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Runaway particle production in de Sitter space

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We examine a particle-production mechanism for scalar $\lambda\varphi^4$ theory in de Sitter space which has the feature that the rate of particle production is proportional to the number of particles present, yielding an exponentially increasing rate of production. General arguments strongly suggest that this process is generic to *any* renormalizable interacting theory on de Sitter space. The interpretation of this process and its relation to the response of freely falling "particle detectors" is discussed.

I. INTRODUCTION

The topic of particle creation in curved space-times has held considerable theoretical interest in recent years, because of its application to cosmological and astrophysical problems as well as its potential for illuminating "zeroth-order" aspects of quantum gravity. As a result, a considerable body of literature devoted to its study has sprung up, the prototypical example of which is Hawking's calculation of black-hole radiance.[1]

This paper will focus on a particular particle-creation process in de Sitter space-time for a self-interacting scalar field with interaction $\lambda\varphi^4$. The mechanism in question has the interesting feature that it is a runaway process which produces an exponentially increasing number of particles. It is therefore expected to dominate other modes of particle production which have been studied for $\lambda\varphi^4$ theory.[2,3] Although the example studied here is that of a self-interacting scalar field, general arguments strongly suggest that the basic process is generic to any interacting field of any spin in de Sitter space.

Because of its generic nature and runaway production rate, this process has important implications for the physics of the very early universe. In particular, it suggests that de Sitter space is dynamically unstable, and a recent examination of the back reaction[4] of this process on the space-time metric shows, modulo the usual uncertainties of semiclassical back-reaction calculations, that it will lead to a decay or diminution of the effective cosmological "constant."

The plan of this paper is as follows. Section II motivates the quantum-field-theory calculation by discussing the treatment by Gibbons and Hawking[5] of a quantum-mechanical model "atom" which interacts with a free scalar field in a de Sitter space. Section III presents the particle-production calculation. Section IV examines the choice of "vacuum" state used in Secs. II and III. Finally, Sec. V concludes the paper and discusses the relation of this particle-production mechanism to radiation from electrons accelerating in Minkowski space.

The units used are such that $\hbar=c=G=k=1$, and the sign conventions follow those of Misner, Thorne, and Wheeler.[6]

De Sitter space is defined as a vacuum solution of Einstein's equations with a cosmological constant Λ which is positive. The properties of de Sitter space are discussed in Refs. 6, 7, and 8. The form of the metric most relevant to this paper is

$$ds^2=-dt^2+(3/\Lambda)\cosh^2[(\Lambda/3)^{1/2}t]d\Omega^2 \,,$$

where $d\Omega^2$ is the metric on a three-sphere. However, the results derived in the following sections do not specifically depend on this choice. We remark in passing that the de Sitter space can be physically viewed as a four-dimensional hyperboloid embedded in a five-dimensional flat Lorentzian space. Under the "Euclidean-type" transformation $t\to -i\tau$, this simply becomes a four-sphere in R^5, as can be seen from the metric above.

II. MODEL "ATOMS"

As a first step toward understanding particle production in de Sitter space we can consider the effects of placing a simple model "atom" in a de Sitter space in which a free scalar quantum field φ has been defined. We shall assume that the "atom" is a nonrelativistic quantum system with the following properties:

(a) Its Hamiltonian has a discrete set of energy levels E_n. A good example of this sort of system would be a simple harmonic oscillator.

(b) The "atom" moves semiclassically along the prescribed world line $x(t)$.

(c) The "atom" couples linearly to the scalar field via the interaction Lagrangian $L_{\text{int}}=\lambda Z\varphi$, where λ is a coupling constant and Z corresponds to the monopole moment of the "atom."

The details of constructing a scalar quantum field in de Sitter space have been treated at length[7-12] by several authors, and will only be briefly outlined here.

