A Schrödinger fit of the spectra of light and heavy mesons by the common potential $V_0+Ar^{0.1}$

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A simultaneous Schrödinger fit of the spectra of both light and heavy mesons by an empirical power-law potential of the form $V(r)=-8.0248+6.757r^{0.1}$ (in GeV units) is obtained by extending a previous fit of the spectra of self-conjugate mesons to that of non-self-conjugate mesons containing up, down, and strange quarks. In spite of the smallness of the constituent quark masses involved, the nonrelativistic fit for the light mesons is astonishingly good.

I. INTRODUCTION

The non-Coulombic power-law potential model has been used recently by various authors[1] with remarkable success in nonrelativistic potential-model studies of heavy mesons. In a recent paper we have shown[2] that the spectra of light and heavy self-conjugate mesons $q\bar{q}$ can be simultaneously fitted by such a simple empirical potential of the form

$$V(r)=V_0+Ar^{\nu},\quad \nu=0.1\ , \tag{1.1}$$

with $A>0$. In spite of the smallness of the constituent quark masses involved, our nonrelativistic fit for the light mesons of ρ^0 and ϕ systems has been found excellent. This inspires us to extend these considerations to lighter non-self-conjugate mesons to find their bound-state masses. Work in this direction, giving a simultaneous Schrödinger fit of $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, and $c\bar{s}$ spectra by such a power-law potential, has already been done by Martin.[3] The present work aims to extrapolate our previous nonrelativistic fit of Ref. 2 to the bound states of non-self-conjugate mesons. The light and heavy non-self-conjugate mesons which are studied in this work are D^0, D^+, F^+, B^-, B^0, G^0, and H^- mesons corresponding to different quark-antiquark configurations $q_1\bar{q}_2$ such as $c\bar{u}$, $c\bar{d}$, $c\bar{s}$, $b\bar{u}$, $b\bar{d}$, $b\bar{s}$, and $b\bar{c}$, respectively.

In the present paper our chief interest is to obtain a simultaneous nonrelativistic fit of the spectra of both light and heavy mesons with the power-law potential (1.1) in a unified manner by using same set of potential parameters and quark masses. Our fundamental assumption on the confining potential is that the potential parameters are flavor independent. We are interested only in the gross structure of the energy levels but not in the fine-hyperfine splittings. Here it must be pointed out that the usual nonrelativistic Schrödinger-type approach for heavy quarkonia is justified on the basis of the large quark masses involved, but the same approach may be unsuitable for the ordinary light mesons due to relativistic effects expected to be significant in these cases. On the other hand, the relativistic approaches attempted in some limited senses by some authors[4] are by no means simple and straightforward. Therefore nonrelativistic potential-model studies are often extended[5] to investigate light hadrons, which give , if not a quantitative, at least a qualitative understanding of their spectra. As a matter of fact, in Ref. 2 we found that ignoring the relativistic corrections does not spoil the quantitative results in case of light mesons of ρ^0 and ϕ systems. Hence in the present work we do not attempt any relativistic formulation. We rather believe that the nonrelativistic fit of Ref. 2 can be successfully extended to the case of light non-self-conjugate mesons which would ultimately provide a simultaneous Schrödinger fit of the spectra of both light and heavy mesons in the framework of our power-law potential model.

II. BASIC THEORY FOR THE SCHRÖDINGER FIT

The expression for the bound-state mass of a two-body system $q_1\bar{q}_2$ of masses m_{q_1},m_{q_2} and reduced mass $m/2$ is obtained from the Schrödinger equation ($\hbar=c=1$)

$$\frac{d^2U(r)}{dr^2}+\left[m[E-V(r)]-\frac{l(l+1)}{r^2}\right]U(r)=0\ , \tag{2.1}$$

where the symbols have their usual meaning with $m=2m_{q_1}m_{q_2}/(m_{q_1}+m_{q_2})$. Taking $V(r)$ as given in (1.1) and substituting $\rho=(r/r_0)$ with the scale factor r_0 chosen conveniently as

$$r_0=(mA)^{-1/(\nu+2)}\ , \tag{2.2}$$

Eq. (2.1) reduces to the form

$$\frac{d^2U(\rho)}{d\rho^2}+\left[\epsilon-\rho^{\nu}-\frac{l(l+1)}{\rho^2}\right]U(\rho)=0\ , \tag{2.3}$$

