PHYSICAL REVIEW D VOLUME 28, NUMBER 8 15 OCTOBER 1983

Testing for a cosmological influence on local physics using atomic and gravitational clocks

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(Received 24 February 1983)

The existence of a possible influence of the large-scale structure of the Universe on local physics is discussed. A particular realization of such an influence is discussed in terms of the behavior in time of atomic and gravitational clocks. Two natural categories of metric theories embodying a cosmic influence exist. The first category has geodesic equations of motion in *atomic units*, while the second category has geodesic equations of motion in *gravitational units*. Equations of motion for test bodies are derived for both categories of theories in the appropriate parametrized post-Newtonian limit and are applied to the Solar System. Ranging data to the Viking lander on Mars are of sufficient precision to reveal (i) if such a cosmological influence exists at the level of Hubble's constant, and (ii) which category of theories is appropriate for a description of the phenomenon.

I. INTRODUCTION

In the last few decades several theories have been proposed which couple cosmology with local physics. One characteristic which these theories have in common is that they predict a variation of Newton's constant with cosmic time, an effect usually referred to as "G dot." In Secs. II and III of this paper we discuss the fact that these theories fall naturally into two distinct categories. In Secs. IV and V we analyze each of the categories of theories in turn and calculate equations of motion for test bodies in the post-Newtonian limit. The major objective of this paper is to discuss and compare the predictions of these two categories of theories for the motion of Solar System bodies. Such an analysis is particularly timely in view of the recent acquisition of ranging data to the Viking lander on Mars, which are an order of magnitude more accurate than any Solar System astrometric data gathered thus far. In Sec. VII, we find that this increased accuracy, together with the larger time baseline of astrometric data, should increase the sensitivity of the Solar System dynamic model to G-dot effects by almost two orders of magnitude. For the first time it should now be possible to directly and unambiguously detect such a cosmic influence on local physics if it exists at the level of Hubble's constant. Moreover, these data will also allow one to distinguish between the two categories of theories discussed in this paper on a purely observational basis.

II. MINIMAL COUPLING OF COSMOLOGY WITH LOCAL PHYSICS

Every complete dynamical theory defines its own natural set of units standards. For example, the natural time unit of any gravity theory is the orbital period of a macroscopic body about a central mass, while the natural time unit of quantum electrodynamics is the frequency of a photon emitted in an atomic transition. Such natural generators of time-units standards are called clocks. A central question of physics is whether or not all such fundamental clocks are mutually commensurable,[1,2] i.e., whether or not the ratio of their rates is constant (for example, independent of cosmic epoch). This question can be answered directly by comparison of the time measures of two such clocks. This paper provides a framework for such a comparison.

If cosmology does directly influence local physics, then one would expect the coupling coefficients between different physical fields to vary on a cosmic time scale, thus making the fundamental clocks of Nature incommensurable. A minimal form of such incommensurability is one in which all microscopic clocks, i.e., atomic clocks, nuclear clocks, etc., are commensurate with each other while gravitational clocks are not commensurable. This choice is consistent with the assumption that gravitation is the only significant interaction on a cosmological scale. One writes[1,2]

$$\frac{ds_E}{ds_A}=\varphi \, , \tag{1}$$

where ds_E is a physical time interval measured with a gravitational clock (Einstein clock or E clock), while ds_A is the same physical time interval measured with an atomic clock (A clock). The natural time unit of an E clock is called an E unit, while the natural time unit of an A clock is called an A unit.

Constant φ means that the two clocks are commensurate, while variable φ means that the two clocks are not commensurate. While the dynamics of φ may be treated like a simple scalar field, as is done in some of the A theories mentioned below, the dynamics of φ may also be so inherently complex as, for example, the dynamics of a coefficient of friction.[2] We make no assumptions concerning the dynamics of φ in this paper.

