Four-quark operators in hyperon radiative decays and Hara's theorem

A. N. Kamal and Riazuddin*

Theoretical Physics Institute and Department of Physics, University of Alberta, Edmonton, Alberta T6G 2J1, Canada

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We demonstrate that in the effective Hamiltonian for the process $s+u \to u+d+\gamma$, one can identify two types of local four-quark operators; type I operators violate Hara's theorem and type II respect it. By an explicit calculation we demonstrate that the quark-quark bremsstrahlung process generates both types of operators but only type I survive in the SU(3) limit.

I. INTRODUCTION

In the radiative decays of hyperons a net flavor change $s \to d$ occurs. In a single-quark-transition model, $s \to d+\gamma$, involving two-quark operators, it was demonstrated by Gilman and Wise[1] that the known bound[2] on $\Gamma(\Xi^- \to \Sigma^-\gamma)$ is violated and that $\Gamma(\Lambda \to n\gamma)$ and $\Gamma(\Omega^- \to \Xi^-\gamma)$ are probably too high by an order of magnitude. Later, Kamal and Verma[3] considered a two-quark transition, $s+u \to u+d+\gamma$, which gives rise to four-quark operators. It was demonstrated[3] that a model involving both single-quark and two-quark transitions could be consistent with all known data.[2] Eckert and Morel[4] subsequently demonstrated that a model involving two-quark transitions alone did not produce the correct sign for the asymmetry parameter[2] for $\Sigma^+ \to p+\gamma$.

References 1, 3, and 4 demonstrate that even though the single-quark transition may not be the dominant[4] contributor to the hyperon-radiative-decay rates its inclusion is necessary to get the correct asymmetry for $\Sigma^+ \to p+\gamma$. In a recent paper,[5] hyperon radiative decays have been discussed in the MIT bag model. In this paper,[5] local two-quark, four-quark, and six-quark operators are derived. It is unlikely, though, that the six-quark operators will play an important role[3,5] in hyperon radiative decays. Reference 5 also considers hard-gluon corrections to the strangeness-changing four-quark weak interaction. References 1, 3, and 4 do not consider gluon corrections.

In Sec. II of this paper we calculate the matrix element for $s+u \to u+d+\gamma$ in a nonrelativistic quark model to lowest order in electroweak coupling. We classify the effective local four-quark operators into two types. We demonstrate that type I operators violate Hara's theorem[6] and type II obey Hara's theorem. We demonstrate that the source of type I operators is the bremsstrahlung process. In Sec. III we summarize the results and comment on the works of Refs. 3 and 5.

II. CALCULATIONS

The two-quark transition, $s+u \to u+d+\gamma$, contributes to hyperon radiative weak decays, $\Sigma^+ \to p\gamma$, $\Xi^0 \to \Sigma^0\gamma$, $\Xi^0 \to \Lambda\gamma$, $\Sigma^0 \to n\gamma$, and $\Lambda \to n\gamma$, but not to $\Xi^- \to \Sigma^-\gamma$ and $\Omega^- \to \Xi^-\gamma$ since Ξ^- and Ω^- do not contain a combination of s and u quarks.

Nonrelativistically, to *zero order* in photon momentum, one can write down two types of four-quark operators. For each type, there are both parity-conserving (PC) and parity-violating (PV) operators.

Type I:

$$\text{PC:}\quad i(\vec{\epsilon}\times\hat{k})\cdot(u^\dagger\vec{\sigma}s d^\dagger u - u^\dagger s d^\dagger\vec{\sigma}u)\ ; \tag{1}$$

$$\text{PV:}\quad \vec{\epsilon}\cdot(u^\dagger\vec{\sigma}s)\times(d^\dagger\vec{\sigma}u)\ . \tag{2}$$

Type II:

$$\text{PC:}\quad (u^\dagger\vec{\sigma}\cdot\vec{\epsilon}s d^\dagger\vec{\sigma}\cdot\hat{k}u - u^\dagger\vec{\sigma}\cdot\hat{k}s d^\dagger\vec{\sigma}\cdot\vec{\epsilon}u); \tag{3}$$

$$\text{PV:}\quad (u^\dagger\sigma\cdot\vec{\epsilon}s d^\dagger u - u^\dagger s d^\dagger\vec{\sigma}\cdot\vec{\epsilon}u)\ . \tag{4}$$

Here u, s, etc. represent two-component Pauli spinors with obvious flavors, $\vec{\epsilon}$ is the photon polarization vector, and $\hat{k}$ is the unit vector in the direction of the photon momentum.

