Constraints on proton lifetime and $\sin^2\theta_W$ in SO(10) grand unified theory

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Simple constraints on the proton-decay lifetime are derived in SO(10) grand unified theory for major descents to the standard group $SU(3)_C \times SU(2)_L \times U(1)_Y$. It is shown that the only unambiguous pattern of symmetry breaking that predicts a longer proton lifetime [than SU(5)] goes through the Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$ and distinguishes quarks from leptons [i.e., SU(4) breaks down to $SU(3)_C \times U(1)_{B-L}$] before parity is broken [i.e., $SU(2)_R$ is broken]. The lifetime depends on M_R [the mass scale associated with $SU(2)_R$ breaking] but neither on the other intermediate mass scales nor on $\sin^2\theta_W$ in the limit where Higgs-boson contributions are neglected.

I. INTRODUCTION

The minimal grand unified theory (GUT)[1] based on SU(5) has been remarkably successful both in explaining the quantization of electric charge and in predicting[2] an acceptable value of the weak-neutral-current mixing angle θ_W: In particular, Marciano and Sirlin[3] have obtained, taking into account higher-order corrections,

$$\sin^2\theta_5(M_W) = 0.210 + 0.004 N_H \pm 0.002 , \qquad (1)$$

where $\sin^2\theta_5(M_W)$ is the value of $\sin^2\theta_W(M_W)$ predicted by SU(5), M_W is the mass of W_L, and N_H is the number of light Higgs doublets.[3] The uncertainty in Eq. (1) reflects the uncertainty in the QCD parameter $\Lambda_{\overline{MS}}$ ($\Lambda_{\overline{MS}} = 100-200$ MeV, and MS denotes the minimal-subtraction scheme). Minimal SU(5) GUT also predicts the grand unification mass M_5 where the low-energy coupling constants $\alpha_s(SU(3)_C)$, $\alpha_L(SU(2)_L)$, and $\alpha_Y(U(1)_Y)$ merge:

$$M_5 = (1 \text{ to } 4) \times 10^{14} \text{ GeV} . \qquad (2)$$

The uncertainty in Eq. (2) comes from the uncertainty in $\Lambda_{\overline{MS}}$ (a factor of two) and uncertainties in possible higher-order effects.[4] From M_5 one predicts the proton lifetime[4]:

$$\tau_p = 10^{29 \pm 2} \text{ yr} , \qquad (3)$$

where the uncertainty in Eq. (3) reflects both the uncertainty in M_X and in the calculation of hadronic matrix elements. Note that increasing the number of Higgs doublets or adding Higgs scalars in other representations (in order to explain a possible deviation of ρ from unity[5]) can only decrease M_5 and thus decrease τ_p.[6] Recently, the Irvine-Michigan-Brookhaven (IMB) experiment[7] has failed to observe proton decay in the supposed dominant decay mode $e^+\pi^0$, implying $\tau_p \geq 6.5 \times 10^{31}$ yr. This result raises seroius doubt concerning the validity of minimal SU(5) GUT.

Even if minimal SU(5) GUT is ruled out, the question still remains as to whether some other GUT group can match the desirable properties of SU(5) and still predict an acceptable lifetime for proton decay. SO(10) GUT was first introduced[8] to cure a rather inelegant feature of SU(5), namely the fact that each fermion generation requires two ($\bar{5} \oplus 10$) irreducible representations of SU(5). SO(10) GUT can assign each family of fermions to a single irreducible representation 16, and, indeed, makes provision for a right-handed neutrino and *a fortiori* for a finite neutrino mass. Later, it was shown[9] that only SO(10) shares the property, with SU(5), of containing $SU(2)_L$ and $U(1)_Y$ as local symmetries and not allowing exotics or mirror fermions. If we allow exotic particles which are real as a whole with respect to the color group, $SU(2)_L$, and $U(1)_Y$ and insist on a single irreducible representation, E_6 GUT is permitted in addition to SO(10). The argument why mirror fermions are considered undesirable is that it is difficult to understand the lightness of ordinary fermions if one uses the survival hypothesis. In order to escape these constraints, some people have resorted to reducible representations of SU(n) or to the introduction of gauge groups other than the standard ones (e.g., hypercolor, etc.).[10]

