Electromagnetic corrections to semileptonic decays with a polarized emitted hyperon

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The radiative corrections to the Dalitz plot of hyperon semileptonic decays when the recoil hyperon is polarized are studied. An expression for the transition probability that is suitable for high-statistics experimental analysis is obtained. The model-dependent part of the radiative corrections is retained in general so that the results obtained are not limited to specific model calculations. The asymmetry coefficients are dealt with in the second part of this paper.

I. INTRODUCTION

In a previous publication[1] we studied the radiative corrections to the Dalitz plot and angular correlation and asymmetry coefficients of semileptonic decays of polarized decaying hyperons. We obtained an expression, valid within certain reasonable approximations, that is not restricted to any particular model for strong interactions. The result obtained, although it does not solve the problem of the model dependence does show that modified form factors can be introduced and that it is only such form factors that can be experimentally determined. Therefore, the expressions obtained allow that experimental analysis of Dalitz plots be finished in a model-independent fashion. The experimental values of those modified form factors can later be used for comparison with theoretical predictions once a specific model is chosen to compute the model-dependent part of radiative corrections. In this way one can avoid the risk of comparing model-dependent experimental numbers with predictions based on different models or even on incompatible ones.

For a long time it has already been proposed[2] to measure semileptonic decays with polarized emitted hyperons instead of initially polarized ones. Very recently, the first measurements of the Dalitz plot and asymmetry coefficients of $\Sigma^- \to \Lambda e \nu$ with polarized Λ have been reported.[3] It is therefore timely to calculate the radiative corrections to the case when it is the recoil hyperon that is polarized. General expressions with $V-A$ theory, without radiative corrections, have already been available in the literature for quite some time.[4] We shall address ourselves to the question of radiative corrections and we shall show that essentially the results obtained in Ref. 1 apply also to this case—the differences are only in the details.

In Sec. II we discuss the separation of the model-dependent part of radiative corrections and we give the relevant transition amplitudes. In Sec. III we derive a general expression for the differential decay probability. In Sec. IV we compute the radiative corrections to asymmetry coefficients, and in Sec. V we make some final remarks.

II. RADIATIVE CORRECTIONS WITH A POLARIZED EMITTED HYPERON

The basic approach we shall follow in computing the virtual and inner-bremsstrahlung radiative corrections to first order in α when the final baryon is polarized is the same as in Ref. 1. We separate the troublesome model-dependent part from a finite gauge-invariant model-independent part. The latter can be computed and the former can be incorporated into the form factors already present. We shall neglect the effects of the four-momentum transfer q on the radiative corrections. One can estimate that contributions of order $(\alpha/\pi)q$ are smaller than one half of a percent. Thus our results will be useful if the experimental precision on the determination of form factors is of a few percent. On the other hand, q will be kept to all orders in the uncorrected matrix elements.

The separation procedure applies at the amplitude level[5] and thus, it is valid whether the polarization of the recoil baryon is going to be observed or not. It is clear then that the virtual amplitudes of Ref. 1 can be carried over to this paper without further ado. There is no need to reproduce all the lengthy Feynman integrals. The transition amplitude with virtual radiative corrections is[6]

$$A_v = M_0\left[1+\frac{\alpha}{\pi}\hat{P}_1\right] + \frac{G_v}{\sqrt{2}}\bar{u}_e \not{p}_1 \gamma_\lambda(1+\gamma_5)v_\nu W_\lambda(p_1,p_2)\frac{\alpha}{\pi}\hat{P}_2 + \frac{\alpha}{\pi}\bar{u}_B(c\gamma_\lambda + d\gamma_\lambda\gamma_5)u_A \bar{u}_e\gamma_\lambda(1+\gamma_5)v_\nu \,. \quad (1)$$

Here c and d are two constants that contain all of the model dependence. The hadronic part of the matrix elements is as usual

$$W_\lambda(p_1,p_2) = \bar{u}_B\left[f_1(q^2)\gamma_\lambda + \frac{f_2(q^2)}{M_1}\sigma_{\lambda\rho}q_\rho + \frac{f_3(q^2)}{M_1}q_\lambda + \left[g_1(q^2)\gamma_\lambda + \frac{g_2(q^2)}{M_1}\sigma_{\lambda\rho}q_\rho + \frac{g_3(q^2)}{M_1}q_\lambda\right]\gamma_5\right]u_A \,. \quad (2)$$