We shall assume that the field has the wave equation

$$(\Box+\xi R+M^2)\varphi=0 \,. \tag{2.1}$$

We shall use $\xi=\frac{1}{6}$, which is the value for conformally invariant fields. One can construct solutions to (2.1) without any technical difficulty, and proceed with canonical quantization in the usual fashion.[13-15] When one attempts to construct a Hilbert space and set up a Fock basis of particle states, however, problems arise because there is no generally covariant way to define particle states. This is equivalent to saying that there is no covariant way to define the vacuum. In order to proceed, we shall adopt the de Sitter—invariant vacuum investigated by Nachtmann[7] and Chernikov and Tagirov[8] and used by many other authors.[8–20] This choice is crucial to our results and will be discussed in Sec. IV. This vacuum will be labeled $|0_{\rm inv}\rangle$ to distinguish it from other nonequivalent vacuums.

Having chosen a vacuum we can now decompose φ into positive-and negative-frequency solutions:

$$\varphi(x)=\sum_k (a_k f_{k,p}^- + a_k^\dagger f_{k,p}^+)$$

with

$$p=[(\Lambda/3)M^2-\tfrac{1}{4}]^{1/2}\ .$$

The case when $(\Lambda/3)M^2\le\frac{1}{4}$ is discussed by Tagirov.[9] The two-point or Wightman functions have been calculated by many authors using different techniques[9–19] with the result

$$\begin{aligned}\Delta^-(x_1,x_2)&=i\langle 0_{\rm inv}|\varphi(x_1)\varphi(x_2)|0_{\rm inv}\rangle\\&=i\Lambda[24\pi\cosh(\pi p)]^{-1}(\partial/\partial G)P_{-1/2+ip}(G)\end{aligned}\tag{2.2}$$

with

$$G=\cosh[(\Lambda/3)^{1/2}\sigma(x_1,x_2)]\ .$$

Here, σ is twice the world function, and is equal to the geodesic time interval between x_1 and x_2 in de Sitter space. The function $P_{-1/2+ip}$ is a Legendre function[20,21] of complex order and is related to the conal harmonic functions. Using Eq. (2.2) one can obtain the other Green's functions of the theory in the usual manner.

The response of the model "atom" as it moves along a geodesic world line in de Sitter space has been calculated by Gibbons and Hawking[5] with the result that it becomes excited with a thermal spectrum of excitations at a temperature

$$T_{\text{de Sitter}}=(2\pi)^{-1}(\Lambda/3)^{1/2}\ .$$

Recall, however, that we started with a supposed *vacuum* state for the field.

This appears somewhat paradoxical at first, and has previously been discussed in terms of "observer-dependent" particles and the many-worlds interpretation of quantum mechanics.[5] The question of "observer-dependent" particles is, however, largely semantic in content and arises because the notion of positive and negative frequency, and hence that of a vacuum, is *basis dependent.* Observers which have different rest-frame coordinate systems will in general have Fock bases for the field φ which are related by a frequency-mixing Bogoliubov transformation.[14,15,22,23] A given state, such as that we have *labeled* $|0_{\rm inv}\rangle$, can therefore appear as a vacuum to some observers and as a many-particle state to others. We shall refer to the Fock basis for which $|0_{\rm inv}\rangle$ is a vacuum or no-particle state as the $|0_{\rm inv}\rangle$ vacuum basis.

Sciama and his collaborators[22,23] have elegantly demonstrated that the correct interpretation is that the excitations come from interactions between the "atom" and zero-point fluctuations in the state $|0_{\rm inv}\rangle$. Attempts to naively treat these fluctuations on the same footing as particle states like $|1_{\rm inv}\rangle$, defined with respect to the $|0_{\rm inv}\rangle$ vacuum basis, are doomed from the start.

The "thermal" nature of de Sitter space, which was first discovered in a slightly different context by Figari, Hoegh-Krohn, and Nappi,[11] is often discussed in terms of the periodicity of (2.2) in Euclidean time ($\sigma\to -i\widetilde{\sigma}$). This fact is mathematically responsible for the "atom" result and indeed it makes the situation *formally* identical to a thermal state in flat space. We must reiterate, however, that the field is *not* in a thermal state with respect to the Fock basis we have chosen. These issues will be discussed in Sec. IV; at present it will suffice to point out that the response of the model "atoms" is basis *independent* given a specified initial state and world line $x(t)$. Thus, all observers will agree that the "atom" becomes excited.

