Two-photon decays of heavy neutrinos

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(Received 25 February 1983)

The process $\nu_1 \rightarrow \nu_2 + \gamma + \gamma$ is studied in detail. Formulas for the decay rate are given in the context of the $SU(2) \times U(1)$ model with massive neutrinos and it is shown that this process can dominate over $\nu_1 \rightarrow \nu_2 + \gamma$ for some range of values of m_{ν_1}, depending on the relevant mixing angles. A general, model-independent analysis is also given with particular emphasis on the difference between the cases of Dirac and Majorana neutrinos. In particular, we show how the invariant amplitudes in the Majorana case can be expressed in terms of the amplitudes calculated as if the neutrinos were Dirac particles.

I. INTRODUCTION

The decay mode $\nu_1 \rightarrow \nu_2 + \gamma$ for massive neutrinos has been the subject of much discussion in the recent literature. This has been motivated in part by the implications of this process in a variety of astrophysical problems[1] and by the current theoretical ideas that suggest that neutrinos are massive.[2] The study of this process also has an intrinsic theoretical interest because it reveals some of the peculiar properties of Majorana neutrinos.[3-5] As the detailed calculations of the rates show,[6,7] in most models a leptonic Glashow-Iliopoulos-Maiani (GIM) mechanism suppresses the amplitude by a factor of $(m_\tau/M_W)^2$, which, for $m_\nu \simeq 50$ eV, gives a decay rate of the order of 10^{29} yr. It is possible to avoid the GIM cancellation in some models, which, however, look somewhat artificial unless they can be independently confirmed. An exception is perhaps the Zee model of neutrino masses[8] in which rates of the order of 10^{20} yr are obtained.[9]

Motivated by the same considerations, we have studied the process $\nu_1 \rightarrow \nu_2 + \gamma + \gamma$. Our purpose in this work is to discuss the general features in a model-independent way, with particular attention to the difference between the cases of Dirac and Majorana neutrinos. In addition, in order to gain an idea of the rates that can be expected, we have carried out an explicit calculation in the standard $SU(2) \times U(1)$ model with Dirac neutrino masses. Instead of the GIM suppression factor $(m_\tau/M_W)^4$ for the $\nu_1 \rightarrow \nu_2 + \gamma$ decay in the same model,[6] the rate for the two-photon mode is suppressed by $(m_\nu/m_e)^4$ for $m_\nu < 2m_e$. For $m_\nu \simeq 50$ eV, the two-photon mode is negligible, but for $m_\nu \simeq 2m_e$ it can dominate over the one-photon mode. For $m_\nu > 2m_e$, the channel $\nu_1 \rightarrow \nu_2 + e^- + e^+$ dominates and the radiative decays (with one or two photons) are irrelevant. We also show how the invariant amplitudes in the Majorana case can be expressed in terms of the amplitudes calculated as if the neutrinos were Dirac particles.

In Sec. II, we set down the general form of the amplitude and examine the implications of *CP* invariance and Hermiticity on the invariant amplitudes in four separate cases: Dirac or Majorana neutrinos with $\nu_1 = \nu_2$ or $\nu_1 \neq \nu_2$. In addition, it is shown how the amplitudes in the Majorana case can be expressed in terms of the amplitudes calculated as if the neutrinos were Dirac particles. In Sec. III, we exhibit an explicit calculation of the rate in the context of the standard $SU(2) \times U(1)$ model with Dirac neutrino masses, and Sec. IV contains our conclusions.

II. CONSTRAINTS ON THE INVARIANT AMPLITUDES

We discuss the kinematics in the channel

$$\gamma(k) + \nu_1(p_1) \rightarrow \gamma(k') + \nu_2(p_2) \ . \tag{2.1}$$

If $\nu_1 = \nu_2$, and if the neutrinos are Dirac particles, this process is similar to Compton scattering for which standard analyses exist.[10] Our purpose is to generalize them to include the off-diagonal case $\nu_1 \neq \nu_2$ and the case in which the neutrinos are Majorana particles.

In order to write down the most general form of the amplitude, it is convenient to define the orthogonal vectors

$$\begin{aligned} e_{1\mu} &\equiv \epsilon_{\mu\nu\alpha\beta} P^\nu K^\alpha q^\beta \ , \\ e_{2\mu} &\equiv q^2 P_\mu + K_\mu (P\cdot K) - q_\mu (P\cdot q) \ , \\ e_{3\mu} &\equiv q_\mu \ , \\ e_{4\mu} &\equiv K_\mu \ , \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} q &\equiv k + k' \ , \\ K &\equiv k' - k = p_1 - p_2 \ , \\ P &\equiv p_1 + p_2 \ . \end{aligned} \tag{2.3}$$

Using

$$e_i \cdot K = e_i \cdot q = 0 \quad (i = 1,2) \ ,$$

we obtain

$$e_i \cdot k = e_i \cdot k' = 0 \quad (i = 1,2) \ . \tag{2.4}$$

On the other hand,

$$e_3 \cdot k = e_3 \cdot k' = e_4 \cdot k = -e_4 \cdot k' = q^2 ,$$

so that only e_1 and e_2 are gauge invariant. Therefore, the most general form of the amplitude is

$$M = \epsilon^{\mu *}(k')\epsilon^{\nu}(k)\Gamma_{\mu\nu} , \tag{2.5}$$

where

$$\begin{aligned}\Gamma_{\mu\nu} = {} & G_0(e_{1\mu}e_{1\nu}+e_{2\mu}e_{2\nu})+G_1(e_{1\mu}e_{2\nu}+e_{2\mu}e_{1\nu}) \\ & +G_2(e_{1\mu}e_{2\nu}-e_{2\mu}e_{1\nu}) \\ & +G_3(e_{1\mu}e_{1\nu}-e_{2\mu}e_{2\nu}) .\end{aligned} \tag{2.6}$$