The uncorrected transition amplitude is given by

$$M_0=\frac{G_V}{\sqrt{2}}W_\lambda(p_1,p_2)\bar{u}_e\gamma_\lambda(1+\gamma_5)v_\nu \ . \tag{3}$$

The model-independent part of the virtual correction is contained in the functions $\hat{P}_1$ and $\hat{P}_2$. It is customary to work in the rest frame of the recoil hyperon when its polarization is observed. The expressions for $\hat{P}_1$ and $\hat{P}_2$ in this frame are[7,8]

$$\hat{P}_1=\frac{\alpha}{2\pi}\left[\ln\frac{\lambda}{m}\left[-2+\frac{2}{\beta}\tanh^{-1}\beta\right]-\frac{1}{\beta}(\tanh^{-1}\beta)^2+\frac{1}{\beta}L\left[\frac{2\beta}{1+\beta}\right]+\frac{1}{\beta}\tanh^{-1}\beta+\tfrac{3}{2}\ln\frac{M_1}{m}-\frac{11}{8}\right], \tag{4}$$

$$\hat{P}_2=-\frac{\alpha}{2\pi}\frac{m}{M_1\hat{E}\beta}\tanh^{-1}\beta \ , \tag{5}$$

where λ is an infrared cutoff and L is the Spence function. The constants c and d can be absorbed into the leading form factors, as was the case in Ref. 1; namely,

$$f_1'(0)=f_1(0)+\frac{\alpha}{\pi}c \ , \tag{6}$$

$$g_1'(0)=g_1(0)+\frac{\alpha}{\pi}d \ . \tag{7}$$

The inner-bremsstrahlung amplitude does not introduce any model dependence if terms of order $(\alpha/\pi)q$ are neglected in the transition rate. Its explicit expression is given in Ref. 1, so there is no need to repeat it here. Its contribution to the decay probability will be given in the next section.

It is clear then that the calculation of the radiative corrections to order α when the emitted hyperon is polarized is basically the same one as when the initial hyperon is polarized. The only differences lie in the computation of the traces, especially in the one that arises from the second term in Eq. (1). In passing, let us remark that the above results correspond to a neutral emitted hyperon. If this hyperon were charged then it would be $\not{p}_2$ that appears in the second term of Eq. (1) and $\hat{P}_1$ of Eq. (2) would contain a term $(\alpha/\pi)(\pi^2/\beta)$ coming from the final-state Coulomb interaction.

III. DIFFERENTIAL DECAY RATE WITH POLARIZED EMITTED HYPERONS

As we just mentioned we shall give our results in the rest frame of the emitted hyperon. The differential decay rate when this hyperon is polarized is

$$\begin{aligned} dw(A\to Bl\nu)=&\frac{1}{2}\frac{G_V{}^2}{(2\pi)^5}\frac{(\hat{E}_m-\hat{E})^2\hat{l}\hat{E}\,d\hat{E}\,d\hat{\Omega}_l d\hat{\Omega}_\nu}{[1+(\hat{E}/M_2)(1-\beta\hat{x})]^3}\\ &\times\left\{\hat{D}_1'\left[1+\frac{\alpha}{\pi}(\hat{\phi}_1+\hat{\theta}_1)\right]+\beta\hat{l}^*\cdot\hat{p}_\nu^*D_2'\left[1+\frac{\alpha}{\pi}(\hat{\phi}_2+\hat{\theta}_2)\right]+\beta\hat{S}_2\cdot\hat{l}^*\hat{D}_5'\left[1+\frac{\alpha}{\pi}(\hat{\phi}_2+\hat{\theta}_2)\right]\right.\\ &\left.+\hat{S}_2\cdot\hat{p}_\nu^*\hat{D}_6'\left[1+\frac{\alpha}{\pi}(\hat{\phi}_1+\hat{\theta}_1)\right]\right\}, \end{aligned} \tag{8}$$