A previously neglected consequence of the excitation is that the "atom" will also spontaneously drop in energy and emit a particle. The calculation will only be sketched here as it is similar to that for excitation.[5,13,24,22,23] Assume that the "atom" is a harmonic oscillator with the $L_{\rm int}$ above. We wish to find the transition probability for the "atom" to go from a state $|E_n\rangle$ to a state $|E_m\rangle$ with the excess energy $E=E_n-E_m>0$ being emitted in a particle state $|1_{\rm inv}\rangle$ defined with respect to the $|0_{\rm inv}\rangle$ vacuum Fock basis. This is

$$P_{0\to 1}=\sum_{1_{\rm inv}}\sum_{E_n}\left|\left\langle E_0,1_{\rm inv}\left|\exp\left[-i\int L_{\rm int}(t)dt\right]\right|E_n,0_{\rm inv}\right\rangle\right|^2 .\tag{2.3}$$

The matrix element in (2.3) is understood to be that for the combined system of φ and the "atom." We are only interested in showing that there is a finite transition probability for *some* particle to be produced, hence the sum over all one-particle states. The calculation for a specific particle may be found in Ref. 25. After using first-order perturbation theory to expand (2.3), we can use the well-known transition amplitudes for the forced harmonic oscillator to reduce the problem to

$$\begin{aligned}P_{0\to 1}&=\tfrac{1}{2}\lambda^2\sum_{1_{\rm inv}}\int\int\exp[iE\sigma(x,x')]\langle 1_{\rm inv}|\varphi(x)|0_{\rm inv}\rangle\langle 0_{\rm inv}|\varphi(x')|1_{\rm inv}\rangle(-g)^{1/2}d^4x(-g')^{1/2}d^4x'\\&=\lambda^2(2E)^{-1}\int\int\exp[iE\sigma(x,x')]\langle 0_{\rm inv}|\varphi(x)\varphi(x')|0_{\rm inv}\rangle(-g)^{1/2}d^4x(-g')^{1/2}d^4x'\ .\end{aligned}\tag{2.4}$$

After a simple change of variables and use of (2.2) we obtain a similar integral to that found in the excitation calculation[5,23] with the result (for effectively massless)

$$dP_{0\to 1}/dt=\lambda^2(2E)^{-1}\exp[2\pi E(3/\Lambda)^{1/2}]\{\exp[2\pi E(3/\Lambda)^{1/2}]-1\}^{-1}\ .$$

This is the same as the probability per unit proper time for the "atom" to go from $|E_n\rangle$ to $|E_m\rangle$ at the temperature $T_{\text{de Sitter}}$. This shows that there is probability 1 for a particle to be omitted when this transition occurs. There is no room in the formalism for anything else to occur. If we initially prepare the "atom" in its ground state and the field in the vacuum state $|0_{\text{inv}}\rangle$, then the system will evolve so that in the future the "atom" will be found in an excited state and the field will be in some many-particle state. This demonstrates that "atoms" in the de Sitter "vacuum" $|0_{\text{inv}}\rangle$ will act to dynamically destabilize it by emitting particles.

Once again there is nothing paradoxical going on. Under the Sciama interpretation, the space-time curvature is doing work on the "atom" by bumping it into vacuum fluctuations, and this is dissipated as particles. The situation is quite similar to that of an "atom" which is being accelerated through the Minkowski vacuum state in flat space, which also becomes thermally excited.[26,27] In that case the work is done by the source of energy providing the acceleration[22–24] and the "atom" emits particles defined with respect to the usual Minkowski Fock basis.[24]

Although the above result is important as a matter of principle, it is rather contrived as a physical process and should be considered more as a *Gedanken experiment.* The physics that it has illuminated is that the effect of the geometry of de Sitter space on the field φ is to make its vacuum fluctuations or vacuum polarization interact with freely falling observables. The new feature we have focused on here is that this interaction leads to particle creation.