Hara's theorem[6] states that in the SU(3) limit the parity-violating amplitude for $\Sigma^+ \to p\gamma$ vanishes. Type I operators violate Hara's theorem. The expectation value of the PV operator of Eq. (2), evaluated for $\Sigma^+ \to p\gamma$ with SU(6) interval wave functions for baryons, is nonzero. On the other hand, the expectation value of the PC operator of Eq. (1), evaluated for $\Sigma^+ \to p\gamma$, vanishes. The situation for type II operators is the reverse and, therefore, they respect Hara's theorem.

By an explicit calculation we demonstrate that type I operators do indeed arise from the quark-quark bremsstrahlung processes.

A. Bremsstrahlung process[3,4,7]

Consider the processes shown in Fig. 1. The explicit form of the matrix element arising from Fig. 1(a) is

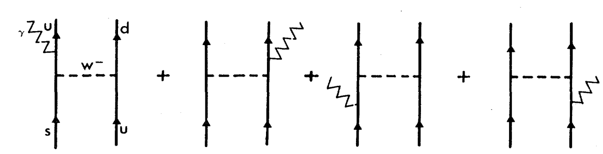

FIG. 1. Quark-quark bremsstrahlung diagrams.

$$M^{(4)}(\text{Fig. 1(a)}) = \frac{e_u G_F \sin\theta_C \cos\theta_C}{\sqrt{2}\, 2p_3\cdot k}\bar{u}(2\epsilon\cdot p_3 + \not\epsilon \not k)\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u \quad , \tag{5}$$

where $e_u = \frac{2}{3}e$ and θ_C is the Cabibbo angle in a four-quark model. As explained in Ref. 3, the $O(1/k)$ term in Eq. (5) makes no contribution to hyperon weak radiative decays due to gauge invariance. The matrix element then reduces to (a factor of $eG_F \sin\theta_C \cos\theta_C/\sqrt{2}$ is being suppressed)

$$M^{(4)}(\text{Fig. 1(a)}) = \frac{2/3}{2p_3\cdot k}\bar{u}\not\epsilon \not k \gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u \quad . \tag{6}$$

To evaluate this matrix element to zero order in k it is sufficient to use the static approximation for the Dirac spinors and approximate the propagator, $1/(p_3\cdot k)\approx 1/(mk)$, where m is the u-quark mass. In our calculation, we use $m_u = m_d = m$ and $m_s \neq m$.

On summing the contribution of all the graphs in Fig. 1, *and adding the Hermitian conjugate,* we get the following matrix element,

$$\begin{aligned} M^{(4)} = {} & \frac{1}{2mk}(1-\xi/3)[\bar{u}\not\epsilon \not k\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)u - \bar{s}\not\epsilon \not k\gamma_\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)d] \\ & + \frac{1}{2mk}[\bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\not\epsilon \not k\gamma_\mu(1-\gamma_5)d - \bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\not\epsilon \not k\gamma_\mu(1-\gamma_5)u] \\ & + \frac{1}{mk}(1-\xi/3)[\bar{d}\not\epsilon(1-\gamma_5)u\bar{u}\not k(1-\gamma_5)s - \bar{d}\not k(1-\gamma_5)u\bar{u}\not\epsilon(1-\gamma_5)s] \\ & + \frac{1}{mk}[\bar{s}\not\epsilon(1-\gamma_5)u\bar{u}\not k(1-\gamma_5)d - \bar{s}\not k(1-\gamma_5)u\bar{u}\not\epsilon(1-\gamma_5)d] \quad , \end{aligned} \tag{7}$$

where $\xi = (m_s - m_d)/m_s$. If one makes use of the identity

$$\gamma^\alpha\gamma^\beta\gamma^\mu = g^{\alpha\beta}\gamma^\mu - g^{\alpha\mu}\gamma^\beta + g^{\beta\mu}\gamma^\alpha + i\gamma_5\epsilon^{\alpha\beta\mu\nu}\gamma_\nu \tag{8}$$

in the first two square brackets of Eq. (7) one obtains

$$\begin{aligned} M^{(4)} = {} & -\frac{i}{2mk}(1-\xi/6)\epsilon_\alpha k_\beta \epsilon^{\alpha\beta\mu\nu}[\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\nu(1-\gamma_5)u - (\mu \leftrightarrow \nu)] \\ & + \frac{\xi}{6mk}\epsilon^\mu k^\nu[\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\nu(1-\gamma_5)u - (\mu\leftrightarrow\nu)] + \text{H.c.} \end{aligned} \tag{9}$$