The term involving G_2 in Eq. (2.6) can be written in a different form. Using the identity

$$g_{\lambda\mu}\epsilon_{\nu\alpha\beta\gamma}-g_{\lambda\nu}\epsilon_{\mu\alpha\beta\gamma}+g_{\lambda\alpha}\epsilon_{\mu\alpha\beta\gamma}-g_{\lambda\beta}\epsilon_{\mu\nu\alpha\gamma}+g_{\lambda\gamma}\epsilon_{\mu\nu\alpha\beta}=0 ,$$

we obtain

$$e_{1\mu}e_{2\nu}-e_{2\mu}e_{1\nu}=P^2\epsilon_{\mu\nu\alpha\beta}K^{\alpha}q^{\beta} . \tag{2.7}$$

The G_i are of the form

$$G_i=\bar{u}(p_2)Q_i u(p_1) , \tag{2.8}$$

where, using the Dirac equation, the Q_i can be written as

$$\begin{aligned} Q_0 &= (F_1+f_1\gamma_5)+(F_2+f_2\gamma_5)\not{q} , \\ Q_1 &= i\gamma_5(F_3+f_3\gamma_5)+(F_4+f_4\gamma_5)\not{q} , \\ Q_2 &= i\gamma_5(F_5+f_5\gamma_5)+(F_6+f_6\gamma_5)\not{q} , \\ Q_3 &= (F_7+f_7\gamma_5)+(F_8+f_8\gamma_5)\not{q} . \end{aligned} \tag{2.9}$$

The invariant amplitudes F_i and f_i are functions of

$$t=K^2$$

and

$$\Delta=P\cdot q . \tag{2.10}$$

For later use, it is convenient to define $Q_{\mu\nu}$ by writing

$$M=\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q_{\mu\nu}(P,q,K)u(p_1) . \tag{2.11}$$

From Eqs. (2.5), (2.6), and (2.8), it follows that $Q_{\mu\nu}$ is given by a formula analogous to Eq. (2.6) with the replacement $G_i \to Q_i$.

We now examine the constraints on the F_i and f_i imposed by Hermiticity[11] and, if applicable, CP invariance. We consider four cases separately: $\nu_1=\nu_2$ and $\nu_1 \neq \nu_2$ when the ν_i are Dirac or Majorana particles.

A. $\nu_1 \neq \nu_2$ Dirac case

In this case, Hermiticity by itself does not imply any restrictions on the invariant amplitudes, but combined with CP invariance it yields some reality conditions. CP invariance implies that the Lagrangian is invariant under the substitution

$$\begin{aligned} \nu_{iL} &\to \eta_i \nu_{iR}^c , \\ \nu_{iR} &\to \eta_i \nu_{iL}^c \quad (i=1,2) , \\ A_\mu &\to -A_\mu , \end{aligned} \tag{2.12}$$

where A_μ is the photon field, and

$$\nu_i^c \equiv i\gamma_2\nu_i^* \tag{2.13}$$

in the Dirac representation for the γ matrices. If the Lagrangian is CP invariant, these transformations imply

$$\begin{aligned} & M(\gamma(k)+\bar{\nu}_1(p_1)\to\gamma(k')+\bar{\nu}_2(p_2)) \\ & \quad =\eta_1\eta_2\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q_{\mu\nu}^{CP}(P,q,K)u(p_1) , \end{aligned} \tag{2.14}$$

where $Q_{\mu\nu}^{CP}$ is obtained from $Q_{\mu\nu}$ by replacing $\gamma_5 \to -\gamma_5$ and $\epsilon_{\mu\nu\alpha\beta}\to -\epsilon_{\mu\nu\alpha\beta}$ (in particular, $e_{1\mu}\to -e_{1\mu}$).

On the other hand, Hermiticity implies that for any diagram D that contributes to $M(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2))$, there is a corresponding diagram D' that contributes to $M(\gamma(k')+\nu_2(p_2)\to\gamma(k)+\nu_1(p_1))$, which is obtained by replacing every vertex in D by its Hermitian conjugate. Therefore,

$$\begin{aligned} & M(\gamma(k')+\nu_2(p_2)\to\gamma(k)+\nu_1(p_1)) \\ & \quad =\epsilon^{\mu}(k')\epsilon^{\nu *}(k)\bar{u}(p_1)\bar{Q}_{\mu\nu}(P,q,K)u(p_2) , \end{aligned} \tag{2.15}$$

where[12]

$$\bar{Q}_{\mu\nu}(P,q,K)=\gamma_0 Q_{\mu\nu}^{\dagger}(P,q,K)\gamma_0 . \tag{2.16}$$

Further, the substitution rule[13] and Eq. (2.15) yield

$$\begin{aligned} & M(\gamma(k)+\bar{\nu}_1(p_1)\to\gamma(k')+\bar{\nu}_2(p_2)) \\ & \quad =(-1)\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{v}(p_1)\bar{Q}_{\mu\nu}(-P,-q,-K)v(p_2) , \end{aligned} \tag{2.17}$$

where

$$v(p)=i\gamma_2 u^*(p) . \tag{2.18}$$

Substituting Eq. (2.18) in Eq. (2.17) and comparing with Eq. (2.14), we finally obtain

$$\begin{aligned} F_i^* &= \eta_1\eta_2^* F_i , \\ f_i^* &= \eta_1\eta_2^* f_i . \end{aligned} \tag{2.19}$$

Thus, if the invariant amplitudes have no absorptive part,[14] then they are relatively real.

