Implications of supersymmetric SO(10) grand unification

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We investigate the properties of a class of supersymmetric SO(10) grand unified models with an intermediate mass scale M_R corresponding to the breaking of left-right symmetry. We consider the possibility that the supersymmetry-breaking scale M_S coincides with M_R and study the constraints imposed on the value of M_R by low-energy parameters such as $\sin^2\theta(M_W)$ and $\alpha_{\text{strong}}(M_W)$ for different plausible scenarios for the mass spectrum of the supermultiplet fields. We find that the allowed values of M_R are greater than 10^{10} GeV. If, however, the supersymmetry-breaking scale $M_S > M_R$ and M_S is chosen as a free parameter, lower intermediate mass scales are possible. Some implications of our model are noted.

I. INTRODUCTION

The subject of supersymmetric unification of elementary-particle interactions[1,2] is of great current interest. Making a unified model supersymmetric not only helps to reduce the number of arbitrary coupling parameters in it but it also raises the hope that it may provide a resolution of the puzzle of large mass ratios often occurring in these theories. In general, global supersymmetry makes it possible to tune parameters[3] in the tree level once and for all, and predict a mass spectrum for fermions and bosons that is stable under radiative corrections. In particular, any desired large mass ratios can be made natural, although rather arbitrarily. From this point of view, global supersymmetry seems particularly well suited for application to grand unified theories.

Application of global supersymmetry to SU(5) grand unified models has been recently carried out by Dimopoulos and Georgi[2] and Sakai.[2] The SU(5) model has two mass scales M_U, the grand unification scale, and M_W, the scale of weak interactions. One assumes that supersymmetry is unbroken down to M_W. This enables one to keep the mass of the Weinberg-Salam Higgs doublet light ($\sim M_W$) by one fine tuning of parameters. The radiative corrections do not alter this result; this makes the gauge hierarchy natural. The implications of this model for baryon nonconservation[3] and cosmology[4] have been studied by various people.

In this paper, we discuss a supersymmetric grand unification based on the SO(10) grand unification group.[5] The main motivation for studying this model is that unlike the SU(5) model, SO(10) allows for intermediate gauge symmetries $G_I \subset \text{SO}(10)$ to exist, such that $G_I \supset \text{U}(1)\times\text{SU}(2)_L\times\text{SU}(3)_c$. Possibilities of new physics[6] associated with this intermediate symmetry depends on the mass scale M_I above which G_I is a good symmetry. As is well known, if we restrict the unification mass M_U to be less than the Planck mass M_P, M_I is restricted by low-energy constraints coming from the phenomenological parameters $\alpha_s(M_W)$ and $\sin^2\theta_W(M_W)$. In this paper, we study the constraints on M_I taking into account the effects of supersymmetry, where the intermediate symmetry is the left-right-symmetric[5] group of electroweak interactions.

The SO(10) grand unified model can be broken via the following symmetry-breaking chains,[7,8] with $\text{SU}(2)_L\times\text{SU}(2)_R$ symmetry at the intermediate stage:

$$\begin{aligned}\text{SO}(10) &\underset{M_U}{\rightarrow} \text{SU}(2)_L\times\text{SU}(2)_R\times\text{SU}(4)_c\ (H_1)\\ &\underset{M_c}{\rightarrow} \text{SU}(2)_L\times\text{SU}(2)_R\\ &\qquad\times\text{SU}(3)_c\times\text{U}(1)_{B-L}\ (H_2)\\ &\underset{M_R}{\rightarrow} \text{U}(1)_Y\times\text{SU}(2)_L\times\text{SU}(3)_c\ (G_{123})\ .\end{aligned}$$

We consider the two cases: (i) $M_c = M_U$ so that

$$\text{SO}(10) \underset{M_U}{\rightarrow} H_2 \underset{M_R}{\rightarrow} G_{123}$$

and (ii) $M_c = M_R$ so that

$$\text{SO}(10) \underset{M_U}{\rightarrow} H_1 \underset{M_R}{\rightarrow} G_{123}\ .$$

The next important question is the following: What is the scale of supersymmetry breaking, M_S? The evolution of gauge coupling constants depends sensitively on M_S since for $\mu > M_S$, both the scalar mesons and fermions of the supersymmetric multiplet contribute whereas below the scale M_S, we are reasonably free to adjust the mass spectrum by including soft supersymmetry-breaking terms. We consider two situations: one in which supersymmetry remains unbroken until the scale M_R and another in which it is broken above the intermediate mass scale M_R. For these classes of theories (with $M_S > M_R$ and $M_S \approx M_R$), we study the constraints on M_R coming from $\sin^2\theta_W(m_W)$ and $\alpha_3(m_W)$. These constraints depend sensitively on the mass spectrum of the supersymmetric multiplets. It is therefore important to know the masses of the various scalar-boson and fermion multiplets in the theory. In this connection, we make specific mention of the particular case when pseudo-Goldstone supermultiplets result from large accidental symmetries of the superpotential. The point is that in contrast with ordinary gauge theories, the existence of nonrenormalization theorems in supersymmetric models implies that the pseudo-Goldstone supermultiplet can have a mass at most of order M_S. Therefore, if pseudo-Goldstone particles arise at the superheavy mass scale, as they do in our case, their contribution to renormalization-group equations must be included starting at $\mu \approx M_S$.

Taking this effect into account, we obtain the values of M_R and M_U allowed by low-energy data. Our main result is that for the case $M_S = M_R$, constraints of supersymmetry rule out any intermediate mass scale below 10^{10} GeV. This value is suggestively close to supersymmetry-breaking scales considered in models with geometric hierarchies.[9,10] These results are given in Tables VI and VII. In addition, we display the mass scales allowed by low-energy constraints in Table III when $M_S >> M_R$.

This paper is organized as follows. In Sec. II, we present the detailed Higgs-multiplet and symmetry-breaking structure of our model. In Sec. III, we present the renormalization-group equations for $\sin^2\theta_W(m_W)$ and $\alpha_3(M_W)$. In Sec. IV, we present a detailed analysis of the superpotential and the fine-tuning conditions necessary to obtain the desired mass spectra. We conclude in Sec. V with a brief discussion of our results and their implications for baryon nonconservation.