$\epsilon^{\alpha\beta\mu\nu}$, being a rank-4 pseudotensor, requires a rank-2 pseudotensor from the first square bracket of Eq. (9) to generate a scalar matrix element. On the other hand, one needs a rank-2 tensor from the second square bracket of Eq. (9) to generate a scalar. Therein lies the difference between these two terms. The matrix element of Eq. (9) is simulated by an effective local Hamiltonian of the form[8]

$$H_{\text{eff}} = g\tilde{F}^{\mu\nu}J_{\mu\nu} + fF^{\mu\nu}J_{\mu\nu} + \text{H.c.} \quad , \tag{10}$$

where g and f are constants of order eG_F, $F^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are the electromagnetic antisymmetric field tensor and its dual, respectively, and

$$J_{\mu\nu} = \bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma_\nu(1-\gamma_5)u - (\mu\leftrightarrow\nu) \quad . \tag{11}$$

A nonrelativistic reduction of Eq. (9) in the Coulomb gauge ($\epsilon_0 = 0$, $\vec{\epsilon}\cdot\vec{k} = 0$) leads to the following PC and PV Hamiltonians,

$$\begin{aligned} H_{\text{eff}}^{\text{PC}} = {} & \frac{i}{m}(1-\xi/6)(\vec{\epsilon}\times\hat{k})\cdot(u^\dagger s d^\dagger \vec{\sigma} u - u^\dagger\vec{\sigma} s d^\dagger u) \\ & + \frac{\xi}{6m}(u^\dagger\vec{\sigma}\cdot\vec{\epsilon} s d^\dagger \vec{\sigma}\cdot\hat{k}u - u^\dagger\vec{\sigma}\cdot\hat{k}sd^\dagger\vec{\sigma}\cdot\vec{\epsilon}u) + \text{H.c.} \end{aligned} \tag{12}$$

and

$$\begin{aligned} H_{\text{eff}}^{\text{PV}} = {} & \frac{i}{m}(1-\xi/6)\vec{\epsilon}\cdot(u^\dagger\vec{\sigma}s)\times(d^\dagger\vec{\sigma}u) \\ & + \frac{\xi}{6m}(u^\dagger\vec{\sigma}\cdot\vec{\epsilon}sd^\dagger u - u^\dagger s d^\dagger\vec{\sigma}\cdot\vec{\epsilon}u) + \text{H.c.} \end{aligned} \tag{13}$$

The bremsstrahlung process, therefore, generates four-quark operators of both type I and type II defined in Eqs. (1)–(4). Note, however, that in the SU(3) limit, $\xi = 0$, only type I operators survive. One is, therefore, led to a violation of Hara's theorem.

In a recent paper,[5] the four-quark operators were evaluated in the MIT bag model. It was found that, with SU(3) breaking, the parity-violating amplitude for $\Sigma^+ \to p\gamma$ was an order of magnitude larger than the parity conserving amplitude. If Hara's theorem were obeyed by the four-quark operators, then the situation would be expected to be the other way round. In Ref. 5, gluon corrections, as calculated by Gilman and Wise,[9] were also included.

B. Internal radiation

Four-quark operators can also be generated by the process shown in Fig. 2. The contribution of this internal-radiation process is suppressed by a factor mk/m_W^2 compared to the bremsstrahlung process of Fig. 1. It is easily shown that

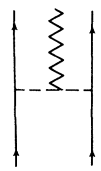

FIG. 2. An internal-radiation diagram.

this internal-radiation process yields a local four-quark operator of type

$$m\epsilon^{\mu}k^{\nu}[\bar{u}\gamma_{\mu}(1-\gamma_5)s\bar{d}\gamma_{\nu}(1-\gamma_5)u-(\mu\leftrightarrow\nu)] \qquad (14)$$

apart from a coefficient $eG_F/m_W{}^2\sin\theta_C\cos\theta_C$. In the notation of Eqs. (10) and (11), the internal radiation yields an $H_{\rm eff}$ of form $F^{\mu\nu}J_{\mu\nu}$ only; that is, it yields only type II four-quark operators which respect Hara's theorem.

III. DISCUSSION

Through an explicit calculation we have demonstrated that the local four-quark operators generated by the bremsstrahlung processes of Fig. 1 are of two types:

$$\text{Type I:}\ \ H_{\rm eff}^{\rm I}=f\tilde{F}^{\mu\nu}J_{\mu\nu} \qquad (15)$$

and

$$\text{Type II:}\ \ H_{\rm eff}^{\rm II}=gF^{\mu\nu}J_{\mu\nu}\ , \qquad (16)$$

where $J_{\mu\nu}$ is defined in Eq. (11). In the nonrelativistic quark model it can be shown explicitly, by using SU(6) baryon wave functions, that the parity-violating amplitude for $\Sigma^+\to p\gamma$ is nonzero for $H_{\rm eff}^{\rm I}$ and the parity-conserving amplitude for $\Sigma^+\to p\gamma$ is zero. The converse is true for $H_{\rm eff}^{\rm II}$. Four-quark operators of type I, therefore, violate Hara's theorem. It is also shown that in the SU(3) limit the four-quark operators generated by the bremsstrahlung process are dominantly (zero order in k) of type I. These statements can also be proven[8] in the form of a theorem using U-spin rotation and CP invariance. The purpose of the present work is to demonstrate that the four-quark operators of type I not only do occur but that they dominate over type II operators.