B. $\nu_1=\nu_2$ Dirac case

The amplitude in this case is of the same form as in the off-diagonal case:

$$\begin{aligned} & M(\gamma(k)+\nu(p_1)\to\gamma(k')+\nu(p_2)) \\ & \quad =\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q_{\mu\nu}(P,q,K)u(p_1) , \end{aligned} \tag{2.20}$$

where $Q_{\mu\nu}$ has an expansion similar to that given in Eq. (2.9). In contrast to the off-diagonal case, Hermiticity and CP invariance can be used independently in the present case. Let us consider Hermiticity first. Repeating the steps that led to Eq. (2.15), we obtain in this case

$$\begin{aligned} & M(\gamma(k')+\nu(p_2)\to\gamma(k)+\nu(p_1)) \\ & \quad =\epsilon^{\mu}(k')\epsilon^{\nu *}(k)\bar{u}(p_1)\bar{Q}_{\mu\nu}(P,q,K)u(p_2) \end{aligned}$$

so that

$$M(\gamma(k)+\nu(p_1)\to\gamma(k')+\nu(p_2))$$
$$=\epsilon^{\mu}(k)\epsilon^{\nu*}(k')\bar{u}(p_2)\bar{Q}_{\mu\nu}(P,q,-K)u(p_1)\ . \quad (2.21)$$

Comparing Eqs. (2.21) and (2.20), we then obtain

$$Q_{\mu\nu}(P,q,K)=\bar{Q}_{\nu\mu}(P,q,-K)\ ,$$

which in terms of the invariant amplitudes implies the conditions

$$\begin{aligned}&F_{1,2,4,5,7,8} \text{ and } f_{2,3,4,8} \text{ real },\\ &F_{3,6} \text{ and } f_{1,5,6,7} \text{ imaginary }.\end{aligned} \quad (2.22)$$

The assumption of CP invariance yields further restrictions. Following the steps leading to Eq. (2.14), and remembering that $\eta_1=\eta_2$ in the present case, we obtain

$$M(\gamma(k)+\bar{\nu}(p_1)\to\gamma(k')+\bar{\nu}(p_2))$$
$$=\epsilon^{\mu*}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q^{CP}_{\mu\nu}(P,q,K)u(p_1)\ , \quad (2.23)$$

where, as in Eq. (2.14), $Q^{CP}_{\mu\nu}$ is obtained from $Q_{\mu\nu}$ by making the replacement $\gamma_5\to-\gamma_5$ and $\epsilon_{\mu\nu\alpha\beta}\to-\epsilon_{\mu\nu\alpha\beta}$. The substitution rule then gives

$$M(\gamma(k')+\nu(p_2)\to\gamma(k)+\nu(p_1))$$
$$=(-1)\epsilon^{\mu}(k')\epsilon^{\nu*}(k)\bar{v}(p_2)Q^{CP}_{\mu\nu}(-P,-q,-K)v(p_1)\ ,$$

and finally

$$M(\gamma(k)+\nu(p_1)\to\gamma(k')+\nu(p_2))$$
$$=(-1)\epsilon^{\mu}(k)\epsilon^{\nu*}(k')\bar{v}(p_1)Q^{CP}_{\mu\nu}(-P,-q,K)v(p_2)\ . \quad (2.24)$$

Substituting Eq. (2.18) in (2.24) and comparing with Eq. (2.20), we obtain

$$F_{3,6}=f_{1,5,6,7}=0\ . \quad (2.25)$$

As it should be, Eq. (2.25) also follows by putting $\eta_1=\eta_2$ in Eq. (2.19) and comparing with Eq. (2.22).

C. $\nu_1\neq\nu_2$ Majorana case

Before proceeding to extend the preceding discussion to the case of Majorana neutrinos, it is useful to review briefly our phase conventions, which are explained in more detail in Ref. 5. In general, the phases of the left-handed neutrino fields ν_{iL} can be chosen such that the Majorana condition is

$$\nu^c\equiv i\gamma_2\nu_i^*=\nu_i \quad (2.26)$$

for every neutrino specie. Together with a phase convention for the charged-lepton fields, Eq. (2.26) implies a definite phase convention for the Kobayashi-Maskawa matrix in the lepton sector. If CP is a symmetry, then the Lagrangian is invariant under the transformation

$$\begin{aligned}&\nu_{iL}\to\eta_i\nu_{iR}\ ,\\ &\nu_{iR}\to\eta_i\nu_{iL}\ ,\\ &A_\mu\to-A_\mu\ ,\end{aligned} \quad (2.27)$$

where, as a consequence of Eq. (2.26),

$$\eta_i=\pm1\ . \quad (2.28)$$

Strictly speaking, the phase factors in the CP transformation law for Majorana neutrinos are $\eta=\pm i$ (see, e.g., Ref. 3 and references therein). However, since fermion fields always occur in pairs in the Lagrangian, only the relative sign between these factors is relevant. Equation (2.26) also implies that the plane-wave decomposition of the Majorana field is

$$\nu=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3p}{2E}\sum_s[u(p,s)e^{-ip\cdot x}a(p,s)+v(p,s)e^{ip\cdot x}a^*(p,s)] \quad (2.29)$$

where $v(p,s)$ is given by Eq. (2.18).

