Nonminimal coupling and Bianchi type-I cosmologies

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Bianchi type-I exact solutions are obtained in two models involving nonminimal coupling of gravitation and other fields.

I. INTRODUCTION

The Lagrangian for interacting gravitational and other fields is generally constructed using the minimal-coupling principle. Only such terms are included that give unambiguously the curved-space generalization of the flat-Minkowski-space form of the equations of motion for nongravitational fields. Terms that contain the curvature of space-time are usually disregarded. The validity of such an assumption is open to question.

Recently, Frøyland[1] and the present group of authors[2] have considered a model of a massless conformally invariant scalar field interacting with gravitation in a nonminimal manner. Static as well as Bianchi type-I cosmological solutions were obtained.

In this paper we obtain Bianchi type-I cosmological solutions for the following cases.

(i) The coupled Einstein-Maxwell—conformally-invariant-scalar field equations with minimal coupling between electromagnetic and gravitational fields.

(ii) The gauge-dependent theory involving nonminimal coupling of the electromagnetic vector potential with gravity.

The order of presentation is as follows. In Secs. II and III, we solve and discuss the above models, Sec. IV presents the conclusions.

II. COUPLED EINSTEIN—MAXWELL—CONFORMALLY-INVARIANT-SCALAR FIELDS

The system of coupled Einstein-Maxwell-conformally-invariant-scalar fields can be described by the Lagrangian[3]

$$L=\sqrt{-g}\left[\frac{1}{2\kappa}\left[1-\frac{\kappa\phi^2}{6}\right]R+\tfrac{1}{2}\partial_\mu\phi\cdot\partial^\mu\phi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{2.1}$$

where R, ϕ, and $F_{\mu\nu}$ denote the curvature scalar, the scalar field, and the electromagnetic field, respectively.

The equations of motion may be written as

$$R^\mu{}_\nu-\tfrac{1}{2}g^\mu{}_\nu R=-\kappa(S^\mu{}_\nu+E^\mu{}_\nu)\,, \tag{2.2}$$

$$F^{\alpha\beta}{}_{;\beta}=0,\quad {}^*F^{\alpha\beta}{}_{;\beta}=0\,, \tag{2.3}$$

$$\phi^\mu{}_{;\mu}+\frac{R}{6}\phi=0\,, \tag{2.4}$$

where the stress-energy tensor of the conformally-invariant-scalar field is given by

$$S^\mu{}_\nu=\partial^\mu\phi\partial_\nu\phi-\tfrac{1}{2}\delta^\mu{}_\nu\partial_\alpha\phi\partial^\alpha\phi+\tfrac{1}{6}[\delta^\mu{}_\nu\Delta_\alpha\Delta^\alpha(\phi^2)-G^\mu{}_\nu\phi^2]\,. \tag{2.5}$$

Here $G^\mu{}_\nu$ is the Einstein tensor, and ∂^μ and ∇^μ are ordinary and covariant derivatives. The stress-energy tensor of the electromagnetic field is given by

$$E^\mu{}_\nu=-(F^{\mu\alpha}F_{\nu\alpha}-\tfrac{1}{4}\delta^\mu{}_\nu F^{\alpha\beta}F_{\alpha\beta})\,. \tag{2.6}$$

Using (2.5), Eq. (2.1) can be written conveniently as

$$G^\mu{}_\nu f(\phi^2)=\kappa(-\mu^\mu\phi\partial_\nu\phi+\tfrac{1}{2}\delta^\mu{}_\nu\partial_\alpha\phi\partial^\alpha\phi)+\delta^\mu{}_\nu\nabla_\alpha\nabla^\alpha f-\nabla^\partial\nabla_\nu f-\kappa E^\mu{}_\nu\,, \tag{2.7}$$

where

$$f(\phi^2)=1-\frac{\kappa}{6}\phi^2\,. \tag{2.8}$$

By forming the trace of Eq. (2.8) and using (2.3), one obtains

$$R=0\,. \tag{2.9}$$

We can now use this relation to rewrite Eqs. (2.8) and (2.3) in the forms

$$R^\mu{}_\nu f=\delta^\mu{}_\nu u_\alpha u^\alpha-4u^\mu u_\nu+2uu^\mu{}_{;\nu}-\kappa E^\mu{}_\nu \tag{2.10}$$