SO(10) is attractive for another reason: if one assumes that the grand unification group contains $SU(2)_L$ and $B - \alpha L$ [α is a numerical constant and B (L) denotes the baryon (lepton) number] as local symmetries and again requires the absence of exotic particles and mirror fermions, then SO(10) is unique[11] and has $B - L$ ($\alpha = 1$) as a generator. For all these reasons, it is desirable to reexamine proton decay within the framework of SO(10).

In the past, a number of papers have dealt with the predictions of GUT's beyond minimal SU(5) for proton decay and $\sin^2\theta_W$. However, a clear connection between SU(5) and other GUT's has not been derived and the special character of SO(10) has not been fully appreciated. In this paper, we show that it is possible to write down simple and explicit relations between the predictions of SU(5) and SO(10) for the unification mass and $\sin^2\theta_W$. Further, we will show that SO(10) predicts a longer proton lifetime [than SU(5)] for one particular pattern of symmetry breaking to the standard group $SU(3)_C \times SU(2)_L \times U(1)_Y$. In Sec. II, we present some details of our calculation and in Sec. III we discuss our results.

II. RENORMALIZATION-GROUP ANALYSIS OF SO(10)

It is well known that if we restrict ourselves to maximal subgroups, then there are only two cases: (A) $SO(10) \to SO(6)\times SO(4)$ [i.e., $SU(4)\times SU(2)\times SU(2)$]; (B) $SO(10)\to SU(5)\times U(1)_X$. Even if we extend candidates for subgroups to symmetric subgroups,[12] we are forced back to cases (A) and (B), since other possibilities, $SO(10)\to SO(p)\times SO(q)$ $(p+q=10,\ p\neq 6)$, cannot have complex representations with respect to $SU(3)_C\times SU(2)\times U(1)$. Furthermore, since the color group is SU(3), case (A) has the unique physical assignment to its simple groups: $SU(4)\times SU(2)_L\times SU(2)_R$, where SU(4) contains $SU(3)_C$ and $U(1)_{B-L}$ and the electric charge operator is given by $Q=T_{3L}+T_{3R}+(B-L)/2$.[13] However, case (B) has two different physical interpretations for SU(5): (a) the Georgi-Glashow SU(5) (Ref. 1); (b) SU(5)′, where the electric-charge operator is given by $Q=T_{3L}-\frac{1}{5}Z+\frac{1}{5}X$ [$U(1)_Z$ is contained in SU(5)′].[14]

As far as the proton lifetime is concerned, case (B) is not interesting, since one can immediately show that $\tau_p\le\tau_p^{SU(5)}$. More explicitly, τ_p for the Georgi-Glashow type is essentially identical with minimal SU(5), whereas SU(5)′ leads to a shorter proton lifetime.[14] It should be noted that for both paths of case (B) at the first stage of symmetry breakdown, both $SU(2)_L$ and $SU(3)_C$ are contained in a simple group [SU(5)] and the $B-L$ generator is obtained only by a linear combination of a generator of SU(5) and $U(1)_X$, i.e., $B-L=-4Y+X$ for the Georgi-Glashow SU(5) and $B-L=(X+4Z)/5$ for SU(5)′. The $U(1)_X$ symmetry is broken at the second stage of symmetry breaking, but this mass scale cannot be constrained, except by Higgs-boson contributions (see the Appendix). The paths in case (A) have just the reverse property in that the very first stage of symmetry breaking yields the separation of $SU(3)_C$ and $SU(2)_L$, and SU(4) contains the explicit $U(1)_{B-L}$ generator.

Before proceeding to our analysis of case (A) for SO(10), it is useful to state explicitly the following boundary condition.