We now adapt the discussion in Sec. II A to the case of Majorana neutrinos. The amplitude is of the same form as in Eq. (2.11) for Dirac neutrinos. However, in contrast to the Dirac case, Hermiticity and CP invariance yield independent conditions on the invariant amplitudes. Let us consider Hermiticity first. Following the analysis leading to Eq. (2.17), and remembering that $\bar{\nu}_i=\nu_i$ in the present case, we obtain

$$M(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2))$$
$$=(-1)\epsilon^{\mu*}(k')\epsilon^{*}(k)\bar{v}(p_1)\bar{Q}_{\mu\nu}(-P,-q,-K)v(p_2)\ . \quad (2.30)$$

Substituting Eq. (2.18) in (2.30) and comparing with Eq. (2.11), we obtain the conditions

$$\begin{aligned}&F_i \text{ real (all } i)\ ,\\ &f_i \text{ imaginary (all } i)\ .\end{aligned} \quad (2.31)$$

CP invariance yields further conditions. If the Lagrangian is CP invariant, then the transformations in Eq. (2.27) imply that

$$M(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2))$$
$$=\eta_1\eta_2\epsilon^{\mu*}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q^{CP}_{\mu\nu}(P,q,K)u(p_1)\ , \quad (2.32)$$

where, as in Eq. (2.14), $Q^{CP}_{\mu\nu}$ is obtained from $Q_{\mu\nu}$ by making the substitution $\gamma_5\to-\gamma_5$ and $\epsilon_{\mu\nu\alpha\beta}\to-\epsilon_{\mu\nu\alpha\beta}$. Equations (2.32) and (2.11) imply

$$Q_{\mu\nu}(P,q,K)=\eta_1\eta_2Q^{CP}_{\mu\nu}(P,q,K)\ ,$$

which yields

$$\begin{aligned}&F_i=\eta_1\eta_2F_i\ ,\\ &f_i=-\eta_1\eta_2f_i\ .\end{aligned} \quad (2.33)$$

[These conditions also follow from Eq. (2.19) with $\eta_2^*=\eta_2$ and Eq. (2.31), as they should.] Therefore, if the initial and final neutrinos have the same (opposite) CP parity, $Q_{\mu\nu}$ behaves as a tensor (pseudotensor). This situation contrasts with the results for the process $\nu_1\to\nu_2+\gamma$. The quantity analogous to $Q_{\mu\nu}$ is Γ_μ defined by

$$M(\nu_1\to\nu_2+\gamma)=\epsilon^{\mu*}(q)\bar{u}(p_2)\Gamma_\mu u(p_1)\ .$$

In this case CP invariance implies that if the initial and final neutrinos have the same (opposite) CP parity, then Γ_μ is of the form $\sigma_{\mu\nu}q^\nu\gamma_5(\sigma_{\mu\nu}q^\nu)$ (Refs. 3–7). In our derivation, the origin of these contrasting results for $Q_{\mu\nu}$ and Γ_μ is the minus sign in the CP transformation law of A_μ in Eq. (2.27), which cancels in the two-photon process.

The result in Eq. (2.33) for the Majorana case also differs from the corresponding result in Eq. (2.19) for the Dirac case. In the Majorana case, CP invariance forbids the simultaneous presence of F_i and f_i terms while both type of terms are allowed in the Dirac case.

D. $\nu_1=\nu_2$ Majorana case

This is an interesting case because the Majorana condition, Hermiticity, and CP invariance yield independent conditions on the invariant amplitudes. The amplitude is of the same form given in Eq. (2.20). Remembering that $\bar{\nu}=\nu$, the substitution rule implies

$$M(\gamma(k')+\nu(p_2)\to\gamma(k)+\nu(p_1)) =(-1)\epsilon^{\mu}(k')\epsilon^{\nu*}(k)\bar{v}(p_2)Q_{\mu\nu}(-P,-q,-K)v(p_1)$$

so that

$$M(\gamma(k)+\nu(p_1)\to\gamma(k')+\nu(p_2)) =(-1)\epsilon^{\mu}(k)\epsilon^{\nu*}(k')\bar{v}(p_1)Q_{\mu\nu}(-P,-q,K)v(p_2)\ . \quad (2.34)$$

The consistency of Eqs. (2.34) and (2.20) requires

$$F_{3,6}=f_{2,3,4,8}=0\ . \quad (2.35)$$

Hermiticity yields additional conditions, which can be most simply obtained by putting $\nu_1=\nu_2$ in Eq. (2.30) for the off-diagonal Majorana case. In this way, we obtain conditions analogous to Eq. (2.31):

$$F_i \text{ real}\ , \qquad f_i \text{ imaginary}\ . \quad (2.36)$$

Equations (2.36) and (2.35) are consistent with Eq. (2.22) as they should be since the requirements of Hermiticity can also be obtained by putting $\bar{\nu}=\nu$ in the diagonal Dirac case.