III. METRIC THEORIES

Analysis of Solar System astrometric data is customarily carried out for "metric theories." By definition,[3] a metric theory is any theory possessing a mathematical representation in which (i) spacetime has a metric, (ii) world lines of test bodies are geodesics of that metric, (iii) nongravitational laws in any freely falling frame reduce to the laws of special relativity. This is equivalent to the assertion that in this representation one has $u^{\alpha}{}_{;\sigma}u^{\sigma}=0$ and $T^{\alpha\sigma}{}_{;\sigma}=0$ where u^{α} and $T^{\alpha\sigma}$ are the four-velocity and stress-energy tensor, respectively. In this representation the equation of motion for test bodies is the geodesic equation

$$\frac{d^2x^{\alpha}}{ds^2}+\Gamma^{\alpha}_{\lambda\sigma}\frac{dx^{\lambda}}{ds}\frac{dx^{\sigma}}{ds}=0 . \tag{2}$$

The geodesic equation (2) is not invariant under a change of units.[4] In the last section we introduced two natural systems of units. Thus one must specify the particular system of units in which the geodesic equation is assumed to hold. "A mathematical representation in which world lines of test bodies are geodesics" is equivalent to "a choice of units in which world lines of test bodies are geodesics." Consequently, all metric theories can be separated into at least two categories, namely, those theories geodesic in E units and those theories geodesic in A units.

The Solar System is currently being used to test for the subtle dynamical effects predicted by different metric theories of gravity.[3] Modern experiments model gravitation in the so-called parametrized post-Newtonian (PPN) formalism, a "theory of theories" in which terms appearing in the spacetime metric depend on undetermined, dimensionless coefficients (parameters) whose values are adjusted to fit the existing Solar-System astrometric data sets. When using Solar System astrometric data to test metric theories in the PPN framework, one assumes the equation of motion to be the geodesic equation. However, one is also implicitly assuming an underlying set of units standards, viz., those units in which the paths of test bodies are geodesics. Since Solar System timing data are gathered using atomic clocks and therefore are in A units, the equations of motion used in the Solar System model will not be the geodesic equation for theories which are metric in E units. This difference, as we discuss below, will allow one to distinguish between the "geodesic in A units" and "geodesic in E units" assumptions using the Solar System astrometric data as the basis for discrimination.

There is a similarity between the PPN formalism and the formalism used in this paper to describe the cosmic influence on local physics. Just as the PPN formalism is independent of the details of the particular metric theory of gravity (i.e., independent of the gravitational field equations), so too is this formalism largely independent of the details of the particular model by which cosmology is coupled to local physics. As we show below, for each case of E theories or A theories, we find one additional parameter to add to the PPN parameter set. Solar System astrometric data can now be used to search for the values of these parameters in addition to the usual PPN parameters. The values of these two parameters are a direct measure of the amount of cosmic influence on local physics.

IV. A THEORIES

A theories are defined to be gravitation theories whose test bodies follow geodesics in A units:

$$A \text{ units: } T^{\alpha\sigma}{}_{;\sigma}=0,\ \ u^{\alpha}{}_{;\sigma}u^{\sigma}=0 . \tag{3}$$

In the last few decades several such theories have been proposed, formulated in such a way as to couple cosmology with local physics. The archtype is the scalar-tensor gravity theory of Brans and Dicke,[5] but examples of other such theories are the scalar-tensor theories of Bergmann,[6] Nordtvedt,[7] and Wagoner,[8] the vector-tensor theories of Hellings, Nordtvedt, and Will,[9,10] and the tensor-tensor theory of Rosen.[11] These theories do belong to the category of A theories since they were all explicitly constructed to satisfy Eqs. (3) for test bodies, in A units. Although the details vary, all these theories share one common feature. They each possess auxiliary fields in addition to the metric tensor field, and the asymptotic values of these auxiliary fields are cosmological in origin. Since the Universe evolves with time, these auxiliary fields vary on a cosmological time scale.

The effect of these auxiliary fields on local physics is to induce a renormalization of some of the parameters entering into the general PPN metric so that such parameters become dependent on the asymptotic field values.[3] This means that these PPN parameters now vary with cosmic time. Since the parameter entering at lowest order in the PPN expansion is the product of Newton's constant G with the mass M of the central body, the product GM can acquire an induced variation with cosmic time. This induced cosmological time variation is what is sought when one seeks to "measure G dot." This is why measuring G dot has become synonymous with "measuring a cosmological influence on local physics."