and

$$u^\mu{}_{;\mu}=0\,, \tag{2.11}$$

where

$$u=\left[\frac{\kappa}{6}\right]^{1/2}\phi \tag{2.12}$$

and

$$f=1-u^2\,. \tag{2.13}$$

We take the metric corresponding to the Bianchi type-I model in the form

$$ds^2=dt^2-A^2dx^2-B^2dy^2-C^2dz^2\,, \tag{2.14}$$

where A, B, and C are functions of t only. We assume that u and $F^{\mu\nu}$ share this symmetry of space-time.

Now $u=u(t)$ and (2.1) give

$$\dot{u}=-\frac{K_2}{ABC}\,, \tag{2.15}$$

where the overdot means d/dt.

Next, introducing the vector potential W_μ by

$$F_{\mu\nu}=W_{\nu;\mu}-W_{\mu;\nu}\,, \tag{2.16}$$

the equation for $^*F^{\mu\nu}$ becomes an identity. Also from (2.10) we see that $E^\mu{}_\nu$ must be diagonal too.

Equations (2.5) then lead to three possible cases:

$$\begin{aligned}&\text{(i)}\ \ F_{01},F_{23}\neq 0\,,\\ &\text{(ii)}\ \ F_{02},F_{31}\neq 0\,,\\ &\text{(iii)}\ \ F_{03},F_{12}\neq 0\,.\end{aligned} \tag{2.17}$$

Without loss of generality, we may consider only case (iii), $F_{03},F_{12}\neq 0$. Also, since the electromagnetic stress-energy tensor is duality invariant,[4] we can choose $F_{12}=0$. From (2.3) we have

$$F^{30}=-\sqrt{2}K_1/ABC\,. \tag{2.18}$$

Substituting (2.18) into (2.6) gives the following nonvanishing components of the stress-energy tensor of the electromagnetic field:

$$E^0_0=-E^1_1=+E^2_2=-E^3_3=\frac{K_1{}^2}{A^2B^2}\,. \tag{2.19}$$

Substituting (2.14), (2.15), and (2.19) into (2.10), one obtains the following explicit form of the field equations:

$$f(\ddot{A}/A+\ddot{B}/B+\ddot{C}/C)=-3\dot{u}^2+\dot{f}(\dot{A}/A+\dot{B}/B+\dot{C}/C)-\kappa\frac{K_1{}^2}{A^2B^2}\,, \tag{2.20}$$

$$f[(\dot{A}/A)^\cdot+\dot{A}/A(\dot{A}/A+\dot{B}/B+\dot{C}/C)]=\dot{u}^2-\dot{f}(\dot{A}/A)+\kappa\frac{K_1{}^2}{A^2B^2}\,, \tag{2.21}$$

$$f[(\dot{B}/B)^\cdot+\dot{B}/B(\dot{A}/A+\dot{B}/B+\dot{C}/C)]=\dot{u}^2-\dot{f}(\dot{B}/B)+\kappa\frac{K_1{}^2}{A^2B^2}\,, \tag{2.22}$$

$$f[(\dot{C}/C)^\cdot+\dot{C}/C(\dot{A}/A+\dot{B}/B+\dot{C}/C)]=\dot{u}^2-\dot{f}(\dot{C}/C)-\kappa\frac{K_1{}^2}{A^2B^2}\,. \tag{2.23}$$

Equations (2.20)–(2.23) are, however, all not independent owing to Eq. (2.9). To obtain the solution we consider the set of equations (2.21)–(2.23). Using (2.15), one obtains after integration

$$ABCf[\ln\sqrt{f}C]^\cdot=(K_3{}^2-\kappa K_1{}^2fC^2)^{1/2}\,, \tag{2.24}$$

where K_3 is an integration constant. Here we are considering the positive root only. The sum of (2.21) and (2.23) yields on integration

$$[\ln(\sqrt{f}A)+\ln(\sqrt{f}C)]^\cdot=-\frac{K_4}{fABC}\,, \tag{2.25}$$

where K_4 is a constant of integration. Also the sum of (2.22) and (2.23) gives on integration

$$[\ln(\sqrt{f}B)+\ln(\sqrt{f}C)]^\cdot=-\frac{K_5}{fABC}\,, \tag{2.26}$$

where K_5 is a constant of integration.