In similar form, the implications of CP invariance in this case can be obtained from the diagonal Dirac case (with $\bar{\nu}=\nu$) or from the off-diagonal Majorana case with $\nu_1=\nu_2$ (and $\eta_1=\eta_2$). Together with Eq. (2.35) these two procedures yield equivalent conditions which are summarized by

$$f_i=0\ . \quad (2.37)$$

Therefore, only $F_{1,2,4,5,7,8}$ are nonzero and, according to Eq. (2.36) are real. If CP is not conserved, then $f_{1,5,6,7}$ can also be nonzero, and according to Eq. (2.36) they are imaginary.

E. The Majorana amplitude in terms of the Dirac amplitude

We now show how the invariant amplitudes in the Majorana case can be expressed in terms of the invariant amplitudes calculated as if the neutrinos were Dirac particles. Let us denote the contribution of a given diagram D by

$$M^D(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2)) =\epsilon^{\mu*}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q^D_{\mu\nu}(P,q,K)u(p_1)\ . \quad (2.38)$$

For each diagram D there is a corresponding diagram D' which is obtained by replacing every vertex in D by its Hermitian conjugate and gives an additional contribution to the amplitude. The contribution of D' can be obtained by the same reasoning leading to Eq. (2.17). (Remembering that $\bar{\nu}_i=\nu_i$.) Thus, we obtain

$$M^{D'}(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2)) =(-1)\epsilon^{\mu*}(k')\epsilon^{\nu}(k)\bar{v}(p_1)\bar{Q}^D_{\mu\nu}(-P,-q,-K)v(p_2)\ . \quad (2.39)$$

The complete amplitude is given by the sum of Eqs. (2.38) and (2.39). Denoting by F_i^M and f_i^M the complete invariant amplitudes in the Majorana case and by F_i^D, f_i^D the corresponding quantities calculated as if the neutrinos were Dirac particles, Eqs. (2.38) and (2.39) yield

$$F_i^M=F_i^D+F_i^{D*}\ , \qquad f_i^M=f_i^D-f_i^{D*}\ . \quad (2.40)$$

It is readily verified that Eq. (2.40) is consistent with our previous results. In particular, the results in Secs. II C and II D for the off-diagonal and diagonal Majorana cases, respectively, follow from Eq. (2.40) and the results in Secs. II A and II B for the corresponding Dirac cases. Equation (2.40) is also very useful in explicit calculations of the amplitude if one wishes to compare the rates in the Dirac and Majorana cases.

F. The Dirac neutrino as two degenerate Majorana neutrinos

It is instructive to recover the results for the diagonal Dirac case as the limiting case in which the Dirac neutrino is regarded as two degenerate Majorana neutrinos of opposite CP. Denoting by ν the Dirac neutrino, the associated Majorana neutrino fields are

$$\nu_1=\frac{1}{\sqrt{2}}(\nu+\nu^c)\ , \qquad \nu_2=\frac{1}{i\sqrt{2}}(\nu-\nu^c)\ . \quad (2.41)$$

The U(1) invariance associated with ν implies that [a similar proof is given in Ref. (5), Eq. (28)]

$$M(\gamma+\nu_1\to\gamma+\nu_1)=M(\gamma+\nu_2\to\gamma+\nu_2)\ , \quad (2.42a)$$

$$M(\gamma+\nu_1\to\gamma+\nu_2)=-M(\gamma+\nu_2\to\gamma+\nu_1)\ , \quad (2.42b)$$

and, therefore,

$$M(\gamma+\nu\to\gamma+\nu)=M(\gamma+\nu_1\to\gamma+\nu_1) +iM(\gamma+\nu_1\to\gamma+\nu_2)\ . \quad (2.43)$$

Defining $Q^{(11)}_{\mu\nu}$ and $Q^{(12)}_{\mu\nu}$ by

$$M(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_1(p_2))$$
$$=\epsilon^{\mu *}(k')\epsilon(k)\bar{u}(p_2)Q^{(11)}_{\mu\nu}(P,q,K)u(p_1)\ , \quad (2.44a)$$

$$M(\gamma(k)+\nu_1(p_1)\to\gamma(k')+\nu_2(p_2))$$
$$=\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{u}(p_2)Q^{(12)}_{\mu\nu}(P,q,K)u(p_1)\ , \quad (2.44b)$$

Eqs. (2.43) and (2.44) imply

$$F_i=F_i^{(11)}+iF_i^{(12)}\ ,$$
$$f_i=f_i^{(11)}+if_i^{(12)}\ , \quad (2.45)$$

where F_i,f_i are the invariant amplitudes for the Dirac neutrino, and $F_i^{(11)},f_i^{(11)}(F_i^{(12)},f_i^{(12)})$ are the corresponding quantities in the diagonal (off-diagonal) Majorana case. In addition to the relations in Eq. (2.31), the $F_i^{(12)}$ and $f_i^{(12)}$ satisfy another set of relations if $m_1=m_2$. These are most easily derived by considering Eq. (2.44b). Remembering that $\bar{\nu}_i=\nu_i$, the substitution rule gives

$$M(\gamma(k')+\nu_2(p_2)\to\gamma(k)+\nu_1(p_1))$$
$$=(-1)\epsilon^{\mu}(k')\epsilon^{\nu *}(k)\bar{v}(p_2)Q^{(12)}_{\mu\nu}(-P,-q,-K)v(p_1)\ ,$$

so that

$$M(\gamma(k)+\nu_2(p_1)\to\gamma(k')+\nu_1(p_2))$$
$$=(-1)\epsilon^{\mu *}(k')\epsilon^{\nu}(k)\bar{v}(p_1)Q^{(12)}_{\nu\mu}(-P,-q,K)v(p_2)\ . \quad (2.46)$$