where

$$\epsilon=m(E-V_0)(mA)^{-2/(\nu+2)}\ . \tag{2.4}$$

Now for $\nu>0$, a bound-state solution to Eq. (2.3) can be obtained by any standard numerical method which would yield a positive-definite $\epsilon=\epsilon_{nl}$ (say) corresponding to a $q_1\bar{q}_2$ bound state of radial and orbital quantum numbers n and l, respectively, with the binding energy $E=E_{nl}$ obtained from (2.4) as

$$E_{nl}=a(a/m)^{\nu/(\nu+2)}\epsilon_{nl}+V_0\ , \tag{2.5}$$

where

$$a=(A)^{1/(\nu+1)}\ . \tag{2.6}$$

Then the Schrödinger bound-state mass of the $q_1\bar{q}_2$ system follows immediately as

$$M_{nl}(q_1\bar{q}_2)=m_{q_1}+m_{q_2}+V_0+a(a/m)^{\nu/(\nu+2)}\epsilon_{nl}\ . \tag{2.7}$$

In the special case of an equal-mass system $q\bar{q}$ with mass m_q the Schrödinger bound-state mass can be obtained as

$$M_{nl}(q\bar{q})=2m_q+V_0+a(a/m_q)^{\nu/(\nu+2)}\epsilon_{nl}\ . \tag{2.8}$$

Equations (2.7) and (2.8) with the ϵ_{nl} values obtained from the numerical solution of (2.3) give obviously the Schrödinger bound-state masses of $q_1\bar{q}_2$ and $q\bar{q}$ systems. Therefore Eqs. (2.7) and (2.8) may be used to obtain a simultaneous Schrödinger fit of the spectra of both non-self-conjugate and self-conjugate mesons in a unified manner.

III. PHENOMENOLOGICAL RESULTS

We take

$$V(r) = -8.0248 + 6.757 r^{0.1} \text{ (in GeV units)} \tag{3.1}$$

and $m_c = 1.8567$ GeV, $m_b = 5.2024$ GeV, $m_s = 0.6192$ GeV, $m_u = 0.345$ GeV, $m_d = 0.366$ GeV to compute the Schrödinger mass spectra for both light and heavy mesons.

In such a framework we have already obtained the Schrödinger mass spectrum and relative leptonic widths for ρ^0, ϕ, ψ, and Υ systems in Ref. 2. However, for the study of the light meson ρ^0 (770) considered to be a $q\bar{q}$ configuration such as $(u\bar{u} - d\bar{d})/\sqrt{2}$, we have taken the effective quark mass $m_q = 0.4228$ GeV. For completeness, we would like to quote some results of Ref. 2 regarding the computed masses of these systems in comparison with the corresponding experimental values. These results are given in Table I. We find that even for the light mesons of ρ^0 and ϕ systems, the nonrelativistic fit is excellent.

Now within the same framework as mentioned in (3.1) we obtain the ground-state-mass values of the non-self-conjugate mesons $D^0(c\bar{u})$, $D^+(c\bar{d})$, and $F^+(c\bar{s})$ (in MeV) as

$$\begin{aligned} M_{1S}(c\bar{u})_{\text{cal}} &= 2006.1, \quad M_{1S}(c\bar{u})_{\text{exp}} = 2006 \pm 1.5 \ , \\ M_{1S}(c\bar{d})_{\text{cal}} &= 2008.7, \quad M_{1S}(c\bar{d})_{\text{exp}} = 2008.6 \pm 1 \ , \\ M_{1S}(c\bar{s})_{\text{cal}} &= 2108.0, \quad M_{1S}(c\bar{s})_{\text{exp}} = 2140 \pm 60 \ . \end{aligned} \tag{3.2}$$

These mass values are obtained with no detailed considerations of the hyperfine splittings. However, we must point out here that the potential parameters and also to some extent the heavy-quark masses used here are the ones obtained in reference to $^3S_1$ bound states of the $c\bar{c}$ and $b\bar{b}$ systems. Therefore we find it appropriate to compare our results obtained here with the triplet ground-state experimental masses[6] corresponding to $D^0(c\bar{u})$, $D^+(c\bar{d})$, and $F^+(c\bar{s})$. In that case our calculated results are found to be in reasonably good agreement with the corresponding experimental mass values written to the right of each result in (3.2). Thus we find that the relativistic effects considered to be significant in the case of light non-self-conjugate mesons do not undermine the quantitative results.

