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CPT and *CP* properties of Majorana particles, and the consequences

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Since a Majorana neutrino is its own antiparticle under *CPT*, rather than *C*, an analysis of the *CPT* and *CP* characteristics of a Majorana particle is performed. The *CPT* transformation properties of a Majorana particle of arbitrary spin are obtained in a very simple way. Implications of these properties for the electromagnetic matrix elements of Majorana particles of spin $\frac{1}{2}$ are derived. Finally, the question of when different Majorana neutrinos will make opposing contributions to neutrinoless double-β decay is answered.

Majorana particles are predicted both by grand unified theories, in which these particles are neutrinos, and by supersymmetric theories, in which they are photinos, gluinos, and other states. Until recently, a Majorana particle has been pictured as one which is its own antiparticle under charge conjugation *C*. However, a *physical* Majorana *neutrino*, dressed as it is by maximally *C*-violating weak interactions, cannot be an eigenstate of *C*.[1] Instead, it is an eigenstate of *CPT*, which presumably is not violated at all. It may also be an approximate eigenstate of *CP*. To explore the physics of this situation, an analysis of the *CPT* and *CP* properties of an arbitrary Majorana particle, and of the consequences of these properties, has been carried out. Here we report the main results; further discussion and details will be presented elsewhere.[2]

The effect of *CPT* ($\equiv\zeta$) on the state of any *CPT*-self-conjugate particle f of momentum $\vec{p}$, spin J, and $J_z=s$ is given by

$$\zeta|f(\vec{p},J,s)\rangle=\eta^s|f(\vec{p},J,-s)\rangle \quad . \tag{1}$$

Here, η^s is a phase factor, and we are allowing for the possibility that, for given J, it may depend on s. What can one say about this phase factor, and what are its consequences? To find out, let us go to the rest frame and define the operator

$$b\equiv e^{-i\pi J_y}\zeta \quad . \tag{2}$$

The effect of b on f is obviously

$$b|f(J,s)\rangle=\mu^s|f(J,s)\rangle \quad , \tag{3}$$

where μ^s is some new phase factor. Bearing in mind that ζ, hence b, is antiunitary, one can show trivially that $b^2=1$ when acting on the states $|f(J,s)\rangle$. Since *CPT* commutes with rotations, the definition of b then implies that

$$\zeta^2 e^{-2i\pi J_y}=1 \quad . \tag{4}$$

Now, independent of conventions, a rotation through 2π reproduces the original state times $(-1)^{2J}$. Thus, ζ^2, applied to any Majorana (i.e., *CPT*-self-conjugate) particle of spin J, does exactly the same thing[3]:

$$\zeta^2=(-1)^{2J} \quad . \tag{5}$$

From Eq. (1), and the antiunitarity of ζ,

$$\zeta^2|f(J,s)\rangle=(\eta^s)^*\eta^{-s}|f(J,s)\rangle \quad . \tag{6}$$

Thus, Eq. (5) implies that

$$\eta^{-s}=(-1)^{2J}\eta^s \quad . \tag{7}$$

Apart from this constraint, the individual phase factors η^s are arbitrary, since the states $|f(J,s)\rangle$ can always be redefined according to

$$|f(J,s)\rangle\rightarrow|f(J,s)\rangle'=e^{i\phi_s}|f(J,s)\rangle \quad ,$$

with ϕ_s an arbitrary phase. Under this redefinition

$$\eta^s=\langle f(J,-s)|\zeta|f(J,s)\rangle\rightarrow(\eta^s)'=e^{-i(\phi_{-s}+\phi_s)}\eta^s \quad ,$$

but Eq. (7) is still obeyed.[4]