II. SUPERSYMMETRIC SO(10) MODELS AND PATTERNS OF SYMMETRY BREAKING

Multiplets and notation

We employ the following generic notation for superfields and their components:

Spin	Vector superfield ($\Psi, \cdots$)	Chiral superfield ($\Phi, \cdots$)
0		A^Φ
$\frac{1}{2}$	λ	ψ^Φ
1	V_μ	
Auxiliary	D	F

(1)

Here λ's are four-component auxiliary Majorana spinors, ψ's are chiral spinors, and A's are complex scalars. However, we often use the superfield symbol to stand for its scalar component when the context makes the usage unambiguous. We use the notation and techniques of Ref. 11 for the SO(10) group and have $\alpha, \beta, \ldots$ as SO(10) indices from 1 to 10; $A,B = 1, \ldots, 6$ are SO(6) indices which give the $SU(4)_c$ representation content, while, $a,b,\ldots = 7, \ldots, 10$ belong to the SO(4) subgroup [which is isomorphic to $SU(2)_L \times SU(2)_R$]. We denote by (x,y,z) and (x',y',z',t') the representation content in a given SO(10) multiplet according to the H_1 and H_2 subgroups, respectively. Recall that

$$H_1 = SU(2)_L \times SU(2)_R \times SU(4)_c$$

and (2)

$$H_2 = SU(2)_L \times SU(2)_R \times SU(3)_c \times U(1)_{B-L} .$$

We display in Table I the multiplets employed.

TABLE I. Multiplets employed in this paper.

Supermultiplet	Dim	Submultiplets
Φ (Higgs field)	210	$s(1,1,1)+a(1,1,15)+w_L(3,1,15)$ $+w_R(1,3,15)+z(2,2,10)+z(2,2,10)$
Σ (Higgs field)	126	$\sigma(1,1,6)+\Delta_L(3,1,10)$ $+\Delta_R(1,3,\overline{10})+\alpha(2,2,15)$
$\overline{\Sigma}$ (Higgs field)	126	$\overline{\sigma}(1,1,6)+\overline{\Delta}_L(3,1,\overline{10})$ $+\overline{\Delta}_R(1,3,10)+\overline{\alpha}(2,2,15)$
H (Higgs field)	10	$c(1,1,6)+d(2,2,1)$
χ (Matter field)	16	$q(2,1,4)+q(1,2,4)$
Ψ (Gauge multiplet)	45	$1(3,1,1)+r(1,3,1)+f(1,1,15)+g(2,2,6)$

TABLE II. The mass spectra used as input for the renormalization-group analyses.

Multiplet	Type of fields	SO(10) index structure	Order of mass
$l(3,1,1)$	λ, V_μ	ab	M_R
$r(1,3,1)$	λ	ab	M_R
$f(1,1,8,0)$	λ	AB	M_R
$f(1,1,1,0)$	λ, V_μ	AB	M_R
$q(1,2,4)+q(2,1,4)$	A		M_R
$q(2,1,1,+1)$	A,ψ		M_R
One $SU(2)_L$ doublet	A,ψ		M_R
One (Higgs) $SU(2)_L$ doublet	A		M_W
$q(2,1,4)+q(1,2,4)$	ψ		Fermion masses
$Z^0, W^\pm$, photon, gluons	V_μ		M_W
Subcase A(i)			
$\Delta_R^1+\bar{\Delta}_R^1(1,3,1,\pm2)$	A	$abAbC$	M_R
$\Delta_L^1+\bar{\Delta}_L^1(3,1,1,\pm2)$	A	$abABC$	M_R
Subcase A(ii) same as (i) plus			
$\Delta_R^6+\bar{\Delta}_R^6(1,3,6,\pm2)$	A	$abABC$	M_R
$\Delta_L^6+\bar{\Delta}_L^6(3,1,6,\pm2)$	A	$abABC$	M_R
Subcase A(iii) same as (ii) plus			
$\Delta_R^3+\bar{\Delta}_R^3(1,3,3,\pm2/3)$	A	$abABC$	M_R
$\Delta_L^3+\bar{\Delta}_L^3(3,1,3,\pm2/3)$	A	$abABC$	M_R

Symmetry-breaking patterns

We shall discuss only the cases where the supersymmetry-breaking scale $M_S \gtrsim M_R$, where M_R is the scale at which the intermediate symmetry group (H_1 or H_2) is broken. We list the cases considered along with the mass spectra in each case. Detailed discussion of the mass spectra is given in Secs. IV and V.

Case A. The symmetry hierarchy is

$$SO(10)\rightarrow H_2\rightarrow SU(3)_c\times SU(2)_L\times U(1)_Y\ (G_{123})\ .$$

The only nonzero vacuum expectation values are assumed to be

$$\langle A^{\Phi}_{1234}\rangle=\langle A^{\Phi}_{3456}\rangle=\langle A^{\Phi}_{1256}\rangle =\langle a(1,1,1,0)\rangle=V_0\ , \quad (3a)$$

$$\langle A^{\Sigma}_{\nu_c\nu}\rangle=\langle \alpha(2,2,1,0)\rangle=\kappa\ ,$$

$$\langle A^{\Sigma}_{\nu_c\nu_c}\rangle=\langle \Delta_R(1,3,1,+2)\rangle=V_R\ , \quad (3b)$$

$$\langle A^{\bar{\Sigma}}_{\nu_c\nu_c}\rangle=\langle \bar{\Delta}_R(1,3,1,-2)\rangle=V_R\ ,$$

$$\langle A^{\bar{\Sigma}}_{\nu_c\nu}\rangle=\langle \bar{\alpha}(2,2,1,0)\rangle=\kappa\ , \quad (3c)$$

$$\langle A^{H}_{9,10}\rangle=\langle d(2,2,1,0)\rangle=\kappa\ . \quad (3d)$$

Case B. In this case,

$$SO(10)\rightarrow H_1\rightarrow G_{123}\ ,$$

$$\langle A^{\Phi}_{78910}\rangle=\langle S(1,1,1,0)\rangle=V_0\ , \quad (4a)$$

$$\langle A^{\Sigma}_{\nu_c\nu_c}\rangle=\langle \Delta_R(1,3,1,+2)\rangle=V_R\ , \quad (4b)$$

$$\langle A^{\bar{\Sigma}}_{\bar{\nu}_c\bar{\nu}_c}\rangle=\langle \bar{\Delta}_R(1,3,1,-2)\rangle=V_R\ , \quad (4c)$$

$$\langle A^{H}_{9,10}\rangle=\langle d(2,2,1,0)\rangle=K\ . \quad (4d)$$

Case C. Same as A(i) (see Table II) except that we assume

$$M_S \gg M_R\ . \quad (5)$$

Mass spectra

The mass spectra used as input for the renormalization-group analyses are displayed in Table II. Details are given later. We display multiplets with mass M_R and we use the notation of Eq. (1) and Table I.

