_{\psi}{}^2)} , \tag{37}$$

where $F_{\pi\pi}(m_V{}^2)$ is the pion electromagnetic form factor evaluated at $q^2 = m_V{}^2$. Substituting the experimental values for the various decay widths, one finds that Eq. (37) requires $F_{\pi\pi}(m_{\psi'}{}^2)/F_{\pi\pi}(m_{\psi}{}^2)$ to be approximately equal to 2.5 which clearly contradicts the expected single-pole behavior of the form factor. From this analysis one must conclude that in the case of $\psi' \to \pi^+\pi^-$ decay, single-photon exchange is not the leading contribution and that the direct $\psi' \to \rho$ transition due to vector-meson mixing is quite appreciable. To estimate the strength of this transition let us assume that $\psi' \to \pi^+\pi^-$ proceeds entirely by this mechanism by virtue of a mixing of light and heavy quarks in the ψ'. Then,

$$\Gamma(\psi'\to\pi^+\pi^-)=\tfrac{2}{3}\epsilon_{\psi'\rho}{}^2\left[\frac{(g_{\rho\pi\pi})^2}{4\pi}\right]\times m_{\psi'}\left[\frac{m_{\psi'}{}^2-4m_\pi{}^2}{m_{\psi'}{}^2}\right]^{3/2}, \tag{38}$$

where $\epsilon_{\psi'\rho}$ is a mixing parameter. From the experimental value[1] $\Gamma(\psi'\to\pi^+\pi^-)=1.7\times10^{-5}$ MeV one finds from Eq. (38) that

$$\epsilon_{\psi'\rho}\simeq1.3\times10^{-4}, \tag{39}$$

which is roughly an order of magnitude larger than the value for $\epsilon_{\psi\rho}$ quoted in Eq. (33). *These results suggest that much larger isospin-violating effects may be present in ψ' hadronic decays than occur in ψ hadronic decays.* For example, one can use Eq. (39) to predict the branching ratio for the isospin-violating mode $\psi'\to\omega\pi$, which in our model only depends on $\epsilon_{\psi\rho}$. One finds that

$$\frac{\Gamma(\psi'\to\omega\pi)}{\Gamma(\psi'\to\text{all})}\simeq0.3\%, \tag{40}$$

which is two orders of magnitude larger than the branching ratio for the same decay mode of ψ. It will be interesting to see if such large isospin-violating effects are found in future experimental data on hadronic decays of ψ'.

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[11]If ω and ϕ are ideally mixed then $\omega=(\frac{2}{3})^{1/2}\omega^0+(\frac{1}{3})^{1/2}\omega^8$ and $\phi=(\frac{1}{3})^{1/2}\omega^0-(\frac{2}{3})^{1/2}\omega^8$. Then, assuming ψ is an SU(3) singlet, exact SU(3) requires ψ to couple to the singlet part only of ω and ϕ, from which it immediately follows that $\epsilon_{\psi\omega}=\sqrt{2}\epsilon_{\psi\phi}$.

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