With the *CPT* transformation properties of the Majorana states known, one can now derive *CPT* constraints on matrix elements of the form $\langle f|Q|f\rangle$, where Q is any Hermitian operator whose *CPT* properties are also known. Consider, for example, the electromagnetic current $J_\mu^{\rm EM}$.[5] The photon field $A_\mu(x=0)$ is *CPT*-odd, so if the electromagnetic interaction $J_\mu^{\rm EM}A_\mu$ is to conserve *CPT*, $J_\mu^{\rm EM}(x=0)$ must be *CPT*-odd also. Thus,

$$\begin{aligned}\langle f(\vec{p}_f,J,s_f)|J_\mu^{\rm EM}(0)|f(\vec{p}_i,J,s_i)\rangle&=-\langle\zeta[f(\vec{p}_i,J,s_i)]|J_\mu^{\rm EM}(0)|\zeta[f(\vec{p}_f,J,s_f)]\rangle\\&=-(\eta^{s_i})^*\eta^{s_f}\langle f(\vec{p}_i,J,-s_i)|J_\mu^{\rm EM}(0)|f(\vec{p}_f,J,-s_f)\rangle \quad .\end{aligned}\tag{8}$$

For the case of greatest interest, $J=\frac{1}{2}$,[6] Lorentz invariance and current conservation imply that

$$\langle f(\vec{p}_f,\tfrac{1}{2},s_f)|J_\mu^{\rm EM}(0)|f(\vec{p}_i,\tfrac{1}{2},s_i)\rangle=i\bar{u}(\vec{p}_f,s_f)[F\gamma_\mu+G(q^2\gamma_\mu-\not{q}q_\mu)\gamma_5+M\sigma_{\mu\nu}q_\nu+Ei\sigma_{\mu\nu}q_\nu\gamma_5]u(\vec{p}_i,s_i) \quad . \tag{9}$$

Here u is a Dirac spinor, $q=p_f-p_i$, and F, G, M, and E are form factors depending on q^2. Writing the analogous expression for the right-hand side of Eq. (8) with $J=\frac{1}{2}$ and using the relation

$$u(\vec{p},-s)=(-1)^{s-1/2}\gamma_1\gamma_3\bar{u}^T(\vec{p},s)\ ,$$

we obtain

$$-(\eta^{s_i})^*\eta^{s_f}\langle f(\vec{p}_i,\tfrac{1}{2},-s_i)|J_\mu^{\rm EM}(0)|f(\vec{p}_f,\tfrac{1}{2},-s_f)\rangle$$
$$=(\eta^{s_i})^*\eta^{s_f}(-1)^{s_i-s_f}i\bar{u}(\vec{p}_f,s_f)[-F\gamma_\mu+G(q^2\gamma_\mu-\not{q}q_\mu)\gamma_5-M\sigma_{\mu\nu}q_\nu-Ei\sigma_{\mu\nu}q_\nu\gamma_5]u(\vec{p}_i,s_i)\ . \quad (10)$$

From Eq. (7) for $J=\frac{1}{2}$, we see that $(\eta^{s_i})^*\eta^{s_f}(-1)^{s_i-s_f}=1$. Thus, comparing Eq. (10) with Eq. (9), we conclude that

$$\langle f(\vec{p}_f,\tfrac{1}{2},s_f)|J_\mu^{\rm EM}(0)|f(\vec{p}_i,\tfrac{1}{2},s_i)\rangle=i\bar{u}(\vec{p}_f,s_f)[G(q^2)(q^2\gamma_\mu-\not{q}q_\mu)\gamma_5]u(\vec{p}_i,s_i)\ . \quad (11)$$

This is the most general expression for the matrix element of the electromagnetic current of any spin-$\frac{1}{2}$ Majorana particle, such as a gluino, photino, or neutrino.[7]

We turn now to CP, whose effect on the Majorana state $|f\rangle$ is given by

$$CP|f(\vec{p},J,s)\rangle=\omega|f(-\vec{p},J,s)\rangle\ , \quad (12)$$

where ω is a phase factor. Specializing to $J=\frac{1}{2}$, we recall that P acting alone on a Majorana fermion introduces a phase factor of $\pm i$.[8] If we define C to have the property $C^2=1$, the phase factor introduced by C acting alone is ± 1. Combining C and P, we conclude that the CP parity of a spin-$\frac{1}{2}$ Majorana particle may be $+i$ or $-i$.[9]