On the other hand, the left-hand side of Eq. (2.46) can also be obtained from Eq. (2.42b). Thus we obtain

$$\bar{u}(p_2)Q^{(12)}_{\mu\nu}(P,q,K)u(p_1)=\bar{v}(p_1)Q^{(12)}_{\nu\mu}(-P,-q,K)v(p_2)\ ,$$

which yields

$$F^{(12)}_{1,2,4,5,7,8}=f^{(12)}_{1,5,6,7}=0\ . \quad (2.47)$$

Equations (2.47), (2.45), and (2.31) allows us to recover the results in Eq. (2.22). [It must be emphasized that, in contrast to Eq. (2.31), Eq. (2.47) is only valid in the limit $m_1=m_2$.] Further, Eq. (2.25) can also be recovered from Eqs. (2.47), (2.45), and (2.33) using the fact that $\eta_1=-\eta_2$ for the two degenerate neutrinos that compose the Dirac neutrino.

III. CALCULATION OF THE RATE FOR $\nu_1\to\nu_2+\gamma+\gamma$

In order to estimate the rates that can be expected for the decay

$$\nu_1(p_1)\to\nu_2(p_2)+\gamma(k)+\gamma(k')\ , \quad (3.1)$$

we consider the standard SU(2)×U(1) model[16] with right-handed singlets ν_R. We assume that the mass terms are of the form $\bar{\nu}_L m\nu_R$ so that the neutrinos are Dirac particles. The rates in the case of Majorana neutrinos can be obtained by using Eq. (2.40). To lowest order, the relevant diagrams are shown in Fig. 1. These diagrams are similar to the diagrams for $s\to d+\gamma+\gamma$.[17] The leading contribution to $\Gamma_{\mu\nu}$, in powers of $(1/M_W)$, is of order $(1/M_W)^2$ and comes from diagram (a). The remaining diagrams, as well as the diagrams with unphysical scalars, give contributions to $\Gamma_{\mu\nu}$ of order $(1/M_W)^4$ at most. Explicit evaluation of the diagrams in Fig. 2 yields[18]

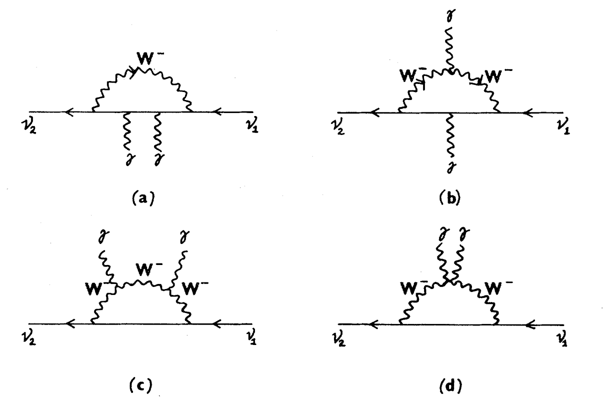

FIG. 1. Diagrams for $\nu_1\to\nu_2+\gamma+\gamma$ in the standard SU(2)×U(1) model with right-handed singlets. In principle, diagrams in which each W line is replaced by an unphysical Higgs scalar should also be included. In addition, there is a set of diagrams with the photon lines crossed.

$$M(\nu_a\to\nu_c+\gamma+\gamma)=\epsilon^{\mu *}(k')\epsilon^{\nu *}(k)\bar{u}(p_2)Q_{\mu\nu}u(p_1) \quad (3.2a)$$

with

$$Q_{\mu\nu}=-2i\epsilon_{\mu\nu\alpha\beta}k'^{\alpha}k^{\beta}(F'_5+f'_5\gamma_5)\ , \quad (3.2b)$$

where

$$f'_5=\left[\frac{2\alpha}{\pi}\right]\left[\frac{G}{\sqrt{2}}\right](m_{\nu_a}-m_{\nu_c})\left[\sum_b\frac{U^*_{bc}U_{ba}I_b(s)}{m_b{}^2}\right]\ ,$$
$$F'_5=\left[\frac{2\alpha}{\pi}\right]\left[\frac{G}{\sqrt{2}}\right](m_{\nu_a}+m_{\nu_c})\left[\sum_b\frac{U^*_{bc}U_{ba}I_b(s)}{m_b{}^2}\right]\ . \quad (3.3)$$

In Eq. (3.3), m_b $(=m_e,m_\mu,m_\tau)$ is the mass of the charged lepton in the loop and

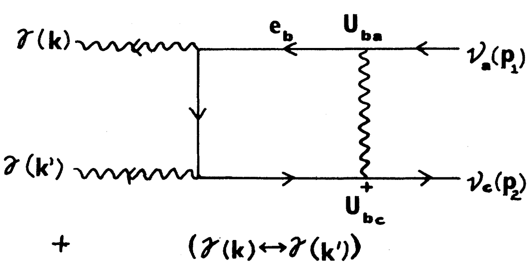

FIG. 2. Diagrams that give the leading contribution to $\nu_1\to\nu_2+\gamma+\gamma$. a, b, and c are family indices, e_b is a charged lepton, and U_{ba} is the Kobayashi-Maskawa matrix in the lepton sector.