Our model calculation disregards gluon effects. In Ref. 5, hard-gluon effects as calculated by Gilman and Wise[9] were incorporated in a relativistic MIT-bag-model calculation. It was found[5] that the parity-violating amplitude for $\Sigma^+\to p\gamma$, arising from the four-quark operators originating in the bremsstrahlung process, was an order of magnitude larger than the parity-conserving amplitude. This demonstrates that the bremsstrahlung processes produce local effective four-quark operators largely of type I. The results of Ref. 5 are consistent with ours. In an earlier paper[3] by one of the authors it was assumed that Hara's theorem was respected by the four-quark operators. This was ensured by an antisymmetrization prescription.[3] We now believe that the prescription of antisymmetrization is incorrect and that Hara's theorem is indeed violated by the local four-quark operators arising from the bremsstrahlung process.

It is worth pointing out that local two-quark operators for the radiative hyperon decays can only be of the form

$$H_{\rm eff}^{(2)}=eG_F\bar{s}\sigma_{\mu\nu}(a+b\gamma_5)dF^{\mu\nu}+\text{H.c.} \qquad (17)$$

That is, they cannot involve the dual tensor $\tilde{F}^{\mu\nu}$ because of the identity

$$\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}=2i\sigma_{\mu\nu}\gamma_5\ . \qquad (18)$$

Having demonstrated that Hara's theorem [vanishing of the parity-violating amplitude in $\Sigma^+\to p\gamma$ in the SU(3) limit] is violated by the four-quark operators arising from the bremsstrahlung process, we examine the assumptions made in the derivation of the theorem. In Hara's original derivation,[6] a general expression for

$$B_i\to B_f+\gamma+U\ , \qquad (19)$$

where U is a weak spurion, was written down under certain restrictive conditions. It was assumed that the weak spurion transforms like a member of an octet, $U_2^3+U_3^2$ in tensor notation or like λ_6 in vector notation. The electromagnetic field transforms like A_1^1 (or equivalently $\lambda_3+\lambda_8/\sqrt{3}$). By following the general method in Ref. 10 one can demonstrate that the most general form of the parity-violating part of the amplitude for the radiative decay of (19), in the SU(3) limit and with CP invariance, is of the form

$$(\bar{B}_1^1B_3^2U_2^3-\bar{B}_2^3B_1^1U_3^2+\bar{B}_1^1B_2^3U_3^2-\bar{B}_3^2B_1^1U_2^3)A_1^1\ . \qquad (20)$$

In deriving Eq. (20) we have used

$$\bar{B}_j^i\underset{CP}{\rightarrow}B_i^j\ ,$$

$$B_j^i\underset{CP}{\rightarrow}\bar{B}_i^j\ , \qquad (21)$$

$$U_2^3\underset{CP}{\rightarrow}-U_3^2,\ \text{for PV spurion}\ .$$

Note that Eq. (20) is symmetric under $(2\leftrightarrow3)$ as Hara[6] requires. As Eq. (20) does not contain $\bar{\Sigma}^+p$ $(=\bar{B}_2^1B_1^3)$ configuration, Hara's theorem follows. The essential ingredient in Hara's derivation is that the weak spurion transforms like λ_6 or $U_2^3+U_3^2$. This is rather a restrictive condition to demand of a current×current product, i.e., $\Delta I=\frac{1}{2}$ dominance.

In more recent derivations (Vasanti[6] and Ahmed and Ross[6]) of the result that the parity-violating part of $\Sigma^+\to p\gamma$ vanishes in the SU(3) limit, the amplitude for the radiative decays is correctly written as the matrix element of a time-ordered product of three currents, two weak and one electromagnetic. The U-spin and CP properties of this time-ordered product are assumed to be those of a simple product of these operators. However, in the matrix element, the time-ordered products do not factorize in this simple manner. Contractions over fermion spinors introduce extra Dirac γ matrices which, in addition to the usual $(V-A)_\mu\times(V-A)_\nu$ current structure, generate tensor-current structures [see Eq. (7)] involving three γ matrices. Once we recast these new current structures into the $(V-A)_\mu\times(V-A)_\nu$ form, one is led to the reversal of the roles of the parity-conserving and parity-violating amplitudes, due to the appearance of the pseudotensor $\epsilon^{\alpha\beta\mu\nu}$ as seen from Eq. (9).

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*Present address: Department of Physics, University of Petroleum and Minerals, Dhahran, Saudi Arabia.

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