Heavy quarks in the jet calculus

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In this paper we explore a method for treating heavy quarks such as c and b quarks within the jet calculus. These quarks are differentiated from the more common u, d, and s quarks by the requirement that the gluons never branch into heavy-quark pairs during the jet development. We compute and discuss the charmed-quark "propagators"; the x distribution of colorless clusters containing a charmed quark, a noncharmed antiquark, and gluons; and the mass distribution of the parent partons giving rise to these colorless clusters.

I. INTRODUCTION

Over the last few years, there has been a great deal of activity in the development of the "jet calculus"—i.e., methods for summing certain classes of QCD graphs in hopes of obtaining some clues to the observed properties of quark and gluon jets. Among the interesting results of this work is the fact that one can keep track of color-singlet "clusters" within the jet composed of a quark, an antiquark, and gluons,[1] and that these hadron precursors have a mass distribution which peaks at values only a few times greater than the masses of the common hadrons.[2]

Recently these jet-calculus expressions have been rewritten[3] to keep track of the momentum of the gluons in the colorless clusters as well as that of the quarks. This means that the momentum of the colorless clusters can be computed explicitly; this is interesting if one wishes to consider their use in the phenomenology of hadrons in jets. The equations of Ref. 3 have been solved by Jones and Migneron[4] to obtain the longitudinal-momentum distribution of the colorless clusters in quark and gluon jets for the case where all the quarks in the jet are light.

This paper is an extension of these calculations to the case where one of the quarks is very heavy (for example, the charmed quark c). It is not reasonable to expect gluons to branch into $c\bar{c}$ pairs in jet development at current Q^2. This means that the equations obeyed by the propagators are different and that there are some different properties of the resulting color-singlet distributions containing the charmed quarks.

In Sec. II, we extend the equations of Ref. 3 to the charmed-quark propagators and compute these propagators as functions of x and Q^2.

Section III is devoted to the computation of the spectrum of parent partons to these colorless clusters, and a comparison of the charmed case with the production of "favored" and "nonfavored" colorless clusters in jet development containing only light quarks. We find that for clusters produced at x values greater than 0.2, the shapes of the mass spectra for the charmed and noncharmed cases are similar; in both cases clusters produced at $x=0.9$ tend to have higher average masses than those produced at $x=0.2$.

However, when the momentum of the cluster is integrated over all x, colorless clusters involving charmed quarks (or indeed the original quark in any quark jet) have a more sharply peaked mass spectrum than those involving two nonleading quarks. This may explain differences in the literature between previous analytic calculations (which involved only singlets including the "first" quark) and Monte Carlo studies.

Section IV contains a preliminary discussion of the problems associated with applications of this work to the computation of fragmentation functions of charmed quarks into charmed hadrons. Section V is a summary and conclusions.

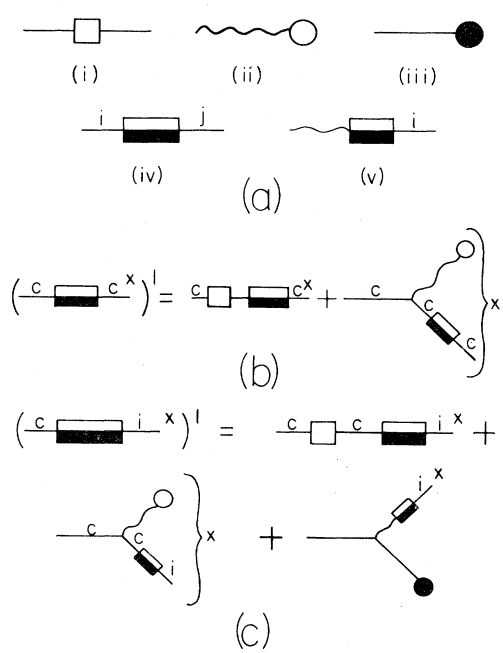

FIG. 1. (a) Symbols used in equations: (i) The quark "potential"

$$V_q=-\int_\epsilon^{1-\epsilon} dx\frac{\alpha(z(1-z)k^2)}{2\pi}[\tfrac{1}{2}\hat{P}_q^{gq}(z)+\tfrac{1}{2}\hat{P}_q^{qg}(z)]\ ,$$