Now, what of the relative CPT and CP parities of *different* Majorana particles and their consequences? For a particle f with $J=\frac{1}{2}$, we may, following Eq. (7), define the CPT parity $\eta(f)$ by writing

$$\eta^s(f)\equiv\eta(f)(-1)^{s-1/2}\ . \quad (13)$$

For the electromagnetic current, we have in analogy with Eq. (9)

$$\langle f_2(\vec{p}_f,\tfrac{1}{2},s_f)|J_\mu^{\rm EM}(0)|f_1(\vec{p}_i,\tfrac{1}{2},s_i)\rangle$$
$$=i\bar{u}_2(\vec{p}_f,s_f)[F_{21}(q^2\gamma_\mu-\not{q}q_\mu)+G_{21}(q^2\gamma_\mu-\not{q}q_\mu)\gamma_5+M_{21}\sigma_{\mu\nu}q_\nu+E_{21}i\sigma_{\mu\nu}q_\nu\gamma_5]u_1(\vec{p}_i,s_i)\ . \quad (14)$$

In this expression, F_{21}, G_{21}, M_{21}, and E_{21} are transition form factors. Hermiticity of $J_\mu^{\rm EM}$ requires that $F_{21}=F_{12}^*$, and similarly for the other form factors. On the other hand, the CPT constraint analogous to Eq. (8) relates F_{21} to F_{12}, etc. From the Hermiticity and CPT relations combined, we find that

$$F_{12}^*=-\xi F_{12},\quad G_{12}^*=\xi G_{12}\ ,$$
$$M_{12}^*=-\xi M_{12},\quad E_{12}^*=-\xi E_{12}\ , \quad (15)$$

where $\xi=\eta^*(f_1)\eta(f_2)$.[10] The significant feature of this result is the fact that each form factor has a phase which does not depend on q^2. If this overall phase is extracted, the form factor becomes a real function suitable for representation by a conventional dispersion relation. However, the actual phase of each form factor, while correlated with the phases of other, nonelectromagnetic amplitudes in the problem,[9] does not have any absolute significance, for, instead of working with the state $|f_1\rangle$, we can always choose to work with $e^{i\phi}|f_1\rangle$. If the transition form factors are still defined by a relation of the form (14), they will then all change by a factor $e^{i\phi}$. Correspondingly, if

$$\zeta|f_1(\vec{p},J,s)\rangle=\eta^s(f_1)|f_1(\vec{p},J,-s)\rangle\ ,$$

then for

$$|f_1(\vec{p},J,s)\rangle'=e^{i\phi}|f_1(\vec{p},J,s)\rangle\ ,$$

we have

$$\zeta|f_1(\vec{p},J,s)\rangle'=e^{-2i\phi}\eta^s(f_1)|f_1(\vec{p},J,-s)\rangle'\ , \quad (16)$$

owing to the antiunitarity of ζ. That is, the states $|f_1\rangle'$ have a CPT parity $\eta(f_1')=e^{-2i\phi}\eta(f_1)$. Hence

$$\xi'=\eta^*(f_1')\eta(f_2)=e^{2i\phi}\xi\ ,$$

so that Eq. (15) implies correctly that the new (21) form factors are $\pm e^{i\phi}$ times the old ones. This discussion makes clear that the relative CPT parities of different Majorana particles do not have any absolute significance. They can be altered by redefinitions of the states.

By contrast, the relative CP parities of different Majorana particles *do* have absolute significance, independent of the definitions of the states. The implications of CP conservation for electromagnetic transitions among spin-$\frac{1}{2}$ Majorana fermions may already be found in the literature.[11] Namely, when CP is conserved, $F_{21}=M_{21}=0$ if f_1 and f_2 have the same CP parity (either $+i$ or $-i$), and $G_{21}=E_{21}=0$ if they have opposite CP parity. For radiative decay involving a real photon, $f_1\to f_2+\gamma$, only the M and E terms contribute [see Eq. (14)]. Thus, when CP is conserved, the radiation will be pure $E1$ (pure $M1$) when f_1 and f_2 have the same (opposite) CP parity.