If one knows the solution of (2.24), then one can determine A and B from (2.25) and (2.26). To obtain the solution, we introduce a new function ψ such that

$$C=\frac{2K_3}{\sqrt{\kappa}}\frac{1}{\sqrt{f}}(\psi+K_1{}^2/\psi)^{-1}\,. \tag{2.27}$$

Substituting (2.27) into (2.24) gives

$$(\ln\psi)^\cdot=K_3/fABC\,. \tag{2.28}$$

Using (2.28), one then obtains easily from (2.25) and (2.26), respectively,

$$A=\frac{\sqrt{\kappa}}{2K_3}K_6\frac{(\psi+K_1{}^2\psi^{-1})}{\sqrt{f}}\psi^{K_4/K_3} \tag{2.29}$$

and

$$B=\frac{\sqrt{\kappa}}{2K_3}K_7\frac{(\psi+K_1{}^2\psi^{-1})}{\sqrt{f}}\psi^{K_5/K_3}\,, \tag{2.30}$$

where K_6 and K_7 are integration constants. Using (2.27), (2.29), and (2.30) in (2.19), one finds

$$3K_1{}^2+K_2{}^2-K_4K_5=0\,. \tag{2.31}$$

From (2.15) and (2.28) one obtains after integration

$$u=\frac{\psi^{K_2/K_3}-\psi^{-K_2/K_3}}{\psi^{K_2/K_3}+\psi^{-K_2/K_3}}\,, \tag{2.32}$$

where we have taken the constant of integration equal to 1. Using (2.32) in (2.17), (2.29), and (2.30), one obtains

$$A=a(\psi^{K_2/K_3}+\psi^{-K_2/K_3})(\psi+K_1{}^2\psi^{-1})\psi^{K_4/K_3}\,, \tag{2.33}$$

$$B=b(\psi^{K_2/K_3}+\psi^{-K_2/K_3})(\psi+K_1{}^2\psi^{-1})\psi^{K_5/K_3}\,, \tag{2.34}$$

$$C=c(\psi^{K_2/K_3}+\psi^{-K_2/K_3})(\psi+K_1{}^2\psi^{-1})^{-1}\,, \tag{2.35}$$

where a, b, and c are constants. Using (2.33)–(2.35) in (2.28), one obtains after integration

$$\begin{aligned}-px+q&=\int[\psi^\alpha+\psi^\beta+K_1{}^2(\psi^{\alpha-2}+\psi^{\beta-2})]d\psi\\ &=\frac{\psi^{\alpha+1}}{\alpha+1}+\frac{\psi^{\beta+1}}{\beta+1}+K_1{}^2\left[\frac{\psi^{\alpha-1}}{\alpha-1}+\frac{\psi^{\beta-1}}{\beta-1}\right],\\ &\alpha,\beta\neq\pm 1\,,\\ &\alpha=\frac{K_2+K_4+K_5}{K_3},\quad \beta=\frac{-K_2+K_4+K_5}{K_3}\,.\end{aligned} \tag{2.36}$$

$p=abc$ and q is a constant of integration. From (3.3) one gets, using (3.19)–(3.21),

$$F^{03}=\frac{\sqrt{2}K_1}{p}\times(\psi^{K_2/K_3}+\psi^{-K_2/K_3})^3(\psi+K_1{}^2\psi^{-1})^{(K_4+K_5)/K_3-1}\ . \tag{2.37}$$

The set of Eqs. (3.19)–(3.21) with (3.18), (3.22), (3.23), and (3.17) gives the complete solution of the Einstein–Maxwell–conformally-invariant-scalar field.

The solutions (2.33)–(2.35) correspond to the spatially homogeneous anisotropic Bianchi-I universe having a conformally-invariant-scalar field and the electromagnetic fields as the energy source. The role of each field is easily distinguishable.