An estimate of the expected magnitude of such cosmologically induced effects can be made by noting that the time scale of such effects should be determined by Hubble's constant H_0. Thus the fractional rate of change of a typical PPN parameter α is expected to be

$$\frac{\dot{\alpha}}{\alpha}\sim H_0=5\times10^{-11}h_{50}\ \text{yr}^{-1} \qquad (h_{50}=H_0/[50\ (\text{km/sec})/\text{Mpc}]) . \tag{4}$$

This places a severe constraint on the meaurement of such an effect. Such effects are not detectable at the level of laboratory physics.

For all the theories mentioned above the predicted value of G dot [actually (GM) dot since this is the relevant parameter in the PPN expansion which is used to test these theories] can be written as

$$\frac{(GM)^{\cdot}}{GM}=fH_0 , \qquad (5)$$

where f is given in terms of the parameters of the theory. For example, in the Brans-Dicke theory one has[3]

$$f\sim\frac{1}{\omega+1}, \quad \dot{M}=0 \qquad (6)$$

with ω being a free parameter of the theory. For all the theories mentioned above the parameters defining f also enter into the renormalized PPN parameters. For example, in Brans-Dicke theory one has[3] the PPN parameters

$$\beta=1, \quad \gamma=\frac{\omega+1}{\omega+2} \qquad (7)$$

with all other PPN parameters vanishing as in general relativity. Because of this theory-dependent coupling between G dot and the other PPN parameters, a measurement of the PPN parameters limits G dot, and conversely. For all the theories mentioned above, current observational limits on the PPN parameters[3] restrict f to be less than 10^{-2}, so that the predictions of these particular theories for G dot are

$$\frac{(GM)^{\cdot}}{GM}<5\times 10^{-13}h_{50}\ \mathrm{yr}^{-1} . \qquad (8)$$

However, from Eqs. (4) and (5) we expect f to be of order unity. Therefore none of the theories mentioned above admit a cosmological influence at the level expected from Eq. (4).

While all of the specific theories mentioned above couple G dot with the PPN parameters, it is conceivable that a gravity theory of sufficient complexity may be formulated in which each of the PPN parameters has an independent cosmologically induced time variation. It is this point of view which has motivated previous G-dot measurements. One takes the standard PPN metric with constant parameters, sets each of these parameters to be an unknown function of cosmic time, and then expands these functions as power series in time. The net result of this procedure is to introduce one more term into the PPN analysis which is proportional to $(GM)^{\cdot}$. All other parameters such as $\dot{\beta}$, $\dot{\gamma}$, etc. enter at higher order and can be ignored. One then treats $(GM)^{\cdot}$ as just one more free parameter to be determined by the Solar-System astrometric data.

Such an analysis has been conducted in the past using radar-ranging measurements to the inner planets. The accuracy available at that time gave the upper bound[12]

$$\frac{(GM)^{\cdot}}{CM}<24\times 10^{-11}\ \mathrm{yr}^{-1} , \qquad (9)$$

which value is just above the relevant range of values indicated by Eq. (4). As discussed below, new very-high-precision ranging data from the Viking lander on Mars are expected to provide a sensitivity of a part in 10^{11} per year, within the relevant range of values indicated by Eq. (4).

These A theories with nonzero G dot are examples of the minimal coupling of cosmology with local physics, discussed in Sec. II above, precisely because $G=G(t)$ in A units. The A theories seek to introduce a minimal coupling of cosmology with local physics by preserving the form of nongravitational physics in A units, but changing the form of the gravitational field equations in A units (compared with general relativity). This coupling of local physics with cosmology is *indirect* through the effect of the metric in expression (3).

