Dispersive contribution to K^0-$\bar{K}^0$ transition and Higgs-boson-exchange model of *CP* violation

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Dispersive π^0, η, and η' pole contributions to *CP*-violating K^0-$\bar{K}^0$ transition are analyzed for the Higgs-boson-exchange model of *CP* violation and are found to produce ϵ_m/ξ much larger than the box-diagram contributions, independent of *CP*-violating $\Delta S=1$ matrix elements. The penguin-type *CP*-violating $\Delta S=2$ transition due to one-gluon exchange, not analyzed previously, is found to be negligible. This indicates that barring accidental cancellation of the low-energy and high-energy dispersive contributions, the Higgs-boson-exchange model of *CP* violation can still be acceptable with present data on ϵ'/ϵ.

Within the gauge theory of weak interactions, the six-quark model with complex coupling constants of gauge bosons with quarks given by the Kobayashi-Maskawa matrix (KM model) seems to be the most economical model for *CP* violations.[1] This model predicts a vanishingly small value for ϵ'/ϵ and is therefore experimentally indistinguishable from the superweak theory at present.[2] However, in general, if there exist more than two Higgs doublets, then *CP* violations can also arise from the complex Yukawa coupling constants of the Higgs bosons to quarks. This is the spontaneous-*CP*-violation model[3,4] of Lee and Weinberg. This model gives a large neutron electric dipole moment[3,4] of the order 10^{-24}–10^{-25} cm, which is on the edge of experimental limits and is about 5 orders of magnitude larger than that given by the KM model.[5] Until there is a much improved experimental upper limit for the neutron electric dipole moment, there seems to be no clearcut evidence in favor of either of these two models and their variations.

Recently, Deshpande and Sanda (D&S) have independently carried out an analysis of *CP*-violation effects in the $K^0\bar{K}^0$ system in the Higgs-boson-exchange model.[6,7] Assuming the existence of charged Higgs bosons which couple to quarks with complex Yukawa coupling constants similar to the complex KM matrix, they obtain a quite large value for ξ/ϵ_m in conflict with experiments. This result was confirmed by Donoghue, Hagelin, and Holstein in a subsequent analysis.[8] Using the MIT bag model to calculate the *CP*-violating matrix elements $\langle\bar{K}^0|\mathscr{L}_w(\Delta S=2)|K^0\rangle$ and $\langle\pi^0\pi^0|\mathscr{L}_w(\Delta S=1)|K^0\rangle$, they found that ξ/ϵ_m is indeed larger than experiment by a factor of more than 20. This large discrepancy between theory and experiment is then used to rule out the Higgs-boson-exchange model of *CP* violation. The large value for ξ/ϵ_m in this model is due mainly to the fact that the coefficient of the $\Delta S=1$ operator is larger than that of the $\Delta S=2$ operator of the box diagrams by a factor $\ln(m_H{}^2/m_c{}^2)$ with $m_H{}^2 \gg m_c{}^2$, and the conclusion reached by these authors is based on the estimate for $\langle\bar{K}^0|\mathscr{L}_w(\Delta S=2)|K^0\rangle$ from the box diagrams alone. However, in this model, because of the suppression of the box-diagram contribution to $\langle\bar{K}^0|\mathscr{L}_w(\Delta S=2)|K^0\rangle$ relative to $\langle\pi^0|\mathscr{L}_w(\Delta S=1)|K^0\rangle$, other contributions to $\langle\bar{K}^0|\mathscr{L}_w(\Delta S=2)|K^0\rangle$ must be included. One such contribution is given by the dispersion part[9] of

$$\int d^4x\,\langle\bar{K}^0|T\{\mathscr{L}_w(x)\mathscr{L}_w(0)\}|K^0\rangle\ ,$$

which is dominated by the low-lying intermediate state (the so-called large-distance contribution) and can be estimated more or less in a model-independent way. The importance of the dispersion contribution to ϵ_m in the Higgs-boson-exchange model of *CP* violation has also been discussed by Chang, who pointed out[10] that a large dispersive part can save the model. In this paper, we present an estimate of this contribution and show that under reasonable assumptions, the π^0, η, and η' intermediate states (pole-dominance approximation) alone gives ϵ_m/ξ much larger than the box-diagram contribution. This indicates that the $\Delta S=2$ K^0-$\bar{K}^0$ transition induced by the mixing of K^0, $\bar{K}^0$ with π^0, η, and η' is large and can produce ϵ'/ϵ compatible with experiment. Barring accidental cancellation between the low-energy (π^0,η,η' pole) and the high-energy (believed to be small) dispersive contributions, it appears that the Higg-boson-exchange model of *CP* violation cannot be ruled out at the moment as claimed by Despande and Sanda and by Donoghue *et al.*

In the following, for convenience we assume standard K^0-$\bar{K}^0$ phenomenology and use the usual notations of Gilman and Wise and others.[11] The usual parameters characterizing *CP* violation in $K\to 2\pi$ decays are defined as follows:

$$\begin{aligned}\eta_{+-}&=\frac{\langle\pi^+\pi^-|\mathscr{L}_w|K_L\rangle}{\langle\pi^+\pi^-|\mathscr{L}_w|K_S\rangle}=\epsilon+\epsilon'\ ,\\ \eta_{00}&=\frac{\langle\pi^0\pi^0|\mathscr{L}_w|K_L\rangle}{\langle\pi^0\pi^0|\mathscr{L}_w|K_S\rangle}=\epsilon-2\epsilon'\ .\end{aligned}\tag{1}$$