Boundary condition: Assume that a grand unified group G has an intermediate step G_1 before being broken into G_0 and the associated mass scales are M_1 and M_0, respectively. In the limit where M_0 coincides with M_1, then both the mass scale and the value of $\sin^2\theta$ are given by the values obtained when G is directly broken into G_0.

Taking into account this condition, it is clearly sufficient to discuss the following two paths for case (A) (Ref. 15):

1. $$SO(10)\underset{M_X}{\to}SU(4)\times SU(2)_R\times SU(2)_L \underset{M_C}{\to} SU(3)_C\times U(1)_{B-L}\times SU(2)_R\times SU(2)_L \underset{M_R}{\to} SU(3)_C\times U(1)_{B-L}\times U(1)_R\times SU(2)_L \underset{M'}{\to} SU(3)_C\times U(1)_Y\times SU(2)_L\ ;$$

2. $$SO(10)\underset{M_X}{\to}SU(4)\times SU(2)_R\times SU(2)_L \underset{M_R}{\to} SU(4)\times U(1)_R\times SU(2)_L \underset{M_C}{\to} SU(3)_C\times U(1)_{B-L}\times U(1)_R\times SL(2)_L \underset{M'}{\to} SU(3)_C\times U(1)_Y\times SU(2)_L\ .$$

Since SU(4) contains $B-L$ as a generator, it is evident that both paths for case (A) proceed through $B-L$ gauge groups[15a] before reaching the standard group. It is also obvious that if $M_C\to M_X$ in path 1 and $M_R\to M_X$ in path 2, the intermediate Pati-Salam group $SU(4)\times SU(2)_L\times SU(2)_R$ is eliminated.

Now, we carry out the renormalization-group analysis for the two paths without Higgs-boson contributions. This enables us to derive some clearcut constraints on proton lifetime and $\sin^2\theta$ for both paths. Later, we will consider how these constraints are modified by the presence of Higgs scalars. We take the view that greater reliance can be placed on those conclusions whose qualitative features are not altered by the Higgs contribution.

If we next invoke the above boundary condition for both paths of case (A) and use renormalization-group equations, the intermediate mass scales M_C, M_R, and M' should satisfy the following equation:

$$0=A\ln(M_X/M_5)+B\ln(M_5/M_j)+C(\sin^2\theta-\sin^2\theta_5)/\alpha_e\ , \tag{4}$$

where M_j denotes the intermediate mass scale and A,B,C are numerical constants. In fact, without Higgs-boson contributions, one obtains

$$\ln(M_X/M_5)+\tfrac{1}{2}\ln(M_5/M_C)=3\pi\Delta/11\alpha_e\ , \tag{5}$$

$$\ln(M_5/M_C)+\ln(M_5/M_R)=6\pi\Delta/11\alpha_e\ , \tag{6}$$

where $\Delta=\sin^2\theta-\sin^2\theta_5$ and the two equations hold for both paths. Note that M', which is the mass scale for the breaking of $U(1)_R$, drops out and there is no constraint on it. [This provides a motivation for the study of a low-energy effective theory based on $SU(3)_C\times SU(2)_L\times U(1)_R\times U(1)_{B-L}$ (Ref. 16).] The fact that the $U(1)_R$-breaking mass scale is not restricted by the renormalization-group analysis has a simple explanation (see the Appendix).

A very interesting relation emerges from Eqs. (5) and (6):

$$\ln(M_X/M_5)=\tfrac{1}{2}\ln(M_5/M_R)$$

or

$$M_X=M_5(M_5/M_R)^{1/2}\ . \tag{7}$$

Equation (7) tells us that there is a simple connection between the strength of charged right-handed currents and the SO(10) unification mass. Note that the unification mass is independent of M_C and $\sin^2\theta$. Similarly, one obtains