As mentioned in Sec. III, modern observational data are gathered and recorded in A units. Therefore explicit comparison of theory with observation is carried out by a straightforward use of the second of Eq. (3) for test bodies in A units. The post-Newtonian coordinates are chosen so that far from the central mass the metric in A units takes the form

$$ds_A{}^2=dt^2-\delta_{ij}dx^i dx^j+h_{\alpha\sigma}(x)dx^\alpha dx^\sigma , \qquad (10)$$

where the $h_{\alpha\sigma}$ term is the local perturbation due to the central mass.[3] Following Will,[3] we write for the PPN metric

$$ds_A{}^2=g_{\alpha\sigma}dx^\alpha dx^\sigma , \qquad (11a)$$

$$g_{00}=1-2U+O_4 , \qquad (11b)$$

$$g_{ij}=-(1+2\gamma U)\delta_{ij}+O_4 , \qquad (11c)$$

$$g_{0i}=O_3 , \qquad (11d)$$

$$U=\frac{GM}{c^2 r}\sim O_2, \quad r^2=x^i x^i , \qquad (11e)$$

where U is the Newtonian potential, γ is a PPN parameter, and O_2 denotes the maximum value of the Newtonian potential. The post-Newtonian coordinate velocity dx^i/dt of an orbiting body is O_1.

As is well known,[3] use of (11) in the geodesic equation (3) gives the coordinate acceleration

$$\frac{d^2x^i}{dt^2}=-\frac{G_0M_0}{r}\frac{x^i}{r^2}+\frac{(\mathrm{PPN}_4)^i}{r} -\frac{(GM)_0^{\cdot}}{(GM)_0}\left[\frac{G_0M_0}{r}\frac{x^i}{r^2}(t-t_0)\right]+\frac{O_6}{r} , \qquad (12)$$

where the $\mathrm{PPN}_4=O_4$ terms contain all the usual PPN corrections and where the $(GM)_0^{\cdot}$ term comes from expanding

$$GM=(GM)_0+(GM)_0^{\cdot}(t-t_0)+\cdots , \qquad (13)$$

since auxiliary fields are assumed to induce a cosmological time dependence in GM. The additional terms in (13) are much smaller than the terms shown. They arise, for example, from the Earth's motion relative to the local cosmic rest frame and from local contributions to the auxiliary fields. Equations (12) can be integrated perturbatively to obtain the coordinate positions of two test bodies (planets) as

$$x_a^i(t)=\bar{x}_a^i(t)-\frac{(GM)_0^{\cdot}}{(GM)_0}\left[\bar{x}_a^i(t)(t-t_0)-\frac{d\bar{x}_a^i(t)}{dt}(t-t_0)^2\right]+\cdots , \tag{14}$$

where $a=1,2$ indexes the bodies. The $\bar{x}_a^i(t)$ are the solutions of (12) for constant GM. The range between the two orbiting bodies equals one-half the round-trip light time between the two bodies. To lowest order this is given by

$$\rho^2(t)=[x_1^i(t)-x_2^i(t)][x_1^i(t)-x_2^i(t)] \tag{15}$$

and using (14) we find

$$\rho(t)=\bar{\rho}(t)-\frac{(GM)_0^{\cdot}}{(GM)_0}\left[\bar{\rho}(t)(t-t_0)-\frac{d\bar{\rho}(t)}{dt}(t-t_0)^2\right]+\cdots , \tag{16}$$

where $\bar{\rho}(t)$ is the range between the two bodies for constant GM.

V. E THEORIES

E theories are defined to be gravitation theories in which test bodies follow geodesics in E units. Therefore,

$$E \text{ units: } T^{\alpha\sigma}{}_{;\sigma}=0,\quad u^{\alpha}{}_{;\sigma}u^{\sigma}=0 . \tag{17}$$

Since test bodies are assumed to follow geodesic paths in the geometry determined using E units, they will not follow geodesic paths in the geometry determined using A units (assuming a nonconstant φ). Therefore in A units one has in general

$$A \text{ units: } T^{\alpha\sigma}{}_{;\sigma}\neq 0,\quad u^{\alpha}{}_{;\sigma}u^{\sigma}\neq 0 . \tag{18}$$

Thus E theories seek to introduce a minimal coupling of cosmology with local physics by preserving the form of gravitational physics in E units, but changing the form of some part of nongravitational physics in A units.[13,14] The coupling of cosmology with local physics is *direct* through the nonvanishing of the right sides of Eq. (18).