There is a particularly interesting consequence of the relative phases of different Majorana particles in neutrinoless double-β decay $[(\beta\beta)_{0\nu}]$. As is well known, the observation of this process would be evidence that neutrinos are of Majorana character.[12] However, the absence of the process is not necessarily evidence that they are not. The reason is that in the core reaction which engenders $(\beta\beta)_{0\nu}$, $W^-+W^-\to e^-+e^-$ via Majorana neutrino exchange, the amplitudes contributed by the different neutrino mass eigenstates ν_m can oppose each other. At first sight this is surprising, since each of these amplitudes involves two identical vertices, and so is proportional to the square of a coupling constant. Nevertheless, opposition is possible, as was made clear by the model of Zee,[13] in which the neutrinos are Majorana particles, but the rate for $(\beta\beta)_{0\nu}$ vanishes completely. Conflicting statements have been made about the circumstances under which the contributions of different ν_m interfere destructively. Doi *et al.*[14] claim that this requires *CP* violation. Wolfenstein[15] states that it occurs when *CP* is conserved but the *CP* parities of the interfering neutrinos are opposite.[16] Halprin, Petcov, and Rosen,[17] in an interesting recent study of cancellation between heavy and light neutrinos, say that the sign of the interference depends on that in a "Majorana condition" which appears to express the *charge-conjugation* properties of the neutrino field. No one has considered the possibility that the character of the interference depends on the *CPT* properties of the neutrinos.

We have explored the nature of this interference assuming only that the charged-current interaction is of the form

$$\mathscr{H}(x)=g\sum_m W_\mu^- \bar{e}\gamma_\mu(1+\gamma_5)U_{em}\nu_m+\text{H.c.} \quad . \tag{17}$$

Here, g is the (real) overall coupling strength, and U is the lepton mixing matrix.[18] It turns out that when *CP* is conserved one can find the conditions for constructive or destructive interference in a way which is plainly independent of any details of formalism, such as the phases of the U_{em}, the definitions of states, phase factors which may occur in the plane-wave expansions of fields, etc.

When *CP* is violated, the contributions of different neutrinos can have *arbitrary* phases relative to one another. The reason is that *CPT* by itself does not constrain the phases of the U_{em}.[19] Since the contribution of ν_m involves two identical vertices, each proportional to U_{em}, the amplitude will be proportional to $U_{em}{}^2$, and can have any phase at all relative to that of another neutrino.

When *CP* is conserved, it is convenient to consider the core process in the cross channel $e^+ + W^- \rightarrow e^- + W^+$. To find out how this reaction depends on the quantum numbers of its contributing neutrinos, we express its S-matrix element,

$$S_{fi}=-\int d^4x_1\int_{t_2<t_1} d^4x_2\langle e^-W^+|\mathscr{H}(x_1)\mathscr{H}(x_2)|e^+W^-\rangle \quad , \tag{18}$$

explicitly in terms of the intermediate neutrino states. There are four ways in which the annihilation and creation of the external particles can be divided between $\mathscr{H}(x_1)$ and $\mathscr{H}(x_2)$, so that in Eq. (18)

$$\begin{aligned}\langle e^-W^+|\mathscr{H}(x_1)\mathscr{H}(x_2)|e^+W^-\rangle = \sum_{\substack{m\\ \vec{p},\lambda}}[\,&\langle e^-W^+|\mathscr{H}(x_1)|\nu_m(\vec{p},\lambda)\rangle\langle\nu_m(\vec{p},\lambda)|\mathscr{H}(x_2)|e^+W^-\rangle\\ &+\langle 0|\mathscr{H}(x_1)|e^+W^-\nu_m(\vec{p},\lambda)\rangle\langle e^-W^+\nu_m(\vec{p},\lambda)|\mathscr{H}(x_2)|0\rangle\\ &+\langle e^-|\mathscr{H}(x_1)|W^-\nu_m(\vec{p},\lambda)\rangle\langle W^+\nu_m(\vec{p},\lambda)|\mathscr{H}(x_2)|e^+\rangle\\ &+\langle W^+|\mathscr{H}(x_1)|e^+\nu_m(\vec{p},\lambda)\rangle\langle e^-\nu_m(\vec{p},\lambda)|\mathscr{H}(x_2)|W^-\rangle\,] \quad .\end{aligned} \tag{19}$$

