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Radiative meson decays in a dual model

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Radiative decay widths of the low-lying mesons are calculated in the framework of a dual-model type of realization of extended vector-meson dominance. Except possibly for $K^{*-} \to K^- \gamma$, the predictions are in reasonable agreement with the data and represent a considerable improvement over naive vector-meson-dominance results.

Considerable effort has been devoted in the past to the understanding of radiative meson decays[1] in the framework of various theoretical models, such as vector-meson dominance (VMD) and the quark model, complemented with some symmetry constraints, such as SU(3) or nonet symmetry. On the other hand, the overall quality of the experimental data has been steadily improving to a point where it is now possible to make more precise quantitative tests of the various theoretical models. An attractive feature of VMD is the correlation it achieves among different photon-mediated processes,[2] serving also as a useful phenomenological alternative to quantum chromodynamics (QCD) in the large-distance domain where perturbative calculations break down.

It has become gradually clear, both from theoretical and experimental evidence, that the radial excitations of the ground-state vector mesons constitute an important component of the electromagnetic current.[2] In connection with radiative meson decays, it has been known for quite some time that these radial excitations should give rise to sizable corrections to naive VMD predictions.[3–7] For instance, if one uses the values of the strong coupling constants $g_{\rho\omega\pi}$ and $g_{\rho\phi\pi}$ that have been extracted from experiment,[3,8] then, e.g., the predicted rate for $\pi^0 \to \gamma\gamma$ in VMD is off by a factor of 2. Disregarding this inconsistency and using $\Gamma(\pi^0 \to \gamma\gamma)$ as an input does not solve the problem completely, as the resulting predictions for $\Gamma(\phi \to \eta\gamma)$ and $\Gamma(K^{*0} \to K^0\gamma)$ turn out too large by roughly a factor of 2.

In this paper we reexamine the radiative decays of "old" mesons, i.e., below charm threshold, in the framework of a dual-model type of realization of extended VMD (EVMD). Our approach differs from other EVMD calculations[3–7] in several respects. First, no attempt will be made to impose on the large-distance phenomenology, embodied in the dual model, the short-distance behavior predicted by perturbative QCD. A recent analysis of the pion form factor $F_\pi(q^2)$ up to $q^2 \simeq -10$ (GeV/c)^2 has shown no indication of any such smooth extrapolation.[9] In any case, such an attempt would only introduce additional assumptions[10] which are not needed here, as the factorization properties of the dual model to be discussed below help to fix the free parameters from the data on $F_\pi(q^2)$. Next, the values of $g_{\rho\omega\pi}$ and $g_{\rho\phi\pi}$ that have been extracted from experimental data on $\omega \to 3\pi$ and $\phi \to 3\pi$ will be used throughout. This allows an unambiguous prediction of the $\pi^0 \to \gamma\gamma$, $\rho \to \pi\gamma$, $\omega \to \pi\gamma$, and $\phi \to \pi\gamma$ decay rates independent of any symmetry considerations. SU(3) arguments will be invoked only for those decays involving the η and X (η') mesons to relate the (experimentally) unknown couplings in the standard fashion. The final difference with previous treatments of the subject lies in the model to be chosen as a realization of EVMD. The choice here is the factorizable dual model for three-point functions in the zero-width approximation,[11–14] whose expression for a general off-mass-shell vertex is

$$F(p_1^2,p_2^2,p_3^2) = g \prod_{i=1}^{3} \Gamma(\beta_i - s_i) \frac{\Gamma(-\alpha'(p_i^2 - M_i^2))}{\Gamma(\beta_i - s_i - \alpha'(p_i^2 - M_i^2))} , \tag{1}$$

where the constant g is fixed by the value of the residue at the fully on-mass-shell point, $\alpha' = 1/2M_\rho^2$ is the Regge slope, s_i is the spin of the ith particle with

$$\alpha(p_i^2) = s_i + \alpha'(p_i^2 - M_i^2) \tag{2}$$

being the Regge trajectory, and β_i is a free parameter governing the asymptotic behavior of the form factor in the spacelike region. This model has been successfully applied to purely hadronic[11–13] as well as electromagnetic[9,14] reactions, with all results being in agreement with its factorization properties.