(ii) σ, the probability that gluons go only to gluons, (iii) the probability that a quark goes into anything$=1$, (iv) H_i^j, propagator for finding a quark in a quark jet, and (v) H_g^i, propagator for finding a quark in a gluon jet. (b) Equation for the charmed-quark favored propagator H_c^c. (c) Equation for the charmed-quark unfavored propagator H_c^i.

II. PROPAGATOR EQUATIONS

Because this paper supplements Refs. 3 and 4, we refer the reader to those works for detailed discussions of our approach. Briefly, the point is to write integrodifferential equations for planar QCD graphs (in leading-logarithm approximation) representing the probability that a particular quark will be the first quark emitted in the evolution. The two basic propagators for the charmed case are H_c^c and H_c^i where i is a "light" quark; these are represented in Fig. 1 by the boxes with one side open and one side filled. These propagators H differ from the Γ's of Bassetto, Ciafaloni, and Marchesini[1] (BCM) in that in the case of the H's the argument x is the sum of the momentum of the first quark and all gluons emitted prior to this quark, whereas for the Γ's it represents only the momentum of the first quark.

In Figs. 1(b) and 1(c) we depict the equations for the two propagators of interest,

$$k^2\frac{dH_c^c(k^2,x)}{dk^2}=V_q(k^2)H_c^c(k^2,x)+\int_0^x\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\sigma(\lambda(z)k^2)H_c^c\left[\lambda(1-z)k^2,\frac{x-z}{1-z}\right], \tag{1a}$$

$$k^2\frac{d}{dk^2}H_c^i(k^2,x)=V_q(k^2)H_c^i(k^2,x)+\int_0^x\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\sigma(\lambda(z)k^2)H_c^i\left[\lambda(1-z)k^2,\frac{x-z}{1-z}\right]$$
$$+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)H_g^i\left[\lambda(z)k^2,\frac{x}{z}\right]. \tag{1b}$$

There is no H_g^c because of our requirement that no heavy-quark pairs be created in the evolution. Hence each c-quark jet contains one and only one c quark at any Q^2 during its evolution.

Equation (1a) has the interesting feature that it can be solved by a δ function at $x=1$. This is a natural solution, since at $Q^2=m_c{}^2$ the propagator must be a δ function. If we write $H_c^c=H(k^2)\delta(1-x)$ the coefficient $H(k^2)$ obeys the equation

$$k^2\frac{d}{dk^2}H(k^2)=-H(k^2)\left\{C_F\int_{Q_0{}^2}^{k^2}\frac{[1-\sigma(k'^2)]\alpha(k'^2)dk'^2}{\pi k'^2}-\frac{3}{4\pi}C_F\alpha(k^2)[1-\sigma(k^2)]\right\}, \tag{2}$$

recognizable as the equation given by BCM for the "flavor-carrying" parton (which indeed the charmed quark is). The difference between our use of H_c^c and the BCM use of ϕ is that they are dealing with the *probability* to find the first quark with gluons only, whereas we are going to explicitly use the x distribution for this case. Of course the probabilities are the same. Be redefining x to include the gluon momentum, we have made the x distribution trivial.