In models with *CP* violation arising solely from the K_S^0-K_L^0 transition, for example, in the superweak theory[12] of Wolfenstein, $\epsilon'=0$ and $\eta_{+-}=\eta_{00}$. In general $\epsilon'\neq 0$ for theories with $\Delta S=1$ interactions responsible for a direct *CP*-violating $K_L^0\to 2\pi$ decay in addition to the $\Delta S=2$ interactions induced by the transition mass matrix. In particular, if the direct *CP*-violating interactions is purely $\Delta I=\frac{1}{2}$ (e.g., penguin-type diagrams), following Gilman and Wise, we have

$$\frac{\epsilon'}{\epsilon}=-\left[\frac{1}{20}\right]\frac{2\xi}{\epsilon_m+2\xi}\ , \tag{2}$$

where

$$\epsilon_m=\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}}\ , \tag{3}$$

$$\xi=\frac{\mathrm{Im}\langle\pi\pi(I=0)\,|\,\mathscr{L}_w\,|K^0\rangle}{\mathrm{Re}\langle\pi\pi(I=0)\,|\,\mathscr{L}_w\,|K^0\rangle}\ , \tag{4}$$

and M_{12} is the K^0-$\bar{K}^0$ transition mass matrix defined as

$$\langle K^0\,|\,\mathscr{L}_w\,|\bar{K}^0\rangle=2m_KM_{12}\ .$$

The ratio ϵ_m/ξ is a measure of the importance of the transition mass matrix relative to the direct interactions. Experimentally we have[13]

$$\frac{\epsilon'}{\epsilon}=-0.003\pm0.015 \tag{5}$$

corresponding to $\xi/\epsilon_m=0.03^{+0.25}_{-0.13}$, showing that *CP*-violating effects are almost exclusively due to the K^0-$\bar{K}^0$ transition mass matrix.

In the simplest version of the Higgs-boson-exchange model with the t quark neglected and $m_H\gg m_c$, the box diagram with one Higgs and one W exchange (WH) is the main contribution to the short-distance $\Delta S=2$ *CP*-violating interactions responsible for the K_S^0-K_L^0 transition and is given by[6–8]

$$\mathscr{L}_-^{\Delta S=2}=\tilde{g}\bar{d}_i\gamma_\mu(1+\gamma_5)s_j\partial_\nu[\bar{d}_j\gamma^\mu\gamma^\nu(1-\gamma_5)s_i]\ , \tag{6}$$

where

$$\tilde{g}=\frac{G_F{}^2}{32\pi^2}m_c{}^2m_s(\cos\theta_C\sin\theta_C)^2\sum_{i=1}^{2}\frac{\mathrm{Im}\gamma_i^*}{m_{H_i}{}^2} \tag{7}$$

with γ_i in general complex having a phase δ_H reponsible for *CP* violation.

The $\Delta S=1$ *CP*-violating direct interaction is given by the penguin-type diagram which gives rise to a direct coupling of the gluon to the s and d quarks (Fig. 1),

$$\mathscr{L}_-^{\Delta S=1}=i\tilde{f}\,\bar{d}\sigma^{\mu\nu}(1-\gamma_5)\lambda_a sF^a_{\mu\nu}\ , \tag{8}$$

where $F^a_{\mu\nu}$ is the gluon field-strength tensor and $\tilde{f}$ is given by

$$\tilde{f}=\frac{G_F}{\sqrt{2}}\frac{g_s}{32\pi^2}m_c{}^2m_s\cos\theta_C\sin\theta_C\times\sum_{i=1}^{2}\frac{\mathrm{Im}\nu_i^*}{m_{H_i}{}^2}\left[\ln\frac{m_{H_i}{}^2}{m_c{}^2}-\frac{3}{2}\right]\ . \tag{9}$$

At the operator level, we see immediately that the coefficient of $\mathscr{L}_-^{\Delta S=1}$ is larger than that of $\mathscr{L}_-^{\Delta S=2}$ by a factor $\ln(m_H{}^2/m_c{}^2)$ which can be substantial for $m_H{}^2\gg m_c{}^2$. Evaluations of the matrix elements of (6) and (8) using vacuum approximation by D&S or the MIT bag model by Donoghue *et al.* give

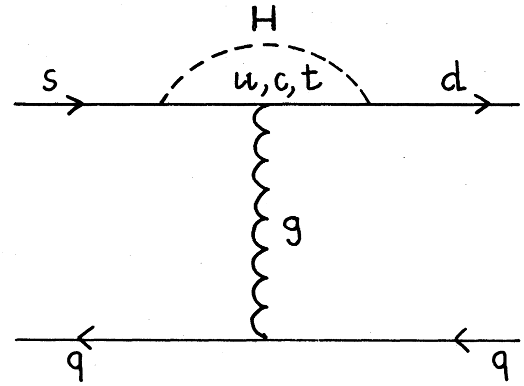

FIG. 1. Higgs-boson penguin contribution to direct, *CP*-violating $\Delta S=1$ transition.

