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Single-spin asymmetries in the Drell-Yan process

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The angular distribution of lepton pairs produced via the $q\bar{q}$ annihilation process is studied through $O(\alpha_s{}^2)$ in perturbative QCD. Asymmetries for single-spin experiments are found to be very small in this channel. This provides an interesting new QCD test in forthcoming $\pi^- p^\uparrow$ experiments. The smallness of of the result is intimately related to the gauge structure of the theory and to the color coefficient associated with the $q\bar{q}$ subprocess.

The treatment of lepton-pair hadroproduction in quantum chromodynamics (QCD) has been the scene of continuous progress over the last years.[1] New experimental data have recently led to fairly comprehensive information on various differential cross sections.[2] The theoretical situation, while becoming vastly more intricate and sophisticated, is not yet settled. For example, testing the Drell-Yan mechanism for $d\sigma/d^4Q$ has become quite involved because of the phenomenological necessity of including higher-twist and intrinsic-transverse-momentum effects. There are single-spin-dependent quantities in $d\sigma/d^4Q\,d\Omega$, however, which vanish identically in the Drell-Yan picture since no imaginary (absorptive) phase is associated with the parton probability distributions. At the Born-term level, i.e., the usual applicable limit of perturbative QCD, these therefore provide a useful null test immune to the values of the parton distributions. The lowest order in which a parity-conserving single-spin dependence can occur in the usual QCD framework can be understood as follows: one power of α_s is needed to provide $Q_T \neq 0$ and another power of α_s comes from loop integrals which can generate imaginary parts. Furthermore, the complete calculation, as presented in Eq. (5), yields a coefficient of $\alpha_s{}^2$ proportional to $(C_F - N_c/2)$ in the $q\bar{q}$ channel. These terms conspire in QCD to practically cancel so the null result of the parton model is maintained for, e.g., $\pi^- p$ lepton-pair production.

Since single-spin proposals are under consideration at CERN and Fermilab, it is useful and urgent to clarify the QCD expectations in some detail. In this paper we study the spin dependence of the differential cross section

$$\frac{d\sigma}{d^4Q\,d\Omega}(A^\uparrow B \to \mu^+\mu^- X) \ ,$$

where Q^μ is the lepton-pair four-momentum and Ω the angles of the leptons in a given frame. Correlations between Ω and a single hadron spin probe directly[3] the imaginary part of interfering amplitudes. We consider the $q\bar{q}$ fusion subprocess, known to dominate in $\pi^- p$ (and $\bar{p}p$) collisions. The reason for this is that the spin-dependent parton distribution functions are only known for quarks.[4] Of course, the calculation can be extended to include the contribution of the qg subprocess with some assumptions for the spin-dependent gluon distribution function.

Let us first remind the reader about the basic formalism we are going to use. Calling $L^{\mu\nu}$ and $W^{\mu\nu}$ the usual leptonic and hadronic tensors such that

$$e^2 W^{\mu\nu} = \int d^4x\, e^{iQ\cdot x} \langle P_A S_A P_B | J^\mu(0) J^\nu(x) | P_A S_A P_B \rangle \ ,$$
$$L^{\mu\nu} = -2Q^2 \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{k^\mu k^\nu}{k^2} \right) \ , \tag{1}$$

where k is the difference between the leptons' momenta k_1 and k_2, one gets

$$\frac{d\sigma}{d^4Q\,d\Omega} = \frac{\alpha^2}{2(2\pi)^4} \frac{1}{Q^2 s} \left(\delta^{ij} - \frac{k_1^i k_1^j}{\vec{k}_1{}^2} \right) W_{ij} \tag{2}$$

in a lepton-pair rest frame. Constructing in this frame the basis[5]

$$Z = P_A Q\cdot P_B - P_B Q\cdot P_A ,$$
$$X = P_A Q^2 Z\cdot P_B - P_B Q^2 Z\cdot P_A + Q(Q\cdot P_B Z\cdot P_A - Q\cdot P_A Z\cdot P_B) \ ,$$
$$Y^\mu = \epsilon^{\mu\alpha\beta\gamma} P_{A\alpha} P_{B\beta} Q_\gamma \ ,$$

