PHYSICAL REVIEW D VOLUME 28, NUMBER 1 1 JULY 1983

New view of quark and lepton mass hierarchy

Yoshio Koide
Laboratory of Physics, Shizuoka Women's University, Yada 409, Shizuoka 422, Japan
(Received 26 January 1983)

The hierarchical structure of realistic quark and lepton masses is investigated on the basis of a three-family model, where masses and mixings of the first- and second-generation quarks are successfully described by lepton masses only. An excellent prediction $m_\tau = 1.7866$ GeV is obtained from the input data m_e and m_μ only.

Investigation of hidden rules, even if they are empirical ones,[1] behind observed quark and lepton mass spectra may provide a very important clue to a unified model of quarks and leptons.

Recently, the author has proposed a composite model[2] of leptons and quarks (hereafter we refer to it as model I), which has provided the following mass matrices of leptons $l_i = (\nu_i, e_i)$ and quarks $q_i = (u_i, d_i)$ $(i = 1,2,3$ is the family number):

$$M^l = \begin{bmatrix} \delta_1^l & 0 & 0 \\ 0 & \delta_2^l & 0 \\ 0 & 0 & \delta_3^l \end{bmatrix} ,$$
$$M^q = \begin{bmatrix} \delta_1^q & 0 & 0 \\ 0 & \delta_2^q & 0 \\ 0 & 0 & \delta_3^q \end{bmatrix} + \Delta^q \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} . \tag{1}$$

Here the diagonal elements δ_i^l and δ_i^q $(\ll \Delta^q)$ are given by

$$\delta_i = (Q_F)^{\text{total}} v_F , \tag{2}$$

where v_F is a flavor-independent parameter with dimension of mass and Q_F ("flavor charges") are defined as coupling constants of preons with a vector boson ϕ_F. The small mass terms δ_i^l and δ_i^q are given explicitly by[3]

$$\delta_i^\nu = 0, \quad \delta_i^u = (\alpha z_i + z_0)^2 ,$$
$$\delta_i^e = (z_i + z_0)^2, \quad \delta_i^d = (\alpha - 1)^2 z_i^2 , \tag{3}$$

where

$$z_1 + z_2 + z_3 = 0 \tag{4}$$

(and $\alpha = \sqrt{3}$ in model I).

The most striking point in the model is that the Δ term appears in the quark mass matrix, while the term is absent in the lepton mass matrix. Diagonalization of the quark mass matrix (1) makes from the quark states (q_1, q_2, q_3) two massless states

$$q_\pi = (q_1 - q_2)/\sqrt{2} ,$$
$$q_\eta = (q_1 + q_2 - 2q_3)/\sqrt{6} , \tag{5}$$

and one massive state (with the mass $3\Delta^q$)

$$q_0 = (q_1 + q_2 + q_3)/\sqrt{3} , \tag{6}$$

in the limit of $\delta_i = 0$. (This scenario for the quark mass matrix was first discussed by Harari, Haut, and Weyers,[4] who assumed an S_3 symmetry for quarks.) Therefore the presence of small mass terms δ_i provides realistic masses for leptons l_i and both the masses and mixing for the first- and second-generation quarks q_π and q_η. In other words, we can predict the masses of the first- and second-generation quarks and the mixing between those by using lepton masses only. In fact, we have successfully obtained the relation[5] for the Cabibbo angle θ_C,

$$\tan\theta_C = \sqrt{3} \frac{(m_\mu)^{1/2} - (m_e)^{1/2}}{2(m_\tau)^{1/2} - (m_\mu)^{1/2} - (m_e)^{1/2}} . \tag{7}$$

Moreover, model I has predicted the mass of the τ lepton $m_\tau = 1.777$ GeV from the input data m_e and m_μ only, where the use of the relation

$$z_0 = [(z_1{}^2 + z_2{}^2 + z_3{}^2)/3]^{1/2} \tag{8}$$

has been essential.

The phenomenological success of the mass matrices (1) is worth noting for all the theorists who intend to build a unified model of quarks and leptons, whether their scenarios are based on a unified gauge theory or on a composite model, because we may forget the original composite model after we have once obtained the successful mass matrices of quarks and leptons. Taking such a phenomenological success seriously, in the present paper, we will give further phenomenological studies based on the mass matrices (1) with the relation (3).

In spite of such successful predictions of m_τ and θ_C, however, there is an unsatisfactory point in model I: The smallness of m_e in comparison with m_μ and m_τ is merely accidental, that is, it relies on an accidental coincidence $z_1 \simeq -z_0$ in model I.