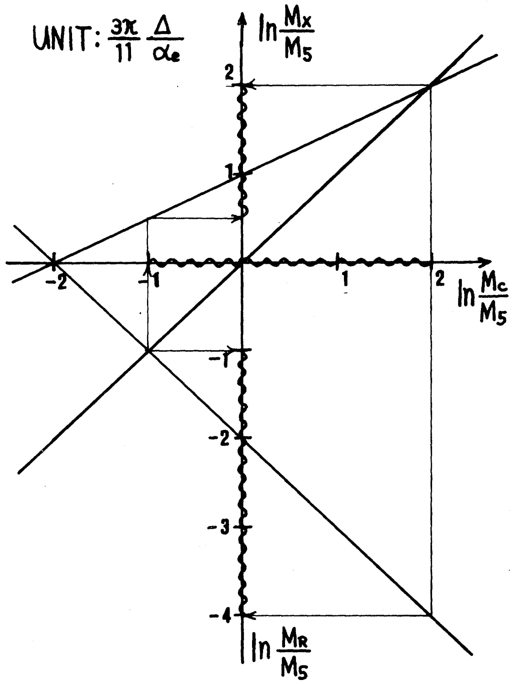

FIG. 1. Mass relation for path 1. Wavy lines denote the allowed regions.

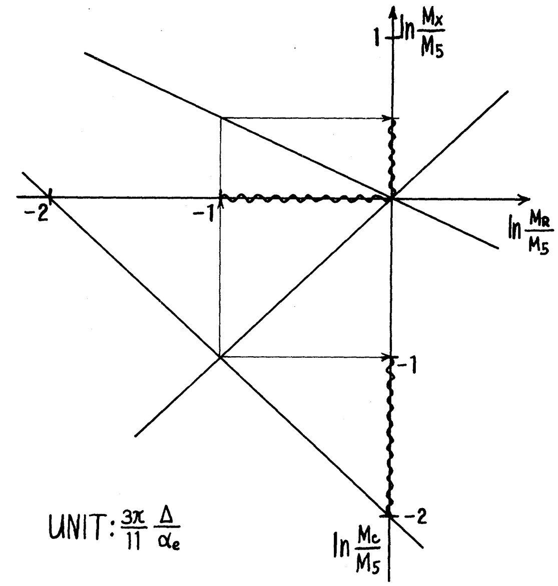

FIG. 2. Mass relation for path 2. Wavy lines denote the allowed regions.

$$\sin^2\theta=\sin^2\theta_5+11\alpha_e/(6\pi)[\ln(M_X/M_C)+\tfrac{1}{2}\ln(M_5/M_R)] \,. \quad (8)$$

Thus, having either M_C or M_R as intermediate mass scales implies an increase of $\sin^2\theta$ beyond its value in minimal SU(5).

One can obtain bounds for various mass scales from Eqs. (5) and (6). One should be careful not to violate the order of mass scales, i.e., $M_X \geq M_C \geq M_R$ for path 1 and $M_X \geq M_R \geq M_C$ for path 2. Furthermore, unless $\sin^2\theta=\sin^2\theta_5$, one cannot have $M_X=M_R=M_C$. These constraints have not been emphasized before.[17] The easiest way of obtaining bounds on the intermediate mass scale is as follows: For path 1, take the horizontal axis to be $\ln(M_C/M_5)$ and the vertical axis to be $\ln(M_X/M_5)$ and $\ln(M_R/M_5)$, since we must have $M_X \geq M_C \geq M_R$. The result is given in Fig. 1. We have

$$M_5\exp\left[\frac{1}{2}\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] \leq M_X \leq M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] , \quad (9)$$

$$M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] \leq M_C \leq M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] , \quad (10)$$

$$M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] \geq M_R \geq M_5\exp\left[-4\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] . \quad (11)$$