which defines the angles θ and ϕ, the differential cross section for $Q_T \neq 0$ may be written in the form[5]

$$\frac{d\sigma}{d^4Q\,d\Omega} = \frac{\alpha^2}{2(2\pi)^4} \frac{1}{Q^2 s} \{ 2(W_{0,0} + Y\cdot S_A T_{0,0}) + \tfrac{1}{3}[(1-3\cos^2\theta)(W_{2,0} + Y\cdot S_A T_{2,0})]$$
$$+ \sin 2\theta \cos\phi (W_{2,1} + Y\cdot S_A T_{2,1}) + \tfrac{1}{2}[\sin^2\theta \cos 2\phi (W_{2,2} + Y\cdot S_A T_{2,2})]$$
$$+ \sin 2\theta \sin\phi (X\cdot S_A T^T_{2,-1} + Z\cdot S_A T^L_{2,-1}) + \sin^2 2\theta \sin 2\phi (X\cdot S_A T^T_{2,-2} + Z\cdot S_A T^L_{2,-2}) \} \ ,$$

where we have averaged over polarizations of hadron B but kept the spin vector S_A of hadron A. It is obvious that to define the angle ϕ one must have $Q_T \neq 0$. Straightforward power counting in QCD reveals that

$$W_{2,2},\ T_{2,2},\ T^L_{2,-2},\ T^T_{2,-2} \propto \frac{Q_T^2}{\hat{s}} ,$$
$$W_{2,1},\ T_{2,1},\ T^L_{2,-1},\ T^T_{2,-1} \propto \frac{Q_T}{\sqrt{\hat{s}}} , \tag{4}$$

when compared to $W_{0,0}$, $\hat{s}$ being the usual subprocess Mandlestam variable. Moreover, usual chiral properties suppress transverse-spin effects by $1/\sqrt{\hat{s}}$ so that $T_{2,1}$ and $T^T_{2,-1}$ should be small. Helicity effects can, on the other hand, be large. Indeed deep-inelastic polarized experiments show that quarks remember fairly well the proton's helicity.[4] We thus have chosen to first calculate the largest-helicity term $T^L_{2,-1}$, at lowest nontrivial order, i.e., at $O(\alpha_s^2)$ for $q\bar{q}$ channel.

Let us now present the main steps of the calculation and outline the basic features of the result. We assume some factorization of the long-distance dynamics from the short-distance perturbatively calculated QCD diagrams. We use the dimensional-regularization procedure and work in the Feynman gauge. The goal is to calculate the imaginary part of the interference between Born graphs [Fig. 1(a)] and higher-order graphs [some of which are drawn in Figs. 1(b) – 1(d)], summing over the antiquark helicity states while taking the difference between the quark helicity states. It is straightforward to see that only graphs with a loop may have an imaginary part. Moreover, one can show that only those in Figs. 1(b) – 1(d) actually contribute. The computation thus requires, apart from some rather large traces which have been performed with the help of the symbolic program REDUCE,[6] the calculation of integrals of the form

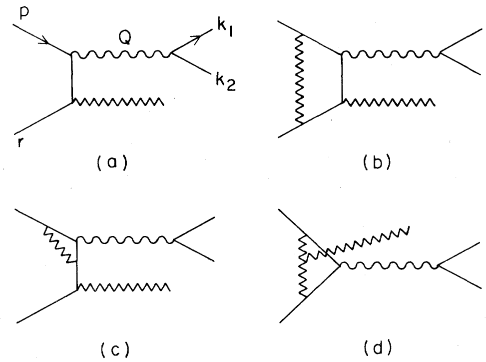

FIG. 1. (a) The lowest-order graph for $q\bar{q} \to \gamma^* g$. (b) The graphs contributing in Feynman gauge to the imaginary part of $W^{\mu\nu}$ at order α_s^2 (crossed graphs have been omitted).