It is easy to show that Eq. (1) leads to the following modification of the VMD propagator,

$$\frac{1}{M_V^2 - q^2} \to \alpha'\Gamma(\beta - 1) \frac{\Gamma(-\alpha'(q^2 - M_V^2))}{\Gamma(\beta - 1 - \alpha'(q^2 - M_V^2))} . \tag{3}$$

Using Eq. (3), the correction to the $VP\gamma$ coupling at $q^2 = 0$ (P being a pseudoscalar meson) turns out to be

$$g_{VP\gamma}|_{\mathrm{EVMD}} = g_{VP\gamma}|_{\mathrm{VMD}} \Gamma(\beta - 1) \frac{\Gamma(1 + \alpha' M_V^2)}{\Gamma(\beta - 1 + \alpha' M_V^2)} . \tag{4}$$

This dimensional coupling determines the radiative decay rates according to

$$\Gamma(V \to P\gamma) = \frac{\alpha}{24} (g_{VP\gamma})^2 \left(\frac{M_V^2 - \mu_P^2}{M_V} \right)^3 , \tag{5}$$

$$\Gamma(P \to V\gamma) = \frac{\alpha}{8} (g_{VP\gamma})^2 \left(\frac{\mu_P^2 - M_V^2}{\mu_P} \right)^3 . \tag{6}$$

On the other hand, the $P \to \gamma\gamma$ decay rate is given by

$$\Gamma(P \to \gamma\gamma) = \frac{\pi\alpha^2}{4} (g_{P\gamma\gamma})^2 \mu_P^3 , \tag{7}$$

where $g_{P\gamma\gamma}$ obeys an equation similar to Eq. (4) except for the obvious doubling of the Γ-function correction term.

Using the factorization properties of the model,[11-14] the parameter β in the ρ-meson propagator is the same as that determined from the pion-form-factor data,[9] i.e., $\beta_\rho = 2.33$. In order to simplify the present analysis it will be assumed that $\beta_\omega = \beta_\phi = \beta_\rho$. This condition follows naturally from the structure of more complicated dual models,[7] and studies of the nucleon-form-factor data indicate that it should be a reasonable approximation.[15] In any case, we have checked that, by allowing β_ρ, β_ω, and β_ϕ to become free parameters and performing a least-squares fit to the whole set of ratiative meson decays, the results do not change appreciably, thus confirming the assumption. An update of the analysis of Ref. 3 to include the most recent values[16] of $\Gamma(\omega \to 3\pi)$ and $\Gamma(\phi \to 3\pi)$, as well as the value

$$(g_{\rho\pi\pi/4\pi})^2 = 3.0 \pm 0.1$$

obtained from $\Gamma(\rho \to 2\pi)$, gives

$$(g_{\rho\omega\pi})^2 = 282 \pm 13 \text{ GeV}^{-2} \ , \qquad (g_{\rho\phi\pi})^2 = 0.55 \pm 0.03 \text{ GeV}^{-2} \ . \tag{8}$$

Such a value of $g_{\rho\omega\pi}$ is in excellent agreement with theoretical expectations[8] and when Eq. (8) is used in the quark-model relation

$$|g_{\rho\phi\pi}| = |g_{\rho\omega\pi}| \tan(\theta_V - \theta_{id}) \ ,$$

where the ideal ω-ϕ mixing angle is $\theta_{id} = \sin^{-1}(1/\sqrt{3})$, one obtains $\theta_V = 37.8° \pm 0.1°$. This result should be compared with $\theta_V = 37.7°$ (39°) obtained from the linear (quadratic) Gell-Mann–Okubo mass formula, and with $\theta_V = 40° \pm 2°$ from the branching ratio[16] $\Gamma(\omega \to e^+e^-)/\Gamma(\phi \to e^+e^-)$. The remaining parameters needed are the $V\gamma$ couplings f_ρ, f_ω, and f_ϕ defined through the decomposition of the electromagnetic (EM) current as

$$j_\mu = e\frac{M_\rho^2}{f_\rho}\rho_\mu + e\frac{M_\omega^2}{f_\omega}\omega_\mu + e\frac{M_\phi^2}{f_\phi}\phi_\mu \ . \tag{9}$$