Note the correlation: the larger M_X and M_C, the smaller M_R. The smallest M_R is possible when $M_X=M_C$ [SO(10$\rightarrow$ $SU(3)_C\times U(1)_{B-L}\times SU(2)_R\times SU(2)_L$]. Note also that since $M_R \geq M_W$, we have $M_X \leq M_5(M_5/M_W)^{1/2}$ for $\sin^2\theta \geq \frac{1}{4}+\frac{1}{3}\alpha_e/\alpha_s$ ($=0.28$ for $\alpha_s=10^{-1}$ and $\alpha_e=128^{-1}$). For $M_X=M_C$, $M_R=M_W$ is allowed for $\sin^2\theta \geq 0.28$. This fact was noticed by Rizzo and Senjanovic.[18] For path 2, we obtain (see Fig. 2 for details)

$$M_5 \leq M_X \leq M_5\exp\left[\frac{1}{2}\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] , \quad (12)$$

$$M_5 \geq M_R \geq M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] , \quad (13)$$

$$M_5\exp\left[-2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] \leq M_C \leq M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] . \quad (14)$$

Thus, it is clear that path 1 of case (A) predicts a longer proton lifetime, while path 2 yields an ambiguous result (since M_X is very close to M_5). As we see later, path 2 is very sensitive to Higgs contributions. Since we argue that the GUT prediction is unreliable if the qualitative features are dependent on Higgs-boson contributions, we discard path 2. It should be noted that for path 1, the longer the proton lives, the smaller M_R is. The value of $\sin^2\theta$ is always larger than its SU(5) value and in general increases as M_R decreases [see Eq. (8)].

Having established that path 1 of case (A) is the only pattern of symmetry breaking that can give an unambiguously longer proton lifetime than SU(5), we next turn on Higgs contributions to ascertain the correction due to Higgs scalars. We consider the effect of those Higgs scalars which contribute between M_C and M_X since they have the most significant effect on the unification mass scale

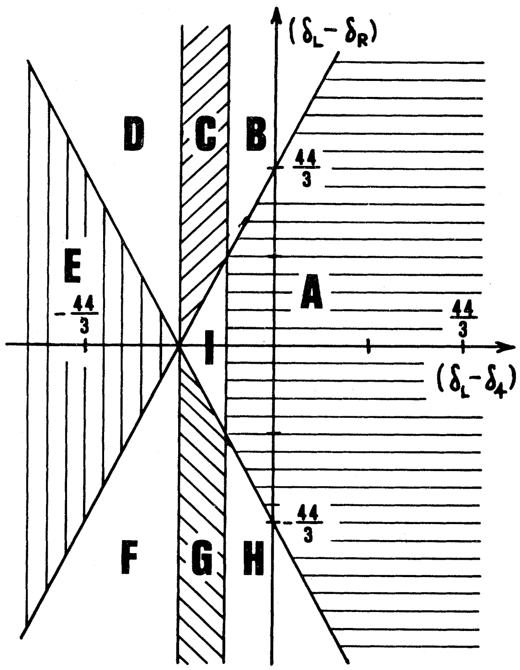

FIG. 3. Nine cases of various Higgs-boson contributions for path 1. The boundary lines are: $\delta_L-\delta_4=-\frac{11}{3}, -\frac{22}{3}$; $2(\delta_L-\delta_4)+(\delta_L-\delta_R)=-\frac{44}{3}$; $2(\delta_L-\delta_4)-(\delta_L-\delta_R)=-\frac{44}{3}$. Case A corresponds to the one discussed in the text.

M_X.

For path 1, we obtain the following equations, similar to Eqs. (5) and (6):

$$[1+\tfrac{3}{22}(\delta_L-\delta_4)]\ln(M_X/M_5)+\tfrac{1}{2}[1+\tfrac{3}{11}(\delta_L-\delta_4)]\ln(M_5/M_C)=3\pi\Delta/(11\alpha_e)\,, \quad (15)$$

$$[1+\tfrac{3}{44}(3\delta_L-\delta_R-2\delta_4]\ln(M_5/M_C)+[1+\tfrac{3}{22}(\delta_L-\delta_4)]\ln(M_5/M_R)=6\pi\Delta/(11\alpha_e)[1+\tfrac{3}{44}(\delta_L+\delta_R-2\delta_4)]\,, \quad (16)$$

where the constants δ_4, δ_L, and δ_R denote Higgs-boson contributions to the groups SU(4), $SU(2)_L$ and $SU(2)_R$, respectively.[19] Eliminating Δ from Eqs. (15) and (16), we get

$$M_X=M_5(M_5/M_R)^{1/2}[(M_5/M_R)^{1/2}(M_5/M_C)]^H\,, \quad (17)$$

where

$$H=\tfrac{3}{44}(2\delta_4-\delta_L-\delta_R)/[1-\tfrac{3}{44}(2\delta_4-\delta_L-\delta_R)]\,. \quad (18)$$