For $K_2=0$, one finds from (2.36) that the scalar field vanishes and $\alpha=\beta$. One obtains then the Bianchi-I electromagnetic universe, the detailed discussion of which is given by Carmeli, Charach, and Malin.[5]

For $K_1=0$, one has a conformally-invariant-scalar field only as an energy source. This problem has been treated by us in a previous paper.[2] When both $K_1=K_2=0$, Eq. (2.36) yields

$$\frac{\psi^{\alpha+1}}{\alpha+1}=\tfrac{1}{2}px+\tfrac{1}{2}q\ , \tag{2.38}$$

when $\alpha=(K_4+K_5)/K_3$. Substituting (2.38) into (2.13)–(2.15), one easily gets the metric

$$ds^2=dt^2-a_1t^{2p_1}dx^2-a_2t^{2p_2}dy^2-a_3t^{2p_3}dz^2\ , \tag{2.39}$$

where

$$\begin{aligned} p_1&=(K_3+K_4)/(K_3+K_4+K_5)\ ,\\ p_2&=(K_3+K_5)/(K_3+K_4+K_5)\ ,\\ p_3&=-K_3/(K_3+K_4+K_5)\ . \end{aligned} \tag{2.40}$$

Since

$$p_1+p_2+p_3=p_1{}^2+p_2{}^2+p_3{}^2=1\ ,$$

Eq. (2.39) with (2.40) represents the well-known Kasner[6] metric.

For the combined electromagnetic and conformally-invariant-scalar field, the structure of singularities depends on the field parameters K_1, K_2, K_3, K_4, and K_5. As $t\to 0$, $u\to 0$, $\psi\to 0$, the volume V of the $t=$constant three-dimensional hypersurface$\to 0$. The electromagnetic field becomes the dominant energy source. As the energy-momentum tensor of the electromagnetic field is spatially anisotropic, the expansion factor C along the electric field decreases while those along the orthogonal directions increase. At this stage the model has Kasner-type behavior. As $t\to\infty$, $u\to\pm1$, $\psi\to\infty$, $V\to\infty$. At this limit, the scalar field is the dominant energy source. The energy-momentum tensor of the scalar field is spatially isotropic and the expansion factors tend to an isotropic limit. The model tends to the open Friedmann type. The model thus evolves from an initial Kasner-type electromagnetic universe to the final isotropic open Friedmann-type universe, through the quasi-isotropic intermediate stages.

III. NONMINIMALLY COUPLED ELECTROMAGNETIC FIELD

Nonminimal coupling of the vector potential with gravity in the spatially homogeneous cosmological models was investigated in an interesting paper by Novello and Salim.[7] A special case of their Lagrangian is

$$L=\sqrt{-g}\left[\frac{1}{\kappa}(1+\lambda W_\mu W_\nu g^{\mu\nu})R-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]\ , \tag{3.1}$$

in which

$$F_{\mu\nu}\equiv W_{\nu,\mu}-W_{\mu,\nu}\ , \tag{3.2}$$

where λ is a constant with the same dimensionality as Einstein's coupling constant κ.

The equations of motion for $g_{\mu\nu}$ and $F_{\mu\nu}$ obtained from (3.1) are

$$(1+\lambda W^2)(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R)-\lambda\Box W^2 g_{\mu\nu}+\lambda W^2{}_{;\mu;\nu}+\lambda R W_\mu W_\nu=-E_{\mu\nu}\ , \tag{3.3}$$

$$F^{\mu\nu}{}_{;\nu}=-\frac{\lambda}{\kappa}RW^\mu\ . \tag{3.4}$$

The trace of (1.3) gives

$$R=-3\lambda\Box W^2\ . \tag{3.5}$$

The wave equation for the vector potential W^μ is given by

$$\Box W^\mu+R^\mu{}_\alpha W^\alpha-(W^\nu{}_{;\nu})^{;\mu}=\frac{3\lambda^2}{\kappa}(W^2)W^\mu\ . \tag{3.6}$$

However, in their analysis of particular models, they considered the case when both the electric and magnetic vectors are null. In other words, their vector potential W^μ is curl-free.[7] Still, the nonminimal coupling is found to be responsible for a Friedmann-type cosmos with a minimum radius.

In this paper we investigate the effect of this nonminimal coupling when the vector potential is not curl-free, but the Maxwell equations remain linear.