III. RUNAWAY PARTICLE PRODUCTION

The basic physics behind the results of the previous section is that inertial (i.e., geodesic) observers effectively "see" a thermal Green's function despite the fact that $|0_{\text{inv}}\rangle$ is a no-particle state with respect to the basis we have chosen. Following Gibbons and Perry,[28] we expect that the periodicity in imaginary time of the propagator will hold true for any renormalizable field theory on de Sitter space, regardless of its spin or interactions. This is

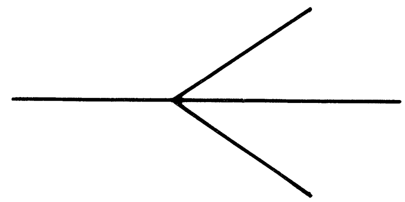

FIG. 1. Runaway $\lambda\varphi^4$ graph in de Sitter space.

because we can use perturbation theory to relate the n-point functions of the theory in question to the free--field two-point function (2.3). This is entirely consistent with the intuitive notion that this is basically a geometric effect.

The next logical generalization to make is to replace the "atom" of Sec. II by a quantum field. To do this we will consider a scalar field with the self-interaction Lagrangian $L_{\text{int}} = :\lambda\varphi^4:$, where λ is a coupling constant, and the normal ordering is done with respect to the $|0_{\text{inv}}\rangle$ vacuum basis. Once again we shall skip over the further details because they are well known.[2,3,12,29]

The preceding heuristic arguments suggest that we consider pair creation catalyzed by a single particle which takes the role of the "atom" of Sec. II. To lowest order in perturbation theory this process is represented by the Feynman graph in Fig. 1, which we will proceed to evaluate. Remarkably, a parallel calculation was done for two-dimensional de Sitter space by Nachtmann[18] in 1967. He was unable to interpret his result in the manner done here because, among other things, the discovery of the de Sitter temperature occurred nearly ten years later. The calculation sketched below follows Nachtmann's treatment with the improvement of working in a four-dimensional de Sitter space.

The first-order matrix element for the process of Fig. 1 is given formally by $\langle 3_{\text{inv}}|S_1|1_{\text{inv}}\rangle$, which is

$$i\int \langle 3_{\text{inv}}|:\lambda\varphi^4:|1_{\text{inv}}\rangle(-g)^{1/2}d^4x = 4i\lambda\int \langle 3_{\text{inv}}|[\varphi^+(x)]^3|0_{\text{inv}}\rangle f_{j,p}^-(x)(-g)^{1/2}d^4x\ , \tag{3.1}$$

where $|1_{\text{inv}}\rangle = a_j^\dagger|0_{\text{inv}}\rangle$, etc., and φ^+ is the positive-frequency component of the field. The transition probability summed over the final three-particle states is

$$\begin{aligned}P_{1\to 3} &= \sum_{3_{\text{inv}}} |\langle 3_{\text{inv}}|S_1|1_{\text{inv}}\rangle|^2 \\ &= 16\lambda^2\int\int\sum_{3_{\text{inv}}}\{\langle 0_{\text{inv}}|[\varphi^-(x)]^3|3_{\text{inv}}\rangle\langle 3_{\text{inv}}|[\varphi^+(x')]^3|0_{\text{inv}}\rangle\} f_{j,p}^+(x)f_{j,p}^-(x')(-g)^{1/2}d^4x(-g')^{1/2}d^4x' \\ &= 96\lambda^2\int\int[-i\Delta^-(x,x')]^3 f_{j,p}^+(x)f_{j,p}^-(x')(-g)^{1/2}d^4x(-g')^{1/2}d^4x'\ .\end{aligned} \tag{3.2}$$