$$\frac{\xi}{\epsilon_m(\mathrm{box})}\simeq2.6\left[\ln\frac{m_{H_1}{}^2}{m_c{}^2}-\frac{3}{2}\right]\sim7\ ,\ \text{for } m_{H_1}\sim10\text{ GeV}\ , \tag{10}$$

which indeed shows that

$$\epsilon_m(\mathrm{box})/\xi\ll1\ .$$

Thus Eq. (10) is clearly in conflict with experiment [Eq. (5)]. This conclusion remains valid when the t-quark contribution and the double-Higgs-boson-exchange diagrams (HH) are included.[8] Without other contributions to the K^0-$\bar{K}^0$ transition mass matrix, it seems that the Higgs-boson-exchange models (and their variation) would be in trouble and could be ruled out as concluded by D&S and by Donoghue *et al.* This is however not the case since the dispersive part of $\langle K^0\,|\,\mathscr{L}_w(\Delta S=2)\,|\bar{K}^0\rangle$ arising from second-order perturbation with respect to $\mathscr{L}_w(\Delta S=1)$ has not been included in their analysis. Because of the suppression of $\epsilon_m(\mathrm{box})/\xi$ in this model, ϵ_m/ξ receives a dominant contribution from the dispersive part which must now be estimated. Using the definitions (3) and (4), the ratio ϵ_m/ξ is given by

$$\frac{\epsilon_m}{\xi}=\left[\frac{\mathrm{Im}\langle K^0\,|\,\mathscr{L}_-^{\Delta S=2}\,|\bar{K}^0\rangle}{\mathrm{Im}\langle\pi\pi(I=0)\,|\,\mathscr{L}_-^{\Delta S=1}\,|K^0\rangle}\right]\times\left[\frac{\mathrm{Re}\langle\pi\pi(I=0)\,|\,\mathscr{L}_+^{\Delta S=1}\,|K^0\rangle}{\mathrm{Re}\langle K^0\,|\,\mathscr{L}_+^{\Delta S=2}\,|\bar{K}^0\rangle}\right]\ , \tag{11}$$

where the $\pm$ subscripts denote, respectively, the *CP*-conserving (even) and *CP*-violating (odd) parts of $\mathscr{L}_w$.

To estimate (11), we shall first show that $\mathscr{L}_\pm^{\Delta S=1}$ being $I=\frac{1}{2}$ operators transform as (1,8) representation of SU(3)×SU(3). This can be easily seen for the *CP*-conserving part $\mathscr{L}_+^{\Delta S=1}$, which is obtained from the penguin diagrams and is of the form

$$\mathscr{L}_+^{\Delta S=1}=C_+\sum_a J_\mu^a\bar{d}\gamma_\mu(1+\nu_5)\lambda^a s\ ,$$

where the J_μ^a are the color-octet gauge vector currents transforming as singlet under SU(3)×SU(3) [flavor

SU(3)]. Since the color-octet $\Delta S=1$ left-handed currents $\sum_a \bar{d}\gamma_\mu(1+\gamma_5)\lambda^a s$ belong to the (1,8) representation, $\mathscr{L}_+^{\Delta S=1}$ contains only (1,8) terms. This is also a reasonable assumption for the *CP*-odd part by noting that the one-gluon-exchange contribution to the $\Delta S=1$ transition induced by the penguin *sd* gluon vertex is given by

$$\mathscr{L}_-^{\Delta S=1}=(i)^2\int d^4x\, D_{\mu\nu}^{ab}(x)T\{K_\mu^a(x)J_\nu^b(0)\}\ ,$$

where

$$K_\mu^a=i\tilde{f}\partial_\nu[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)\lambda^a d]$$

and the J_μ^a are the color-octet gauge-vector currents invariant under SU(3)$\times$SU(3). Since $\mathscr{L}_-^{\Delta S=1}$ contain no ultraviolet-divergent terms, no renormalization is necessary and the hadronic matrix elements of $\mathscr{L}_-^{\Delta S=1}$ are finite (i.e., Lamb-shift contribution). This situation is similar to the regularized part of the one-photon contributions to $\mathscr{L}_{\rm EM}$ in hadron electromagnetic (EM) mass shifts and $\eta\to 3\pi$ decay where the tadpole u_3 term has been subtracted by renormalization.[14,15] Since no anomalous commutator due to a u_6- or u_7-type piece is present, the SU(3)$\times$SU(3) properties of $\mathscr{L}_-^{\Delta S=1}$ are determined by K_μ^a [SU(3)$\times$SU(3) generators commute with J_μ^a]. From the equation of motions for the quark field operators and using equal-time commutation relations,[16] we see that K_μ^a contains only $(3,\bar{3})$ and (1,8) pieces. The (1,8) piece is proportional to the current quark masses since in the limit of vanishing quark masses, the four-momentum operator commutes with the SU(3)$\times$SU(3) generators so that $\partial_\mu[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)d]$ transform like $\bar{s}\sigma_{\mu\nu}(1+\gamma_5)\lambda^a d$ which is a $(3,\bar{3})$ piece. Lorentz invariance suggests that the (1,8) piece must be of the form

$$\sum_a m_s J_\mu^a \bar{s}\gamma_\mu(1+\gamma_5)\lambda^a d\ ,$$

which has been previously found by Hill in an estimate of the contribution to *CP*-conserving *K* decays by a similar magnetic penguin operator.[17] The $(3,\bar{3})$ terms denoted by (u_7',v_6'), etc., transform exactly like the (u_i,v_i) of the $(3,\bar{3})$ quark-mass term under SU(3)$\times$SU(3). In a nonlinear realization of SU(3)$\times$SU(3) they can be given in terms of the pseudoscalar coupling matrix M as[18,19]

$$u_7'={\rm Tr}[\lambda_7(M+M^\dagger)]\ ,$$

$$v_6'={\rm Tr}[\lambda_6(M-M^\dagger)]\ ,$$

with M satisfying the unitary condition $MM^\dagger=1$ and can be expanded in terms of the pseudoscalar meson fields ϕ_i $(i=1,\ldots,8)$ as

$$M(f\phi)=1+2if\phi+2(if\phi)^2+\cdots\ ,$$

where

$$\phi=\sum_{i=1,8}\phi_i\frac{\lambda_i}{\sqrt{2}}\ .$$

f is the inverse of the pion decay constant $(f^{-1}=f_\pi=m_\pi)$. Assuming that SU(3)$\times$SU(3) is broken by the mass term