Since the Maxwell equations are linear, we set $R=0$, which implies that

$$\Box W^2=0\ . \tag{3.7}$$

Using Eq. (3.7), the field equations (3.3) take a very simple form,

$$\Omega R^\mu{}_\nu+\Omega^{;\mu}{}_{;\nu}=-\kappa E^\mu{}_\nu\ , \tag{3.8}$$

where

$$\Omega=1+\lambda W^2\ , \tag{3.9}$$

and the wave equation (3.7) reduces to

$$\Box W^\mu+R^\mu{}_\alpha W^\alpha-(W^\nu{}_{;\nu})^{;\mu}=0\ . \tag{3.10}$$

Equation (3.4) goes over to the usual Maxwell equation

$$F^{\mu\nu}{}_{;\nu}=0\ . \tag{3.11}$$

For the Bianchi type-I model, the calculations are similar to those in the previous section. Using the same notation as in Sec. II for the electromagnetic field, we get

$$\Omega = -\frac{K_2}{ABC} , \tag{3.12}$$

$$\Omega\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right] = -\Omega - \frac{\kappa K_1{}^2}{A^2B^2} , \tag{3.13}$$

$$\Omega\left[\frac{\ddot{A}}{A}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\right] = -\dot{\Omega}\frac{\dot{A}}{A}\frac{\kappa K_1{}^2}{A^2B^2} , \tag{3.14}$$

$$\Omega\left[\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{A}}{BA}+\frac{\dot{B}\dot{C}}{BC}\right] = -\dot{\Omega}\frac{\dot{B}}{B}+\frac{\kappa K_1{}^2}{A^2B^2} , \tag{3.15}$$

$$\Omega\left[\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{A}}{CA}+\frac{\dot{C}\dot{B}}{CB}\right] = -\dot{\Omega}\frac{\dot{C}}{C}-\frac{\kappa K_1{}^2}{A^2B^2} . \tag{3.16}$$

Equations (3.13)–(3.16) are all not independent due to the relation

$$R=0 . \tag{3.17}$$

Thus when we work out the solution we can omit Eq. (3.13).

Equation (3.16) can be expressed, using (3.13), as

$$2\left[\Omega\sqrt{-g}\,\frac{(C\sqrt{\Omega})^{\cdot}}{C\sqrt{\Omega}}\right]^{\cdot} = -\frac{2K_1{}^2}{AB}C ,$$

which on integration yields

$$\Omega\sqrt{-g}\,\frac{(C\sqrt{\Omega})^{\cdot}}{C\sqrt{\Omega}} = [K^2-\kappa K_1{}^2(C\sqrt{\Omega})^2]^{1/2} . \tag{3.18}$$

K is a constant of integration. We take here only the positive sign before the radical. To integrate further we make the substitution

$$C=\frac{2K_1}{\sqrt{\kappa}}\Omega^{-1/2}(v+K_1{}^2v^{-1})^{-1} , \tag{3.19}$$

where $v=v(t)$.

Substituting (3.19) into (3.18) gives

$$\frac{\dot{v}}{v} = -\frac{K}{\sqrt{-g}\,\Omega} . \tag{3.20}$$

Equations (3.14)–(3.16) give

$$\frac{(A\sqrt{\Omega})^{\cdot}}{A\sqrt{\Omega}}+\frac{(C\sqrt{\Omega})^{\cdot}}{A\sqrt{\Omega}} = -\frac{K_3}{\sqrt{-g}\,\Omega} , \tag{3.21}$$

$$\frac{(B\sqrt{\Omega})^{\cdot}}{A\sqrt{\Omega}}+\frac{(C\sqrt{\Omega})^{\cdot}}{A\sqrt{\Omega}} = -\frac{K_4}{\sqrt{-g}\,\Omega} , \tag{3.22}$$

where K_3 and K_4 are constants of integration.