In Fig. 2 we show the k^2 dependence of the coefficient $H(k^2)$ of the δ function, and the behavior in x and k^2 of $H_c^i(x,k^2)$ for the case of three light flavors i. Because we have used only eight moments to reconstruct the H_c^i function, the behavior near $x=0$ is not shown. We have expanded the calculation to 16 moments for a few runs to examine the shape more carefully. The functions H_c^i have a large spike at $x=0$ which (at large k^2) contributes the bulk of the sum rule (see Ref. 3):

$$\int dx\left[\sum_i H_c^i(Q^2,x)+H_c^c(Q^2,x)\right]=1\,.$$

III. PROPERTIES OF COLORLESS CLUSTERS

The equation [see Fig. 3(a)]

$$\frac{1}{\sigma_c}\frac{d\sigma_c}{dx}=\int_{Q_D{}^2}^{Q^2}\frac{dk^2}{k^2}\int_0^1 dx_1\int_0^1 dx_2\,\delta(x_1+x_2-x)$$
$$\times\int_{x_1+x_2}^1\frac{d\xi}{\xi^2}D_c^c(Q^2,k^2;\xi)\int_{x_1/\xi}^{1-x_2/\xi}\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{qg}(z)$$
$$\times H_c^c\left[\lambda(z)k^2,\frac{x_1}{\xi z}\right]H_g^i\left[\lambda(1-z)k^2,\frac{x_2}{\xi(1-z)}\right] \tag{3}$$

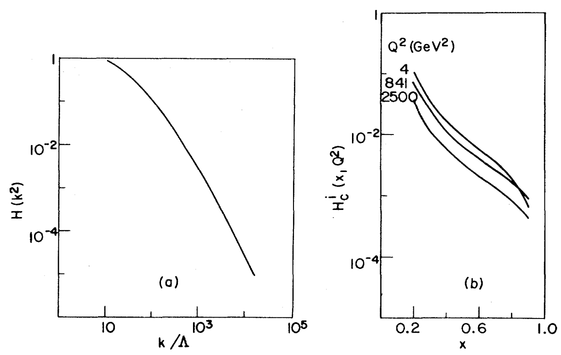

FIG. 2. (a) Behavior of the coefficient of $\delta(1-x)$ in H_c^c. (b) Behavior of H_c^i.

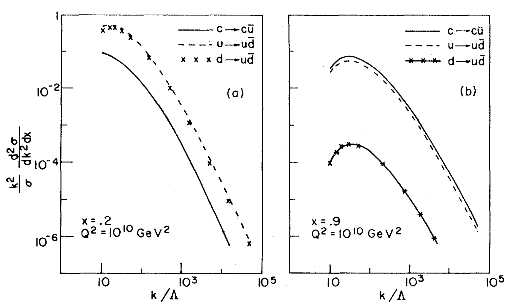

FIG. 4. Parent-parton mass distributions for colorless clusters at particular x values. Three cases are shown: c-$c\bar{u}$, u-$u\bar{d}$, and d-$u\bar{d}$. $Q^2=10^{10}$ GeV^2. (a) $x=0.2$ and (b) $x=0.9$.

can be used to compute the distribution of colorless clusters involving the charmed quark. Here the moments of D_{cc} are given by[5]

$$D_{cc}(Q^2,k^2,m)=\exp[A_q^{qg}(m)(Y-y)]$$

with $Y-y=(1/2\pi b)\ln[\ln(Q^2/\Lambda^2)/\ln(k^2/\Lambda^2)]$.

First we compute the x distribution of all charmed colorless clusters within the charmed-quark jet; this is shown in Fig. 3(b) as a function of Q^2 and x. In preparing this graph, we followed these steps: The standard equations for the light quarks (see Refs. 3 and 4) were integrated from $Q_0{}^2=0.062$ GeV^2 (where $\alpha_s=\pi$) to $Q_c{}^2=(1.5)^2$ GeV^2. Then the equations for H_c^c and H_c^i were added and the resulting set integrated from the charmed-quark mass to the D-meson mass (taken here to be 2 GeV). Finally the equations for the moments of the colorless-cluster distribution were added and the answers integrated from (2 GeV)^2 to Q^2 for the jet.

The point of this procedure is to enforce the heavy mass of the charmed quark in a natural way, by making sure that the boundary conditions for the charmed-quark propagators are applied at the charmed-quark mass, and that no colorless clusters are produced with masses less than the mass of the lightest physical charmed hadron.