$$I_b(s)=\int_0^1 d\alpha_1 d\alpha_2 \theta(1-\alpha_1-\alpha_2)\alpha_1\alpha_2 \frac{1}{1-\dfrac{\alpha_1\alpha_2 s}{m_b^2}} , \tag{3.4}$$

where

$$s=2k\cdot k' , \tag{3.5}$$

In terms of the invariant amplitudes defined in Sec. II, Eq. (3.2) corresponds to $P^2F_5=F'_5$, $P^2f_5=f'_5$ with all the other amplitudes being zero. Equations (3.2) and (3.3) lead to a decay rate

$$\frac{d\Gamma}{ds}=\left[\frac{1}{128\pi^3}\right]\frac{F(s)\lambda(s,m_{\nu_a}{}^2,m_{\nu_c}{}^2)}{m_{\nu_a}{}^3} , \tag{3.6}$$

where

$$F(s)=s^2\{|F'_5|^2[(m_{\nu_a}+m_{\nu_c})^2-s] + |f'_5|^2[(m_{\nu_a}-m_{\nu_c})^2-s]\} \tag{3.7}$$

and, as usual,

$$\lambda(x,y,z)=(x^2+y^2+z^2-2xy-2xz-2yz)^{1/2} . \tag{3.8}$$

In Eq. (3.6), s varies in the range

$$0\le s\le (m_{\nu_a}-m_{\nu_c})^2 . \tag{3.9}$$

In order to obtain a numerical estimate for the decay rate, let us assume that $m_{\nu_a}<2m_e$. In this case the term with $m_b=m_e$ in Eq. (3.3) dominates. Approximating $I_e(s)$ by $I_e(0)=\frac{1}{24}$, and defining

$$r=\frac{m_{\nu_c}}{m_{\nu_a}} ,\quad \hat{s}=\frac{s}{m_{\nu_a}{}^2} ,\quad R=U^*_{ec}U_{ea}(1-r) ,\quad R'=U^*_{ec}U_{ea}(1+r) , \tag{3.10}$$

the decay rate can be expressed as

$$\frac{d\Gamma}{d\hat{s}}=\Gamma_0^{(\gamma\gamma)}f(\hat{s}) , \tag{3.11}$$

where

$$f(\hat{s})=\hat{s}^2\lambda(1,\hat{s},r^2)\{R^2[(1+r)^2-\hat{s}] +R'^2[(1-r)^2-\hat{s}]\} \tag{3.12}$$

and

$$\Gamma_0^{(\gamma\gamma)}=\left[\frac{\alpha G^2 m_{\nu_a}{}^5}{128\pi^4}\right]\left[\frac{\alpha}{288\pi}\right]\left[\frac{m_{\nu_a}}{m_e}\right]^4 =(7.1\times10^{10}\ \text{sec})^{-1}\left[\frac{m_{\nu_a}}{1\ \text{MeV}}\right]^9 . \tag{3.13}$$

On the other hand, the rate for $\nu_a\to\nu_c+\gamma$ in the same model is given by[19]

$$\Gamma(\nu_a\to\nu_c+\gamma)=\Gamma_0^{(\gamma)}(1-r^2)^3(1+r^2)(U_{\tau a}U^*_{\tau c})^2 , \tag{3.14}$$

where

$$\Gamma_0^{(\gamma)}=\left[\frac{\alpha G^2 m_{\nu_a}{}^5}{128\pi^4}\right]\frac{9}{16}\left[\frac{m_\tau}{M_W}\right]^4 =(5.6\times10^{13}\ \text{sec})^{-1}\left[\frac{m_{\nu_a}}{1\ \text{MeV}}\right]^5 . \tag{3.15}$$

Therefore, the basic difference between the one- and two-photon decays is that the GIM supression factor $(m_\tau/M_W)^4$ in $\Gamma_0(\gamma)$ is replaced by $(m_{\nu_a}/m_e)^4$ in $\Gamma_0^{(\gamma\gamma)}$. For m_ν in the eV range this makes $\Gamma_0^{(\gamma\gamma)}$ negligible compared with $\Gamma_0^{(\gamma)}$. However, for $m_{\nu_a}>0.2$ MeV, the two-photon process can dominate. For example, for $m_{\nu_a}\simeq1$ MeV we have $\Gamma_0^{(\gamma\gamma)}\simeq(2\times10^3\ \text{yr})^{-1}$, while $\Gamma_0^{(\gamma)}$ is three orders of magnitude smaller. Notice, however, that these estimates can change drastically when the effect of the mixing angles are taken into account. In particular, the rate for $\nu_a\to\nu_c+\gamma+\gamma$ is controlled by the coupling to the lightest charged lepton while $\nu_a\to\nu_c+\gamma$ is controlled by the coupling to the heaviest charged lepton. If ν_a and ν_c are not strongly coupled to the τ lepton, the two-photon decay becomes more important even for masses $m_{\nu_a}<0.2$ MeV.