One gets from Eqs. (3.20)–(3.22), after further integration,

$$\Omega AC = K_5 v^{K_3/K} , \tag{3.23}$$

$$\Omega BC = K_6 v^{K_4/K} , \tag{3.24}$$

where K_5 and K_6 are constants of integration. The equations (3.19), (3.29), and (3.24) will satisfy Eq. (3.14) if $R=0$ is satisfied. From $R=0$ one obtains the relation of constraints,

$$4K^2-4K_3K_4+3K_2{}^2=0 . \tag{3.25}$$

From Eqs. (3.12) and (3.20) we obtain Ω explicitly as

$$\Omega = v^{K_2/K} = v^{\lambda/K} . \tag{3.26}$$

Since $\Omega=1+\lambda W^2 \to 1$ as $\lambda\to 0$, we have chosen $K_2=\lambda$ and the constant of integration equal to 1. It is worth noticing here that the coupling constant is independent of the constant K_1 related with the electromagnetic field. To complete the solution we determine the function $v(t)$ from Eq. (3.20), using (3.23), (3.24), and (3.26) as

$$+\alpha t+\beta = \int dv\, v^{[2(K_3+K_4-K)-\lambda]/2K}(v+K_1{}^2v^{-1}) , \tag{3.27}$$

where β is a constant of integration.

The system of equations (3.20), (3.23), and (3.24) with Eq. (3.27) completely determines the model. Equation (2.18) specifies the electric field.

As is evident from Eq. (3.27), there exist different possible models depending on

$$\mu = \frac{2(K_3+K_4+K)-\lambda}{2K} \neq -2,0$$

or $\mu=-2,0$.

When $\lambda=0$, the solutions correspond to the Bianchi type-I electric universe.[3] When $K_1=0$, the electric field vanishes. In this case the four-potential W_μ is the gradient of a scalar field. We shall consider all possible cases here.

Case 1. $\mu\neq 2,0$: Then, from Eq. (3.27), one obtains

$$v \sim t^{1/(\mu+2)} = t^{2K/[2(K_3+K_4+K)-\lambda]} . \tag{3.28}$$

Equation (3.26) yields

$$\Omega \sim t^{2\lambda/[2(K_3+K_4+K)-\lambda]} . \tag{3.29}$$

From Eqs. (3.20), (3.23), and (3.24), we find, using Eqs. (3.28) and (3.29),

$$A \sim t^{2(K_3+K)-\lambda/[2(K_3+K_4+K)-\lambda]} , \tag{3.30}$$

$$B \sim t^{2(K_4+K)-\lambda/[2(K_3+K_4+K)-\lambda]} , \tag{3.31}$$

$$C \sim t^{-(\lambda+2K)/[2(K_3+K_4+K)-\lambda]} . \tag{3.32}$$

When $\lambda\to 0$, Eqs. (3.30)–(3.32) correspond to Kasner's[6] models. If we write

$$\begin{aligned} p_1 &= 2(K_3+K)-\lambda/[2(K_3+K_4+K)-\lambda] , \\ p_2 &= 2(K_4+K)-\lambda/[2(K_3+K_4+K)-\lambda] , \\ p_3 &= -(\lambda+2K)/[2(K_3+K_4+K)-\lambda] , \\ p_4 &= 2\lambda/2(K_3+K_4+K)-\lambda . \end{aligned} \tag{3.33}$$

Then, one finds that, using (3.25),

$$\sum_{i=1}^{4} p_i = \sum p_i{}^2 = 1 . \tag{3.34}$$

Novello and Salim[8] found an equivalent solution using a Kasner-type ansatz.

If $K_3 = K_4 = -2K = -\lambda/2$, one obtains t = constant hypersurfaces conformally flat.

The model corresponds to the flat Friedman-type with isotropic expansion.

Case 2. $\mu = -2$: In this case, from Eq. (3.27) one gets

$$\begin{aligned} v &\sim e^{\alpha t} , \\ \Omega &\sim e^{(\lambda\alpha/2K)t} , \\ A &\sim e^{(1+K_3/K)\alpha t} , \\ B &\sim e^{(1+K_4/K)\alpha t} , \\ C &\sim e^{-[(\lambda+2K)/2K]\alpha t} . \end{aligned} \tag{3.35}$$

The expansion factor along z depends on the coupling constant λ, while those along the orthogonal directions are independent of λ.

The case $\mu = 0$ does not occur when $K_1 = 0$.

IV. CONCLUSION

Bianchi type-I cosmologies for nonminimal coupling between gravitation and other fields were obtained for two models. The physical consequences in each case have already been discussed in the earlier sections.

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