Cases A(i), A(ii), and A(iii). For all these cases, in addition to the multiplets in Table II, there are pseudo-Goldstone supermultiplets

$$P_A=\omega_L^3(3,1,3(\tfrac{4}{3})+\bar{3}(-\tfrac{4}{3}))\oplus\omega_R^3(1,3,3(\tfrac{4}{3})+\bar{3}(-\tfrac{4}{3}))\oplus t^6(2,2,6,\tfrac{2}{3})\oplus\bar{t}^6(2,2,6,-\tfrac{2}{3}) . \tag{6}$$

Their masses are, at most, of order M_S and thus they must be reckoned into the renormalization-group equations above M_R. We call this case A(a). When P_A is included in the superheavy set, we call it case A(b). The point of including them in the superheavy set is that if the Higgs sector is further complicated by including 45- and 54- dimensional irreducible representations, the set P_A may be made superheavy.

Case B. This corresponds to case A(iii) with gauge group H_1 above M_R. The pseudo-Goldstone supermultiplets are

$$P_B=w_R(1,3,15)+w_L(3,1,15)+t(2,2,10)+t(2,2,10) . \tag{7}$$

Subcases (a) and (b) correspond to intermediate and superheavy masses for P_B, respectively.

Case C. In this case we assume $M_S \gg M_R$. We display the results of the renormalization-group analysis under this assumption for cases A(b)(i) as Table III. In the other subcases the constraint of perturbative unification, i.e., that α_{strong} remain bounded by a number less than 1, does not allow any significant lowering of the value of M_R.

III. CONSTRAINTS ON THE MASS SCALES FROM $\alpha_s(M_W)$ AND $\sin^2\theta(M_W)$

In this section, we will write down the $\sin^2\theta_W(M_W)$ and the QCD coupling $\alpha_3(M_W)$ in terms of the intermediate mass scales. The formulas are identical to these in the nonsupersymmetric case, except that the coefficients in the evolution of coupling constants will be different for the supersymmetric case. For generality, let us consider, the following symmetry-breaking chain:

$$\mathrm{SO}(10) \underset{M_U}{\longrightarrow} H_1 \underset{M_c}{\longrightarrow} H_2 \underset{M_W}{\longrightarrow} G_{123} . \tag{8}$$

Let us recall the important steps in the derivation of the equations for $\sin^2\theta_W$ and $\alpha_3(M_W)$. First we note that all generators of the SO(10) group defined in the 16-dimensional spinor representation, for example, must be normalized in the same way and if we take I_{3L}, I_{3R}, I_{3c}, etc., defined as physical $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$, and $\mathrm{SU}(3)_c$ generators, respectively, we obtain

$$\mathrm{Tr}I_{2L}{}^2=\mathrm{Tr}I_{2R}{}^2=\mathrm{Tr}I_3{}^2=\mathrm{Tr}I_{B-L}{}^2=\mathrm{Tr}I_Y{}^2=\mathrm{Tr}I_{\mathrm{SO}(10)}{}^2=2. \tag{9}$$

We note, however, that the physical $B-L$ and Y generators over the whole 16-dimensional representation do not satisfy Eq. (10). Taking

$$\mathrm{Tr}\left[\frac{B-L}{2}\right]^2\Bigg|_{\underline{16}} \quad \text{and} \quad \mathrm{Tr}Y^2|_{\underline{16}} ,$$

we conclude that

$$\frac{B-L}{2}=(\tfrac{2}{3})^{1/2}I_{B-L} , \quad \frac{Y}{2}=(\tfrac{5}{3})^{1/2}I_Y . \tag{10}$$

Since $Y/2=I_{3R}+(B-L)/2$, we obtain

$$\frac{Y}{2}=I_{3R}+(\tfrac{2}{3})^{1/2}I_{B-L} , \tag{11}$$

$$I_Y=(\tfrac{3}{5})^{1/2}I_{3R}+(\tfrac{2}{5})^{1/2}I_{B-L} . \tag{12}$$

From Eqs. (11) and (12), we conclude that

$$\frac{1}{g_Y{}^2(M_R)}=\frac{3}{5}\frac{1}{g_{2R}{}^2(M_R)}+\frac{2}{5}\frac{1}{g_{B-L}{}^2(M_R)} \tag{13}$$

and

$$g'(m_W)=(\tfrac{3}{5})^{1/2}g_Y(m_W) . \tag{14}$$

Now, let us write

$$\frac{dg_i}{dt}=\frac{Ai}{16\pi^2}g_i{}^3 , \tag{15}$$

where g_i goes over g_{2L} for $\mathrm{SU}(2)_L$, g_{2R} for $\mathrm{SU}(2)_R$, g_{B-L} for I_{B-L} and g_Y for I_Y coupling, respectively. Now, we remember the following additional boundary conditions representing the grand unification of coupling constants:

TABLE III. Intermediate mass scales for case C when $M_S \gg M_R$. Masses in GeV.

