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Understanding of spin $\frac{1}{2}$, family structure, and mass pattern of leptons and quarks in a composite model

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A relativistic three-preon interaction with a spin dependence $\Sigma^a \equiv \sigma^{(1)}_{\mu\nu}\sigma^{(2)}_{\nu\lambda}\sigma^{(3)}_{\lambda\mu}$ is proposed for an SU(2) preon (t,v) model. The Σ^a operator (i) gives zero force for $S=\frac{3}{2}$ states, thus rendering them dynamically irrelevant, and (ii) has the *complex* combinations $\chi'' \pm i\chi'$ of the two spin-$\frac{1}{2}$ functions (χ',χ'') as eigenstates with eigenvalues of *opposite* signs so that only one combination is relevant for confinement. This feature in turn leads to just *three* S_3-symmetry classes (out of six available ones) of spatial wave function, again in complex form which are sought to be identified with the generation structures. Their spatial functional forms are such as to obey severe selection rules preventing electromagnetic transitions such as $\mu \rightarrow e\gamma$. Apart from these qualitative features which are shown to remain valid within a fairly general (Bethe-Salpeter-type) dynamical framework, a quantitative model of confinement with a very steep potential $(\sim R^{12})$ is proposed for a unified description of lepton and quark spectra. This leads to a universal (l,q) mass formula for all three generations with only two free parameters, in rather good agreement with the observed pattern. In particular the lepton mass ratios which do not involve any free parameter are predicted as $m_\mu/m_e = 196.7$ and $m_\tau/m_\mu = 14.3$.

I. INTRODUCTION

Composite or preon models[1–15] have made almost instant impact on theoretical fancies ever since they made their first appearance.[1–4] Their preference over grand unified theories[16] (which have also been formulated at the preon level[17]) stems partly from their intrinsic appeal based on the successive lessons on the history of elementarity and partly from the economy expected in a description in terms of more elementary constituents than quarks (q) and leptons (l), though opinion may differ on the latter count in the absence of a consensus on the origin (intrinsic vs dynamical) of certain crucial degrees of freedom such as color. Composite models which have been extensively investigated in recent years are mostly of a formulational nature[4–12] in which certain theoretical requirements, especially a self-consistent reproduction, at the composite (q,l) level, of the absence of anomalies at the preon level,[5] have received much attention.[5–7] There is also some limited evidence of investigations of observational constraints imposed by low-energy physics, especially the role of the anomalous magnetic moments $(g-2)$ of e and μ in determining the energy scale of the preonic forces.[13–15]

Formulations of preon models are largely based on the theoretical concepts already developed at the hadron and quark levels, such as gauge fields, current algebra, and so on. The models so far suggested can be classified under two broad headings.

(i) Models[4,7,10] in which the preons are essentially chosen as synonymous with certain chosen attributes (color, flavor, generation index) of the composites (q,l) themselves. In such models the constituents (preons) do not have any independent status apart from the respective degrees of freedom they are intended to convey. An extreme example of this situation is envisaged in Ref. 4 with a corresponding degree of indifference to the understanding of the fundamental attributes in a dynamical fashion.

(ii) Models[8,9] in which the preons play a role more akin to nucleons in nuclei or quarks in hadrons, so that these constituents are recognized to have an existence independent of the attributes they are supposed to possess. Such models have the potential to provide a dynamical understanding of at least some (if not all) of the composite attributes by facilitating a fuller exploitation of the mathematics of permutation symmetries which naturally come into play whenever *identical* particles are involved. An extreme example of this situation is the Harari-Shupe[8] model wherein even the color attribute was sought to be understood as a permutation-symmetry problem.

Eventually, both types of models have come nearer each other, the first[10–12] by adopting an intermediate philosophy demanding certain attributes (especially the generation index) to be dynamically generated, and the second[9] by allowing certain attributes (color and flavor) to be intrinsic to the preons. However, there is little evidence so far for concrete dynamical formulations beyond the qualitative levels of field-theoretic principles.[5,9,17,18] On the other hand, there are important issues connected with composite functions which (at the present state of the art) would seem to need more concrete forms of dynamics at least for a first-order understanding, with a view to providing clues to the desirable features that a good theory of the future should incorporate. Two such issues that we feel need more serious attention than hitherto evidenced are the following.

(A) Why are there only spin-$\frac{1}{2}$ leptons and quarks all the way up to the third generation, with no trace of $S=\frac{3}{2}$ so far?

(B) What quantum number if any provides a meaningful distinction among successive generations which look observationally alike and yet seem to maintain their relative stabilities against mutual transitions (e.g., $\mu \not\rightarrow e\gamma$)? In this respect, the radial-excitation picture[10,11] appears unattractive, but other possibilities[12,19] exist.

In this paper, we wish to address specifically questions (A) and (B) and look for a suitable dynamical basis intended to provide some definitive answers to these questions. To that end it is first necessary to fix on a definite preon picture from among several available candidates,[4] in particular, type (ii) models since permutation symmetries are expected to play a vital role in our analysis. We choose this to be the later version[9] of the Harari-Shupe model[8] in which (a) the SU(2) preons (t,v) are taken to be almost massless (to facilitate confinement in a chiral fashion[6,15]) and (b) color is taken as an intrinsic attribute[9] (not dynamically generated). The 't Hooft conditions[5] are trivially satisfied[4] in this SU(2) case,[5,6] while the color representations $\underline{3}$ ($\underline{3}^*$) for t (v) are capable of imparting the desired color structures[9,17] (singlet for ttt and vvv; $\underline{3}$ for vtt and $\underline{3}^*$ for tvv) among other possibilities (to be discussed in Secs. II and V).

As to the essential dynamics, the preons are supposed to be confined by hypercolor forces, for which, however, we have nothing to offer except an effective description through a suitable ansatz on the overall spin and momentum dependence for the preon interaction kernel within the framework of a three-preon Bethe-Salpeter (BS) equation. The details thereof are described in Secs. II and III, but the crucial assumption which seems to hold the key to the main points of the answers to questions (A) and (B) is the following spin dependence for the effective three-preon interaction:

$$\Sigma^a \equiv \sigma^{(1)}_{\mu\nu}\sigma^{(2)}_{\nu\lambda}\sigma^{(3)}_{\lambda\mu}\ , \tag{1.1}$$

while the more quantitative issues such as the mass spectra of lepton and quark generations require a knowledge of the specific form of confinement (harmonic or otherwise). The paper is so arranged as to separate the general aspects bearing on issues (A) and (B) from the more quantitative questions such as the (l,q) mass spectra. To that end, Sec. II describes the classification of ttt wave functions according to S_3 symmetry and Σ^a dynamics and shows how the general *algebraic* structure of any reasonable form of a dynamical equation containing Σ^a as a kernel incorporates the qualitative answers to (A) and (B). Section III outlines the derivation of Eq. (2.11), starting from a three-preon Bethe-Salpeter equation in the instantaneous approximation. This equation, though ultrarelativistic in character (involving massless preons), is Schrödinger-type in appearance with the traditional roles of coordinates and momenta interchanged. Section IV sketches the S_3 structures of the spatial wave functions compatible with the form of Eq. (2.11) as well as confinement. These (complex) functions which are eigenstates $\exp(\frac{1}{2}iN\lambda)$ of an operator $\hat{\Lambda}=-i\delta_\lambda$ break up into just *three* classes ($N=3n\pm1$, $3n$) which are identified with the three generations. Absence of such functions in the quark model of baryons is discussed. Sections V and VI are taken up with the solution of the radial equation for a definite model of confinement. With some further similarity assumptions on the dynamics of ttt and vtt states (and the analogous vvv and tvv states), a universal mass formula for (l,q) states is derived. Comparison with the data reveals unexpectedly good agreement on the whole, but at the cost of a rather extended structure of these composites. In this agreement, a crucial role is played by a mass formula of the form $M \sim B^{21}$ (where β is a slowly varying function of the quantum numbers) as a natural outcome of the postulated dynamics. Section VII gives a critical discussion of the "good" (qualitative) and "bad" (quantitative) features of the model, and the extent to which the latter can stand modifications without affecting the former.