As in the case of A theories discussed in Sec. IV, it is easiest to understand the results of comparing theory with observation if all the equations are written in A units. However, this is more difficult for E theories than it was for A theories since the relevant equations are only known in E units and therefore must be converted to A units. We do this in two parts. First we obtain the PPN metric for E theories in E units and then transform it into A units. Then we obtain the Newtonian acceleration of a test body about a central mass and the range between two orbiting bodies, also in A units.

A. PPN metric in A units

The starting assumption is that gravitational physics is unchanged in E units. Thus the field equations of the gravitational theory will give components $g^E_{\alpha\sigma}$ of the E-metric tensor, and the E-time interval measured along the world line of some test body will be given by

$$ds_E{}^2=g^E_{\alpha\sigma}dy^\alpha dy^\sigma , \tag{19}$$

where (y) are coordinates (dimensionless spacetime markers). Since gravitational physics is unchanged in E units, it is possible to choose the coordinates (y) such that $g^E_{\alpha\sigma}$ assumes the standard PPN form [the (y) are a PPN coordinate system in E units]. Thus the (y) are chosen so that far from the central mass, $ds_E{}^2$ assumes the form of Eq. (10). The PPN metric $g^E_{\alpha\sigma}$ takes the standard form (11) in E units.

From the fundamental equation (1), the A-time interval measured along the world line of some test body is given by

$$ds_A{}^2=g^A_{\alpha\sigma}dy^\alpha dy^\sigma=\varphi^{-2}g^E_{\alpha\sigma}dy^\alpha dy^\sigma , \tag{20}$$

where $g^A_{\alpha\sigma}$ is a metric deduced from ranging data timed with A clocks. However, the (y) coordinates are no longer PPN coordinates for the A geometry, since far from the central mass, Eqs. (10) and (20) give

$$ds_A{}^2=\varphi^{-2}(dy^0)^2-\varphi^{-2}\delta_{ij}dy^i dy^j+\varphi^{-2}h_{\alpha\sigma}(y)dy^\alpha dy^\sigma , \tag{21}$$

which is not of the required PPN form (10). In order to write (21) in asymptotic PPN form the coordinate transformation

$$y^0=t+\tfrac{1}{2}\dot{\varphi}_0(t-t_0)^2+\tfrac{1}{2}\dot{\varphi}_0 x^k x^k+\cdots ,\qquad y^i=x^i+\dot{\varphi}_0(t-t_0)x^i+\cdots , \tag{22a}$$

$$t=y^0-\tfrac{1}{2}\dot{\varphi}_0(y^0-t_0)^2-\tfrac{1}{2}\dot{\varphi}_0 y^k y^k+\cdots ,\qquad x^i=y^i-\dot{\varphi}_0(y^0-t_0)y^i+\cdots \tag{22b}$$

is used which allows the A metric (21) to be written as

$$ds_A{}^2=dt^2-\delta_{ij}dx^i dx^j+\cdots , \tag{23}$$

where the extra terms are either local perturbations due to the central mass, or are of second order in $\dot{\varphi}_0$. Therefore the (x) constitute PPN coordinates for the A geometry to the required level of accuracy. Equations (22) allow one to pass between PPN coordinates for the A geometry (x) and PPN coordinates for the E geometry (y).

In order to obtain the additional terms in (23), we expand φ as

$$\varphi=1+\dot{\varphi}_0(t-t_0)+\cdots . \tag{24}$$

Since φ represents the local manifestation of global effects on local dynamics, this is valid to the same order of accuracy as the expansion (13) for GM in the A theories. $\varphi_0=1$ was chosen by normalizing Eq. (1) at $t=t_0$. From Table I we see that inside the Solar System

TABLE I. Orbital radius r, Newtonian potential U, and $H_0 r/c$ for the five inner planets.