$\alpha_S(M_W)$	$\sin^2\theta_W(M_W)$	$r=\log_{10}M_R$	$s=\log_{10}M_S$	$u=\log_{10}M_U$
0.100	0.248	6.5	12.9	14
0.109	0.257	4	12.3	14
0.12	0.244	6.5	11.8	14
0.121	0.261	2.5	11.7	14

$$g_Y(M_U)=g_{2R}(M_U)=g_{2L}(M_U)=g_{B-L}(M_U)=g_3(M_U)=g_{\mathrm{SO(10)}}(M_U)\ . \tag{16}$$

Integrating Eq. (12) between various mass scales and using boundary condition equations (10), (11), and (13), we obtain the following equations (we assume $M_X=M_U$ and use the fact that, $\sin^2\theta_W=e^2/g_{2L}{}^2$):

$$\sin^2\theta_W\,|_{M_W}=\frac{3}{8}+\frac{5}{8}\frac{\alpha}{2\pi}\left[(A_{2L}-A_Y)\ln\frac{M_R}{M_W}+(A_{2L}{}^S-\tfrac{3}{5}A_{2R}{}^S-\tfrac{2}{5}A_{B-L}{}^S)\ln\frac{M_U}{M_R}\right] \tag{17}$$

and

$$1-\frac{8\alpha(M_W)}{3\alpha_3(M_W)}=\frac{5}{3}\frac{\alpha(M_W)}{2\pi}\left[(\tfrac{3}{5}A_{2L}+A_Y-\tfrac{8}{5}A_3)\ln\frac{M_R}{M_W}+[\tfrac{3}{5}(A_{2L}+A_{2R})+\tfrac{2}{5}A_{B-L}-\tfrac{8}{5}A_3]\ln\frac{M_U}{M_R}\right], \tag{18}$$

where A_{2L} and A_3 stand for $\mathrm{SU(2)}_L$ and $\mathrm{SU(3)}_c$ contributions to Eq. (12) for $M_W<\mu<M_R$ and $A_{2L}{}^S$, and $A_3{}^S$, etc., denote the same contributions for the mass range $M_S\approx M_R<\mu<M_U$. For case C the relevant formulas are obtained by replacing $A_{B-L}{}^S$ and $A_3{}^S$ everywhere by $A_4{}^S$.

For case A, the formulas for A_{2L}, A_Y, and A_3 are

$$\begin{aligned}&A_{2L}=-\tfrac{22}{3}+\tfrac{4}{3}N_G+\tfrac{1}{6}\ ,\\&A_Y=\tfrac{4}{3}N_G+\tfrac{1}{10}\ ,\\&A_3=-11+\tfrac{4}{3}N_G\ .\end{aligned} \tag{19}$$

We have assumed that only one complex scalar $\mathrm{SU(2)}_L$ Higgs doublet contributes between M_W and M_R. N_G is the number of fermion generations. We point out that to calculate A_Y we use the formula

$$A_Y=\tfrac{3}{5}\sum_i\left[\frac{Y^i}{2}\right]^2, \tag{20}$$

where i runs over fermions and scalar bosons. The rest of the coefficients appearing in Eqs. (14) and (17), i.e., $A_{2L}{}^S$, $A_{2R}{}^S$, $A_3{}^S$, and $A_{B-L}{}^S$ receive contributions from whole supermultiplets since they were in the region of energy $\mu>M_S>M_R$. The relevant formulas[12] are

$$\begin{aligned}&A_{2L}{}^S=-6+2N_G+\sum_i T_L(R_i)\ ,\\&A_{2R}{}^S=-6+2N_G+\sum_i T_R(R_i)\ ,\\&A_{B-L}^S=+2N_G+\sum_i T_{B-L}(R_i)\ ,\\&A_3{}^S=-9+2N_G+\sum_i T_3(R_i)\ ,\\&A_4{}^S=-12+2N_G+\sum_i T_4(R_i)\ ,\end{aligned} \tag{21}$$

where i goes over supermultiplets other than those coming from $\chi^a(16)$ and the gauge vector supermultiplets $\psi_3(1,1,80)$, $\Psi_4(1,1,15)$, $\Psi_R(1,3,1)$, $\Psi_L(3,1,1)$.

For D complex weak isodoublets and T isotriplets,

$$\begin{aligned}&T_{L,R}=\tfrac{1}{2}(D_{L,R}+4T_{L,R})\ ,\\&T_{B-L}=\tfrac{3}{2}\sum_i\frac{(B-L)_i{}^2}{4}\ ,\end{aligned} \tag{22}$$

where i goes over all members of the multiplet (either the boson or the fermion part) with appropriate transformation properties. In Tables IV and V, we give the values of the T's for various representations.

For case C ($M_S>M_R$),

$$\begin{aligned}\sin^2\theta_W(M_W)=\frac{3}{8}+\frac{5}{8}\frac{\alpha(M_W)}{2\pi}\Bigg[&(A_{2L}-A_Y)\ln\frac{M_R}{M_W}+(A'_{2L}-\tfrac{3}{5}A_{2R}-\tfrac{2}{5}A_{B-L})\ln\frac{M_R}{M_W}\\&+(A_{2L}{}^S-\tfrac{3}{5}A_{2R}{}^S-\tfrac{2}{5}A_{B-L}{}^S)\ln\frac{M_U}{M_S}\Bigg],\\1-\frac{8\alpha(M_W)}{3\alpha_3(M_W)}=\frac{5\alpha(M_W)}{3(2\pi)}\Bigg[&(\tfrac{3}{5}A_{2L}+A_Y-\tfrac{8}{5}A_3)\ln\frac{M_R}{M_W}+(\tfrac{3}{5}A'_{2L}+\tfrac{3}{5}A_{2R}+\tfrac{2}{5}A_{B-L}-\tfrac{8}{5}A'_3)\ln\frac{M_S}{M_R}\\&+(\tfrac{3}{5}A_{2L}{}^S+\tfrac{3}{5}A_{2R}{}^S+\tfrac{2}{5}A_{B-L}{}^S-\tfrac{8}{5}A_3{}^S)\ln\frac{M_U}{M_S}\Bigg],\end{aligned} \tag{23}$$

TABLE IV. Contributions of various multiplets to the β functions for $SU(2)_\Delta \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$.