$$\mathscr{L}_m=\frac{1}{8f^2}{\rm Tr}[(a+b\lambda_8)(M+M^\dagger)]\ ,$$

then after a straightforward calculation one finds that u_7' and v_6' are proportional to the divergences of the vector and axial-vector currents and are changes of the strong-interaction Lagrangian induced by small SU(3)$\times$SU(3) rotations[16] of the order $O(G_F m_p^2)$. Hence by the same rotation one can eliminate these $\Delta S=1$ $(3,\bar{3})$ terms from the Lagrangian without changing other terms (i.e., mass term) by any appreciable amount. This important result was first given by Coleman and Glashow and by Callan, as explained by S. L. Adler and R. F. Dashen [see *Current Algebra* (Benjamin, New York, 1969), p. 132] and is clarified by Cantor in Ref. 19 (the second paper). The reason for performing SU(3)$\times$SU(3) rotations is that any off-diagonal term such as $\phi_K\phi_\pi$ does not describe the physical Lagrangian and must be diagonalized with respect to hypercharge. (The argument of Callan based on the properties of the divergence is rather involved but is not needed, however.) Note that the ability to throw away u_7',v_6' terms depends crucially on the nonlinear realization of SU(3)$\times$SU(3) symmetry since at the quark level (u_i',v_i') and (u_i,v_i) $(i=0,1,\ldots,8)$ are two distinct sets of $(3,\bar{3})$ and $(3,\bar{3})$ operators. This again reminds us of the phenomenological-Lagrangian approach as a powerful technique in dealing with current-algebra and soft-pion processes.

We can thus assume that the effective Lagrangian for direct *CP*-violating *K* decays involves only (1,8) terms to first order in the SU(3)$\times$SU(3)-symmetry-breaking parameter (current quark mass), although we do not know how to separate this piece from $K_\mu^a J_\mu^a$ in a simple manner. This is sufficient for our purpose since all we need is the SU(3)$\times$SU(3) transformation properties of $\mathscr{L}_-^{\Delta S=1}$. We can now write the nonlinear phenomenological Lagrangian for $\mathscr{L}_-^{\Delta S=1}$ in terms of M:

$$\mathscr{L}_-^{\Delta S=1}=\tilde{f}\,{\rm Tr}(\lambda_7\partial_\mu M\partial_\mu M^\dagger)\ ,$$

since only derivative coupling is allowed for (1,8) piece. This follows from a beautiful theorem due to Coleman, Wess, and Zumino on the nonlinear realization of chiral symmetry.[19,20] The K-π transition and $K\to 2\pi$ decay amplitudes are then quadratic in momenta for both the *CP*-conserving and *CP*-violating direct decays. In this case the Callan-Treiman relation should be also valid for the *CP*-violating $K_S\to 3\pi$ and $K_L\to 2\pi$ decay amplitudes in the Higgs-boson models. Measurements of $K_S\to 3\pi$ decay are needed to confirm these SU(3)$\times$SU(3) transformation properties for $\mathscr{L}_-^{\Delta S=1}$. Equation (11) can now be expressed in terms of the matrix elements $\langle\pi^0|\mathscr{L}_\pm^{\Delta S=1}|K^0\rangle$ defined in the soft-pion limit $(p_{\pi^0}=p_{K^0},\, p_{K^0}^2=m_K^2)$ and is given by

$$\frac{\epsilon_m}{\xi}=\left[\frac{{\rm Im}\langle K^0|\mathscr{L}_-^{\Delta S=2}|\bar{K}^0\rangle}{{\rm Im}\langle\pi^0|\mathscr{L}_-^{\Delta S=1}|K^0\rangle}\right]\left[\frac{{\rm Re}\langle\pi^0|\mathscr{L}_+^{\Delta S=1}|\bar{K}^0\rangle}{{\rm Re}\langle K^0|\mathscr{L}_+^{\Delta S=2}|\bar{K}^0\rangle}\right]. \tag{12}$$

The *CP*-conserving matrix element $\langle\pi^0|\mathscr{L}_+^{\Delta S=1}|K^0\rangle$ can be obtained from the $K_S^0\to 2\pi$ decay amplitude. For $p_{\pi^0}=p_{K^0}$, we have

$$a^{+}_{K^0\pi^0}=\langle \pi^0 \,|\, \mathscr{L}^{\Delta S=1}_{+} \,|\, K^0\rangle=\left[\frac{cG_F}{\sqrt{2}}\right]\frac{m_K{}^2 f_\pi{}^2}{\sqrt{2}}\ , \qquad (13)$$

where f_π is the pion decay constant ($f_\pi = m_\pi$) and c is a parameter calculated from the $K^0_S \rightarrow 2\pi$ decays rates by Cronin as[18]

$$c = 1.1 \pm 0.1\ .$$

Note that the off-shell $K^0_S(k) \rightarrow \pi^0(p)+\pi^0(q)$ amplitude is given by

$$A\,(00) = -\frac{icG_F}{2}(2k^2-p^2-q^2)f_\pi\ . \qquad (14)$$

The matrix element $\langle K^0 \,|\, \mathscr{L}^{\Delta S=2}_{+} \,|\, \bar{K}^0\rangle$ is given by the K^0_S-K^0_L mass difference taken from experiment:[21]

$$\mathrm{Re}\langle K^0 \,|\, \mathscr{L}^{\Delta S=2}_{+} \,|\, \bar{K}^0\rangle = -3.52\times 10^{-5} \times (G_F m_p{}^2)^2 m_K \ \mathrm{GeV}\ . \qquad (15)$$