Planet	r (10^8 km)	$U\sim O_2$ (10^{-8})	$H_0 r/c$ ($10^{-16}h_{50}$)
Mercury	0.55	2.68	3.0
Venus	1.08	1.37	5.8
Earth	1.50	0.98	8.1
Mars	2.26	0.65	12
Jupiter	7.77	0.19	42

$$\dot{\varphi}_0 r \sim H_0 r \sim O_4 , \tag{25}$$

where r is the orbital distance from the Sun.

Applying the coordinate transformation (22) to the metric (20) and using (24) gives

$$ds_A{}^2 = [g^E_{00} + 2g^E_{0i} x^i \dot{\varphi}_0] dt^2 + 2[g^E_{0i} + (g^E_{ij} + g^E_{00}\delta_{ij}) x^j \dot{\varphi}_0] dx^i dt + [g^E_{ij} + 2g^E_{0i} x^j \dot{\varphi}_0] dx^i dx^j . \tag{26}$$

Upon taking account of (11) and (25), this becomes

$$ds_A{}^2 = g^E_{\alpha\sigma}(y(x)) dx^\alpha dx^\sigma + O_6 \tag{27}$$

inside the Solar System. One concludes that through PPN order O_5 the form of the required PPN metric tensor for the A geometry inside the Solar System is identical to the form of the PPN metric tensor for the E geometry, provided $y = y(x)$ given by (22a).

B. Equations of motion in A units

Consistent with our underlying assumption that gravitational physics is unchanged in E units, we assume that test bodies follow geodesics in E units. As discussed in Sec. III, such test bodies do not follow geodesics in A units. Use of Eqs. (1) and (20) in the geodesic equation (17) yields the equation of motion in A units as

$$\frac{d^2x^\alpha}{ds_A{}^2} + \Gamma^\alpha_{\lambda\sigma}(g^A) \frac{dx^\lambda}{ds_A} \frac{dx^\sigma}{ds_A} = \frac{\varphi_{,\sigma}}{\varphi} \left[g^{\alpha\sigma} - \frac{dx^\alpha}{ds_A} \frac{dx^\sigma}{ds_A} \right] , \tag{28}$$

which is no longer a geodesic equation except for the trivial case of $\varphi_{,\sigma} = 0$. In order to obtain the PPN coordinate acceleration of a test body in A units, one can either use the A metric (27) with (24) in the equation of motion (28), or apply the coordinate transformation (22) to the PPN coordinate acceleration of a test body in E units. We adopt the latter approach.

The PPN coordinate acceleration in E units has the same form as Eq. (12) with $(GM)^{\cdot}_0 = 0$ [with (x,t) replaced by (y, y^0)]. Use of (22a) in this equation then gives the PPN coordinate acceleration equation in A units as

$$\frac{d^2x^i}{dt^2} = -\frac{G_0 M_0}{r} \frac{x^i}{r^2} + \frac{(\mathrm{PPN})^i_4}{r} + \dot{\varphi}_0 \left[\frac{G_0 M_0}{r} \frac{x^i}{r^2}(t - t_0) - \frac{dx^i}{dt} \right] + \frac{O_6}{r} . \tag{29}$$

Since for $t - t_0 \sim 1$ yr,

$$\left[\frac{c(t - t_0)}{r} \right] U \sim 10^4 U = O_1 , \tag{30}$$

we see that both terms in the coefficient of $\dot{\varphi}_0$ in (29) are of comparable magnitude for one year of Solar System astrometric data.