Multiplet	T_{2L}	T_{2R}	T_{B-L}	T_Y	T_C (when relevant)
(2,2,0,1)	1	1	0	$\frac{3}{5}$	0
(3,1,2,1)	2	0	$\frac{9}{2}$		0
(1,3,2,1)	0	2	$\frac{9}{2}$		0
$(3,1,\frac{2}{3},6)$	12	0	3		$\frac{15}{2}$
$(3,1,-\frac{2}{3},3)$	6	0	$\frac{3}{2}$		$\frac{3}{2}$
$(1,3,-\frac{2}{3},3)$	0	6	$\frac{3}{2}$		$\frac{3}{2}$
$0(3,1,\frac{4}{3},3)$	6	0	6		$\frac{3}{2}$
$(1,3,\frac{4}{3},3)$	0	6	6		$\frac{3}{2}$
$(2,2,\frac{2}{3},6)$	6	6	4		10
$(1,1,3,\frac{2}{3})$	0	0	1		$\frac{1}{2}$

where the coefficients ${A_{2L}}^S$, ${A_{2R}}^S$, ${A_{B-L}}^S$, and ${A_3}^S$ include the effect of the whole chiral superfield (i.e., both ψ and A), whereas the rest of the coefficients are the same as in nonsupersymmetric theories. We examine the consequences of the Higgs-multiplet spectrum of case A(i) and list the results in Table III. The results of the renormalization-group analysis for cases A, B, and C are listed as Tables VI and VII.

Let us discuss our results. As is clear from Tables VI and VII, the scale of M_R is greater than 10^{10} GeV. Thus, if $M_S \approx M_R$, in supersymmetric SO(10) models, there is no room for low-mass parity restoration, unlike the case of nonsupersymmetric SO(10).[3] We also note that unlike the SU(5) case,[14] where the unification masses are of order 10^{17} GeV, our unification masses in the one-loop approximation are of order 10^{15} GeV.

One point worth emphasizing is that, due to the existence of a large number of Higgs supermultiplets above $\mu > M_R \approx M_S$, the $SU(2)_{L,R}$ and $SU(3)_c$ gauge couplings start increasing beyond $\mu \approx M_R$. If we want to apply the perturbative approximation for $\beta(g)$, we have to check in each of these cases that $g_i(M_U)$, $i = L,R,3$ does not become too large. We find that this rules out the existence of intermediate mass scales in some cases. In Table VI, we display only those cases for which the perturbative approximation holds till $\mu = M_U$.

Finally, we wish to note some uncertainties in our estimates. The first obvious source is that of higher-loop effects in the β function. Another source of uncertainty is the possible large splitting between the fermionic and bosonic partners within a supermultiplet below M_S as well as that due to the mass of superheavy bosons being below M_U.[15] However, we do not expect these uncertainties to change our general conclusion drastically.

TABLE V. Contributions of the various multiplets to the β functions for H_1.

Multiplet	T_{2L}	T_{2R}	T_4
Gauge	-6	-6	-12
(3,1,10)	20	0	9
(1,3,10)	0	20	9
(2,2,10)	10	10	12
(1,1,15)	0	0	4

IV. FEASIBILITY OF ASSUMED SYMMETRY BREAKING

In this section, we analyze our model in some detail to note how the cases mentioned in Sec. II can be realized by natural fine tuning of parameters in the model and also to assure that supersymmetry remains unbroken down to $\approx M_R$. Our discussion will be carried out only at the tree level. The nonrenormalization theorems of supersymmetry then guarantee that the resulting mass spectra below and above M_S do not get mixed up due to radiative corrections.

Our strategy will be to show that the chosen symmetry-breaking patterns and tuning conditions (see below) remove all contributions of order ${M_U}^4$ and ${M_U}^2{M_R}^2$ to the vacuum energy. Terms of order ${M_R}^4$ are then left over, which, together with supersymmetry-breaking negative (mass)^2 for scalar bosons, will generate the remainder of the symmetry breaking due to the vacuum expectation values (VEV's) V_R, K with

$$V_R \gg K . \quad (24)$$

TABLE VI. Values of M_R and M_U for representative allowed values of $\sin^2\theta_W$ and α_{strong} for case A. Masses in GeV.

$\alpha_S(M_W)$	$\sin^2\theta_W$	$r=\log_{10}M_R$	$u=\log_{10}M_U$
		Case A(a)(i)	
0.100	0.214	13.5	14.1
0.110	0.21	12.7	14.1
0.120	0.2075	12.3	14.1
		Case A(a)(ii)	
0.10	0.205	13.6	15.5
0.11	0.203	14.1	15.6
0.12	0.205	14.8	15.2
		Case A(a)(iii)	
0.10	0.224	13.1	14.5
0.11	0.221	13.2	14.8
0.12	0.219	13.5	15.1
		Case A(b)(i)	
0.10	0.219	13.5	14.1
0.11	0.219	12.5	14.0
0.12	0.219	12.0	14.0
		Case A(b)(ii)	
0.10	0.216	13.3	16.1
0.11	0.212	13.7	16.3
0.121	0.209	13.9	16.6
		Case A(b)(iii)	
0.100	0.245	11.9	14.1
0.110	0.242	12.2	14.8
0.121	0.239	12.4	15.1

Preliminaries

As a preliminary, we note that the potential in supersymmetric theories is given by

$$V=\sum_i F_i^\dagger F_i+\tfrac{1}{2}\sum_\alpha D_\alpha{}^2\ , \tag{25}$$

where

$$F_i^\dagger=-\frac{\partial W(A)}{\partial A_i} \tag{26}$$

and

$$D_\alpha=-gA_i^\dagger T_\alpha^i A_i\ , \tag{27}$$

W is the superpotential, i goes over the chiral multiplets and goes over $=1,\ldots,45$ of the group; T^i are the generators in the representation of the ith chiral multiplet. In case A, the superpotential is given by

$$W=W_1+W_2+W_3+W_4\ ,$$

TABLE VII. Same as Table VI, for case B.