Consider now the dispersive part of $\langle K^0 \,|\, \mathscr{L}^{\Delta S=2}_{-} \,|\, \bar{K}^0\rangle$. Assuming that $\mathscr{L}^{\Delta S=1}_{+}$ are local operators, second-order perturbation in $\mathscr{L}^{\Delta S=1}_{\pm}$ gives

$$\langle K^0 \,|\, i\mathscr{L}^{\Delta S=2}_{-} \,|\, \bar{K}^0\rangle = (i)^2 \int d^4x \langle K^0 \,|\, T\{\mathscr{L}^{\Delta S=1}_{w}(x)\mathscr{L}^{\Delta S=1}_{w}(0)\} \,|\, \bar{K}^0\rangle \qquad (16)$$

with

$$\mathscr{L}^{\Delta S=1}_{w} = \mathscr{L}^{\Delta S=1}_{+} + \mathscr{L}^{\Delta S=1}_{-}\ .$$

Equation (16) can now be evaluated using a covariant perturbation theory or dispersion relation. It can be conveniently considered as the scattering amplitude of zero-momentum spurion $\mathscr{L}^{\Delta S=1}_{w}(x)$ on K^0 and is assumed to be dominated by low-lying intermediate states which are the π^0,η,η' pole contributions (pole-dominance approximation) (Fig. 2). We have

$$\mathrm{Im}\langle K^0 \,|\, \mathscr{L}^{\Delta S=2}_{-} \,|\, \bar{K}^0\rangle = +2 \sum_{i=\pi^0,\eta,\eta'} \frac{\mathrm{Im}a^{-}_{iK^0}\mathrm{Re}a^{+}_{\bar{K}^0 i}(\text{penguin})}{(m_K{}^2-m_i{}^2)}\ , \qquad (17)$$

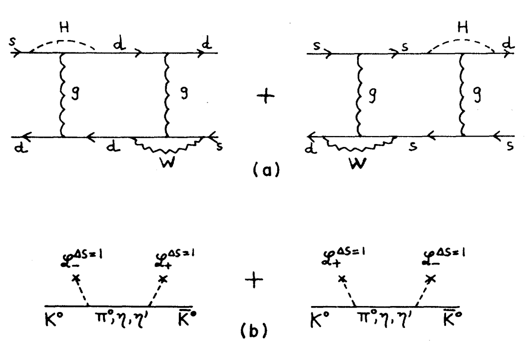

FIG. 2. (a) Dispersive part of the K^0-$\bar{K}^0$ transition due to $d\bar{d}$ and $s\bar{s}$ intermediate states. (b) π^0,η,η' pole contributions to the dispersive part represented by (a).

where

$$a^{-}_{iK^0} = \langle i \,|\, \mathscr{L}^{\Delta S=1}_{-} \,|\, K^0\rangle$$

is the CP-violating $i-K^0$ transition matrix elements of $\mathscr{L}^{\Delta S=1}_{-}$ given in (8). The contribution from low-lying intermediate states in (17) can be represented by $d\bar{d}$ and $s\bar{s}$ intermediate states with strong QCD radiative corrections due to the exchange of soft gluons (which give rise to these hadronic boundstates), as shown in Fig. 2. We have assumed that the CP-violating $a^{-}_{iK^0}$ is caused by the penguin diagrams for the $s \rightarrow d+\text{gluon}$ transition. Thus the $u\bar{u}$ and $c\bar{c}$ intermediate states do not contribute to (17) and the CP-conserving $a^{+}_{\bar{K}^0 i}$ are given by the penguin contributions alone. For these contributions we have the following relations:

$$a^{\pm}_{K^0\eta} = \frac{1}{\sqrt{3}}a^{\pm}_{K^0\pi^0}\ ,$$
$$a^{\pm}_{K^0\eta'} = -\frac{2\sqrt{2}}{\sqrt{3}}a^{\pm}_{K^0\pi^0}\ . \qquad (18)$$

Note that the pseudoscalar-meson nonet is defined in terms of the $q\bar{q}$ states as follows:

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})\ ,$$
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})\ ,$$
$$\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})\ .$$

Because of large cancellation between the π^0 and η pole in Eq. (17) in the exact SU(3) limit, the pseudoscalar meson pole contributions are quite sensitive to η-η' mixing as well as deviations from exact-SU(3) relations between $a^{\pm}_{K^0\pi^0}$ and $a^{\pm}_{K^0\eta}$ [Eq. (18)]. These effects must now be taken into account. This can be most easily done by noting that since the $d\bar{d}$ and $s\bar{s}$ valence quarks participate in penguin interactions (Fig. 1), the K^0-$(q\bar{q})_0$ transitions can be obtained from the following phenomenological Lagrangian:

$$\mathscr{L}^{\Delta S=1}_{\pm} = C^{\pm}\frac{G_F}{\sqrt{2}} f_K f_{q\bar{q}} \partial_\mu K^0 \partial_\mu \varphi_{q\bar{q}}\ , \qquad (19)$$

consistent with SU(3)$\times$SU(3) properties.

