Ground-state and P-wave b-flavored baryons in a consistent quark model with hyperfine interactions

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Masses of the various ground-state and P-wave baryons containing one b quark and u and d quarks are calculated using a parameter set developed for the noncharmed baryons. The sole additional parameter needed, m_b, is obtained from a previous fit to ϕ, ψ, and Υ systems using the same parameter set.

This is one of a series of papers[1-7] which concern an attempt to develop a consistent quark model as an outgrowth of the Isgur-Karl model.[8-10] As a part of this effort Kalman and Pfeffer[7] applied this model to a calculation of the masses of the charmed baryons. A similar calculation is made here for the b-flavored baryons. Aside from the values of parameters already obtained in the fit to strange and nonstrange baryons,[6] the only additional parameter required is that of the b-quark mass. This value can be obtained from a paper by Kalman and Barbari[11] which is devoted to applying baryon parameters to the calculation of the spectrum of quarkonium. The only free parameter used is the effective quark mass parameter, which is thought to have a different value in mesons and baryons.[12] The values of this parameter calculated by Kalman and Barbari[11] are compared in Table I with the values calculated in the baryon fits of Kalman,[6] and Kalman and Pfeffer.[7] Since the s- and c-quark masses are roughly 20 MeV higher in the mesons than in the baryons, it is natural to assume that the b-quark mass is also 20 MeV higher in the mesons. Thus a mass of 5240 MeV is taken as the effective b-quark mass parameter in this paper.[13]

The model employs a Hamiltonian of the form

$$H=\sum_{i=1}^{3} m_i + H_0 + H_{\text{hyp}} \ , \tag{1}$$

where m_i are the quark masses,

$$H_{\text{hyp}}=\sum_{i<j}^{3}\frac{2\alpha_s}{3m_im_j}\left\{\frac{8\pi}{3}\delta^3(\vec{r}_{ij})(\vec{S}_i\cdot\vec{S}_j)+\frac{1}{r_{ij}^3}\left[\frac{3(\vec{S}_i\cdot\vec{r}_{ij})(\vec{S}_j\cdot\vec{r}_{ij})}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j\right]\right\} \ , \tag{2}$$

where $\vec{r}_{ij}$ is the separation between a pair of quarks, $\vec{S}_i$ is the spin of the ith quark, and α_s is the quark-gluon fine-structure constant,

$$H_0=\sum_i P_i^2/2m_i+\sum_{i<j}V^{ij}-\left(\sum_i P_i\right)^2\bigg/ 2\sum_i m_i \ , \tag{3}$$

$$V^{ij}=\tfrac{1}{2}kr_{ij}^2+U(r_{ij}) \ , \tag{4}$$

and $U(r_{ij})$ is some unknown potential, which incorporates an attractive potential at short range (a Coulomb-type piece derived from quantum chromdynamics) and deviations from the harmonic-oscillator interaction at large distances. U and H_{hyp} can be treated by first-order perturbation theory using the harmonic-oscillator wave functions. In the $U=H_{\text{hyp}}=0$ limit, in terms of Jacobi relative coordinates

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2) \ , \tag{5a}$$

$$\vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3) \ , \tag{5b}$$

Eq. (3) decouples:

$$H_0\rightarrow \tilde{H}_0=\frac{P_\rho^2}{2m_\rho}+\frac{P_\lambda^2}{2m_\lambda}+\tfrac{3}{2}k(\rho^2+\lambda^2) \ . \tag{6}$$

The Hamiltonian is thus reduced to a description of two independent harmonic oscillators with the same spring constant k. For convenience the b quark is considered to be at position $\vec{r}_3$. Employing the convention that $m_u\simeq m_d$, the reduced masses of the ρ and λ oscillators have the values

$$m_\rho=m_u, \quad m_\lambda=3m_um_b/(2m_u+m_b) \ . \tag{7}$$

Excluding hyperfine interactions, the total energy of the ground-state and P-wave b-flavored baryons is then

$$E_0(S)=2m_u+m_b+\tfrac{3}{2}(\omega_\rho+\omega_\lambda)+\langle U\rangle_s \ , \tag{8}$$

$$E_0(\rho)=2m_u+m_b+\tfrac{5}{2}\omega_\rho+\tfrac{3}{2}\omega_\lambda+\langle U\rangle_\rho \ , \tag{9}$$

$$E_0(\lambda)=2m_u+m_b+\tfrac{3}{2}\omega_\rho+\tfrac{5}{2}\omega_\lambda+\langle U\rangle_\lambda \ , \tag{10}$$

where

$$\omega_{\rho,\lambda}^2=3k/m_{\rho,\lambda} \ . \tag{11}$$

TABLE I. The effective quark mass parameter in mesons and in baryons.