II. THREE-PREON DYNAMICS: QUALITATIVE CONSIDERATIONS

As stated in Sec. I, we start with the preon model of Ref. 9 wherein the (almost massless) t and v preons are taken to have $\underline{3}$ and $\underline{3}^*$ color representations, respectively, so that the possible color contents of the various lepton and quark composites are as follows:

$$e^{+}(ttt):\ \underline{3}\times\underline{3}\times\underline{3}=\underline{1}+\underline{8}'+\underline{8}''+\underline{10}\ ,$$

$$\nu_e(vvv):\ \underline{3}^*\times\underline{3}^*\times\underline{3}^*=\underline{1}+\underline{8}'+\underline{8}''+\underline{10}^*\ ,$$

$$u(vtt):\ \underline{3}^*\times\underline{3}\times\underline{3}=\underline{3}+\underline{3}+\underline{6}^*+\underline{15}\ ,$$

$$\bar{d}(tvv):\ \underline{3}\times\underline{3}^*\times\underline{3}^*=\underline{3}^*+\underline{3}^*+\underline{6}+\underline{15}^*\ .$$

While the desired multiplicities are no doubt present, the problem of the "unwanted" color representations would presumably have to be viewed as one requiring dynamical treatment, possibly at the level of the preonic interactions (see Sec. VI for a further discussion on this point).

The hypercolor degree of freedom for preons may be treated in formal analogy[9] to the color attribute for quarks vis-à-vis (color-singlet) hadrons. Thus the usual (l,q) states are hypercolorless (singlets) by design, while the formal existence of hypercolored (l,q) states must be recognized as inevitable consequences of a composite model. These states, which are described in Ref. 9, are best regarded as "confined" ones in hypercolor space. For the purposes of the present paper, hypercolor is a "hidden" degree of freedom for the singlet (l,q) states, except that for pure l states (ttt, vvv) this degree of freedom has a vital role in determining the overall antisymmetry of the three-preon wave function in all the degrees of freedom taken together under the assumption of Fermi statistics. (For quark states this role is limited only to the two "like" preons). With this understanding we shall henceforth suppress the hypercolor degrees of freedom from our discussion. This leaves us with mainly two degrees of freedom, viz. spin and momentum for active dynamical consideration, since flavor symmetry is also trivial in this simple (t,v) model. However, the possible existence of an additional degree of freedom (such as a second spin[9] arising out of a chiral symmetry) need not be ruled out at this stage, and its effect on the overall symmetry may be taken into account through a suitable extension to the list of "allowed" three-preon symmetries in the (truncated) spin-cum-momentum space. Because of their higher degree of symmetry, l states (ttt or vvv) will claim our primary attention. The main points of the argument will be later adapted without much difficulty to q states which have more restricted symmetries (see Sec. VI). The first task is to construct ttt wave functions of appropriate symmetries in the various available degrees of freedom on lines already familiar for three-nucleon[20] or three-quark[21,22] states, and in the same notation as far as possible. Let the momentum space ttt wave functions be denoted by $(\psi^s;\psi',\psi'';\psi^a)$—totally symmetric ($s$), mixed symmetric ($m',m''$) and antisymmetric ($a$) combinations, respectively. Similarly, the spin functions of corresponding symmetries are $\chi^s;\chi',\chi'';\chi^a\equiv 0$. In the same notation, the color and hypercolor functions for ttt states must appear as c^a and h^a, respectively. However, to *effectively* accommodate other unspecified degrees of freedom (e.g., a second spin[9]), it is enough to consider the extended choice (h',h'') for the h-function symmetries. The choice h^s does not give rise to any new configuration in $(\psi\chi)$ space, because of a simple duality relation $(\psi'\rightarrow\psi'',\psi''\rightarrow -\psi')$ between s and a symmetries.[21]

The ttt wave function Ψ in (ψ,χ) space associated with the $c^a h^a$ combination has the a form[20–22]

$$\Psi^a=\psi'\chi''-\psi''\chi'+\psi_1^a\chi^s\ . \tag{2.1}$$

Similarly, the $(\psi\chi)$ functions associated with (h',h'') symmetries have the two m forms[20]

$$\Psi''=\psi^s\chi''-\psi^a\chi'+\psi'\chi'-\psi''\chi''+\psi_1''\chi^s\ , \tag{2.2}$$

$$\Psi'=\psi^s\chi'+\psi^a\chi''+\psi'\chi''+\psi''\chi'+\psi_1'\chi^s\ , \tag{2.3}$$

so that the complete wave function in this extended case is

$$c^a(\Psi'h'+\Psi''h'')\ . \tag{2.4}$$

For further analysis, it is enough to consider (2.4), which is richer in structure, and hence includes more possibilities than (2.1). Also, for subsequent purposes it will be convenient to put (2.2) and (2.3) together in the *complex* form[23]

$$\Psi''\pm i\Psi'=\chi^s(\psi_1''\pm i\psi_1')+\sqrt{2}\chi^{\pm}(\psi^s\pm i\psi^a)+\sqrt{2}\chi^{\mp}(-\psi_1''+i\psi_1')\ , \tag{2.5}$$

where

$$\sqrt{2}\chi^{\pm}=\chi''\pm i\chi' \tag{2.6}$$

are two independent complex combinations of the spin-$\frac{1}{2}$ functions (χ) of m symmetry. Likewise, we define the complex space (ψ) functions

$$\sqrt{2}\theta_M^{\pm}=\psi''\mp i\psi'\ ,\qquad \sqrt{2}\theta_S^{\pm}=\psi^s\mp i\psi_a\ , \tag{2.7}$$

by virtue of which (2.5) is compactly expressible as

$$\Psi''\pm i\Psi'=\chi^s(\psi_1''\pm i\psi_1')-2\chi^{\mp}\theta_M^{\pm}+2\chi^{\pm}\theta_S^{\mp}\ . \tag{2.8}$$