$\alpha_S(M_W)$	$\sin^2\theta_W$	$r=\log_{10}M_R$	$u=\log_{10}M_U$
		Case B(a)	
0.1	0.222	14.1	15.8
0.110	0.219	14.4	16.1
0.120	0.215	14.6	16.3
		Case B(b)	
0.100	0.25	12.8	14.5
0.110	0.25	12.9	14.8
0.118	0.25	13.0	15.0

where

$$\Phi^2=\Phi_{\mu\nu\alpha\beta}\Phi_{\mu\nu\alpha\beta}\ ,\quad \Phi^3=\Phi_{\mu\nu\alpha\beta}\Phi_{\alpha\beta\gamma\delta}\Phi_{\gamma\delta\mu\nu}\ ,$$

$$W_1=M_1\Phi^2+a\Phi^3\ , \tag{28}$$

$$W_2=M_2\Sigma\bar{\Sigma}+\sum_k b_k(\Phi\Sigma\bar{\Sigma})_k\ , \tag{29}$$

$$W_3=M_3(H^2)+c_1(\Phi H\Sigma)+c_2(\Phi H\bar{\Sigma})\ , \tag{30}$$

where

$$(\Sigma\bar{\Sigma})=\Sigma_{\mu\nu\alpha\beta\gamma}\bar{\Sigma}_{\mu\nu\alpha\beta\gamma}\ ,$$

$$(\Phi H\overset{(-)}{\Sigma})=\Phi_{\alpha\beta\gamma\delta}H_\lambda\overset{(-)}{\Sigma}_{\alpha\beta\gamma\delta\lambda}\ ,$$

$$(H^2)=H_\mu H_\mu\ ,$$

k runs over the various SO(10) invariants that can be formed from Φ, Σ, and $\bar{\Sigma}$. For example,

$$(\Phi\Sigma\bar{\Sigma})_1=\Phi_{\mu\nu\alpha\beta}\Sigma_{\mu\nu\gamma\delta\lambda}\bar{\Sigma}_{\alpha\beta\gamma\delta\lambda}\ ,$$

$$W_4=d_1\chi^TB^{-1}\Gamma_\mu\chi H_\mu +d_2\chi^TB^{-1}\Gamma_{\mu\nu\alpha\beta\gamma}\chi\bar{\Sigma}_{\mu\nu\alpha\beta\gamma}\ , \tag{31}$$

B is the conjugation matrix for SO(10).[10]

In case B, W_1 is modified to

$$W_1=\lambda X(\Phi^2-24M_1{}^2)+a\Phi^3\ , \tag{32}$$

where X is a left-handed chiral gauge senplet superfield.

In addition to the supersymmetric Lagrangian, we include terms which break supersymmetry softly.[16] Such terms can be of two types: mass terms for scalar bosons (A) and mass terms for gauge fermions (λ)

The presence of scalar mass terms in addition to those coming from the supersymmetric Lagrangian does not lead to quadratic divergencies. Hence the nonrenormalization theorems which are vital to supersymmetric models are violated only by terms of order $\alpha m_s \ln\Lambda$, where Λ is the cutoff. Since we only employ tree-level tunings accurate to first order in M_S, the corrections $O(\alpha M_S\ln\Lambda)$ will not affect these conditions drastically. Thus we add

$$-\mu_\Sigma{}^2(A^{\Sigma\dagger}A^{\Sigma}+A^{\bar{\Sigma}\dagger}A^{\bar{\Sigma}})-\mu_H{}^2A^{H\dagger}A^H+\mu_\chi{}^2A^{\chi\dagger}A^\chi\ , \tag{33}$$

where the μ's are real masses $\sim M_S$.

The other type of supersymmetry-breaking term that is employed is a mass term for gauge fermions,

$$M_4\bar{\lambda}_{\mu\nu}\lambda_{\mu\nu}\ , \tag{34}$$

where M_4 is (M_S). It has been shown[16] that this term too is "soft". It leads to no quadratically divergent radiative corrections. We note that, strictly speaking, this term is not necessary since we expect the breaking of supersymmetry by mass terms for the scalars to lead to one-loop masses for the gauge fermions of order $\sqrt{\alpha}M_S$. Next we note the following formulas for the D term and gauge Yukawa coupling $\mathscr{L}_{gY}$ in our model:

$$D_{\mu\nu}=ig[4A^{\Phi\dagger}_{\mu\gamma\alpha\beta}A^{\Phi}_{\nu\gamma\alpha\beta}+5A^{\Sigma\dagger}_{\mu\gamma\alpha\beta\lambda}A^{\Sigma}_{\nu\gamma\alpha\beta\lambda}-5A^{\bar{\Sigma}\dagger}_{\mu\gamma\alpha\beta\lambda}A^{\bar{\Sigma}}_{\nu\gamma\alpha\beta\lambda}+A^{H\dagger}_{\mu}A^{H}_{\nu}+iA^{x\dagger}\Sigma_{\mu\nu}A^{x}-(\mu\leftrightarrow\nu)]\ , \tag{35}$$

$$\mathscr{L}_{gY}=\sqrt{2}g[\,4A^{\Phi\dagger}_{\mu\gamma\alpha\beta\lambda}\bar{\lambda}_{\mu\nu},\psi^{\Phi}_{\nu\alpha\beta\gamma}+5A^{\Sigma\dagger}_{\mu\gamma\alpha\beta\sigma}\bar{\lambda}_{\mu\nu}\psi^{\Sigma}_{\nu\gamma\alpha\beta\sigma} -5A^{\bar{\Sigma}\dagger}_{\mu\gamma\alpha\beta\sigma}\bar{\lambda}_{\mu\nu}\psi^{\bar{\Sigma}}_{\nu\gamma\alpha\beta\sigma}+A^{H\dagger}_{\mu}\bar{\lambda}_{\mu\nu}\psi^{H}_{\mu}+iA^{\chi\dagger}\bar{\lambda}_{\mu\nu}\Sigma_{\mu\nu}\psi^{\chi}]+\text{H.c.}\ , \tag{36}$$

where $\Sigma_{\mu\nu}$ are the SO(10) generators in the spinorial representation.

Now, we are ready to discuss the details of gauge and supersymmetry breaking. Since we want the first stage of the gauge symmetry breaking to preserve supersymmetry, we have to show that both the F and D terms vanish at the first stage.