$\varphi_{q\bar{q}}$ is the pseudoscalar field operator associated with the $(q\bar{q})_{0^-}$ state. As a rough estimate of SU(3)-violation effect, let us use the quark model and assume that

$$f_\pi = f_{d\bar{d}}\ ,$$
$$f_K = f_{d\bar{s}} = f_{d\bar{d}}(1+\epsilon)\ , \qquad (20)$$
$$f_{s\bar{s}} = f_{d\bar{s}}(1+\epsilon)\ ,$$

we have

$$\frac{f_K}{f_\pi} = 1+\epsilon \qquad (21)$$

and

$$\frac{a^{\pm}_{K^0 s\bar{s}}}{a^{\pm}_{K^0 d\bar{d}}}=\frac{f_{s\bar{s}}}{f_{d\bar{d}}}=1+2\epsilon+O(\epsilon^2)\ . \qquad (22)$$

Note that to first order in the SU(3)-breaking parameter ϵ, Eqs. (20) reproduce the Gell-Mann—Okubo-type relation for the pseudoscalar meson decay constants[22]:

$$4f_K-f_\pi=3f_{\eta_8}+O(\epsilon^2)\ . \qquad (23)$$

With $f_K/f_\pi=1.28$, we obtain

$$\epsilon=0.28\ , \qquad (24)$$

resulting in an important deviation from the SU(3) relation between $a^{\pm}_{K^0\pi^0}$ and $a^{\pm}_{K^0\eta}$. Using (22) we find

$$a^{\pm}_{K^0\eta_8}=\frac{1}{\sqrt{3}}a^{\pm}_{K^0\pi^0}(1+4\epsilon)\ ,$$
$$a^{\pm}_{K^0\eta_0}=\frac{2\sqrt{2}}{\sqrt{3}}a^{\pm}_{K^0\pi^0}(1+\epsilon)\ . \qquad (25)$$

From the $\eta'\to 2\gamma$ decay rate we know[23] that the system π^0-η-η' can be described by the pseudoscalar meson nonet with an η-η' mixing angle $\theta_P=10.5°$ and a possible negligibly small glueball component in the η'. Thus to a good approximation we have

$$\eta=\eta_8\cos\theta_P+\eta_0\sin\theta_P\ ,$$
$$\eta'=-\eta_8\sin\theta_P+\eta_0\cos\theta_P\ . \qquad (26)$$

Using (17) we get

$$\frac{\mathrm{Im}\langle K^0|\mathscr{L}_-^{\Delta S=2}|\bar{K}^0\rangle}{\mathrm{Im}\langle\pi^0|\mathscr{L}_-^{\Delta S=1}|K^0\rangle}=2f\left[\frac{\mathrm{Re}a^{+}_{K^0\pi^0}}{m_K{}^2-m_\pi{}^2}\right]\left[1+\frac{m_K{}^2-m_K{}^2}{3(m_K{}^2-m_\eta{}^2)}(1+\delta)^2+\frac{8}{3}\frac{m_K{}^2-m_\pi{}^2}{m_K{}^2-m_{\eta'}{}^2}(1+\delta')^2\right]\ , \qquad (27)$$

where

$$\delta=4\epsilon-2\sqrt{2}\sin\theta_p(1+\epsilon)\ ,$$
$$\delta'=\epsilon+\frac{\sin\theta_P}{2\sqrt{2}}(1+4\epsilon)\ . \qquad (28)$$

From (27) we see that for $\sin\theta_P>0$, the effect of SU(3) violation and η-η' mixing tend to cancel out largely, however, with $\epsilon=0.28$ the overall effect is still large and increases the pole contributions.

Numerically, we find

$$\delta=0.54\ ,$$
$$\delta'=0.41\ , \qquad (28')$$

which increase the η and η' contributions to (27) by a factor of 2.

The expression (27) is independent of the detailed form of the *CP*-violating $a^{-}_{\pi^0K^0}$ matrix element. f is the fraction of the penguin contribution to the total $a^{+}_{K^0\pi^0}$ given by (13). Calculations using PCAC (partial conservation of axial-vector current)[24] and the MIT bag model[25] indicate that f can be a large fraction of the total amplitude ($f=0.75$ according to Gilman and Wise[11]).

Using experimental values for the *CP*-conserving matrix elements, we obtain numerically.

$$\frac{\epsilon_m(\pi^0,\eta,\eta')}{\xi}=\frac{f}{7}c^2\times 10^5\times\left[\frac{f_\pi{}^2}{m_p{}^2}\right]^2\times\left[\frac{m_K}{1\ \mathrm{GeV}}\right]\times 4.17=17.3f\ . \qquad (29)$$

The above result [Eq. (29)] should be taken as a crude estimate for ϵ_m/ξ since other effects not included in the nonlinear phenomenological Lagrangian [i.e., SU(3) $\times$SU(3)-symmetry-breaking effects other than those given by the mass term and $\pi\pi$ final-state interactions, etc.] can modify the current-algebra Callan-Treiman-type relations. The successful calculation[18] of $K\to 3\pi$ decay amplitude in terms of the $K\to 2\pi$ decay rates (to within 20%) shows that such effects are small for the *CP*-conserving part. For the *CP*-violating part, these effects could be large, but it is not expected in any case to modify by a large factor the current-algebra relation between $K\to 2\pi$ and K-π amplitudes which is all we need to arrive at Eq. (29).