Role of the spin operator Σ^a

The significance of the spin functions χ^s and $\chi^{\pm}$ is most succinctly expressed in terms of the effect on them of the operator (1.1) whose space components have the simple form $\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{\sigma}_3$. The crucial relations are

$$\begin{aligned}&\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{\sigma}_3\chi^s=0\ ,\\&\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{\sigma}_3\chi^{\pm}=\pm2\sqrt{3}\chi^{\pm}\ ,\end{aligned}\tag{2.9}$$

which show that χ^s and $\chi^{\pm}$ are eigenstates of this (fully antisymmetric) operator, while the real-form functions (χ',χ'') are *not*. In particular, the null eigenvalue for χ^s implies that if the operator Σ^a represents the entire spin-dependence of the kernel, then as an immediate consequence, all $S=\frac{3}{2}$ three-preon states would be effectively *force free* and hence not of physical interest as composite (l,q) states. In a similar vein, the fact that the complex spin functions $\chi^{\pm}$ are eigenstates of Σ^a suggests, via (2.8), that the dynamical equations would get decoupled in terms of the corresponding spatial (momentum) functions $\theta^{\pm}_{M,S}$, again in complex form. In particular, the opposite signs of the eigenvalues for $\chi^{\pm}$ [see (2.9)], would suggest that out of the two sets of functions $\theta^{\pm}_{M,S}$ only one set, say $\theta^{+}_{M,S}$ would exhibit the right sign for *confinement* and hence of direct relevance to the (l,q) states, while the other set $\theta^{-}_{M,S}$ (corresponding to unconfined preons) can be left out of further consideration. And since each set consists of just *three* types of functions (two for θ_M and one for θ_S), we see here the interesting possibility of associating these three functions with just the three-tier generation structure of (l,q) states which has by and large come to be accepted by consensus.

The foregoing contain the essential features of what we believe could constitute possible answers to the two issues (A) and (B) raised in Sec. I, even before going into any details of three-preon dynamics. The crucial role is that of the spin operator Σ^a, which (i) by its null effect on $S=\frac{3}{2}$ states, straightaway renders them irrelevant, and (ii) by projecting out just three of the six available space functions (in complex θ form) as having the right sign for a confining kernel, offers the possibility of a viable generation identification. For more quantitative ideas on the concrete forms of the θ functions representing the generation structures consistent with their relative stability against electromagnetic (EM) decay, as well as their mass spectra, it is necessary to have a more quantitative formulation of three-preon dynamics centered around the spin operator Σ^a. Our concrete proposal in this regard boils down to a spatial dynamic of the standard form,

$$[-\nabla_\xi{}^2-\nabla_\eta{}^2+V(\rho,M)-E(M)]\theta^{+}_{M,S}(\vec{\xi},\vec{\eta})=0\ ,\tag{2.10}$$

where $\vec{\xi},\vec{\eta}$ are the two independent *internal* coordinates, V is a function of $\rho=(\xi^2+\eta^2)^{1/2}$, and possibly also of the composite mass M, while the eigenvalue E is itself a function of M. This equation, though strongly reminiscent of a three-body Schrödinger equation of the nonrelativistic type, nevertheless admits of an alternative interpretation as an (ultra)relativistic equation with the traditional roles of position and momentum coordinates reversed. The latter picture which is more in conformity with the (almost) massless character of the preons held together by chiral confinement,[6,9,15] seems to emerge from some plausible confining assumptions on the structure of the effective interpreon forces within the overall context of a Bethe-Salpeter equation for the three-preon system. The necessary steps leading from a three-preon Bethe-Salpeter equation to the form (2.10) are outlined in the next section. The derivation is admittedly pedagogical and involves certain assumptions (to be specified in context) on the (parametric) structure of the kernel consistent with the three-body symmetry of the *ttt* system, and its justification, if any, lies in the obvious simplicity and intuitive appeal of Eq. (2.10), which must stand out on intrinsic grounds irrespective of the limitations or otherwise of the derivation itself.

III. A BETHE-SALPETER BASIS FOR EQ. (2.10)

In this section, we shall seek a justification for Eq. (2.10) within a Bethe-Salpeter (BS) framework which is believed to be the most conventional dynamical basis for a relativistic system. Specifically, we shall be concerned with the following theoretical aspects.

(a) A simple field-theoretic mechanism as a possible candidate for the effective spin dependence Σ^a for the three-preon kernel.

(b) A three-dimensional (instantaneous or null-plane) formulation of the three-preon BS equation, and its adaptation to a "harmonic" kernel.

(c) A justification for the form $V(\rho)\sim\rho^{1/3}$ based partly on aspect (b) and partly on a three-way factorizability of the BS kernel.

Mechanism for Σ^a structure

Since, according to the qualitative analysis of Sec. II, the spin structure Σ^a plays a central role in the understanding of the main observational features of (l,q) composites, we must look for a basically spin-flip mechanism for preon forces. A spin-flip mechanism in turn is expected to be magneticlike, in contrast to the situation at the quark level where the effective $q-q$ or $q-\bar{q}$ interactions viz., $\gamma_\mu^{(1)}\gamma_\mu^{(2)}$ have an electric character and are dominantly nonflip in content. Spin-flip preon interactions can be conceived in both pairwise ($\sigma_{\mu\nu}^{(1)}\sigma_{\nu\mu}^{(2)}$) and three-way ($\Sigma^a$) forms. While our discussion in Sec. II was based on the three-way form Σ^a, it may be noted that the pairwise form is also capable of generating qualitatively similar features, though at the cost of more assumptions and less elegance. Our present preference for the three-way form at the preon level stems partly from consideration of elegance and simplicity and partly from a desire to seek a maximal degree of departure from the pairwise forms already so familiar at the quark or nucleon levels.

A possible mechanism for generating a Σ^a structure is the following. Consider a hypergluon field $F_{\mu\nu}$ $(=-F_{\nu\mu})$ which is an *intrinsically* antisymmetric tensor (not of the usual $\partial_\mu V_\nu - \partial_\nu V_\mu$ type) and couples to a preon field ψ in the Pauli form $\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}$, suppressing the hypercolor labels. If the $F_{\mu\nu}$ field in turn has a self-coupling of the form $F_{\mu\nu}F_{\nu\lambda}F_{\lambda\mu}$, then it is not difficult to see that a three-way mechanism like Fig. 1(a) can generate a Σ^a structure through standard field contraction (Feynman gauge). More formal questions such as renormalizability of these basic interactions, or how many more types of effective preon couplings of similar strength can be generated by them, are best relegated to a later stage, contingent on the success of the present investigation. At this stage, the Σ^a structure will be taken as an effective description, notionally supported by a three-way picture like Fig. 1(a), together with its implied factorizability aspects. We are thus led to consider a three-way kernel of the form

$$K(p,p') = i^3 G F_{123} \Sigma^a R(p_i, p_i') \ , \tag{3.1}$$

where the four-momenta (p_i, p_i') are shown in the diagram, G is a coupling constant, and F_{123} represents the overall color dependence of the three-preon interaction. R is a scalar function of the momenta whose form will be specified below.

At this stage it is useful to record the crucial property of γ_5 invariance of the $\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}$ interaction (since $\sigma_{\mu\nu}$ commutes with γ_5) which helps preserve the chiral character ($\psi_{L,R}$) of the preon fields. Since this statement holds separately at each of the three-preon interaction vertices of Fig. 1(a), the above property of chiral invariance with respect to Σ^a as a whole is immediately seen to hold for the three-preon composite as well. This feature ensures that the chiral $SU(2)\times U(1)$ symmetry of the standard electroweak interaction at the (l,q) composite level will not be disturbed by the three-preon interaction (3.1) despite its strongly spin-flip character.