$$\mathrm{Im}\frac{1}{i}\int d^n m \frac{(1, m^\mu, m^\mu m^\nu, m^\mu m^\nu m^\rho)}{m^2(p-m)^2(Q-m-r)^2(m+r)^2} ,$$

where $n = 4 - \epsilon$. The resulting expression at the subprocess level, which is the coefficient of $\delta((p+r-Q)^2)$, may be cast into the form

$$Z \cdot S_A \hat{T}^L_{2,-1} = 2\pi^2 e_q^2 \alpha_s^2 \frac{C_F}{N_c}\left(C_F - \frac{N_c}{2}\right)\left[A\left(\frac{2}{\epsilon} - \ln\frac{Q^2}{\mu^2}\right) + B\ln\frac{\hat{s}}{Q^2} + C\ln\left(1 + \frac{Q_T^2}{Q^2}\right) + D\ln\left(\frac{Q\cdot p}{Q\cdot r}\right) + E\right] . \tag{5}$$

The infrared finiteness of the result at this order requires that A be zero, which we indeed find. This is in itself a nontrivial result ensuring the applicability of perturbative QCD. An imaginary divergence would not have been factorizable into the usual quark distribution. The $B - E$ coefficients have fairly compact forms:

$$B = \frac{-2\Delta Q^4}{\hat{s} Q_T^3 \sqrt{Q^2 + Q_T^2}},\quad C = -B,\quad D = \frac{Q^2[Q^2(\hat{s} - Q^2) + 2Q_T^2 \hat{s}]}{\hat{s} Q_T^3 \sqrt{Q^2 + Q_T^2}} ,$$
$$E = \frac{-\Delta[\hat{s}^2(13Q^2 + 9Q_T^2) - 12\hat{s}(Q^4 + 4Q^2Q_T^2 + 3Q_T^4) - Q^2(Q^4 - 15Q^2Q_T^2 - 12Q_T^4)]}{\hat{s} Q_T(\hat{s} - Q^2)(Q^2 + Q_T^2)\sqrt{Q^2 + Q_T^2}} , \tag{6}$$

with $\Delta = Q\cdot p - Q\cdot r$, $\ln(Q\cdot r/Q\cdot p)$ being twice the rapidity of the pair in the subprocess c.m. system.[7]

For this discussion, we define an integrated asymmetry $\mathfrak{A}$ for the physical process by

$$\mathfrak{A} = \frac{\int_0^1 d\cos\theta\left[\int_0^\pi d\phi - \int_\pi^{2\pi} d\phi\right] d\sigma^\uparrow/d^4Q\, d\Omega}{\int_0^1 d\cos\theta\left[\int_0^\pi d\phi + \int_\pi^{2\pi} d\phi\right] d\sigma/d^4Q\, d\Omega} , \tag{7}$$

where $d\sigma^\uparrow$ indicates positive helicity λ for proton $\mathfrak{A}$ [since $\mathfrak{A}$ is linear in λ, one can replace $d\sigma^\uparrow$ by $(d\sigma^\uparrow - d\sigma^\downarrow)/2$]. As a preliminary step one can define $\hat{\mathfrak{A}}$, the asymmetry at the subprocess level, by an obvious modification of Eq. (7). It is easy to show that

$$\mathfrak{A} = \frac{2Z\cdot S_A T^L_{2,-1}}{3\pi W_{0,0}} , \tag{8}$$

and a similar expression holds for $\hat{\mathfrak{A}}$.

In Fig. 2, we plot the subprocess asymmetry as a function of the lepton-pair transverse momentum Q_T for $Q^2 = 25$ GeV^2, $\hat{s} = 200$ and 400 GeV^2, and the photon rapidity $y > 0$. [Note that, at this level, the differential cross section $d\sigma/d^4Q\, d\Omega$ is proportional to a $\delta(|y| - y_0)$ term and that $\hat{\mathfrak{A}}$ is odd in y.] This asymmetry is quite small; indeed this might have been anticipated for the $q\bar{q}$ channel, as we will now show.