Case A

The vanishing of the D terms is immediate since $\langle A^{\Phi}\rangle$ leaves all diagonal generators of the SO(10) group unbroken, i.e., the rank of the group remains conserved.[17] This is also easily seen by substituting Eq. (3a) in Eq. (35).

It is easy to check that if

$$V_0=-\frac{m_1}{a}\ , \tag{37}$$

then it follows that the vacuum energy of the model receives no contribution order $M_U{}^4$ from $F^{\Phi\dagger}F^{\Phi}$ and the group is now broken down to

$$\text{SU}(2)_L\times\text{SU}(2)_R\times\text{U}(1)_{B-L}\times\text{SU}(3)_c\ .$$

However, the existence of the couplings b_k, C_1, and C_2 (i.e., of $\Phi\Sigma\bar{\Sigma}$, $\Phi H\Sigma$, and $\Phi H\bar{\Sigma}$ type terms in the superpotential) raises the possibility that there may be contributions to the vacuum energy of order $O(M_U{}^2M_R{}^2)$, contrary to our requirement that supersymmetry is unbroken at energies above M_R. Eliminating $O(M_U{}^2M_R{}^2)$ contributions requires the imposition of fine-tuning constraints on the parameters in the theory. These constraints come from the requirement that F^{Σ}, $F^{\bar{\Sigma}}$, and F_H at the minimum are at most of the order $M_R{}^2$ and are given by

$$c_1=c_2=c \ , \tag{38a}$$

$$M_2-3V_0B_\Delta=0+O(M_R) \ , \tag{38b}$$

$$5M_2-3V_0B_\alpha+6C_1V_0=O(M_R) \ , \tag{38c}$$

$$M_3+12CV_0=O(M_R) \ , \tag{38d}$$

where B_Δ and B_α are linear combinations of b_k. It then follows that the potential at the classical minimum can be written as

$$V=\sum_{n=1}^{4} a_n V_R{}^n K^{4-n}-\mu_\Sigma{}^2 V_R{}^2-\mu_H{}^2 K^2 \ , \tag{39}$$

where

$$V_R=\langle A_\Sigma\rangle=\langle A_{\bar{\Sigma}}\rangle \text{ and } K=\langle A_H\rangle$$

with $\mu_\Sigma,\mu_H \lesssim M_R$; a_n are arbitrary parameters. It is clear that there exist ranges of these parameters, for which we obtain the desired hierarchy between V_R and K, i.e., $V_R >> K$. However, since supersymmetry is already broken at this level, the fine-tuning conditions have to be imposed order by order in perturbation theory.

Case B

Recall that in this case A_{78910} breaks SO(10) to H_1. As in case A, A does not contribute to $\langle D_\alpha\rangle$. The vanishing of F^Φ and F^X implies that

$$\langle A^X\rangle=0\langle A^\Phi_{78910}\rangle=V_0=m \ . \tag{40}$$

Furthermore, since

$$\langle A_{78910A}\rangle=\langle A_{78910A}\rangle=\langle H_A\rangle=0 \ , \tag{41}$$

the invariants $\langle\Phi\rangle H\Sigma$ and $\langle\Phi\rangle H\bar{\Sigma}$ make no superheavy contributions to $\langle F^\Sigma\rangle$. Just as in case A we tune

$$M_2+B_\Delta V_0=O(M_R) \ . \tag{42}$$

This also has the effect of making the Δ submultiplets light on the superheavy scale.

V. MASS SPECTRA IN SO(10) SUPERSYMMETRIC MODEL

The mass spectrum of this theory is quite rich since preservation of supersymmetry at the superheavy scale entails a pseudo-Goldstone phenomenon, by which certain submultiplets of the superfield Φ get no superheavy mass at all. Since nonrenormalization theorems prevent a radiative mass from developing until supersymmetry is broken, these particles get mass at most of order $\sqrt{\alpha}M_S$. By further complicating the model one may destroy the global symmetries responsible for the pseudo-Goldstone phenomenon. However, in view of the high intermediate scales found by us we have not carried this out.

In this section, we present details of the mass spectra in the 10, 210, 126, and H. Since we are interested only in masses of order M_U and those protected from being of $O(M_U)$, our approach will be to concentrate on the *fermionic* mass terms since at energies $O(M_U)$, supersymmetry is valid and automatically provides the boson masses. This results in an enormous simplification since the fermion mass terms are of two types only.

(i) Those coming from the superpotential

$$\tfrac{1}{2}\psi_i^T C^{-1}\frac{\partial^2 W(A)}{\partial A_i \partial A_j}\psi_j+\text{H.c.} \ , \tag{43}$$

where C is the Dirac conjugation matrix.

(ii) Contributions from the gauge Yukawa couplings

$$ig\sqrt{2}(\langle A_i^\dagger\rangle\bar{\lambda}^\alpha T^i_\alpha\psi_i-\bar{\psi}_i\lambda_\alpha T^\alpha_i\langle A_i\rangle) \ . \tag{44}$$

We remind the reader that in the supersymmetric version of the Higgs mechanism, a massless scalar from a chiral multiplet is absorbed by a massless vector boson. At the same time the fermionic partners of the two join to form a Dirac particle of mass equal to that of the massive vector boson.

Case A. Masses in the 210. Using the VEV assigned in Eqs. (3a) and in the formulas (35) and (36), together with the form of the superpotential W_1, it is straightforward, if tedious, to deduce the masses of the (H_2) submultiplets of the 210.

(a) The (1,1,8,0) and (1,1,1,0) in $a(1,1,15)$ get masses $O(M_U)$ from W_1 but the $(1,1,3,(\frac{4}{3})+\bar{3}(-\frac{4}{3}))$ components of $a(1,1,15)$ get no such mass. However, these chiral supermultiplets are precisely those "eaten" by the gauge supermultiplet which becomes massive in the breaking of SU(4) invariance. This is easily verified using Eqs. (36) or (44). The singlet $s(1,1,1)$ (Φ_{78910}) and $b(2,2,6)$ (Φ_{abcA}) get no contribution from the cubic term in W_1, and hence get masses $O(M_U)$ from M_1.