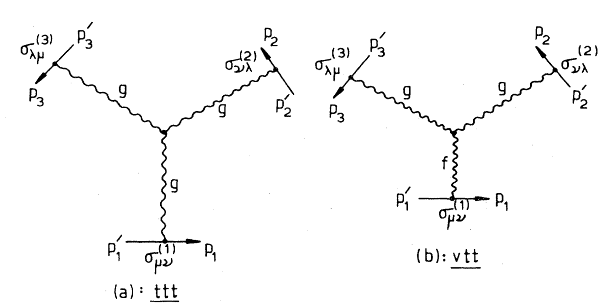

FIG. 1. Three-preon interaction model for (a) *ttt* and (b) *ttv* states. For a significance of the numbers *f* and *g*, see text. The complementary cases *vvv* and *tvv* are similar.

BS equation in instantaneous form

For the three-preon dynamics we consider a BS equation similar to one proposed recently for a three-quark system in the instantaneous approximation[24] and applied to baryon spectra under the assumption of harmonic confinement.[25] Most of the considerations employed in Refs. 24 and 25 may be directly adapted to the present situation, except for certain simplifications arising out of the replacement of the sum of three pairwise kernels in the quark case by a single three-way kernel (3.1) in the preon case. Thus the three-body BS equation may be written as

$$(2\pi i)^2 \prod_1^3 S_F{}^{-1}(p_i)\Psi^{\rm BS}(p_i) = -\int \prod_1^3 d^4p_i^1 \delta^4(P-P')K(p,p')\Psi^{\rm BS}(p_i') \ , \tag{3.2}$$

where

$$S_F{}^{-1}(p_i) = i(m_i + i\gamma^{(i)}\cdot p_i) \ , \tag{3.3}$$

m_i are the (small) preon masses if any, and the unexplained notations are as in Refs. 24 and 25. Reduction of Eq. (3.2) to the three-dimensional form via the instantaneous approximation closely follows the procedure already outlined in Ref. 24 and subsequently,[26] except for modifications due to the difference in the spin structure of the present kernel from the earlier (quark) case. The main points of the recipe, as adapted to the present three-way kernel may be stated without further explanation as

(i) Elimination of Dirac matrices from the left-hand side (LHS) through the ansatz[24,26]

$$\Psi^{\rm BS}(p_i) = \prod_1^3 S_F{}^{-1}(-p_i)\Phi^{\rm BS}(p_i) \ . \tag{3.4}$$

(ii) Effective replacement of each factor $m_i - i\gamma\cdot p_i$

on the right-hand side (RHS) by $2\omega_i$ (where ω_i is the energy corresponding to three-momenta $\vec{p}_i$) in preparation for the instantaneous approximation (ignoring certain commutators with Σ^a). This gives rise to a resultant factor $(8\omega_1\omega_2\omega_3)$ on the RHS.

(iii) Introduction of the instantaneous approximation through the relation[24]

$$\psi(\vec{p}_i)=\int\prod_1^3 dp_{i0}\delta(p_{10}+p_{20}+p_{30}-M)\Phi^{BS}(p_i) \tag{3.5}$$

and the assumption[27] that the kernel involves the time components p_{i0} at most as a sum of these quantities that is to say, M [see Eq. (3.5)].

Insertion of Eq. (3.5) in (3.2) now involves the following integral on the RHS:

$$(2\pi i)^2 I=\int \delta(p_{10}+p_{20}+p_{30}-M)\times\prod_1^3 [dp_{i0}(\omega_i{}^2-p_{i0}{}^2-i\epsilon)^{-1}] , \tag{3.6}$$

which works out straightforwardly as

$$I^{-1}=8\omega_1\omega_2\omega_3(\omega_1+\omega_2+\omega_3-M) . \tag{3.7}$$

Substitution of Eqs. (3.4)—(3.7) in (3.2), and taking note of the factor $8\omega_1\omega_2\omega_3$ arising on the RHS from step (ii) gives the three-dimensional equation

$$i^3(\omega_1+\omega_2+\omega_3)\Psi(\vec{p}_i) =-\int\prod_1^3 d^3p_i'\,\delta(\vec{p}_1'+\vec{p}_2'+\vec{p}_3')\times K(\vec{p}_i,\vec{p}_i',M)\Psi(\vec{p}_i') , \tag{3.8}$$

where the Σ^a in (3.1) may be read simply as $\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{\sigma}_3$, and the three-dimensional wave function in (3.8) may be directly identified as the complex wave functions $\Psi''\pm i\Psi'$ defined in (2.8). Substitution of (2.8) in (3.8) and the use of (2.9) now gives three types of uncoupled θ equations appropriate to the confined states:

$$(\omega_1+\omega_2+\omega_3-M)\theta(\vec{p}_i) =2\sqrt{3}F_{123}G\int\prod_1^3 d^3p_i'\,\delta(\vec{p}_1'+\vec{p}_2'+\vec{p}_3')\times R(\vec{p}_i,\vec{p}_i';M)\theta(\vec{p}_i') . \tag{3.9}$$

Factorizability of the kernel R

Further reduction of (3.9) is facilitated through the use of two independent internal momenta $\vec{\xi},\vec{\eta}$, defined by

$$\sqrt{2}\vec{\xi}=\vec{p}_3-\vec{p}_2 ,\quad \sqrt{6}\vec{\eta}=-2\vec{p}_1+\vec{p}_2+\vec{p}_3 , \tag{3.10}$$

and a suitable ansatz on the R function in keeping with harmoniclike confinement. The latter form, which was defined in Refs. 24 and 25 for pairwise interaction only, now needs to be suitably generalized to meet the requirements of a three-way interaction, in the spirit of Fig. 1(a). A possible harmonic form consistent with three-body symmetry as well as factorizability is

$$\prod_1^3 (A-B\vec{\nabla}_{p_i}\cdot\vec{\nabla}_{p_i'})\delta^3(\vec{\xi}-\vec{\xi}')\delta^3(\vec{\eta}-\vec{\eta}') , \tag{3.11}$$

where A and B are constants, depending at most on M. On the other hand, Eq. (3.11) is still not of the desired form suitable for the language of the $\vec{\xi},\vec{\eta}$ variables. An alternative parametrization which also retains the features of three-body symmetry as well as factorizability is expressed by

$$R=f(\nabla_\xi{}^2+\nabla_\eta{}^2)\delta^3(\vec{\xi}-\vec{\xi}')\delta^3(\vec{\eta}-\vec{\eta}') , \tag{3.12}$$

where

$$f(\nabla^2)=(C+A\nabla^2)(C+\omega A\nabla^2)(C+\omega^2 A\nabla^2) \tag{3.13}$$

and 1, ω, ω^2 are the three cube roots of unity. Substitution of (3.13) in (3.9) and integration over $\vec{\xi}'$, $\vec{\eta}'$ yields the same equation in a differential form, namely,

$$(\omega_1+\omega_2+\omega_3-M)\theta(\vec{\xi},\vec{\eta}) =2\sqrt{3}F_{123}Gf(\nabla_\xi{}^2+\nabla_\eta{}^2)\theta(\vec{\xi},\vec{\eta}) . \tag{3.14}$$