In this paper, we try to clarify the meaning of $z_1 \simeq -z_0$ and we present a new scenario for masses of quarks and leptons.

The mass matrices are given via the following three hierarchial steps:

Step I. There are only the terms Δ^q (Δ^u and Δ^d). Then only the third-generation quarks $q_\sigma = (t,b)$ have $3\Delta^q$.

Step II. The terms δ_i appear. Two leptons $e_2 = \mu$ and $e_3 = \tau$ have masses (so that quarks q_π and q_η also have masses), but lepton $e_1 = e$ remains massless because of $z_1 = -z_0$. Then the relation[6]

$$\tan^2\theta_C = \frac{m_\mu}{m_\tau} = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 \tag{9}$$

is satisfied.

Step III. The electron mass m_e is generated from a very small correction to δ_i given in step II, and the other leptons and quarks also gain realistic masses. Then relation (9) is revised: the Cabibbo angle θ_C is given by formula (7) and the predicted mass m_τ from the input data m_e and m_μ becomes

$$m_\tau = 1.7866 \text{ GeV} \ , \tag{10}$$

which is in excellent agreement with the observed value[7] $m_\tau = 1.7842 \pm 0.0032$ GeV (cf. the old prediction $m_\tau = 1.777$ GeV in model I).

Step I has already been discussed in the short review of model I. The greatest interest in the present paper is the role of δ_i in steps II and III. The appearance of the small mass terms δ_i^q causes mixing between quarks q_π and q_η:

$$\tan 2\theta^q \simeq -\sqrt{3}\frac{\delta_2^q - \delta_1^q}{2\delta_3^q - \delta_2^q - \delta_1^q}\left[1 + \frac{2\delta_3^q - \delta_2^q - \delta_1^q}{6\Delta^q}\right] \ . \tag{11}$$

If δ_i^d is given by $\delta_i^d \propto z_i^2$, with the help of Eq. (4) we can derive

$$\tan\theta^d \simeq \sqrt{3}\frac{z_2 - z_1}{2z_3 - z_2 - z_1} \ , \tag{12}$$

where we drop the negligibly small terms δ_i^d/Δ^d in Eq. (11). Therefore we can obtain the formula (7) under the relation $\theta^u \simeq 0$ which is provided by an appropriate choice of the parameter α in Eq. (3).

In preparation for step II, let us study the limit $z_1/z_0 \to -1$ under the conditions (4) and (8). We can easily find that the limit $m_e \to 0$ in model I means

$$\frac{z_1}{z_0} \to -1, \quad \frac{z_2}{z_0} \to -\frac{\sqrt{3}-1}{2}, \quad \frac{z_3}{z_0} \to \frac{\sqrt{3}+1}{2} \ . \tag{13}$$

Now we can immediately get step II in our scenario as follows: We propose vector bosons π_F, η_F, and σ_F whose interactions with fermions are

$$g_F\left[\frac{1}{\sqrt{2}}(\bar{e}^2e_2 - \bar{e}^3e_3)\pi_F + \frac{1}{\sqrt{6}}(\bar{e}^2e_2 + \bar{e}^3e_3 - 2\bar{e}^1e_1)\eta_F + \frac{1}{\sqrt{3}}(\bar{e}^2e_2 + \bar{e}^3e_3 + \bar{e}^1e_1)\sigma_F\right] \ , \tag{14}$$

where we omit the Lorentz indices and, for simplicity, we describe the currents in terms of quarks and leptons (even if they are composite ones) and moreover denote only the part related to down leptons e_i. The vector boson ϕ_F which plays an essential role to yield δ_i terms is given by a subsequent 45°-mixing state among the bosons π_F, η_F, and σ_F,

$$\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\pi_F - \frac{1}{\sqrt{2}}\eta_F\right] - \frac{1}{\sqrt{2}}\sigma_F \ . \tag{15}$$

We assume that contributions from the other two mixing states are negligibly small. Thus we get the parametrization (13) and we can derive the relation (9) from Eq. (12) and $\theta_C \simeq \theta^d$.