As a check, we note that the zeroth moment of Eq. (3) can be calculated separately:

$$\left.\frac{k^2}{\sigma_c}\frac{d\sigma_c}{dk^2}\right|_{c\bar{u}}=\frac{-k^2}{N_f}\frac{d}{dk^2}\int_0^1 H_c^c(k^2,x)dx$$
$$=\frac{-k^2}{N_f}\frac{d}{dk^2}H(k^2) \qquad (4)$$

with $H(k^2)$ computed from Eq. (2). (This is the equivalent of the equation for the "first" singlet given in Ref. 2.) In Fig. 3(c), we show this zeroth moment superimposed on the x distribution. It is a reasonable approximation to the actual area under the curve obtained from the formulas of Yndurain[6] using eight moments $n=1,\ldots,8$.

Next we proceed to examine the mass distribution of the parent partons of colorless clusters. By use of Eq. (3), we can compute the masses of the parent partons for colorless clusters seen at particular x values. These are shown in Fig. 4, and compared with the identical distributions calculated for two different types of light-quark colorless clusters—the so-called favored ones u-$u\bar{d}$ and the unfavored ones d-$u\bar{d}$. The point of taking these different cases is that the favored propagators also show a definite peaking at large x associated with the "leading-quark effect," whereas the unfavored clusers are concentrated at small x.

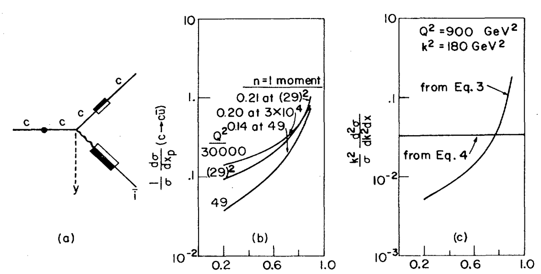

FIG. 3. (a) Depiction of Eq. 3. The bifurcation point

$$y=\frac{1}{2\pi b}\ln[\ln(k^2/\Lambda^2)/\ln(Q_0{}^2/\Lambda^2)]$$

is allowed to vary between $m_D{}^2$ and Q^2. (b) Behavior of $c\bar{u}$ colorless clusters produced in a charmed-quark jet. (c) Check of the procedure: the x-dependent colorless-cluster distribution (computed from the moments $n=1$ through 8) is compared with the separately computed zeroth moment (which should be the integral of the x dependence). This calculation was done for $Q^2=900$ GeV^2 and for a parent-parton mass squared of 180 GeV^2.

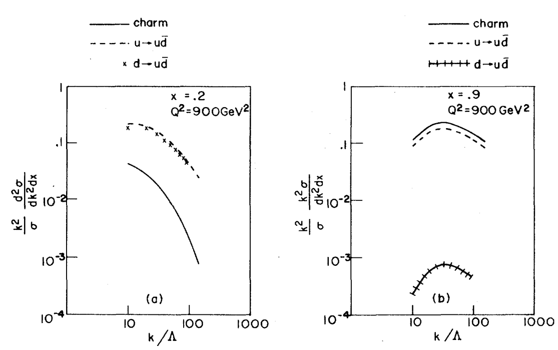

FIG. 5. Same as Fig. 4 except at $Q^2=900$ GeV^2.

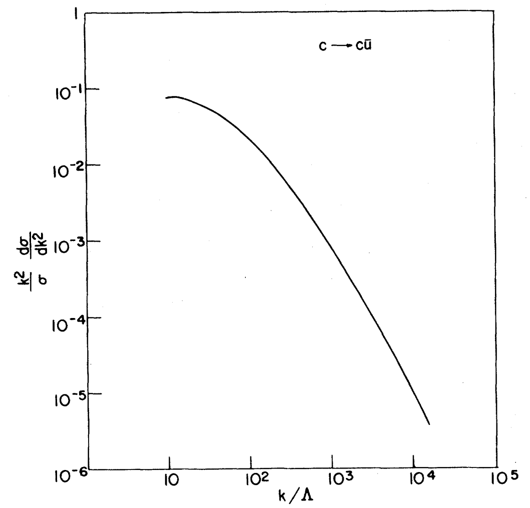

FIG. 6. The parent-parton mass distribution for the charmed case, integrated over all x. This is independent of Q^2.