In order to proceed we will use the following spectral representation:

$$[-i\Delta^-(x,x')]^n = \int_0^\infty \sigma_n(p^2)[-i\Delta^-(x,x';p)]dp^2\ . \tag{3.3}$$

The derivative of $\sigma_n(p^2)$ for the Green's function (2.2) involves the generalized Mehler-Fock integral transform[20,21] and is omitted due to its length. The answer for odd n is

$$\sigma_{2m+1}(p^2)=(\tfrac{1}{4}+p^2)^{2m+1}2^{-1}\sinh(\pi p)[\cosh(\pi p)]^{-2m-1}(\Lambda/24\pi)^{2m}\int_1^\infty [P_{-1/2+ip}{}^{-1}(z)]^{2m+2}(z^2-1)^{-m}dz \tag{3.4}$$

with

$$P^{-1}{}_{-1/2+ip}(z)=(z^2-1)^{-1/2}\int_1^z P_{-1/2+ip}(z)dz \ .$$

Equation (3.3) can be used to write (3.2) as

$$P_{1\to 3}=97\lambda^2\int_0^\infty dp'^2\sigma_3(p'^2)\int\int\sum_k [f^-_{k,p'}(x)f^+_{k,p'}(x')]f^+_{j,p}(x)f^-_{j,p}(x')(-g)^{1/2}d^4x(-g')^{1/2}d^4x' \ . \tag{3.5}$$

The eigenfunctions f^+, f^- are orthonormal in both j and p when p is real.[11] This produces the expression for $P_{1\to 3}$:

$$P_{1\to 3}=\int_0^\infty dp'^2\sigma_3(p'^2)[\delta(p^2-p'^2)]^2 \ .$$

The transition probability for unit proper time is therefore

$$dP_{1\to 3}/dt=48\lambda^2\pi^{-1}\sigma_3(p^2) \ , \tag{3.6}$$

where $\sigma_3(p^2)$ is given by Eq. (3.4).

Since $P_{-1/2+ip}(y)$ is real for all $y\in[1,\infty)$, σ_3 is real and non-negative. If we take the limit as $\Lambda\to 0$, then $p\to\pm i/2$ and σ_3 goes to zero. This simply tells us that no pairs will be created in the flat-space limit. We can also follow Nachtmann[18] and look at the limit in which Λ is large compared to the mass M of scalar particles and $(\Lambda M)/3 \gg \frac{1}{4}$. Then for the case of a single pair being created we have

$$dP_{1\to 3}/dt \approx \lambda^2\exp[-2\pi M(3/\Lambda)^{1/2}] \ . \tag{3.7}$$

This is a Boltzmann distribution at the temperature $T_{\text{de Sitter}}$. This result reproduces the heuristic picture one would expect on the basis of the results of Sec. II.

The most important feature of this process is that it leads to runaway particle production. This is immediately apparent from Fig. 1, since each of the three particles left will itself undergo the process leading to nine particles in the next generation. As long as the probability per unit proper time given by Eqs. (3.6) or (3.7) is nonzero, we can expect the total rate of particle production in the space-time to be proportional to the total number of particles present. Physically this means that de Sitter space is unstable in the presence of a particle and the rate of particle production in the space-time will grow exponentially with time. A more detailed interpretation of this process will be considered in Secs. IV and V.

Mention should be made, however, of the time-reversed process to Fig. 1 which could be called particle destruction. Although the transition amplitudes for the two processes are the same, the actual rates are quite different because of the phase-space factors involved with getting the three particles to come together.

The creation process will obviously be more vigorous than destruction in the early stages when the particle density is negligible. Strangely enough, this will also be the case at *all* times since the destruction rate depends upon the *density* of particles whereas the creation rate depends upon the total *number* of particles. The density, however, is subject to gravitational red-shifting with the expansion of the Universe so it is not possible to form an equilibrium state where the two rates will be equal. The creation rate will always dominate the destruction rate.