If $m_{\nu_a}>2m_e$ (strictly speaking it should be $m_{\nu_a}>2m_e+m_{\nu_c}$), the dominant decay mode is

$$\nu_a\to\nu_c+e^++e^- , \tag{3.16}$$

which occurs through the diagram in Fig. 3. However, if for some reason $U_{e\nu_a}$ and/or $U_{e\nu_c}=0$, this process does not occur at the tree level and the radiative decays are relevant. In this case the term with $m_b=m_\mu$ in Eq. (3.3) dominates and the decay rate for the two-photon decay is given by the same formulas as above with the substitution of the subscript e by μ. Thus, in this case

$$\Gamma_0^{(\gamma\gamma)}=(1.2\times10^{20}\ \text{sec})^{-1}\left[\frac{m_{\nu_a}}{1\ \text{MeV}}\right]^9 . \tag{3.17}$$

For m_{ν_a} much smaller than m_μ, $\Gamma_0^{(\gamma\gamma)}$ is negligible compared with $\Gamma_0^{(\gamma)}$. However, for $m_{\nu_a}>40$ MeV the two-

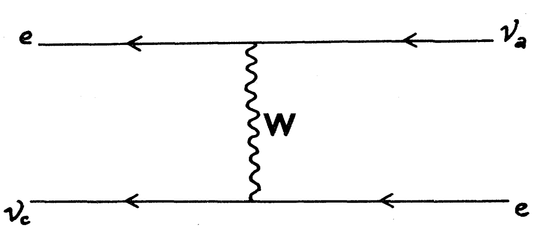

FIG. 3. Tree-level diagram for $\nu_1\to\nu_2+e^-+e^+$.

photon process can be the dominant mode: For example, if $m_{\nu_a}=100$ MeV, then $\Gamma_0^{(\gamma\gamma)}=(4\times10^{-6}\ \text{yr})^{-1}$, while $\Gamma_0^{(\gamma)}=(1.7\times10^{-4}\ \text{yr})^{-1}$. The situation for $m_{\nu_a}>2m_\mu$ can be analyzed in an analogous fashion with obvious modifications.

In summary, the rate for $\nu_a\rightarrow\nu_c+\gamma+\gamma$ is not suppressed by the GIM factor $(m_\tau/M_W)^4$ as is the rate for $\nu_a\rightarrow\nu_c+\gamma$, but rather by $(m_{\nu_a}/m_e)^4$ (assuming $m_{\nu_a}<2m_e$). As a result, the two-photon process is irrelevant if m_{ν_a} is in the eV range, but it can dominate if m_{ν_a} is of the order of a few tenths of MeV. Equation (3.13) provides an estimate of the rates that can be expected for this process.

IV. CONCLUSIONS

The present paper consists of two main parts: a general analysis of the amplitude for $\nu_1\rightarrow\nu_2+\gamma+\gamma$, given in Sec. II, and an explicit calculation of the rate for this process in the SU(2)$\times$U(1) model with Dirac neutrino masses. In addition to illustrating some of the interesting properties of Majorana neutrinos, the analysis of Sec. II serves as a useful guide in explicit calculations of the amplitude. Moreover, the relations in Eq. (2.40) are useful for comparing the rates in the cases of Dirac vs Majorana neutrinos.

The main result of Sec. III is Eq. (3.13), which provides an estimate of the rate in the standard SU(2)$\times$U(1) model with Dirac neutrino masses.

Finally, we mention a recent calculation of gluino pair production by e^-e^+ collisions, by Nelson and Osland.[20] This process occurs through the gluino-gluino-photon vertex, which is analogous to the $\nu\nu\gamma$ vertex for Majorana neutrinos. The analysis of the present paper may be relevant also in that context.

After this manuscript was completed we received a paper by Ghosh[21] in which the rate for $\nu_1\rightarrow\nu_2+\gamma+\gamma$ is calculated in the standard SU(2)$\times$U(1) model.

ACKNOWLEDGMENT

This work is supported in part by the U.S. National Science Foundation.

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[1]The extensive literature on this subject can be traced from the review by F. W. Stecker, *Neutrino 81,* proceedings of the International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, edited by R. J. Cence, E. Ma, and A. Roberts (University of Hawaii High Energy Physics Group, Honolulu, 1981).

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[11]This is sometimes referred to as Hermitian analyticity. See, for example, R. J. Edem, P. V. Landshoff, D. I. Olive, and J. C. Poldinghorne, *The Analytic S Matrix* (Cambridge University Press, Cambridge, England, 1966), pp. 17 and 209. J. D. Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), p. 271.

[12]In general, the F_i and f_i contain the $i\epsilon$ factors to define the branch cuts. Strictly speaking, $\overline{Q}_{\mu\nu}$ is given by Eq. (2.19) if, at the same time, the substitution $i\epsilon\rightarrow -i\epsilon$ is made. The structure of any diagram D is $g\Gamma I$ where, schematically, g represents a product of coupling constants, Γ a product of Dirac matrices, and I is a loop integral. However, the structure of D' is $g^*(\gamma_0\Gamma^\dagger\gamma_0)I$, with the same loop integral. The substitution $i\epsilon\rightarrow -i\epsilon$ in Eq. (2.19) would take this fact into account. Thus, whenever F^* and f^* appear, it is implicitly understood that the substitution $i\epsilon\rightarrow -i\epsilon$ is made. In what follows, we shall say that an invariant amplitude is real (imaginary) if it satisfies $F^*=F$ ($F^*=-F$). We are motivated to use this terminology by our main interest of applying these results to the decay process $\nu_1\rightarrow\nu_2+\gamma+\gamma$ in the case $m_\gamma<2m_e$, in which the loop integral is real.

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[14]Strictly speaking, Eq. (2.19) should read $F_1(\Delta+i\epsilon,t)=\eta_1\eta_2^*F_1^*(\Delta-i\epsilon,t)$, etc. See Ref. 12.

[15]The proof of Eq. (2.39) is identical to the proof given in Ref. 5 for the analogous quantities $\overline{\nu}_1J_\mu^{(\text{EM})}\nu_2$ and $\overline{\nu}_1J_\mu^{(\text{EM})}\nu_2$.

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