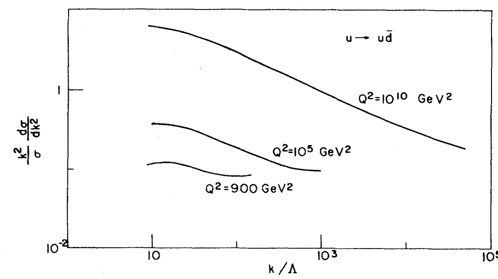

FIG. 7. Parent-parton mass distribution for $u\bar{d}$-gluon colorless clusters in the u-quark jet, at various Q^2. The multiplicity equations of Amati *et al.* (Ref. 1) were used. If the improved multiplicities of Ref. 7 are used, we expect flatter distributions.

To be assured of asymptoticity, we have taken $Q^2=10^{10}$ GeV^2. Notice that although the magnitudes of the curves for the different cases are different, the shapes in k^2 (mass of the parent parton) are identical at a given x value for the charmed and noncharmed cases. In both cases, the average mass of the colorless clusters seen at large x is larger than that of the colorless clusters seen at small x. We conclude that the large-x colorless clusters will have to decay into smaller clusters before turning into hadrons in both the charmed and the noncharmed cases.

To see whether the distributions would change with Q^2, we have also calculated the $k^2d^2\sigma/dk^2dx$ distributions at $Q^2=900$ GeV^2, a currently accessible energy. These are shown in Fig. 5. We see that the general features found at asymptotic Q^2 are clearly present here, and that the peaks of the distributions occur at about the same values of parent-parton masses for the two cases.

For the charmed-quark case, the parent-parton distribution integrated over all x can also be calculated by using Eq. (4). Because the zeroth moment of D_{cc} is 1, this distribution has no Q^2 dependence. The result is shown in Fig. 6. We see it has the same general shape as the distributions at particular x values.

The corresponding distributions (to Fig. 6) for the noncharmed cases depicted in Figs. 4 and 5 cannot be computed without knowing the multiplicities within the jets. These have been studied intensively[7]; the proper expressions can be obtained only by going to higher order. Because these multiplicities do depend on Q^2, the parent-parton curves for the noncharmed case will have a Q^2 dependence. We expect them to be somewhat different from Fig. 6.

For a first look at these distributions, we have used the multiplicity equations of Amati *et al.*[1] to compute the parent-parton mass spectrum for the case of favored colorless clusters u-$u\bar{d}$. The results are given in Fig. 7. We see that the spectrum obtained is not nearly as damped as is the calculated spectrum in Fig. 6. In other words, when *all* the color singlets are included, the average mass is apparently larger than that obtained from the first singlet.

The improved multiplicity distributions of Ref. 7 have the feature that they grow less rapidly with Q^2 than those calculated from the Amati equations. Since the appropriate multiplicity is $n(Q^2,k^2)$, we see that this will make the large-k^2 tail even flatter.