A further reduction of this equation requires a modest approximation which is best suited to the case of negligible preon masses ($\omega_i\approx|\vec{p}_i|$) and consists in the replacement

$$\tfrac{1}{3}\sum_i |\vec{p}_i| \Longrightarrow \left[\tfrac{1}{3}\sum_i \vec{p}_i{}^2\right]^{1/2}=\frac{1}{\sqrt{2}}\rho , \tag{3.15}$$

where $\rho^2=\xi^2+\eta^2$ and use has been made of Eq. (3.10) in the last step. As a result, the energy operator on the LHS of (3.14) takes the simpler form $(3/\sqrt{2})\rho-M$, which can be factored as in (3.13) by virtue of the identity

$$a^3-b^3=(a-b)(a-\omega b)(a-\omega^2 b) \tag{3.16}$$

with the identification

$$a^3=\frac{3}{\sqrt{2}}\rho,\quad b^3=M . \tag{3.17}$$

Taking account of the (factor-by-factor) correspondence of (3.13) and (3.16), a solution of Eq. (3.14) is recognized as

$$\left[\left[\frac{3}{\sqrt{2}}\rho\right]^{1/3} - M^{1/3}\right]\theta(\vec{\xi},\vec{\eta}) = (2\sqrt{3}GF_{123})^{1/3}(C + A\nabla_\xi^2 + A\nabla_\eta^2)\theta(\vec{\xi},\vec{\eta}) , \tag{3.18}$$

in conformity with the "standard form" envisaged in Eq. (2.10).

The foregoing "derivation" has its obvious limitations, apart from the semi-intuitive ansatz on the kernel, such as neglect of certain correction terms arising out of (i) space-time components of Σ^a (which are of order $|\vec{p}_i|/M$) and (ii) nonzero commutators of Σ^a with $S_F^{-1}(-p_i)$ which once again affect is space-time components. The only possible defence for their neglect is that such details would not be too meaningful in the context of an *effective* preon kernel adopted in this paper pending the availability of a more ambitious (field-theoretic?) workable framework. As a comment on the instantaneous approximation, our experience with quark-level dynamics[26] suggests that a strong resemblance to the former comes from the null-plane approximation[28] as well. Therefore, inasmuch as the kernel is taken in an effective parametric form, a physical distinction between these two types of approximations is not possible merely on the basis of the algebraic forms of their respective equations.

Nature of confinement

It is of interest to assess the nature of the extent of the confining mechanism implied in Eq. (3.14) or its "cube-root" counterpart Eq. (3.18). In this connection, it is necessary to remember that $\vec{\xi},\vec{\eta}$ are *momentum* coordinates, so that the kinetic energy (arising from inverse propagators on the LHS) and potential energy (arising from the kernel on the RHS) have effectively interchanged their roles in (3.18) with respect to the "standard" (coordinate space) description. Now in momentum space the confining "force" behaves like $\rho^{1/3}$, as measured with respect to the six-dimensional Laplacian in Eq. (3.18). Translated in coordinated space, the above statement would mean that with respect to the "Laplacian" $\rho^2 = -\nabla_X^2 - \nabla_Y^2$, the confining force has a huge power, that is to say R^{12}, where $R^2 = X^2 + Y^2$. This is far too steep compared to the modest linear or harmonic confinements usually considered at the quark level, and need not be *a priori* unwelcome considering the tightness requirement at the preon level.

IV. THE GENERATION STRUCTURE

Our next task is to spell out the structures of the three generations which have been identified in Sec. II as θ_M^+ (two) and θ_S^+ (one), in terms of appropriate solutions of Eqs. (2.10) or (3.18). For the orbitally unexcited ($L=0$) states, these θ functions depend on three scalars (ρ,γ,λ) defined by[23,29]

$$\xi^2+\eta^2=\rho^2, \quad \xi^2-\eta^2=-\gamma\cos\lambda , \quad 2\vec{\xi}\cdot\vec{\eta}=\gamma\sin\lambda , \tag{4.1}$$

or, equivalently, in terms of the complex vectors

$$\vec{z},\vec{z}^{\,*}=(\vec{\xi}\pm i\vec{\eta})/\sqrt{2} \tag{4.2}$$

as

$$2\vec{z}^{\,*}\cdot\vec{z}=\rho^2, \quad 2z^2, \quad 2z^{*2}=\gamma e^{\pm i\lambda} . \tag{4.3}$$

Since the potential V in (3.18) is a function of ρ only, the (γ,λ) dependence comes entirely from the Laplacian operators which are expressible as

$$\nabla_\xi^2+\nabla_\eta^2=2\vec{\nabla}_{z^*}\cdot\vec{\nabla}_z . \tag{4.4}$$

Taking account of the permutation symmetries of the $\theta_{M,S}^+$ functions, it is not difficult to see that the equations are satisfied with the following z or z^* powers which carry the correct symmetries:

$$\theta_{M1}^+ \sim z^{3n-1}, \quad \theta_S^+ \sim z^{*3n}, \quad \theta_{M2}^+ \sim z^{*3n+1} , \tag{4.5}$$

($n=0,1,2,\ldots$), omitting an overall function of ρ in each case. These three classes of functions which may also be interpreted as eigenstates of the operator[23] $\hat{\Lambda}=-i\partial_\lambda$ with eigenvalues $\frac{1}{2}(3n\mp1)$ and $\frac{3}{2}n$, represent our concrete proposal for the spatial structures of the successive generations, the lowest members corresponding to $n=0$ in each case. These lowest members may be identified as the three basic generations e^+,μ^+,τ^+, whose (λ,γ) dependence comes entirely from (4.5) and corresponds to the respective ground states ($n=0$) of the three series (4.5). Thus

$$\theta(e^+)\sim\gamma^{-1/2}e^{-i\lambda/2}, \theta(\mu^+)\sim\gamma^0e^0 , \quad \theta(\tau^+)\sim\gamma^{+1/2}e^{-i\lambda/2} . \tag{4.6}$$

The higher values of n correspond to a class of (vertical) excitations of these (horizontal) generations. Other classes of vertical excitations are (i) radially excited states (corresponding to successive radial functions of ρ) for each generation and (ii) orbitally excited ($L>0$) states, again for each generation. Such a proliferation of vertical excitations, for each horizontal generation is a necessary consequence of any composite model whose eventual success if any must depend on the details of dynamics, an essential condition being that the first excited state of e^+

should not lie below the ground state of τ^+.

The λ dependence (4.6) of the lepton wave functions, together with the orbitally unexcited nature ($L_\xi = L_\eta = 0$) of the three basic states e,μ,τ, have the following effect on EM transitions involving these states. The *electric* dipole operator which is a linear combination of $\vec{\nabla}_\xi$ and $\vec{\nabla}_\eta$ (i.e., contains one unit of L_ξ and/or L_η) will clearly give a zero matrix element for an electromagnetic (EM) transition involving any two of the states e,μ,τ (each of which has $L_\xi = L_\eta = 0$) after integration with respect to the angular variables $d\Omega_\xi d\Omega_\eta$. This selection rule is operative against all the transitions $\mu\rightarrow e\gamma$, $\tau\rightarrow\mu\gamma$, and $\tau\rightarrow e\gamma$ involving orbitally unexcited states, though it will fail for their orbitally excited counterparts.