As in Sec. IV, Eq. (29) can be integrated perturbatively to give the coordinate positions of two orbiting test bodies (planets) as

$$x^i_a(t) = \bar{x}^i_a(t) - \dot{\varphi}_0 \left[\bar{x}^i_a(t)(t - t_0) - \frac{1}{2} \frac{d\bar{x}^i_a(t)}{dt}(t - t_0)^2 \right] + \cdots , \tag{31}$$

where $a = 1,2$ indexes the bodies. The $\bar{x}^i_a(t)$ are the solutions of (29) for constant φ. As in Sec. IV, the range between the two orbiting bodies equals one-half the round-trip light travel time between the two bodies, and satisfies

$$\rho(t) = \bar{\rho}(t) - \dot{\varphi}_0 \left[\bar{\rho}(t)(t - t_0) - \frac{1}{2} \frac{d\bar{\rho}(t)}{dt}(t - t_0)^2 \right] + \cdots , \tag{32}$$

where $\bar{\rho}(t)$ is the range between the two bodies for constant φ. Similar but less general equations have also been obtained by Shapiro.[15]

VI. COMPARISON OF A THEORIES AND E THEORIES

Comparison of the acceleration equations (12) and (29) reveals that the two categories of theories are really very different both physically and conceptually. This difference is manifested by the presence in (29) of a "dissipative" term proportional to the velocity that is absent in (12), and is an example of the direct influence of cosmology on local physics we alluded to earlier.

This direct influence of cosmology on local physics is also manifest at the level of the equation of motion for test bodies given in (28). The fact that the right side of (28) is nonzero means that in general one has $T^{\alpha\sigma}{}_{;\sigma} \neq 0$ in A units. This means that standard nongravitational physics must be modified by the cosmological influence. The formalism presented here has the advantage that one does not need a detailed model of the cosmic interaction with local physics in order to test for its existence. In this sense the formalism presented here should be viewed as an extension of the PPN formalism used to test for different theories of gravity.

One further point is in order. In the usual definition of "metric theory" given above, condition (iii) requires that nongravitational laws in any freely falling frame reduce to the laws of special relativity. In E units, E theories satisfy $T^{\alpha\sigma}{}_{;\sigma} = 0$ and in that sense E theories are formally metric theories. However, condition (iii) is often stated as requiring that in any freely falling frame all of standard physics should hold. This latter form must clearly be false for E theories since for these theories $T^{\alpha\sigma}{}_{;\sigma} \neq 0$ in A units. Further, should a "freely falling frame" be specified in A units or in E units? The answer must come from observation or from a complete theory, not from this formalism. An analogous question arises when one uses coordinate covariance of special relativity in order to naively insert gravitational effects in the absence of a complete theory, viz., should a freely falling frame be specified relative to the flat background geometry of special relativity or relative to the curved geometry induced by the metric field?

VII. DETECTABILITY

Inspection of the acceleration equations (12) and (29) or the range equations (15) and (32) shows that the two categories of theories produce the same size effect and would be detectable in the data at the same level. These equations can be rewritten in a form which allows the *data* to determine which category of theories, if either, is realized in Nature. We introduce two parameters A and B and write the PPN coordinate acceleration equation in atomic units as

$$\frac{d^2x^i}{dt^2}=-\frac{G_0M_0}{r}\frac{x^i}{r^2}+\frac{(\mathrm{PPN}_4)^i}{r}+(B-A)\left[\frac{G_0M_0}{r}\frac{x^i}{r^2}(t-t_0)\right]-B\frac{dx^i}{dt}+\frac{O_6}{r}\ . \quad (33)$$

Integrating (33) perturbatively, one finds the range between two orbiting test bodies (planets) to be given by

$$\rho(t)=\bar{\rho}(t)-(A+B)\bar{\rho}(t)(t-t_0)+(A+\tfrac{1}{2}B)\frac{d\bar{\rho}(t)}{dt}(t-t_0)^2+\cdots\ . \quad (34)$$

These equations reduce to the acceleration and range equations for A theories when $A=(G\dot{M})_0/(GM)_0$ and $B=0$, and for E theories when $A=0$ and $B=\dot{\varphi}_0$.

Results from radar ranging to Mercury, Venus, and Mars, and radio ranging to the Mariner 9 spacecraft in orbit around Mars have been published by two research groups, Reasenberg and Shapiro[12] of MIT and Anderson *et al.*[16] of JPL, with similar results. Both groups assumed $B=0$ and solved for A in Eqs. (33) and (34).