Because of the large factor $(M_5/M_R)^{1/2}$, Higgs scalars do not change the relation $M_X>M_5$. The bounds for various mass scales depend on the choice of Higgs scalars (see Fig. 3). For the typical case (case A in Fig. 3), the bounds are given as follows:

$$M_5\exp\left[\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\frac{1+\frac{3}{22}(2\delta_4-\delta_L-\delta_R)}{2+\frac{3}{44}(5\delta_L-\delta_R-4\delta_4)}\right]\le M_X\le M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,, \quad (19)$$

$$M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\frac{1-\frac{3}{44}(2\delta_4-\delta_L-\delta_R)}{1+\frac{3}{88}(5\delta_L-\delta_R-4\delta_4)}\right]\le M_C\le M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,, \quad (20)$$

$$M_5\exp\left[-\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\frac{1-\frac{3}{44}(2\delta_4-\delta_L-\delta_R)}{1+\frac{3}{88}(5\delta_L-\delta_R-4\delta_4)}\right]\ge M_R\ge M_5\exp\left[-4\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,. \quad (21)$$

For example, as emphasized by Del Aguila and Ibanez,[20] we take into account the contribution from 126 and employ the extended survival hypothesis.[21] That is, for path 1, we include a single Higgs scalar (10, 1, 3) of $SU(4)\times SU(2)_L\times SU(2)_R$ and put its mass equal to M_C. The bounds change as follows:

$$M_5\exp\left[\frac{10}{8}\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\le M_X\le M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,, \quad (22)$$

$$M_5\exp\left[-\frac{23}{8}\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\le M_C\le M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,, \quad (23)$$

$$M_5\exp\left[-\frac{23}{8}\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\ge M_R\ge M_5\exp\left[-4\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right]\,. \quad (24)$$

Thus, Higgs scalars tend to increase M_X and lower M_C and M_R, so that the proton lifetime continues to be longer than the SU(5) value.

Compared with path 1, we show that path 2 is very sensitive to Higgs-boson contributions. As an example, consider $SO(10)\to SU(4)\times SU(2)_L\times U(1)_R$. Then, the unification mass is given by

$$\ln(M_X/M_5)=\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\frac{2\delta_4-\delta_L-\delta_R}{1+\frac{1}{22}(5\delta_L-\delta_R-4\delta_4)}\,, \quad (25)$$

where δ_R now denotes the Higgs-boson contribution to $U(1)_R$.[22] Thus, the unification mass very easily changes the direction of inequality from M_5 through the Higgs-boson contribution $2\delta_4-\delta_L-\delta_R$. Depending on which Higgs scalars are used, one obtains $M_X>M_5$ (longer proton lifetime)[23] or $M_X<M_5$ (shorter proton lifetime).[20]

For path 1 of case (A), Eqs. (22)–(24) can be used to make numerical estimates for proton lifetime and $\sin^2\theta$.