TABLE I. Schrödinger mass spectrum in GeV for ρ^0, ϕ, ψ, and Υ systems. Within parentheses are the available experimental values which are given for comparison.

nl	$M_{nl}(\rho^0)$	$M_{nl}(\phi)$	$M_{nl}(\psi)$	$M_{nl}(\Upsilon)$
$1S$	0.770 (0.770)	1.0196 (1.0196)	3.097 (3.097)	9.4336 (9.4336)
$2S$	1.402 ($\sim$1.25)	1.640 (1.65)	3.686 3.686)	9.9944 (9.9944)
$3S$	1.772 ($\sim$1.60)	2.004 ($\sim$1.90)	4.031 (4.030)	10.3230 (10.3231)
$4S$	2.037	2.264	4.278	10.5581 (10.5476)
$5S$	2.224	2.467	4.471 (4.417)	10.7418
$1P$	1.225 (1.31)	1.466 (1.44)	3.521 (3.521)	9.8369
$2P$	1.649	1.883	3.916	10.2135
$1D$	1.529	1.765	3.864 (3.772)	10.1068

Then using the same potential model and quark masses as discussed here, we predict the Schrödinger masses (in GeV) for the $1S$ ground states of b-flavored mesons as

$$\begin{aligned} M_{1S}(b\bar{u}) &= 5.312, \quad M_{1S}(b\bar{d}) = 5.313 \ , \\ M_{1S}(b\bar{s}) &= 5.390, \quad M_{1S}(b\bar{c}) = 6.307 \ . \end{aligned} \tag{3.3}$$

Our calculated mass levels for $B^-(b\bar{u})$ and $B^0(b\bar{d})$ mesons are almost degenerate and are compatible with the experimental values[7] 5.16–5.17 GeV.

Finally within the same framework we predict some higher-mass levels of $c\bar{u}$, $c\bar{d}$, and $c\bar{s}$ systems along with those of $b\bar{u}$, $b\bar{d}$, and $b\bar{s}$ systems. The calculated values for

TABLE II. Schrödinger mass spectrum in GeV for $D^0(c\bar{u})$, $F^+(c\bar{s})$, $B^-(b\bar{u})$, $G^0(b\bar{s})$, and $H^-(b\bar{c})$ families.

nl	$M_{nl}(c\bar{u})$	$M_{nl}(c\bar{s})$	$M_{nl}(b\bar{u})$	$M_{nl}(b\bar{s})$	$M_{nl}(b\bar{c})$
$1S$	2.0061	2.1080	5.3123	5.3899	6.3070
$2S$	2.6286	2.7167	5.9316	5.9936	6.8853
$3S$	2.9933	3.0734	6.2945	6.3474	7.2241
$4S$	3.2542	3.3286	6.5541	6.6004	7.4664
$5S$	3.4581	3.5280	6.7569	6.7982	7.6558
$1P$	2.4538	2.5458	5.7577	5.8241	6.7229
$2P$	2.8717	2.9545	6.1735	6.2295	7.1111
$1D$	2.7533	2.8387	6.0557	6.1146	7.0011

the Schrödinger bound-state mass spectra of $D^0(c\bar{u})$, $F^+(c\bar{s})$, $B^-(b\bar{u})$, $G^0(b\bar{s})$, and $H^-(b\bar{c})$ families are displayed in Table II. All these predictions are shortly going to be tested in future experiments at CERN and Fermilab.

Thus we obtain the mass spectra of both light and heavy mesons in the Schrödinger equation with the power-law potential (3.1) in a unified manner using same set of potential parameters and quark masses. In fact, the amount of experimental data explained by our potential model through such a nonrelativistic approach is quite amazing.

IV. CONCLUSION

In this work, we find that it is possible to obtain a simultaneous nonrelativistic fit of the spectra of both light and heavy mesons by a phenomenological power-law potential. This potential is found to provide an adequate description of the Schrödinger mass spectra of ψ, Υ, ρ^0, and ϕ, as well as those of the charmed and b-flavored mesons in a flavor-independent way. The relativistic effects which are considered to be significant in the case of light mesons do not spoil the results of ρ^0, ϕ, D, and F systems. Thus the relativistic effects which are known to be nonnegligible even in the $c\bar{c}$ case are found to be phenomenologically less important in the case of light mesons, due to some unknown reasons. Such observations were also made by many other authors[5] while studying the meson and baryon spectra in nonrelativistic potential models.

In conclusion, we point out that the spectra of both light and heavy mesons can be studied in a unified manner by a nonrelativistic power-law potential model.

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