Thus the dispersive part due to π^0,η,η' intermediate states (the large-distance contribution) gives a large contribution to ϵ_m/ξ. With $f=0.75$, we then get

$$\frac{\epsilon'}{\epsilon}=-0.007\ , \qquad (30)$$

which is somewhat larger than experiment but of the correct sign. However, within experimental errors [Eq. (5)], our value for ϵ'/ϵ is not inconsistent with measurements and therefore the Higgs-boson-exchange model of *CP* violation is still acceptable

In obtaining (20) we have assumed pole dominance for the dispersive contribution to $K^0\bar{K}^0$ transition. High-mass-state contributions (other than η') should be suppressed by the factor $m_K{}^2/m_\phi{}^2$, etc., and need not to be included. The ρ^0 and A_1 contributions to the K^0-$\bar{K}^0$ transition given by Greenberg[26] actually vanish when the field-current identity is used. This is because the effective Lagrangian used in his calculation is of the current-current form[18] and contains a term

$$\mathscr{L}_+^{\Delta S=1}=C\frac{G_F}{\sqrt{2}}f_\pi\partial_\mu K^0V_{3\mu}\ ,$$

which on account of the conservation of the isovector vector currents ($\partial_\mu V_{3\mu}=0$) can be written as

$$\mathscr{L}_+^{\Delta S=1}=C\frac{G_F}{\sqrt{2}}f_\pi\partial_\mu(K^0V_{3\mu})\ .$$

Using the field-current identity (i.e., vector-meson dominance),

$$V_{3\mu}=\frac{m_\rho{}^2}{f_\rho}\rho_\mu^0+\text{other terms}\ .$$

The coupling of $\mathcal{L}_+^{\Delta S=1}$ to K^0 and ρ_μ^0 is then given by

$$\mathcal{L}_+^{\Delta S=1}=C\frac{G_F}{\sqrt{2}}f_\pi\frac{m_\rho{}^2}{d_\rho}\partial_\mu(K^0\rho_\mu^0)\ ,$$

which is a four-divergence and therefore can be discarded from the Lagrangian without affecting the physics. The A_1 contribution vanishes in the same manner. The K^0-ρ^0 transitions, if they exist, cannot be obtained from the field current identity and the Cronin effective Lagrangian,[18] and Greenberg's analysis is inconsistent since current conservation and the field-current identity imply that

$$\partial_\mu\rho_\mu^0=0\ ,$$

as shown by Wess and Zumino and by others.[27] Note that our proof does not rely on this condition. In any case, high-mass-state dispersive contributions to the *CP*-conserving part of the K^0-$\bar{K}^0$ transition cannot be as large as he claimed since *a posteriori* these terms should be relatively small compared to the *c*-quark contribution to the box diagram which constitutes the bulk of the K_S^0-K_L^0 mass difference as we now believe. The 2π intermediate states extended from $s=4m_\pi{}^2$ to 1 GeV^2 (a typical cutoff momentum) are also expected to give a small contribution because of the cancellation between contributions below and above the K^0 mass.[28]

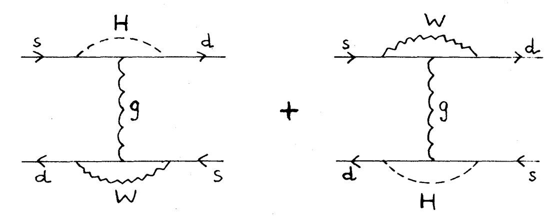

FIG. 3. One-gluon-exchange penguin contribution to *CP*-violating $\Delta S=2$ transition.

An additional short-distance contribution to $\mathrm{Im}\langle\bar{K}^0|\mathcal{L}_-^{\Delta S=2}|K^0\rangle$ not included in the analysis of D&S and of Donoghue *et al.* comes from the *CP*-violating $\Delta S=2$ transition induced by one-gluon exchange with the *CP*-conserving part of the *sd* gluon vertex given by the *W* exchange and the *CP*-violating part by the Higgs-boson exchange as given by (8). The *CP*-conserving part of the *SD* gluon vertex gives rise to the usual penguin interaction[24,29]

$$\mathcal{L}_+^{\Delta S=1}=-\left[\frac{G_F}{\sqrt{2}}\right]\left[\frac{g_s}{12\pi^2}\ln\frac{m_c{}^2}{\mu^2}\right]\bar{d}\gamma_\mu(1+\gamma_5)\lambda_a s(g_{\mu\nu}\partial^2-\partial_\mu\partial_\nu)A_\nu^a+\text{H.c.} \tag{31}$$

Second-order perturbation calculation with respect to the total $\Delta S=1$ interactions then gives the one-gluon-exchange contribution to the *CP*-violating K^--$\bar{K}^0$ transition represented by diagrams of Fig. 3. The contribution to ϵ_m/ξ from these diagrams can be expressed as

$$\frac{\epsilon_m}{\xi}(\text{one-gluon exchange})=-\left[\frac{1}{12\pi^2}\right]\left[\frac{G_F}{\sqrt{2}}\ln\frac{m_c{}^2}{\mu^2}\right]\frac{\mathrm{Re}\langle\pi^0|\mathcal{L}_+^{\Delta S=1}|K^0\rangle}{\mathrm{Re}\langle K^0|\mathcal{L}_+^{\Delta S=2}|\bar{K}^0\rangle}$$
$$\times 2(\sin\theta_C\cos\theta_C)\frac{\langle K^0|[i\bar{d}\sigma_{\mu\nu}(1-\gamma_5)\lambda_a s][\bar{d}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)(1+\gamma_5)\lambda_a s]|\bar{K}^0\rangle}{\langle\pi^0|[i\bar{d}\sigma_{\mu\nu}(1-\gamma_5)\lambda_a s]\frac{1}{q^2}[\bar{d}(q_\nu\gamma_\mu-q_\mu\gamma_\nu)\lambda_a d]|K^0\rangle}\ , \tag{32}$$

where summation over colors and integration over virtual-gluon momenta q are understood.