Using the experimental widths[16] for ρ, ω, and ϕ into e^+e^- one finds

$$f_\rho^2 = 26 \pm 3, \quad f_\omega^2 = 246 \pm 24, \quad f_\phi^2 = 174 \pm 8 \ . \tag{10}$$

It must be pointed out that the radial excitations have been absorbed into the $VP\gamma$ coupling making it a form factor, so that the EM current retains its usual form. As discussed in Ref. 9, and in the context of this particular realization of EVMD, this procedure is totally equivalent to modifying the EM current to include the radial excitations V_n while fixing the ratio g_{VPV_n}/f_{V_n} to obtain a Γ-function type of vertex. Finally, the η-X mixing angle will be given the usual value $\theta_P = -10°$. Within the present framework the possibility[17] $\theta_P = 0°$ does not give reasonable results. It should be added that the full effects of ω-ϕ and η-X mixing have been taken into account in the calculation of the radiative decay rates, and also that the derivation of the relevant formulas for the amplitudes has been made without the approximation $\theta_V = \theta_{id}$. As usual, a degenerate ρ-ω Regge trajectory has been assumed with $\alpha' = 0.83$ GeV^{-2}, although physical masses have been used in the calculation of phase-space factors.

The results, rounded to the first significant figures, are displayed in Table I [column (a)] together with the naive VMD predictions and the experimental data.[16] Also shown [column (b)] is the result of a χ^2 fit to the data regarding the asymptotic rate coefficients β_ρ, β_ω, and β_ϕ as free parameters; this yields $\beta_\rho = 2.4$, $\beta_\omega = 2.4$, and $\beta_\phi = 2.36$. As expected, this last solution is quite close to the first one which invoked factorization ($\beta_\rho = 2.33$) plus the assumption $\beta_\omega = \beta_\phi = \beta_\rho$. Theoretical uncertainties are of the order of 15% from experimental-error propagation alone.

Except possibly for $K^{*-} \to K^-\gamma$, the overall quality of

TABLE I. Radiative meson decay widths in keV predicted by VMD and EVMD (rounded off). Column (a) results from using factorization ($\beta_\rho = 2.33$) plus the assumption $\beta_\omega = \beta_\phi = \beta_\rho$. Column (b) is the result of a χ^2 fit to the data regarding these parameters as free, yielding $\beta_\rho = 2.4$, $\beta_\omega = 2.4$, and $\beta_\phi = 2.36$. Experimental data is taken from Ref. 16.

Decay	VMD	EVMD (a)	EVMD (b)	Experiment
$\pi^0 \to \gamma\gamma$ (eV)	16	8.3	7.3	8.0 ± 0.6
$\rho \to \pi\gamma$	129	92	87	68 ± 8
$\omega \to \pi\gamma$	1448	1025	966	861 ± 56
$\phi \to \pi\gamma$	6	5	4	6 ± 2
$\eta \to \gamma\gamma$	0.781	0.388	0.345	0.324 ± 0.047
$X \to \gamma\gamma$	13	6	5	5 ± 2
$\rho \to \eta\gamma$	91	63.5	60	55 ± 14
$\omega \to \eta\gamma$	12	8	8	3 ± 2.5
$X \to \rho\gamma$	194	137	129	84 ± 30
$X \to \omega\gamma$	18	12	12	8 ± 3
$\phi \to \eta\gamma$	290	57	55	63 ± 9
$K^{*0} \to K^0\gamma$	368	72	69	78 ± 36
$K^{*-} \to K^-\gamma$	90	18	17	62 ± 14
χ^2	1206	40	31	

the predictions is fairly good, representing a considerable improvement over naive VMD. This provides additional evidence in support of the factorizable-dual-model type of realization of EVMD which has been recently applied with success to describe the pion form factor.[9]

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