Finally, step III is given by a very small correction to the "ideal" mixing (15):

$$\cos\epsilon\left[\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\pi_F - \frac{1}{\sqrt{2}}\eta_F\right] - \frac{1}{\sqrt{2}}\sigma_F\right] + \sin\epsilon\left[\frac{1}{\sqrt{2}}\pi_F + \frac{1}{\sqrt{2}}\eta_F\right], \tag{16}$$

which leads to

$$\frac{z_1}{z_0} = -(1-\delta), \quad \frac{z_2}{z_0} = -\left[\frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}\delta\right] \ , \quad \frac{z_3}{z_0} = \frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}\delta \ , \tag{17}$$

where $\delta = \sqrt{2}\tan\epsilon$. The parametrization (17) modifies the old relation (8) as a new relation

$$z_0 = [(z_1{}^2 + z_2{}^2 + z_3{}^2)/3(1+\delta^2)]^{1/2} \ , \tag{18}$$

and exchanges the old prediction $m_\tau = 1.777$ GeV for the further successful prediction (10), $m_\tau = 1.7866$ GeV (where $\delta = 0.040\,26$). (Another case of slight deviation from the "ideal" mixing (15),

$$\frac{1}{\sqrt{2}}\left[\cos\left[\frac{\pi}{4} - \epsilon\right]\pi_F - \sin\left[\frac{\pi}{4} - \epsilon\right]\eta_F\right] - \frac{1}{\sqrt{2}}\sigma_F \ , \tag{19}$$

with $\epsilon > 0$ provides the relation (8) and the old prediction $m_\tau = 1.777$ GeV. All the other cases except for (16) and (19) fail to predict the realistic value of m_τ.)

Now we turn our attention to quark masses. In the present paper, the parameter α may be determined phenomenologically. The choice[8] $\alpha = 1.5 - 1.8$ can provide plausible values of θ_C and quark masses. For example, model I with $\alpha = \sqrt{3}$ has predicted

$$\begin{aligned} &\theta^u = 0.014 \quad (0.80°) \ , \\ &m_u = 0.080 \text{ GeV}, \quad m_c = 2.434 \text{ GeV} \ , \\ &m_d = 0.084 \text{ GeV}, \quad m_s = 0.254 \text{ GeV} \ . \end{aligned} \tag{20}$$

The new parametrization (17) scarcely modifies the old estimates (20). Hereafter, for convenience, we take $\alpha = \sqrt{3}$. The predicted value of m_c is somewhat large and, therefore, we need some suppression mechanism for quark masses. We express the suppression factor in terms of $R \equiv v_F^{\text{quark}}/v_F^{\text{lepton}}$.

Since, in our scenario, there is no principle which predicts Δ^u/Δ^d, we cannot estimate the absolute value of m_t. But it is possible to give the upper limit of m_t by using recent data on b decays. The weak-mixing matrix elements related to b decays are given by

$$\begin{aligned} &U_{b\to u} \simeq \sqrt{2}\lambda^d \tan 2\theta_C \ , \\ &U_{b\to c} \simeq \sqrt{2}(\lambda^u - \lambda^d) \ , \end{aligned} \tag{21}$$

where

$$\lambda^q = \frac{2\delta_3^q - \delta_2^q - \delta_1^q}{18\Delta^q} \simeq \frac{(m_\eta - m_\pi)/2}{m_\sigma - (m_\eta + m_\pi)/2} \ , \tag{22}$$

and $m_\pi = (m_u, m_d)$, $m_\eta = (m_c, m_s)$, and $m_\sigma = (m_t, m_b)$. Putting $x \equiv \lambda^d/\lambda^u$, we can obtain the relations

$$\frac{\Gamma(b\to u)}{\Gamma(b\to c)} \simeq \left|\frac{U_{b\to u}}{U_{b\to c}}\right|^2 \simeq \left[\frac{x}{1-x}\right]^2 \tan^2 2\theta_C \tag{23}$$

and

$$\begin{aligned} m_t &\simeq x\frac{m_c - m_u}{m_s - m_d}\left[m_b - \frac{m_s + m_d}{2}\right] + \frac{m_c + m_u}{2} \\ &\simeq x\frac{m_c - m_u}{m_s - m_d}m_b \ . \end{aligned} \tag{24}$$

The final expression of (24) is independent of the choice of the suppression factor R. Hereafter, for a rough estimate of m_t, we use the final expression of (24) with $m_b \simeq 4.7$ GeV. If $m_t > 66$ GeV ($x > 1$), we get a lower limit on $\Gamma(b\to u)/\Gamma(b\to c)$,

$$\Gamma(b\to u)/\Gamma(b\to c) > \tan^2 2\theta_C = 0.227 \ .$$

Recent data[9] on b decays seem to rule out this case $x > 1$, and to prefer the case[10] $\Gamma(b \to u)/\Gamma(b \to c) < \tan^2 2\theta_C$, which leads to an upper limit on m_t, $m_t < 33$ GeV ($x < 0.5$). Precise measurement of $\Gamma(b \to u)/\Gamma(b \to c)$ in the near future will give a further restricted upper limit of m_t. Conversely, from the present experimental lower limit[11] of m_t, ~ 20 GeV, we can predict the lower limit of $\Gamma(b \to u)/\Gamma(b \to c)$,

$$\Gamma(b \to u)/\Gamma(b \to c) \gtrsim 0.04 \quad .$$

In general, models based on the mass matrices (1), whether there is the relation (3) or not, must satisfy the relations (23) and (24), so that such models can be checked by observations of m_t, and $\Gamma(b \to u)/\Gamma(b \to c)$.