In order to show that the mass distribution of the average singlet could well be larger than that of the first singlet, regardless of the behavior of the multiplicities, consider the evaluation of $(k^2/\sigma)(d\sigma/dk^2)$ at $k^2=Q^2$. At this point the expression for the $q\bar{q}$-gluon color singlets, Eq. (5) of Ref. 3, takes the form (for a quark jet)

$$\frac{1}{\sigma_\alpha}\frac{k^2 d^2\sigma_\alpha}{dk^2 dx}\bigg|_{k^2=Q^2} = \int dx_1 \int dx_2\, \delta(x-x_1-x_2) \times \int_{x_1+x_2}^{1} \frac{d\xi}{\xi^2}\delta_{\alpha B}\delta(1-\xi)\int_{x_1/\xi}^{1-x_2/\xi}\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_{\beta}^{\gamma\delta}(z)H_{\gamma}^{q_l}\left[\lambda(z)k^2,\frac{x_1}{\xi z}\right] \times H_{\delta}^{\bar{q}_m}\left[\lambda(1-z)k^2,\frac{x_2}{\xi(1-z)}\right], \quad (5)$$

so when we integrate over all x, we find in general

$$\frac{1}{\sigma}\frac{k^2 d\sigma}{dk^2}\bigg|_{k^2=Q^2} = \frac{\tilde{H}_{\alpha}^{q_l}(Q^2;0)}{N_f}\left[\frac{-3}{4\pi}C_F\alpha(Q^2)[1-\sigma(Q^2)]+\frac{C_F}{\pi}\int dz\frac{\alpha((1-z)Q^2)}{1-z}[1-\sigma(\lambda(1-z)k^2)]\right],$$

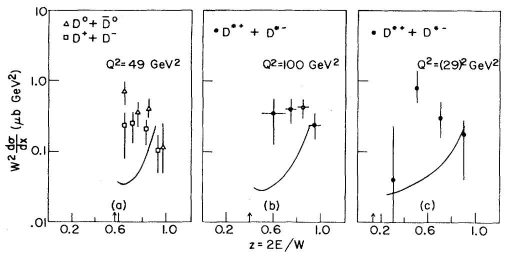

FIG. 8. Charmed-colorless-cluster distributions [the same as shown in Fig. 3(b)] transformed to $x_E=2E/Q$ and compared with currently available data. (a) $Q^2=49$ GeV^2 (data is from Ref. 9), (b) $Q^2=100$ GeV^2 (data is from Ref. 10), and (c) $Q^2=(29)^2$ GeV^2 (data is from Ref. 11).

where

$$\widetilde{H}(Q^2,0)=\int_0^1 H(Q^2,x)dx \ .$$

The factor which multiplies $\widetilde{H}\,^j_i(Q^2,0)$ rises slowly like $\ln[\ln(Q^2/\Lambda^2)]$. Clearly, therefore, the behavior of the "last point" in the mass spectrum (the point at $k^2=Q^2$), will behave roughly like the zeroth moment of the appropriate H^j_i. For the first singlet (as used by Ref. 2) or the charmed singlets we are discussing here, this is just $H(k^2)$, which [as can be seen from Fig. 2(a)], falls rapidly with Q^2.

For the ordinary case, however, $\widetilde{H}\,^i_j(Q^2,0)$ approaches $1/N_f$ as Q^2 approaches infinity. Hence the last point on the mass spectrum of the colorless clusters does not drop off as one might hope; in fact, $(k^2/\sigma)(d\sigma/dk^2)$ at $k^2=Q^2$ rises slowly as Q^2 rises (this value is 0.078 at $Q^2=10$ GeV^2, rising to 0.139 at $Q^2=10^9$ GeV^2). This is presumably an indication of a wide spectrum for the colorless clusters in this case.

In Ref. 8 a Monte Carlo calculation was done to study all colorless clusters; they find a rather broad mass spectrum (for the clusters themselves). In Ref. 2 the distribution for the parent parton of the first color singlet was calculated analytically and shown to drop off rapidly. These authors (Bertolini and Marchesini) state that the other color singlets must also drop off rapidly because they are related to the first one by energy-momentum sum rules.

As we have seen here, it is the *probability* sum rules which force the average singlets to be fatter than the first ones. At large Q^2 it is the spikes at $x=0$ in the H distributions which saturate the probability sum rules

$$\widetilde{H}\,^i_i(Q^2,0)+\sum_{j\neq i}\widetilde{H}\,^i_j(Q^2,0)=1$$

and

$$\widetilde{H}\,^c_c(Q^2,0)+\sum_{i}\widetilde{H}\,^i_c(Q^2,0)=1 \ .$$

These spikes are not present in the H function for the "incident" or first quark. Hence the zeroth moment of the distribution of the first singlet falls off, and the others do not, according to Eq. (6).