The physical reason for the different dependence of the two rates is that the apparent "heat bath" which drives the creation is actually a bath of vacuum fluctuations which does not red-shift.[22,23] The phase factors for destruction on the other hand depend on "real" particles (i.e., those defined with respect to the $|0_{\text{inv}}\rangle$ vacuum basis) which *do* red-shift.

IV. THE VACUUM STATE

The particle-creation calculations in Secs. II and III depend upon a particular choice for the initial "vacuum" state of the field. Since this is perhaps the most basic and crucial assumption in this paper, it bears further scrutiny. There are several well-known ways to approach the construction of a vacuum state in flat space-time, and it is equally well known that all of these methods fail when they are applied to curved space-times. An extensive, and often contradictory, literature has sprouted concerning these issues (see Ref. 15 for a review). We shall only present the briefest summary here, as a more complete treatment of particle and vacuum definition will appear elsewhere.[30]

The essential physics behind these difficulties is that any attempt to define particles or vacuum ultimately depends on a decomposition of the field into positive- and negative-frequency modes, and unfortunately the notion of positive and negative frequency is *not* covariant.[14,15] Mathematically this is reflected in the fact that decompositions based on different coordinate systems for the *same* space-time manifold are in general related by *frequency-mixing* Bogoliubov transformations. A "vacuum" for one coordinate system, which is a state defined to have no positive-frequency modes, can appear as a many-particle state possessing positive-frequency modes in a second coordinate system. Thus a given "vacuum" state will be a no-particle state only with respect to a particular set of coordinate systems or observers.

Sciama and his co-workers[22,23] have shown that such behavior is neither paradoxical nor pathological, but instead is an intrinsic property of relativistic quantum fields in flat or curved space-time. The new "particles" found in the second coordinate system of the above example are simply the physical manifestation of vacuum fluctuations in the "vacuum" of the first coordinate system. Although these vacuum fluctuation "particles" can appear mathematically in a Fock basis, or even physically act to excite appropriately moving model "atoms," they do not possess all of the properties of ordinary Minkowski particles. This is amply demonstrated by the "particles" observed by the model "atoms" of Sec. II, or by constantly accelerated "Rindler" observers in Minkowski space,[27] since in both cases the apparent "particles" do *not* Doppler shift.[22,23]

The main lesson to be learned from the Rindler-space field-theory calculations is that the two main methods of defining particles—operationally using model "particle" detectors or "atoms"[5,27] or mathematically via canonical quantization[14,15,26]—are *insufficient* conditions for defining the properties of ordinary particles *even in Minkowski space.* In order to obtain a suitable generalization of the usual Minkowski-space concept of particle one must evidently make the further requirement that particles transform properly under infinitesimal Lorentz transformations.[30]

From the discussion above it is clear that the question of which "vacuum" to use for de Sitter space is not simply answered by finding a no-"particle" state for a given coordinate system. The most physically reasonable and useful choice would be a state which is closely analogous to the Minkowski vacuum, and which reduces to it in the limit $\Lambda \to 0$. This means that the candidate "vacuum" state should satisfy the following requirements:

(a) It be invariant under the de Sitter group in accord with the invariance of the Minkowski vacuum under the Poincaré group.

(b) Particles defined with respect to the Fock basis for which the "vacuum" is a no-particle state should be "real" in the sense that they have the infinitesimal transformation properties of ordinary Minkowski particles.

These two conditions are satisfied by the state labeled $|0_{\rm inv}\rangle$ in Sec. II. This state has been defined from different points of view by several authors[7–12,16] and applied to various calculations by many authors.[5,18] The state $|0_{\rm inv}\rangle$ is a member of the one-parameter family of states which are invariant under the de Sitter group.[7,8,14] It may be specified up to nonmixing Bogoliubov transformation by using any one of its other properties such as follows:

(1) Covariance and analyticity.[16]

(2) The wave functions $\langle 0_{\rm inv} | \varphi | 1_{\rm inv}\rangle$ move along timelike ($M>0$) or null ($M=0$) geodesics in the geometric optics limit.[8]

(3) The positive- and negative-frequency modes associated with $|0_{\rm inv}\rangle$ reduce to $\exp(\pm i\omega t)$ in the flat-space limit $\Lambda \to 0$.[9,14]

(4) It is the natural vacuum or no-particle state for Gaussian normal coordinates on de Sitter space.[14]

(5) It is the natural vacuum for Euclidean field theory done on the Euclidean version of de Sitter space[3,5,12] which is the four-sphere S^4.