Here λ is the helicity of the neutrino. Let us now apply *CP* to the first of the two matrix elements in each of the four terms of this expansion. For example,

$$\begin{aligned}\langle e^-W^+|\mathscr{H}(x_1)|\nu_m(\vec{p},\lambda)\rangle &= \langle CP[e^-W^+]|CP\mathscr{H}(\vec{x}_1,t_1)(CP)^{-1}|CP[\nu_m(\vec{p},\lambda)]\rangle\\ &=\omega(\nu_m)\omega^*(e^-)\omega^*(W^+)\langle e^+W^-|\mathscr{H}(-\vec{x}_1,t_1)|\nu_m(-\vec{p},\lambda)\rangle \quad .\end{aligned} \tag{20}$$

Here, *CP* invariance,

$$CP\mathscr{H}(\vec{x}_1,t_1)(CP)^{-1}=\mathscr{H}(-\vec{x}_1,t_1) \quad ,$$

is being assumed. The factor $\omega(\nu_m)$ is the *CP* parity of $|\nu_m\rangle$ defined in Eq. (12), and $\omega(e^-)$ and $\omega(W^+)$ are analogous *CP* phase factors for the (non-self-conjugate) electron and W^+ states.[20] Application of *CP* to the first matrix element in any of the four terms on the right-hand side of Eq. (19) yields a similar result. The *CP* parity $\omega(\nu_m)$ always appears, and the *CP*-transformed matrix element always corresponds to creation of an e^+ or annihilation of an e^-, and so is proportional to U_{em}^* [see Eq. (17)]. By contrast, in each of the four terms the second matrix element, to which we do not apply *CP*, is proportional to U_{em}. Turning to kinematics, we note that if all ν_m have masses M_m which are small compared with characteristic values of $\vec{p}$, then the only kinematical dependence of the terms in Eq. (19) on m is a helicity-selection-induced proportionality to M_m. This reflects that fact that for any given value of λ, either the *CP*-transformed matrix elements [see Eq. (20)], or their companions in Eq. (19) to which *CP* has not been applied, involve a left-handed neutrino where a right-handed one would be favored.[21] From Eqs. (19) and (17), it is clear that we have now found all dependences of S_{fi} on m.[22] Thus, when *CP* is conserved, the amplitude for neutrinoless double-β decay, $A((\beta\beta)_{0\nu})$, may be written in the form

$$A((\beta\beta)_{0\nu})=\sum_m\omega(\nu_m)|U_{em}|^2M_m\bar{A} \quad , \tag{21}$$

where $\bar{A}$ is independent of m. The contributions of the different neutrinos are real relative to one another, and the interference between any two of them is constructive if these neutrinos have the same *CP* parity, but destructive if they have opposite CP parity. We see that Wolfenstein's assertion[15] was correct. Note that the analysis surrounding Eq. (20) singles out CP as opposed to C. It could not have been done with C replacing CP because $\mathscr{H}$ is not invariant under C.

Since the relative CP parity of two neutrinos ν_1 and ν_2 determines the electric or magnetic character of the radiation in the decay $\nu_1 \rightarrow \nu_2 + \gamma$, there is a model-independent correlation between the behavior found in this decay and that found in $(\beta\beta)_{0\nu}$. Namely, if CP is conserved, and $\nu_1 \rightarrow \nu_2 + \gamma$ yields pure $E1$ radiation, then the contributions of the intermediate ν_1 and ν_2 states in $(\beta\beta)_{0\nu}$ will add, and if the decay yields pure $M1$ radiation, then they will subtract.

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[1]B. Kayser, Phys. Rev. D 26, 1662 (1982).

[2]B. Kayser (unpublished).