The λ dependence of these states is irrelevant for this argument. For *magnetic* dipole transition on the other hand, which involve the spin operators $\vec{\sigma}_i$ (independent of $\vec{\xi}$ and $\vec{\eta}$), the λ dependence (4.6) of e,μ,τ is crucial for the EM selection rule, since the corresponding matrix elements would now survive the angular integrations over $\hat{\xi}$ and $\hat{\eta}$. Noting that the λ integration ranges over $0 \le \lambda \le z\pi$, it is immediately seen that the $\mu\rightarrow e\gamma$ and $\tau\rightarrow\mu\gamma$ transitions are straight away forbidden. The transition $\tau\rightarrow e\gamma$, while not forbidden by the λ effect, is nevertheless heavily suppressed due to poor matching of their wave functions (corresponding to very different masses). Taking both the electric and magnetic dipole arguments together, it is thus seen that the overall EM selection rule against radiative leptonic decays is rather strongly obeyed.

Comparison with *qqq* wave functions

Before ending this section, it is of interest to note that the complex *ttt* wave functions in both spin ($\chi^\pm$) and spatial ($\theta_{M,S}$) degrees of freedom have no formal analog with the real *qqq* wave functions in the same variables which characterize baryon spectroscopy. (The contrast is of course traceable to the role of the spin operator Σ^a which has necessitated the complex description). An important fallout of the complex representations $\theta^\pm$ has been the emergence of certain λ harmonics with half-integral ($\frac{1}{2}N$) quantum numbers which in turn have been identified with the generation structures. Though known in the formal three-body literature,[23] such λ harmonics have not been in evidence in the standard harmonic-oscillator classification of *qqq* states, the nearest analogs being certain low-lying $(70,0^+)$-type states[29] arising from the insertion of space-exchange terms[29] in the usual (harmonic-oscillator) quark model. In the present preon context these λ harmonics, which correspond to successive half-unit excitations, are seen to provide a welcome interpretation of the generation structures. The fact that such functions have not found a place in the standard baryon spectroscopy so far need not be an adequate reason for ruling them out at the preon level where a richer spectroscopy is now predicted. In particular, the e^+ wave function with its singularity structure ($\sim\gamma^{-1/2}$) has no partner in *qqq* spectroscopy, but such a "mild" singularity is mathematically permissible within a six-dimensional $(\vec{\xi},\vec{\eta})$ space.[30] Perhaps the nearest baryonic equivalent is the spatial structure of the μ^+ state which lies above e^+ by a half unit of λ excitation. On the other hand, the τ^+ state which lies another half-unit above μ^+ again has no conventional baryonic counterpart.

V. MASS SPECTRA OF *ttt* STATES

We now come to the last phase of this investigation, viz., an approximate solution of Eq. (3.18) to obtain an explicit formula for the *ttt* mass spectrum. This part of the program is provisional insofar as it involves some speculative assumptions on the M dependence of the parameters (C,A,g). This exercise is intended more as an illustration of the feasibility of the method to yield quantitatively realistic numbers than as a rigid attempt to define a complete model. As such no formal justification (except dimensional considerations) need be given for the parametric assumptions involved, viz.

$$A = a^2M^2,\quad G^{1/3} = g^3\Lambda^{1/3},\quad C = -(M/m_e)^{1/3}\,, \tag{5.1}$$

where (a,g) are dimensionless constants and Λ is a (large) mass scale. A fixed (electron) mass m_e has been introduced in C for convenience without loss of generality since its multiplying factor $G^{1/3}$ already has an arbitrary constant g with it. We now make use of Eq. (4.5) to write

$$\theta(\vec{\xi},\vec{\eta}) = (z \text{ or } z^*)^N F(\rho)\,, \tag{5.2}$$

where $N = 3n \pm 1$, $3n$, and substitute in (3.18) to obtain the radial equation

$$G^{1/3}A\left[\rho^{-5}\frac{d}{d\rho}\left[\rho^5\frac{dF}{d\rho}\right] + \frac{2N}{\rho}\frac{dF}{d\rho}\right] + \left[CG^{1/3} + M^{1/3} - \left[\frac{3}{\sqrt{2}}\rho\right]^{1/3}\right]F = 0\,, \tag{5.3}$$

where we have absorbed the factor

$$(2\sqrt{3}F_{123})^{1/3} \tag{5.4}$$

in a redefinition of A and C before using the parametrization (5.1). We now transform Eq. (5.3) by the successive substitutions

$$\rho = t^{9/7} , \tag{5.5}$$

$$F(\rho) = R(t)t^{-\gamma-1} , \tag{5.6}$$

and a subsequent scaling with $x = \alpha t$, where

$$\alpha^3 = \frac{81}{49} G^{-1/3} A^{-1} \left[\frac{3}{\sqrt{2}} \right]^{1/3} . \tag{5.7}$$

The resulting equation is

$$\frac{d^2 R}{dx^2} + [-x - \gamma(\gamma+1)x^{-2} + Bx^{4/7}]R = 0 , \tag{5.8}$$

where

$$2\gamma + 2 = (18N + 43)/7 , \tag{5.9}$$

$$\beta = \tfrac{81}{49} m \alpha^{3/7} , \tag{5.10}$$

$$m = M^{1/3} - CG^{1/3} . \tag{5.11}$$

So far, no approximation has been made during the reduction from (3.18) to (5.8). We now seek an explicit solution of (5.8) by reducing it to the Airy form through an expansion about $x = x_0$, where

$$x_0 - \beta x_0^{4/7} + \gamma(\gamma+1)x_0^{-2} = 0 . \tag{5.12}$$

An approximate solution is then

$$R(x) \approx \mathrm{Ai}[b(x - x_0)] , \tag{5.13}$$

where

$$b^3 = 1 - \tfrac{4}{7}\beta x_0^{-3/7} - 2\gamma(\gamma+1)x_0^{-3} . \tag{5.14}$$

The eigenvalues are now given by the standard condition $R(0) = 0$, viz.,

$$\mathrm{Ai}(-bx_0) = 0 , \tag{5.15}$$

which has the sequence of solutions[31]

$$bx_0 = s_r \ (r = 0, 1, 2, \ldots) . \tag{5.16}$$

Because of the rather high-order vanishing of $R(x)$ at $x = 0$, we suggest that $r = 1$ is a physically better candidate than $r = 0$ for the first zero of (5.15). Substitution of (5.16) in (5.12) with the use of (5.14) yields the approximate formula

$$\beta \approx s_r^{3/7} \delta_{\gamma r}^{1/7} [1 + \gamma(\gamma+1)s_r^{-3} \delta_{\gamma r}^{-1}] , \tag{5.17}$$

$$\delta_{\gamma r} = 1 + 2\gamma(\gamma+1)s_r^{-3} . \tag{5.18}$$

The γ values of the first three generations ($e\mu\tau$) which correspond to $N = -1$, 0, $+1$ are given from (5.9), whence their β values are found from (5.17) in terms of[31]

$$s_r(r = 1) = 1.02 \tag{5.19}$$

as

$$\beta(e^+) = 1.653 \equiv \beta_e , \quad \beta(\mu^+) = 2.126, \quad \beta(\tau^+) = 2.413 . \tag{5.20}$$