Reasenberg and Shapiro provide a particularly illuminating example of the subtleties involved in interpreting the results. Table II shows their results for the A parameter when Mercury ranging data alone is used, when Venus ranging data alone is used, when Mars ranging data alone is used, and when all the data sets are used simultaneously. The final entry shows the result of a simple weighted averaging of the first three entries. As emphasized by Reasenberg and Shapiro, only the fourth entry obtained by using all the data sets simultaneously in the data reduction is statistically meaningful. This result is significantly less accurate than a simple averaging might lead one to expect. The reason is that in fitting the first three data sets separately, it is assumed that the orbits of the Earth and the ranged planet are independently adjustable. The fourth entry recognizes the fact that the three data sets are actually tied together since all ranging is done from Earth and the Earth's orbit must be adjusted to fit all the data sets simultaneously. The reason for the discrepancy between the sensitivities shown in entries 4 and 5 is that small systematic errors in the individual data sets might be partially fit by adjusting the Earth's orbit for that data set alone. However, when the Earth's orbit is required to fit all the data sets simultaneously, then the error shows up more clearly and the uncertainty in the value of A increases to its correct value. Consequently, while data from any single-ranged object (for example, the Mercury ranging data) may yield some particular value for the A or B parameters (an upper bound of $A<10^{-10}$ yr^{-1} for the Mercury data), such values invariably degrade when the complete set of ranging data is used. In fact, since the data indicate the existence of systematic errors, Reasenberg and Shapiro find that their results are "not inconsistent" with $A=0$. Thus all entries in Table II are consistent with the bound given in Eq. (9).

TABLE II. Values for the A parameter of Eq. (33) from radar ranging to the inner planets, from Reasenberg and Shapiro (Ref. 12). The B parameter was assumed to be zero in the data reduction. Only the fourth entry labeled "combined" is statistically realistic.

	Planet	A (10^{-11} yr^{-1})
1.	Mercury	6 ± 4
2.	Venus	6 ± 6
3.	Mars	25 ± 33
4.	Combined	15 ± 9
5.	Average	6.2 ± 3.3

Since 1976, ranging data from the Viking lander on Mars have been accumulating. These data are so accurate that they allow one to model Mars' orbit to within ±10 m. With the four-year baseline from the end of the Mariner 9 data (1972) to the beginning of the Viking data (1976), it now appears possible to determine either A or B to a sensitivity of about 1×10^{-11} yr^{-1} using the complete set of ranging data. This is the first unambiguous opportunity to look for cosmologically induced effects on local physics at the level at which they might be expected to exist.

From Eq. (34), the parameters A and B are independently determinable only if the effects of the term linear in t and the term quadratic in t can be separated by the data. The linear term will dominate only for less than the first synodic period (Mars' synodic period is 780 days), and the effect is dominated by the quadratic term thereafter. Thus, if the data are sufficiently accurate to determine an unambiguous value for either A or B alone after one relative orbit, then given a sufficiently long time baseline of data it should be possible to determine both A and B simultaneously. The simultaneous determination of both parameters will not reach the expected sensitivity of 1×10^{-11} yr^{-1} for each parameter individually, but it does appear to be possible to separate A and B at a simultaneous sensitivity of about 4×10^{-11} yr^{-1}.

VIII. SUMMARY

We have presented a framework for the observational detectability and distinguishability of two categories of theories which incorporate cosmological influence into local physics. The advantage of the formalism presented here is that no detailed model of such an interaction is necessary in order to test for its existence. We have found that ranging data to the Viking lander on Mars are sufficiently accurate so that in conjunction with the other Solar System ranging data sets one can attain a physically significant level of detectability for such theories. For the first time it has become possible to directly and unambiguously detect such a cosmic influence on local physics if it exists at the expected level of Eq. (4). These new data from Viking also allow one to observationally distinguish between the two categories of theories at a physically significant level.

ACKNOWLEDGMENT

The work of P.J.A. was supported in part by a grant from NAS-NRC.

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