[3]The result $\zeta^2 = (-1)^{2J}$ already appears in P. Carruthers, Phys. Lett. 26B, 158 (1968), and *Spin and Isospin in Particle Physics* (Gordon and Breach, New York, 1971), and in E. Wigner, in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, New York, 1964), p. 37. See also G. Feinberg and S. Weinberg, Nuovo Cimento 14, 571 (1959). Our own derivation of this result demonstrates, however, that is does not depend on field-theoretic assumptions, or on any conventions, such as those pertaining to Dirac γ matrices, Dirac spinors, or representations of the angular momentum operators. This derivation also shows how surprisingly simply the result can be obtained.

[4]It must be understood that field-theory calculations typically entail choices of states and other quantities which fix the η^s at values which are no longer arbitrary. See Ref. 2.

[5]That a spin-$\frac{1}{2}$ Majorana fermion cannot have a magnetic or electric dipole moment was shown in Ref. 1 by a trivial application of CPT invariance not requiring any knowledge of the CPT phases of the states.

[6]The idea of using the CPT constraint of Eq. (8) to determine the complete electromagnetic matrix element of a $J=\frac{1}{2}$ Majorana fermion is due to J. Nieves [Phys. Rev. D 26, 3152 (1982)] and to B. McKellar [Los Alamos Report No. LA-UR-82-1197 (unpublished)]. However, their calculations need revision to take into account the nontrivial CPT phase factors η^s in this constraint.

[7]This result agrees with that obtained by other means by Kayser (Ref. 1), by Nieves (Ref. 6), and by R. Shrock [Nucl. Phys. B206, 359 (1982)]. See also J. Schechter and J. Valle, Phys. Rev. D 24, 1883 (1981); 25, 283(E) (1982).

[8]E. Majorana, Nuovo Cimento 14, 171 (1937); G. Racah, *ibid.*, 14, 322 (1937); Carruthers, *Spin and Isospin in Particle Physics* (Ref. 3); Kayser (Ref. 1).

[9]For further discussion, see Ref. 2.

[10]For the special case $\xi = 1$, the relations (15) have been obtained previously from CPT by McKellar (Ref. 6), and through another method by Nieves (Ref. 6). These earlier treatments did not uncover the fact that ξ, which in general is not unity, is present in these relations.

[11]McKellar (Ref. 6); Nieves (Ref. 6); P. Pal and L. Wolfenstein, Phys. Rev. D 25, 766 (1982).

[12]S. P. Rosen, in *Neutrino 81*, proceedings of the International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, 1981, edited by R. Cence, E. Ma, and A. Roberts (University of Hawaii, Honolulu, 1981), Vol. 2, p. 76.

[13]A. Zee, Phys. Lett. 93B, 389 (1980).

[14]M. Doi *et al.*, Phys. Lett. 102B, 323 (1981).

[15]L. Wolfenstein, Phys. Lett. 107B, 77 (1981).

[16]In the erratum cited in Ref. 7, Schechter and Valle note that in view of the CP signs pointed out by Wolfenstein (Ref. 15), the CP-conserving lepton mixing matrix is actually not real in their formalism. From the phases in this matrix, it follows that they concur with Wolfenstein's statement.

[17]A. Halprin, S. Petcov, and S. P. Rosen, Phys. Lett. 125B, 335 (1983).

[18]The assumed left-handed structure of the interaction is, of course, not crucial.

[19]The CP-violating phases which may occur in the U matrix in the Majorana case have been analyzed by J. Schechter and J. Valle [Phys. Rev. D 22, 2227 (1980)], and by J. Bernabéu and P. Pascual [CERN Report No. TH.3393-CERN (unpublished)], who also discuss the CP-conserving limit.

[20]We are suppressing the momentum variables of the external e and W states, which change, of course, under CP.

[21]Interference between *heavy* and light neutrinos is discussed in Ref. 17.

[22]Plane-wave expansions of Majorana fields may contain unfamiliar phase factors, as discussed in Ref. 2. However, any such factors would cancel out in Eq. (19), since the field ν_m creates the neutrino ν_m in the matrix elements not treated by CP, but the field $\bar{\nu}_m$ annihilates it in the CP-transformed matrix elements which multiply the untreated ones.