The next higher β value corresponding to $N = 2$ is

$$\beta(l^+) = 2.631 . \tag{5.21}$$

To relate these β values to the lepton masses, substitution of (5.1) in (5.10) and (5.11) gives

$$\beta = \frac{81}{49} \left[\frac{3M}{\Lambda\sqrt{2}} \right]^{1/21} \left[\frac{81}{49a^2 g^3} \right]^{1/7} \times \left[1 + \left[\frac{\Lambda}{m_e} \right]^{1/3} g^3 \right] , \tag{5.22}$$

which shows that M is proportional to β^{21} in this model. Substituting for β values from (5.20) yields the mass ratios

$$M_\mu / m_e = 196.7 \ (206.7) , \quad M_\tau / m_\mu = 14.3 \ (16.9) , \tag{5.23}$$

in unexpectedly good accord with the experimental numbers (in parentheses). The next charged-lepton state is predicted by the mass ratio

$$M_l / M_\tau = \left[\frac{2.631}{2.413} \right]^{21} = 6.15 . \tag{5.24}$$

This is a vertical excitation ($n = 1$) of e^+ belonging to the $N = (3n - 1)$ series, and its safe height above the third (τ^+) generation should be regarded as a welcome feature of the model. Other types of vertical excitations are omitted for brevity. Because of the confining nature of the interaction (see Sec. II), there is no preon pair-production threshold.

For neutrino (vvv) states, identical considerations to the above would lead to a mass formula similar to (5.22), but with the replacement $g^3 \rightarrow f^3$, signifying a different vvv coupling strength via the hypergluons. In this model the other two parameter (Λ,a) have been kept fixed.

VI. MODEL FOR q STATES: A UNIVERSAL MASS FORMULA

In this section, we suggest a model for quarks as vtt (and tvv) states as a simple extension of the ttt model developed in the foregoing. This is depicted in Fig. 1(b) for vtt states in which the strength (f) of the coupling to v lines is taken to be different from the corresponding strength (g) for the t lines. The spin structure of the kernel remains the same as before. Though there is manifestly less symmetry in a vtt system compared with ttt, the classification of spin (χ) and spatial (ψ) functions for *vtt* can nevertheless be made on parallel lines to *ttt*, in much the same way as, e.g., the *qqq* wave functions of Λ, Σ states with unequal mass kinematics admit of (albeit in a broken fashion) S_3 classifications[32] similar to those of N, Δ states where the S_3 symmetry is unbroken. This convention facilitates a classification of quark generations formally similar to lepton generations, but the broken S_3 symmetry in the quark case now helps realize the possibility of the mixing of generations [in the manner of Cabibbo or Kobayashi-Maskawa[33] (KM)]. The spin mechanism Σ^a for keeping out spin-$\frac{3}{2}$ states remains valid.

As to the quark-mass spectrum, we again envisage a simple extension of the parametrization (5.1), wherein a and Λ are kept "universal," but the constant g^3 is replaced by $g^2 f$ for *vtt* states [Fig. 1(b), in accordance with the (empirical) rule that each t line couples through a g factor and a v line through an f factor].

Similarly, gf^2 for *tvv* states. This prescription extrapolates rather naturally to the replacement $g^3 \to f^3$ already suggested for *vvv* states. We are, at this stage, unable to offer any deep motivation for this prescription except for its simplicity and factorable property. Using this prescription, the mass formula for *uct* states can be read from (5.22) with $g^3 \to g^2 f$, for *dsb* states with $g^3 \to gf^2$, and for $\nu_e \nu_\mu \nu_\tau$ with $g^3 \to f^3$. In each case the parameters (a,Λ) have been left unchanged. Note that since β depends only on the generation parameter $\gamma(N)$ [see Eq. (5.17)] and not on g or f, the β value would be the *same* for all quarks and leptons of the *same generation* (e.g., $e^+ u \bar{d} \nu_e$).

Using the above considerations, the mass formula (5.22) can be recast in a universal form (I), normalized to the electron mass, viz.,

$$\text{(I)}\quad M/m_e = \left[\frac{\beta}{\beta_e} \left\{ \frac{1+k}{1+k\alpha^p} \right\} \alpha^{p/7} \right]^{21}, \tag{6.1}$$

where

$$\alpha = F/g, \quad k = g^3(\Lambda/m_e)^{1/3}, \tag{6.2}$$

and $p=0,1,2,3$ for e^+, u, $\bar{d}$, ν_e, respectively. β is the β value [Eq. (5.20)], appropriate to the generation under study. Note that neither k nor α enters the charged-lepton series ($e\mu\tau$) corresponding to $\alpha=1$, whose mass ratios are already given by Eq. (5.23). As to the other composites, the mass predictions of formula (I) [Eq. (6.1)] are listed in Table I for the (illustrative) values

$$k=0.60, \quad \alpha=0.25 . \tag{6.3}$$

Unfortunately, this formula gives too large values for the neutrino masses. Considerable improvement in this regard can be effected by relaxing the universality restriction on the parameters (a,Λ). For example, if the parameter a for the e^+ case is assumed proportional to g^2, viz.,

$$a(g) = a_0 g^2 , \tag{6.4}$$

then its functional forms for $u,\bar{d},\nu$ are determined by the respective replacements

$$g^2 \to g^{4/3}F^{2/3}, \quad g^{2/3}F^{4/3}, \quad F^2 . \tag{6.5}$$

With these replacements, one now arrives at the alternative formula

$$\text{(II)}\quad M/m_e = \left[\frac{\beta}{\beta_e} \left\{ \frac{1+k}{1+k\alpha^p} \right\} \alpha^{p/3} \right]^{21}, \tag{6.6}$$

where α,k continue to be given formally by (6.2), but their values which give reasonable fits to the masses are now different,

$$k=2.95, \quad \alpha=0.605 . \tag{6.7}$$

The mass predictions of (II) are listed in Table I, below those of (I) in each case. One now finds a significant improvement in the neutrino masses, without affecting the quality of the results for the (*uct*) and (*dsb*) series. While the *uct* spectrum looks rather good, the *dsb* series is not so satisfactory, d being too low, b fairly low, and s rather high. On the other hand, it is precisely for the *dsb* series that one expects the Cabibbo or KM[33] mixing to come into play, as a result of which an upward shift in d,b and an opposite (downward) shift in s would be a distinct possibility. The effect of Cabibbo or KM mixing on the *dsb* mass spectra which depends on certain technical assumptions on the mass operator in this model, will be taken up in a later communication.