Of course, the parent-parton spectrum is not the same as the spectrum of the actual colorless clusters. The cluster mass may well be considerably smaller. An actual analytic calculation of the cluster masses must await the solution of the transverse-momentum equations of Ref. 3.

IV. RELATION TO DATA

In Fig. 8 we show the $c\bar{u}$ (or $c\bar{d}$) colorless-cluster distributions predicted by Eq. (3) plotted with currently available data. The calculated distributions peak much more strongly at large x_E than do the data. Of course, our study above of the parent-parton masses, which suggests that the colorless clusters at large x may have masses several times that of a D meson, indicates that we must first allow the colorless clusters to decay into lumps of the appropriate hadronic mass before making a serious comparison. This will produce particles at lower x, where the data peak, but the exact distribution awaits further work.

Because of Eq. (4), we can integrate the parent-parton mass distribution over all k^2 to obtain

$$\int_{m_D{}^2}^{Q^2}\frac{dk^2}{\sigma_c}\frac{d\sigma_c}{dk^2}\bigg|_{c\bar{u}}=\int_0^1 dx\,D_c^{c\bar{u}}(Q^2,x)\bigg|_{\text{perturbative}}$$

$$=\frac{1}{N_f}[H(m_D{}^2)-H(Q^2)] \ .$$

That is, the probability for producing charmed colorless clusters by the perturbative branching studied in the jet calculus rises from zero at $Q_0{}^2=4$ GeV^2 to reach

$$\frac{1}{N_f}H(m_D{}^2)<\frac{1}{N_f}$$

at very large Q^2.

We would expect probabilities for the fragmentation of the charmed quarks into all possible flavors of colorless clusters to add to 1 at asymptotic Q^2 (or to a number higher than 1 if one eventually allows the gluons to split into $c\bar{c}$ pairs). This suggests that there is room for an "intrinsic" fragmentation function at $Q^2=m_D{}^2$. It is not easy to guess the shape of this intrinsic fragmentation function without further work.

This intrinsic fragmentation function will evolve with Q^2 according to the relation

$$\frac{d}{dY}D_c^{c\bar{u}}(Q^2,m)=A_q^{qg}(m)D_c^{c\bar{u}}(Q^2,m) \ .$$

We see that its contribution to the probability will remain constant, although all higher moments will drop with Q^2. If the intrinsic fragmentation function is constrained by the requirement that it integrate to $[1-H(m_D{}^2)]/N_f=0.126/N_f$, its contribution at the values of Q^2 shown in Fig. 8 will be at least an order of magnitude smaller than the data regardless of the shape. Proper comparison with data thus rests heavily on the fragmentation of the $q\bar{q}$-gluon colorless clusters into hadrons.

V. SUMMARY AND CONCLUSIONS

The jet-calculus equations for heavy quarks can be written and solved straightforwardly. The colorless clusters carrying the heavy quark are exactly those called the first clusters by other authors. These charmed colorless clusters tend to carry most of the momentum of the charmed-quark jet.

The mass distributions of the parent partons producing colorless clusters with definite nonzero momentum have similar shapes for charmed and noncharmed clusters.

However, the mass distribution (for parent partons) obtained by integrating over all x is rather more peaked for the charmed than for the noncharmed case.

Because the mass of the parent-parton distribution tends to peak at 6—8 GeV, several times the mass of the lowest charmed hadron, we expect that the decays of the clusters into hadrons must be included before comparison with data can be meaningful. This is borne out by examining the data; the clusters are peaked at much larger x than is the data. Decay distributions are under study and will be reported elsewhere.

ACKNOWLEDGMENTS

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