(b) The representations $w_L(3,1,15)+w_R(1,3,15)$ have index structure $Abab$. By reasoning analogous to (a) above, only the color 3 and $\bar{3}$ in the 15 get no mass from W_1. However, in this case these supermultiplets are not consumed by vector supermultiplets and hence remain without mass to $O(M_U)$.

(c) Finally, $t(2,2,10)+\bar{t}(2,2,\overline{10})$ (Φ_{aABC}) have a mass matrix of form

$$M_U\text{diag}(\eta_2\otimes I_6,-\eta_4,-\eta_4) \ , \tag{45}$$

where η_n is the $n \times n$ matrix with all entries equal to unity. This results in the $(2,2,6,\frac{2}{3})+(2,2,\overline{6},-\frac{2}{3})$ submultiplets having no mass to $O(M_U)$.

The pseudo-Goldstone set in case A is thus that given as (6). This set (P_A) of supermultiplets gets mass at most $\sqrt{\alpha}M_S$.

Case A. Masses in $\Sigma_{(126)}$ *and* $\overline{\Sigma}_{(\overline{126})}$. The tuning condition (38) removes superheavy contributions to the color-singlet masses in the Δ submultiplets of Σ and $\overline{\Sigma}$. Because of the left-right symmetry of the potential and of the superheavy (VEV), this applies to both $\Delta(3,1,10)$ and $\Delta(1,3,10)$.

In cases A(ii) and A(iii), we tune the color triplets and sextets in the Δ submultiplets to have mass $O(M_R)$ by conditions exactly analogous to Eq. (38). For this purpose we introduce additional pairs of $\Sigma,\overline{\Sigma}$ multiplets with independent couplings b_k.

The colored submultiplets in (2,2,15) are superheavy for the same reason as the triplets and sextets in case A(i): the tuning conditions do not remove their superheavy mass terms.

The SU(2) doublet color singlet $\alpha(2,2,1,0)+\overline{\alpha}(2,2,1,0)$ from Σ and $\overline{\Sigma}$ mix with the $d(2,2,1,0)$ from the $H(10)$ in a fermionic mass matrix

$$M=\begin{bmatrix} M_3 & 6cV_0 & 6cV_0 \\ 6cV_0 & 0 & 5M_2-13V_0B \\ 6cV_0 & 5M_2-13V_0B & 0 \end{bmatrix} . \tag{46}$$

Using the tuning conditions (38), we find two superheavy masses $(M_3/2, 3/2M_3)$ and one mass zero to $O(M_U)$.

The color triplets in $\sigma(1,1,6)$, $\overline{\sigma}(1,1,6)$, and $c(1,1,6)$ mix together in the mass matrix

$$\begin{bmatrix} 2M_3 & CV_0 & CV_0 \\ CV_0 & 0 & M_2+V_0B \\ CV_0 & M_2+V_0B & 0 \end{bmatrix} , \tag{47}$$

and thus all get masses $O(M_U)$.

Case B. In this case, by reasoning similar to that followed above, the superheavy VEV

$$\langle A^{\Phi}_{78910}\rangle \sim O(M_U) ,$$

along with the superheavy masses M_1 and M_2, lead to the following spectrum.

(a) There are pseudo-Goldstone supermultiplets

$$P_B:\ t(2,2,10)+\overline{t}(2,2,\overline{10})+a(1,1,15) .$$

(b) All the Δ submultiplets remain at mass M_R, since SU(4) is unbroken at M_U. The rest of the submultiplets in Σ and $\overline{\Sigma}$ are superheavy.

(c) $b(2,2,6)$ in $\Phi(210)$ is the multiplet absorbed by $g(2,2,6)$.

(d) $s(1,1,1)$, $w_R(1,3,15)$, and $w_L(3,1,15)$ get masses from the cubic term.

(e) The $c(1,1,6)$ in $H(10)$ becomes superheavy due to the mass term

$$C\langle A_{abcd}\rangle(C_A\sigma_{Aabcd}+C_A\overline{\sigma}_{Aabcd}) . \tag{48}$$

VI. PHENOMENOLOGICAL IMPLICATIONS AND CONCLUSION

In this section, we summarize our findings with a brief discussion of the implications. Our original aim was to investigate the possible existence of an intermediate mass scale corresponding to left-right-symmetry breaking in the simple supersymmetric SO(10) model. We studied two possible symmetry-breaking chains:

$$SO(10)\rightarrow SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C \rightarrow G_{123} ,$$

and

$$SO(10)\rightarrow SU(2)_L\times SU(2)_R\times SU(4)_C\rightarrow G_{123} ,$$

where G_{123} is the standard model. We found by a detailed renormalization-group analysis that the intermediate mass scale is very high (above 10^{12} GeV in most cases). Thus, observation of any evidence for such a scale either as m_{ν_e} in the eV range, or observation of $(\beta\beta)_{0\nu}$ decay, or N-$\overline{N}$ oscillation, etc., would be an indication against supersymmetric unification based on the SO(10) group [and, of course, against SU(5) grand unification].

As far as proton-decay amplitudes are concerned, we have the following remarks. The dangerous box graphs discussed by Weinberg and reanalyzed in Ref. 18 become suppressed, because, according to Ref. 18, the required condition is

$$M_{\lambda_W}M_{\psi_H} \gg 10^{18}\ \text{GeV}^2 , \tag{49}$$

and this is easily satisfied in all the cases considered since M_{λ_W} is $O(M_R)$ and M_{ψ_H} is $O(M_R)$ or greater. Since we find M_R is greater than 10^{12} GeV in almost every case, it is clear that there is no gross conflict with the observational limits on proton decay. Thus the inequality (49) is easily satisfied.

It is worth noting that a supersymmetry-breaking scale of $10^{10}-10^{12}$ GeV may be of interest from a different point of view where one tries to understand the grand unification mass from M_W and M_S as a radiative-correction effect.[9] Our results therefore would appear to single out SO(10) with an intermediate mass scale as one possibility for understanding the gauge hierarchy problem. It is in this context that case D may prove to be of some interest since it combines $M_S \sim 10^{12}$ GeV with much lower values of M_R.

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