We have so far devoted our studies to seeking a phenomenologically preferable "form" of the mass matrices (1), and have not mentioned the origins which settle the "form." In order to understand the origin of the "ideal" mixing (15) and to explain the values of our adjustable parameters, α, R, Δ^u/Δ^d, and so on, we need a further specific model. For example, the idea of extended hypercolor[12] may be promisingly utilized for the interpretation of the parameters δ_i^l and δ_i^q which are connected only by the square of the parameters Q_F.

We hope that the mass matrices (1) with the relations (3) and (17), which are merely empirical ones so far, cast a new light on the unified model of quarks and leptons, especially on the generation problem.

The author would like to thank Professor K. Matumoto and Professor A. Murayama for calling his attention to Ref. 4.

---

[1]For example, Barut's formula [A. O. Barut, Phys. Rev. Lett. 42, 1251 (1979)]

$$m_n = m_e + m_e \frac{3}{2}\alpha^{-1}[1^4 + 2^4 + \cdots + (n-1)^4]$$

is in excellent agreement with the observed values of m_μ and m_τ. If a fourth charged lepton with the mass $m_4 = 10.29$ GeV were discovered, the formula might be compared with Rydberg's equation for Bohr's theory. However, the present experiments [D. P. Barber *et al.*, Phys. Rev. Lett. 43, 901 (1979); R. Brandelik *et al.*, Phys. Lett. 99B, 163 (1981); Ch. Berger *et al.*, *ibid.* 99B, 489 (1981)] have reported no evidence for such a charged lepton with mass lower than 16 GeV.

[2]Y. Koide, Phys. Lett. 120B, 161 (1983). Also see, Y. Koide, Lett. Nuovo Cimento 34, 201 (1982).

[3]Since the quantum numbers which quarks and leptons can family-independently possess are only three, that is, weak isospin I_3, baryon number B, and lepton number L, the most general form of Q_F under the conditions $Q_F(\nu_i) = 0$ and $Q_F(d_i) \propto z_i$ which lead to $m(\nu_i) = 0$ and the relation (12), respectively, is given by

$$Q_F = (Q_E + B)(z_i + z_0) + 3(\alpha - 1)Bz_i \quad ,$$

where $Q_E = I_3 + (B-L)/2$ and $Q_F(e_i)$ is normalized as $Q_F(e_i) = -(z_i + z_0)$. In other words, the expression (3) of δ_i is the most general form which leads to $m(\nu_i) = 0$ and the formula (7) under $\theta^u \simeq 0$.

[4]H. Harari, H. Haut, and J. Weyers, Phys. Lett. 78B, 495 (1978).

[5]This formula (7) was first found on the basis of another specific model of composite quarks and leptons. See Y. Koide, Phys. Rev. Lett. 47, 1241 (1981).

[6]The prediction $\theta_C = 15°$ [the relation (9) except for the leptonic part] was first derived from a somewhat different scenario by Harari, Haut, and Weyers. See Ref. 4.

[7]Particle Data Group, Phys. Lett. 111B, 1 (1982).

[8]The value of α which gives $\theta^u = 0$ is $\alpha = 2z_0/z_3 = 1.4485$. However, a small positive value ($\sim 10^{-2}$) is preferred to the exact $\theta^u = 0$, because the small θ^u favorably cancels a small deviation from the formula (7) caused by the second term δ_i^d/Δ^d in Eq. (11).

[9]L. J. Spencer *et al.*, Phys. Rev. Lett. 47, 771 (1981); A. Brody *et al.*, *ibid.* 48, 1070 (1982); W. Bartel *et al.*, Phys. Lett. 114B, 71 (1982).

[10]In Ref. 2, only the case $x > 1$ ($m_t >> m_b$) has been investigated and the study of the case $x < 1$ has been dropped.

[11]D. P. Barber *et al.*, Phys. Lett. 108B, 63 (1982).

[12]S. Dimopoulos and L. Susskind, Nucl. Phys. B155, 237 (1979); E. Eichten and K. D. Lane, Phys. Lett. 90B, 125 (1980).