If, for example, we assume the observability of proton decay with the present generation of experiments,[24] i.e., $\tau_p \leq 10^{33}$ yr, and note that[4]

$$\tau_p = (0.6 \text{ to } 25)(M_X/5\times 10^{14})^4\ 10^{30} \text{ yr} ,$$

we obtain the bound

$$M_X \leq 3.2\times 10^{15} \text{ GeV} .$$

Then, using Eqs. (22)–(24) and taking account of the fact that $M_5 = (1 \text{ to } 4)\times 10^{14}$ GeV, we conclude that for the observability of proton decay in SO(10) GUT, M_R should satisfy

$$M_R \geq 3.8\times 10^{10} \text{ GeV for } M_5 = 10^{14} \text{ GeV} ,$$

$$\geq 3.5\times 10^{12} \text{ GeV for } M_5 = 4\times 10^{14} \text{ GeV} .$$

On the other hand, if proton decay is not observable, i.e., $\tau_p \geq 10^{33}$ yr, and if SO(10) GUT is the correct group, one should have

$$\sin^2\theta - \sin^2\theta_5 \geq 0.025 \text{ for } M_5 = 10^{14} \text{ GeV} ,$$

$$\geq 0.015 \text{ for } M_5 = 4\times 10^{14} \text{ GeV} .$$

Let us recall that at present we have[4,25]

$$\sin^2\theta_{\text{expt}} = 0.215\pm 0.012 ,$$

$$\sin^2\theta_5 = 0.215\pm 0.002 .$$

Clearly a major improvement in the precision of $\sin^2\theta_{\text{expt}}$ is required.[25]

III. DISCUSSION

We have seen that the proton decay lifetime predicted by SO(10) GUT cannot exceed the SU(5) value when the descent at the first stage of symmetry breaking is through a simple group of the $V-A$ type, i.e., a group like SU(5) or SU(5)′ which contains both the weak left-handed chiral group $SU(2)_L$ and the vectorlike color SU(3) group. These $V-A$ paths of symmetry breaking must therefore be discarded if one is to explain the IMB experiment. The underlying reason for this result is that both the $SU(5)\times U(1)$ and $SU(5)'\times U(1)$ subgroups can generate proton decay through dimension-6 operators *QQQL* with leptoquark-boson masses comparable to those of SU(5) GUT, and consequently the lifetime cannot exceed the SU(5) GUT lifetime.

The situation is quite different for the parity-restoration path where the first stage of symmetry breaking is of the form $G \to G_C \times G_L \times G_R$, where G_C, G_L, G_R contain $SU(3)_C, SU(2)_L, SU(2)_R$, respectively. In the case of SO(10), $G_C = SU(4), G_L = SU(2)_L, G_R = SU(2)_R$. In this case, leptoquark bosons of G_C do not carry $SU(2)_L$ quantum numbers, and thus cannot induce proton decay through dimension-6 operators *QQQL*. Hence, proton decay only occurs through leptoquark bosons of G, which are not contained in G_C.

We have studied two major parity-restoration paths. Path 1 descends from the Pati-Salam group by differentiating between quarks and leptons [by breaking SU(4) into $SU(3)_C\times U(1)_{B-L}$], then breaks parity through a right-handed charged weak boson W_R [so that $SU(2)_R$ is broken into $U(1)_R$] and in the final stage reaches the standard group $SU(3)_C\times SU(2)_L\times U(1)_Y$ [by combining $U(1)_R$ with $U(1)_{B-L}$]. This pattern of symmetry breaking predicts a longer proton decay lifetime both without and with Higgs scalars being taken into account. It is important to note that the value of τ_p is not sensitive to the value of the "right-handed" Z-boson mass M' so that M' can be quite low (of the order of the "left-handed" Z-boson mass), and therefore detectable in the next generation of accelerators. If we set $M' = M_R$, we eliminate the intermediate group $U(1)_R$ and the breaking of parity is immediately related to the breaking of $B-L$ symmetry.

Path 2 has the intermediate group $SU(4)\times SU(2)_L \times U(1)_R$, and thus we have parity violation before the quarks and leptons are separated [since SU(4) remains unbroken]. In this case, the proton lifetime is very sensitive to the Higgs-boson contribution. Consequently, path 2 cannot be regarded as satisfactory.

In summary, we have shown that it is impossible to predict a satisfactory proton decay lifetime on the basis of SO(10) GUT [i.e., longer than the lifetime predicted by SU(5) GUT] unless parity is restored at some intermediate energy. The most plausible scenario if SO(10) GUT is to explain the longevity of the proton is for the quarks and leptons to be separated before parity is broken. Whether this parity breaking is directly connected with the breaking of $B-L$ local symmetry is not fixed by the proton-decay experiment. Other experiments will have to decide between these two possibilities.