Let us consider the theoretical expression for $\hat{T}^L_{2,-1}$, Eq. (5). The coefficients B, C, and D are imaginary parts of

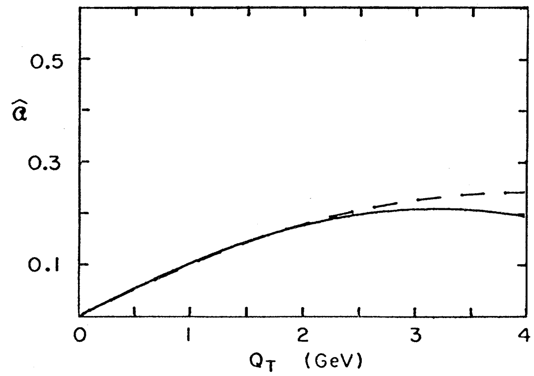

FIG. 2. The asymmetry at the subprocess level, as defined in Eq. (7), in units of $(2C_F-N_c)\alpha_s(Q^2)$ for $\hat{s}=200$ GeV^2 (full curve) and $\hat{s}=400$ GeV^2 (dashed curve).

doubly logarithmic terms while E is the imaginary part of a single logarithm, coming from relations like

$$\mathrm{Im}\ln^2(-s-i\eta)=-i\pi\ln s \ ,$$

$$\mathrm{Im}\ln(-s-i\eta)=-i\pi \ .$$

It is well known that the leading (i.e., $\ln^2 s$) real corrections can be obtained order by order by soft-gluon approximations, i.e., by integrating gluon internal momenta from $O(\sqrt{s})$ down to fixed values as $Q^2, s\to\infty$. Similarly, infrared-divergent real terms of single-logarithm order come from the various collinear regions determined by the external legs. For the purposes of locating imaginary terms, one can replace the original Feynman integrals by their approximations in these regions, neglecting the loop-momentum dependence of propagators that become far off shell.

The analytic properties of this replacement can then be easily pinpointed. For instance, the soft and collinear regions of Fig. 1(d) have no thresholds, except for a gauge term that annihilates with Figs. 1(b) and 1(c) using the Ward identity. There remains only hard-gluon-exchange regions from this diagram which are expected to contribute only to the nonleading coefficient E. This (somewhat oversimplified) reasoning leads to the association of the leading coefficients from the remaining diagrams with the color factor of Fig. 1(b), i.e., $(C_F-N_c/2)$. Thus the color factor of A, B, C, and D in the complete calculation, Eq. (5), seems to have a simple explanation.

The fact that $C_F-N_c/2=-\frac{1}{6}$ in QCD has important consequences. The results of the complete $O(\alpha_s^2)$ calculation can be summarized in the limit Q^2/s, Q_T^2/Q^2, and y fixed and $s\to\infty$ by the estimate[8]

$$|\mathfrak{a}|\simeq\left|C_F-\frac{N_c}{2}\right|\frac{\alpha_s^2\left\langle\left(Q_T/\sqrt{\hat{s}}\right)\lambda_q(x)q(x)\bar{q}(x)\right\rangle}{\alpha_s\langle q(x)\bar{q}(x)\rangle}\sim 5\alpha_s\% \ , \tag{9}$$

where the brackets indicate the parton convolutions. The near cancellation between the color factors therefore suppresses the QCD contribution to $\mathfrak{a}$ substantially. Numerical integration of our result, using the NA3 parton distributions[9] and $\lambda_q(x)=0.94\sqrt{x}$ from the SLAC-Yale experiment,[4] gives $\mathfrak{a}=-0.4\alpha_s\%$ $(-0.9\alpha_s\%)$ at $Q^2=25$ GeV^2, $Q_T=4$ GeV (6 GeV), and $y=0$ for π^-p experiments at $\sqrt{s}=27$ GeV.

We conclude that the null value for $\mathfrak{a}$ of the naive Drell-Yan model including intrinsic effects is not upset at $O(\alpha_s^2)$ in QCD because of a fortunate cancellation in the color algebra for the $q\bar{q}$ subprocess. The $q\bar{q}$ channel, in turn, is known to dominate unpolarized $\pi^-p\to\mu\bar{\mu}+X$ and $p\bar{p}\to\mu\bar{\mu}+X$ for moderate Q_T. Color factors of the quark-gluon Compton-subprocess contribution,[10] which should be of order $N_c/(2C_F-N_c)$ times the color factor in Eq. (9), might tend to compensate for the well known valence suppression of this channel,[11] but at most a few $\alpha_s\%$ contribution to $\mathfrak{a}$ could arise in this way. Parity-violating weak-interaction effects are at the level of a fraction of a percent.[12] We conclude that observation of an asymmetry greater than, e.g., 3% in the usual experimental region would be incompatible with the present framework.

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