TABLE I. Mass prediction (in MeV) for leptons and quarks with e^+ as input. The upper and lower figures in each case correspond to the mass formulas (I) and (II), respectively. See text for details.

	e^+	u	$\bar{d}$	ν_e
(I)	0.511	8.20	1.11	$(31)\times10^{-3}$
(II)	0.511	9.45	1.08	1.8×10^{-3}
	μ^+	c	$\bar{s}$	ν_μ
(I)	100.5	1614	219	6.1
(II)	100.5	1859	212	0.35
	τ^+	t	$\bar{b}$	ν_τ
(I)	1435	23030	3127	87
(II)	1435	26530	3030	5.05

We end this section with some comments on the color factor F_{123} for leptons and its counterpart G_{123} for quarks. As listed in the beginning of Sec. II, in general colored *ttt* states ($\underline{8}$,$\underline{10}$) as well as *vtt* states of the "wrong" color ($\underline{6}^*$,$\underline{15}$) must necessarily appear along with the desired multiplicities ($\underline{1}$ for *ttt*; $\underline{3}$ for *vtt*). To keep out the unwanted color states, one would presumably need some dynamical mechanism which may be introduced through a suitable ansatz on the color factors F_{123} (for *ttt*) or G_{123} (for *vtt*) noted above. A simple factorizable form for F_{123} consistent with Fig. 1(a) is[34]

$$F_{123}=P_{12}P_{31}P_{23}\ ,\quad P_{ij}=\tfrac{1}{3}-\tfrac{1}{2}\vec{\lambda}_i\cdot\tfrac{1}{2}\vec{\lambda}_j\ , \tag{6.8}$$

where P_{ij} is the projection operator for a color-antisymmetric ($\underline{3}^*$) *tt* pair. The operator *F*, which (like Σ^a) is totally antisymmetric in color space, gives "zero force" in all but the color-singlet *ttt* states for which its eigenvalue is unity. Likewise for *vtt* states [Fig. 1(b)] we suggest the following factorable form for G[34]:

$$G_{123}=Q_{12}Q_{31}P_{23}\ ;\quad Q_{12}=\tfrac{1}{9}-\tfrac{1}{6}\vec{\bar{\lambda}}_1\cdot\vec{\lambda}_2\ , \tag{6.9}$$

where Q_{ij} is the *color-singlet* projection operator for a *vt* pair. This gives unit eigenvalue for the $\underline{3}$ representation of *vtt* and zero for 15 and 6*. For *vvv* states P_{ij} must be interpreted as the projection operator for an antisymmetric $\underline{3}$ representation of a *vv* pair, while the definition (6.9) for Q_{ij} remains unchanged. The individual factors of *F* and *G* may be indicated cyclically[34] in Figs. 1(a) and 1(b), respectively. At this stage we are unable to offer any field-theoretic insight into these *ad hoc* constructions which must be regarded as an effective description with a certain degree of cyclic symmetry appeal (like the spin operator Σ^a).

VII. DISCUSSION AND SUMMARY

In the foregoing, we have tried to present a concrete model of preon dynamics in which the emphasis is first on a general understanding of certain crucial features of leptons and quarks (spin $\frac{1}{2}$, generation structure), pending a deeper (field-theoretic) level of formulation, many of which are under way.[35] The model, whose chief ingredient is a totally antisymmetric spin operator Σ^a in all the three preons taken together has been developed in a stagewise fashion starting from a qualitative level and going into successively quantitative details. Already in the first stage (Sec. II), it was found that the operator Σ^a (i) gives zero-force for $S=\frac{3}{2}$ states, thus rendering them dynamically irrelevant, and (ii) singles out only three classes of $S=\frac{1}{2}$ states (out of six available ones) for the "confining interaction," thus providing a natural set of candidates for generation identification. These qualitative results followed directly from the properties of the Σ^a operator without any further assumptions on the details of dynamics. This operator also brought out the special relevance of certain complex combinations of pairs of spin functions (χ',χ'') as well as space functions ($\psi',\psi'';\psi^s,\psi^a$) which exhibit a duality property with respect to S_3 symmetry. Such complex wave functions do not seem to have had any counterpart in conventional *qqq* spectroscopy characterized by real wave functions in individual degrees of freedom, though they have been known in the three-body literature.

In the second stage (Secs. III and IV), we have suggested a more quantitative structure for the three different generations as λ eigenfunctions of the operator $\hat{\Lambda}=-i\partial_\lambda$ with three distinct classes of eigenvalues $\frac{1}{2}N$ ($N=3n\pm1$, $3n$). Even such structures are sufficiently general, insofar as these representations require only a certain form of three-preon dynamics, viz. a three-body Schrödinger-type equation (2.10) in which the "potential" is a function of ρ only. This form was sought to be justified in the sense of an ultrarelativistic equation with the traditional roles of coordinates and momenta interchanged, which was obtainable through a reduction of a three-preon Bethe-Salpeter equation in the instantaneous approximation. Even before going into the precise form of the confining dynamics, the three types of λ harmonics characterizing as many generations were already found to provide very strong selection rules against EM transitions among lepton generations.

It is only in the third stage of this investigation (Secs. V and VI) that more specific assumptions on the model have been made. In particular, the form of confinement, as interpreted in the coordinate representation, is extremely steep, viz. a 12th power, which partly accounts for the high powers[21] evidenced in the mass formulas (6.1) or (6.6). The other factors responsible for such a strong power dependence of *M* are the *ad hoc* assumptions (5.1) on the *M* dependence of the main parameters of the confining interaction. The universality of the (l,q) mass formulas (6.1) or (6.6) has come about from another kind of *ad hoc* assumption, namely an implied property of factorizability in the total strength of the three-way preon interaction wherein the constants *g* and *f* appear in the following combinations for the various cases:

$$g^3\ (ttt),\quad g^2f\ (vtt)\ ,$$
$$gf^2\ (tvv),\quad f^3\ (vvv)\ .$$

With these (speculative) assumptions, though it has been possible to obtain a reasonable mass spectrum for (l,q) states in units of the electron mass in a very economical manner (with just two parameters k and α), these results must nevertheless be regarded as illustrative of the basic applicational potential of the model, rather than as a serious fit to the data. Indeed the parametrization (5.1) is already in trouble from another angle, that is to say, the mass scale of the (l,q) form factors. For, according to the physical content of Eqs. (5.5) and (5.7), this mass scale Λ_M is proportional to $\alpha^{-9/7}$, which when substituted from (5.1) gives

$$\Lambda_M = (\tfrac{49}{81}g^3)^{3/7}\left[\frac{\sqrt{2}}{3}a^6\Lambda M^6\right]^{1/7}$$
$$= (m_e M^6)^{1/7}(\tfrac{324}{49}\beta^{-1})^3 ,$$

the last form being the result of normalization of the parameters to the electron mass, according to Eq. (5.22). This formula predicts for the mass scale of the electron form factor a ridiculously low value of 65 MeV, which is directly traceable to the very parametrization (5.1) which had led to a fairly successful mass spectrum in the first place.

Such problems (serious as they are) are not beyond remedy, especially since the parametric structure of the kernel is rich enough to admit of many alternative forms of parametrization. However, in the language of this section, such alternative forms of parametrization would affect only the third stage of our program, without vitiating the more general features brought out in the first two stages. Therefore, there is much to be said in favor of a deeper formulation of the Σ^a dynamics which not only controls the spin-$\frac{1}{2}$ and generation stability aspects of (l,q) states, but also has other desirable features. One such feature, already noted in Sec. III, stems from the commutativity property

$$[\gamma_5^{(i)},\sigma_{\mu\nu}^{(i)}]=0 \quad (i=1,2,3) ,$$

as a result of which the chiral structure of a three-preon wave function is not destroyed by the spin-operator Σ^a characterizing the full three-preon kernel, thus ensuring the preservation of chiral $SU(2)\times U(1)$ symmetry of the standard electroweak model at the composite (l,q) level.

Our efforts at a more fundamental formulation of Σ^a dynamics are still premature. In the meantime, other related investigations including alternative forms of parametrization of the confining interaction so as to conform more consciously to the mass scale constraints, as well as suitable applications involving more direct use of the preon wave functions, are under way.

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