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APPENDIX

The reason why sometimes the renormalization-group analysis does not give any restriction on mass scales is the following lemma.

Lemma. Let us suppose that $G_1\times G_2$ is broken into G_3 at the mass scale M where G_1 and G_2 are orthogonal to each other (i.e., $\text{Tr}X_1X_2 = 0$ where X_j denotes the Lie-algebra element of G_j). Furthermore, assume that the group structures are the same for G_j ($j = 1,2,3$). Then, the mass scale cannot be constrained, except by Higgs scalars.

Proof. In essence the result follows from the fact that the slopes for the coupling constants α_j^{-1} are the same for properly normalized coupling constants, if the group structures are the same. In detail, it is enough to show that for $X_3 = aX_1 + bX_2$, the equation

$$\alpha_3^{-1}(M) = a^2\alpha_1^{-1}(M) + b^2\alpha_2^{-1}(M) \tag{A1}$$

is actually independent of M under the above assumptions. Because of the renormalization-group equation

$$\alpha_j^{-1}(M) = \alpha_j^{-1}(M')(2\pi)^{-1}B_j \ln(M'/M) ,$$

the lemma is proved if the following is true:

$$B_3=a^2B_1+b^2B_2 \; . \qquad \text{(A2)}$$

Using $X_3=aX_1+bX_2$, the coefficient C_j for the properly normalized Lie algebra element $T_j=C_jX_j$ satisfies

$$C_3{}^{-2}=a^2C_1{}^{-2}+b^2C_2{}^{-2} \; , \qquad \text{(A3)}$$

since $\mathrm{Tr}X_3{}^2=a^2\mathrm{Tr}X_1{}^2+b^2\mathrm{Tr}X_2{}^2$.

Further, since the properly normalized coupling constant g_{0j} is given by $g_{0j}=C_j{}^{-1}g_j$, the proper B_{0j} is equal to $C_j{}^2B_j$. Because the group structures are the same, B_{0j} is independent of j, i.e., $B_{0j}=B_0$. Multiplying Eq. (A3) by B_0 yields Eq. (A2). The Higgs-boson contribution makes the B_{0j} different from each other.

Now, it is clear why the $\mathrm{U}(1)_R$ mass scale is not constrained. It is due to the fact that $\mathrm{U}(1)_R\times\mathrm{U}(1)_{B-L}$ is broken into $\mathrm{U}(1)_Y$. Furthermore, in the case where $\mathrm{SO}(10)\rightarrow\mathrm{SU}(4)\times\mathrm{U}(1)_R\times\mathrm{SU}(2)_L$ directly, the unification mass is given by M_5, if we do not include Higgs scalars. In the case where

$$\begin{aligned}\mathrm{SO}(10)&\rightarrow\mathrm{SU}(5)'\times\mathrm{U}(1)_X\\ &\rightarrow\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Z\times\mathrm{U}(1)_X\\ &\rightarrow\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y \; ,\end{aligned}$$

the $\mathrm{U}(1)_Z$ mass scale cannot be constrained except by Higgs scalars, since $\mathrm{U}(1)_Z\times\mathrm{U}(1)_X$ is broken into $\mathrm{U}(1)_Y$.

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and

$$\sin^2\theta_5=\frac{1}{6}+\frac{5}{9}\frac{\alpha_e}{\alpha_s} \; (=0.21) \; ,$$

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and

$$M_5'=M_5\exp\left[2\frac{3\pi}{11}\frac{\Delta}{\alpha_e}\right] \; ,$$

where $\Delta=\sin^2\theta-\sin^2\theta_5$, and we have dropped the subscript W in θ_W. Since M_X must be larger than M_5', both $\sin^2\theta$ and M_5' are smaller than those of SU(5).

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