Using the identities[7]

$$\bar{d}(\gamma_\mu q_\nu-\gamma_\nu q_\mu)s=\left[\frac{m_s+m_d}{2}\right]\bar{d}\sigma_{\mu\nu}s+\frac{i}{2}\epsilon_{\lambda\mu\nu\tau}(k'+k)_\lambda\bar{d}\gamma_\tau\gamma_5 s\ , \tag{33a}$$

$$\bar{d}(\gamma_\mu q_\nu-\gamma_\nu q_\mu)\gamma_5 s=\left[\frac{m_d-m_s}{2}\right]\bar{d}\sigma_{\mu\nu}\gamma_5 s+\frac{i}{2}\epsilon_{\lambda\mu\nu\tau}(k'+k)_\lambda\bar{d}\gamma_\tau s\ , \tag{33b}$$

where k',k are, respectively, the d and s quark momenta and m_s,m_d are the constituent quark masses. If one neglects the second terms in (33a) and (33b) and SU(3)-violation effects, the operator $\bar{d}(\gamma_\mu q_\nu-\gamma_\nu q_\mu)\gamma_5 s$ will not contribute to the K^0-$\bar{K}^0$ transition which is then given by the same operator responsible for the *CP*-violating direct K^0-π^0 transition apart from the nonlocality of the latter due to the gluon propagator. To estimate (32) we shall assume that these matrix elements are peaked at some value of q^2 given by the mean transverse momentum squared of quark in K and π:

$$\langle q^2\rangle=-\langle p_T{}^2\rangle\ .$$

The ratio of the matrix elements in (32) is then roughly given by $\langle p_T{}^2\rangle$ independent of the detailed form of the matrix elements. We have

$$\frac{\epsilon_m}{\xi}(\text{one-gluon exchange})$$

$$\simeq -\left[\frac{1}{12\pi^2}\ln\frac{m_c^2}{\mu^2}\sin\theta_C\cos\theta_C\right]\left[\frac{c}{3.52}\right] \times\left[\frac{f_\pi^2}{m_p^2}\right]\left[\frac{\langle p_T^2\rangle}{m_p^2}\right]\left[\frac{m_K}{1\text{ GeV}}\right]\times 10^5$$

$$\simeq -0.2\ , \tag{34}$$

for $m_c = 1.5$ GeV, $\mu^2 = 1$ GeV^2, and a typical $\langle p_T^2\rangle = 0.25$ GeV^2. This gives a very small contribution to ϵ_m/ξ of the order 1–2% relative to the dispersive contribution obtained above [Eq. (29)] but of the opposite sign. Note that our expression for ϵ_m/ξ given in (34) is independent of α_s as well as QCD enhancement factors and the *CP*-violating parameters [the quantity $\tilde{f}$ defined in Eq. (9)].

Thus in models with *CP* violation due to Higgs-boson exchange, the short-distance contributions to ϵ_m/ξ are quite small and ϵ_m/ξ is given mainly by the large-distance dispersive contribution obtained as in (29). Allowing for other possible high-energy dispersive contributions, as far as order of magnitude is concerned, we can say that ϵ_m/ξ can be quite large and can produce ϵ'/ϵ consistent with present data unless accidental cancellation occurs between the low-energy and high-energy dispersive contributions.

[1]M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976).

[3]T. D. Lee, Phys. Rev. D 8, 1226 (1973); Phys. Rep. 9, 143 (1979).

[4]S. Weinberg, Phys. Rev. Lett. 31, 657 (1976).

[5]M. B. Gavela *et al.*, Phys. Lett. 109B, 215 (1982) and references to previous works cited therein.

[6]N. Deshpande, Phys. Rev. D 23, 2647 (1981).

[7]A. Sanda, Phys. Rev. D 23, 2647 (1981).

[8]J. F. Donoghue, J. S. Hagelin, and B. R. Holstein, Phys. Rev. D 25, 195 (1982).

[9]Dispersive treatment of K^0-$\bar{K}^0$ transition is given in R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

[10]D. Chang, Phys. Rev. D 25, 1318 (1982).

[11]F. J. Gilman and M. B. Wise, Phys. Rev. D 20, 2392 (1979).

[12]L. Wolfenstein, Phys. Rev. Lett. 13, 180 (1964).

[13]For a review on the measurements on *CP* violation, see K. Kleinknecht, Annu. Rev. Nucl. Sci. 26, 1 (1976).

[14]K. G. Wilson, Phys. Rev. 179, 1499 (1969).

[15]S. Raby, Phys. Rev. D 13, 2594 (1976).

[16]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); V. de Alfaro, S. Fubini, G. Furlan, and C. Rossetti, in *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973), pp. 110 and 375.

[17]C. T. Hill and G. C. Ross, Nucl. Phys. B171, 141 (1980).

[18]J. A. Cronin, Phys. Rev. 161, 1483 (1969).

[19]A. J. Cantor, Phys. Rev. D 3, 3195 (1969); 3, 3205 (1969).

[20]S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969). See also, S. Coleman, in *Theory and Phenomenology in Particle Physics,* proceedings of the 1968 International Summer School, "Ettore Majorana," Erice, Italy, edited by A. Zichichi (Academic, New York, 1969), Part B, p. 649.

[21]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[22]S. Raby, remark given in footnote 37 of Ref. 15.

[23]G. S. Abrams *et al.*, Phys. Rev. Lett. 43, 477 (1979).

[24]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Nucl. Phys. B120, 315 (1977).

[25]J. F. Donoghue, E. Golowich, W. A. Ponce, and B. R. Holstein, Phys. Rev. D 21, 186 (1980).

[26]D. F. Greenberg, Nuovo Cimento 56A, 597 (1968).

[27]J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967) and other references cited therein.

[28]This is confirmed by a recent calculation of Barger *et al.* See the discussion given in Ref. 20 of V. Barger, W. F. Long, E. Ma, and A. Pramudita Phys. Rev. D 25, 1860 (1982).

[29]M. B. Wise and E. Witten, Phys. Rev. D 20, 1216 (1979).