In view of those characteristics and the arguments above, $|0_{\rm inv}\rangle$ appears to be the most reasonable initial state for the calculations in Secs. II and III. We should mention that it is possible to define a state $|0_{\rm static}\rangle$ on de Sitter space such that a particular observer carrying a proper "atom" will not experience excitations in the "atom" so the mechanism of Sec. II fails to occur.[23] This case requires, however, that one abandon the viewpoint espoused above, and instead try to make sense of "observer-dependent particles" which carry no energy (since they do not Doppler shift), and a stress-energy tensor which will diverge to negative infinity at any point in de Sitter space for some observer.[19,23]

V. CONCLUSION

The basic result of this paper is to exhibit a runaway-particle-production mechanism for interacting field theories in de Sitter space using the specific example of scalar $\lambda\varphi^4$ theory. The general arguments about the periodic nature of the propagator mentioned previously strongly suggest that this sort of behavior is generic to a wide range of theories. Investigation of black-hole—de Sitter spaces[5] suggests futher that the process does not crucially depend on the space-time being exactly de Sitter in form. The situation for a "decaying de Sitter" space-time with a time-varying scalar curvature and effective cosmological constant, may be guessed at by the closely similar case of "atoms" in Minkowski space which are given nonconstant accelerations. The result is that the spectrum of excitations loses its exact thermal form, but particles continue to be created.[24,31]

It is interesting to compare the de Sitter space particle production with the somewhat less esoteric situation of particle production in a constant electric field. A charged particle falling in a constant electric field has many formal and physical similarities to a freely falling particle in a de Sitter space:

(a) In both cases there is an *apparent* "event horizon" associated with the classical world lines of the particles.

(b) The classical world line of the charged particle has constant acceleration. Thus a QED treatment of this case is a fully field-theoretic version of the accelerated "atom" in the same way that the $\lambda\varphi^4$ example is to the de Sitter "atom."

These similarities lead one to expect that runaway particle production should also occur for a charged particle in a constant electric field. Recent calculations confirm that this is indeed the case.[25] The QED analog of the process of Sec. III is depicted in Fig. 2. Here Schwinger's exact electron Green's function in a constant electric field (represented by the double lines) takes the place of the de Sitter Green's function (2.2). The formal relation between the two has been explored in Ref. 16.

The simple photon emission from the electron can be shown to have a thermal component at a temperature compatible with that measured by the model "atom"

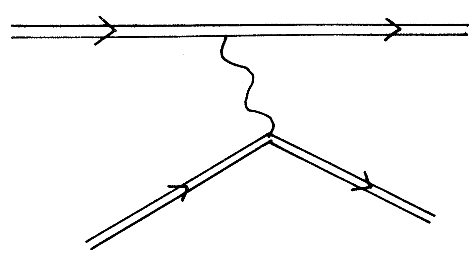

FIG. 2. Runaway QED graph in constant E field.

above. Since any photon in a constant electric field is unstable to pair production, one will get a runaway production of electrons and positrons via the graph of Fig. 2. This example shows that "real" particles are produced as a result of interactions with vacuum fluctuations, lending credence to the interpretations of the de Sitter case in previous sections of the paper. A calculation of the QED graph in Fig. 2 for de Sitter space will be the subject of a future publication.

The runaway nature of the de Sitter—particle-production process argues strongly for its importance in any calculation of the dynamics and evolution of the early universe with a cosmological constant. A study of the back reaction of the runaway particle production on the space-time metric is presented elsewhere.[4]

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