Quark	Mass in baryons (MeV)	Mass in mesons (MeV)
s	665	687
c	1930	1948
b		5263

Calculations of the contributions from the nonharmonic part of the potential have been discussed by Kalman, Hall, and Misra.[1] An exact calculation of the integrals used in calculations of baryon spectra is found in Kalman and Hall.[5] Using the parameters found in the baryon fit of Kalman,[6] it follows that

$$\langle U\rangle_s=\tfrac{1}{3}a(1)+\tfrac{2}{3}a(t)\ , \quad (12)$$

$$\langle U\rangle_\rho=\tfrac{2}{9}b(1)+\tfrac{1}{2}ta(t)(m_\rho/m_\lambda)^{1/2}+\tfrac{1}{9}tb(t)\ , \quad (13)$$

$$\langle U\rangle_\lambda=\tfrac{1}{3}a(1)+\tfrac{1}{6}ta(t)+\tfrac{1}{3}tb(t)(m_\rho/m_\lambda)^{1/2}\ , \quad (14)$$

where

$$t=4/[1+3(m_\rho/m_\lambda)^{1/2}]\ , \quad (15)$$

and $a(t)$ and $b(t)$ are evaluated from the quadratic approximations

$$a(t)=-198.0-733.0t+83.94t^2\ , \quad (16)$$

$$b(t)=-297.0-366.5t-41.97t^2\ . \quad (17)$$

The hyperfine matrix elements needed to complete the calculations of the energy spectrum are found in Isgur and Karl[8] and Kalman and Hall.[5] Isgur and Karl[8] note that to fit the masses of the ground state, the mixing between the ground state and the first excited positive-parity baryons must be included. We derive the masses of the excited positive-parity b-flavored baryons needed for this calculation from the masses of the excited positive-parity charmed baryons obtained by Copley, Isgur, and Karl.[10] Their parameters were reused with the substitution of the value of 5240 MeV as the mass of the effective b-quark mass for their value of the effective c-quark mass. The values of the ground-state and P-wave b-flavored baryons and the hyperfine mixing terms were calculated using $m_u=386$ MeV, $\omega_\rho=274$ MeV, and

$$\delta=\frac{4\alpha_s(m_u\omega_\rho)^{3/2}}{3(2\pi)^{1/2}m_u{}^2}=265\text{ MeV}\ ,$$

as found in the baryon fit of Kalman.[6] The final results for the ground-state and P-wave b-flavored baryons are found in Table II. There are some experimental indications[14] of a Λ_b near the value given in this table. It will be interesting to see if further experimentation confirms the calculated values of the masses of the b-flavored baryons.

TABLE II. The predicted masses of the ground-state and P-wave b-flavored baryons.

State	Calculated mass (GeV)
$\Lambda_b\frac{1}{2}^+$	5.48
$\Lambda_b\frac{1}{2}^-$	5.91
	6.13
	6.25
$\Lambda_b\frac{3}{2}^-$	5.91
	6.20
	6.27
$\Lambda_b\frac{5}{2}^-$	6.21
$\Sigma_b\frac{1}{2}^+$	5.59
$\Sigma_b\frac{3}{2}^+$	5.74
$\Sigma_b\frac{1}{2}^-$	6.03
	6.08
	6.25
$\Sigma_b\frac{3}{2}^-$	6.04
	6.16
	6.25
$\Sigma_b\frac{5}{2}^-$	6.09

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[13]For small changes in the b-quark mass, the predicted mass of the b-flavored baryons changes by the same amount. Thus if the b-quark mass was only 10 MeV higher in the mesons than in the baryons or even had an indicated value in both mesons and baryons, there would only be a marginal change in the results (Table II).
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