where $\hat{S}_2$ denotes the emitted hyperon polarization direction, $\hat{l}^*$ and $\hat{p}_\nu^*$ are the emission directions of the charged lepton and neutrino, $\hat{x}=\hat{l}^*\cdot\hat{p}_\nu^*$, and $\hat{E}_m=(M_1{}^2-M_2{}^2-m^2)/2M_2$. The coefficients $\hat{D}_i'$, with $i=1,2,5,6$, are quadratic functions of the form factors of Eq. (2); the prime indicates that the effective form factors, Eqs (6) and (7), have been replaced into them. $\hat{D}_1'$ and $\hat{D}_2'$ correspond to the unpolarized Dalitz plot; they are the same as $\hat{D}_1'$ and $\hat{D}_2'$ of Ref. 1, except that they are now given in the rest frame of the emitted hyperon. We do not need their explicit form here; it can be found in Refs. 4 and 9. The coefficients $\hat{D}_5'$ and $\hat{D}_6'$ are[10]

$$\begin{aligned} \hat{D}_5'=&|F_1'|^2\left[1-\frac{M_1}{M_2}+\frac{\hat{E}}{M_2}-\beta\frac{\hat{E}}{M_2}\hat{x}\right]+|G_1'|^2\left[1+\frac{M_1}{M_2}+\frac{\hat{E}}{M_2}-\beta\frac{\hat{E}}{M_2}\hat{x}\right]+2\,\mathrm{Re}F_1'G_1'^*\left[1+\frac{\hat{E}}{M_2}-\beta\frac{\hat{E}}{M_2}\hat{x}\right]\\ &+(\mathrm{Re}F_1'F_2^*+\mathrm{Re}G_1'G_2^*)\left[-\frac{\hat{E}_\nu}{M_1}-\beta\frac{\hat{E}}{M_1}\hat{x}\right]+\mathrm{Re}F_1'G_2^*\left[-1+\frac{M_2}{M_1}+\frac{\hat{E}_\nu}{M_1}-\beta\frac{\hat{E}}{M_1}\hat{x}-\frac{m^2}{M_1M_2}\right]\\ &+\mathrm{Re}G_1'F_2^*\left[1+\frac{M_2}{M_1}+\frac{\hat{E}_\nu}{M_1}-\beta\frac{\hat{E}}{M_1}\hat{x}-\frac{m^2}{M_1M_2}\right]+\mathrm{Re}F_2G_2^*\frac{M_2}{M_1}\left[-\frac{\hat{E}}{M_1}-\beta\frac{\hat{E}}{M_1}\hat{x}-\frac{2m^2}{M_1M_2}-\frac{m^2\hat{E}}{M_1M_2{}^2}+\beta\frac{m^2\hat{E}}{M_1M_2{}^2}\hat{x}\right] \end{aligned}$$

$$+(\mathrm{Re}F_1'G_3^*+\mathrm{Re}G_1'F_3^*)\left[-\frac{m^2}{M_1M_2}\right]+(\mathrm{Re}F_2G_3^*+\mathrm{Re}F_3G_2^*)\frac{m^2}{M_1^2}\left[-1-\frac{\hat{E}}{M_2}+\beta\frac{\hat{E}}{M_2}\hat{x}\right]$$

$$+\mathrm{Re}F_3G_3^*\frac{m^2}{M_1^2}\left[-\frac{\hat{E}}{M_2}+\beta\frac{\hat{E}}{M_2}\hat{x}\right] \tag{9}$$

and

$$\hat{D}_6'=|F_1'|^2\left[-1+\frac{M_1}{M_2}-\frac{\hat{E}_\nu}{M_2}+\beta\frac{\hat{E}_\nu}{M_2}\hat{x}-\frac{m^2}{M_2\hat{E}}\right]+|G_1'|^2\left[-1-\frac{M_1}{M_2}-\frac{\hat{E}_\nu}{M_2}+\beta\frac{\hat{E}_\nu}{M_2}\hat{x}-\frac{m^2}{M_2\hat{E}}\right]$$

$$+2\,\mathrm{Re}F_1'G_1'^*\left[1+\frac{\hat{E}_\nu}{M_2}-\beta\frac{\hat{E}_\nu}{M_2}\hat{x}+\frac{m^2}{M_2\hat{E}}\right]+(\mathrm{Re}F_1'F_2^*+\mathrm{Re}G_1'G_2^*)\left[\frac{\hat{E}}{M_1}+\beta\frac{\hat{E}_\nu}{M_1}\hat{x}-\frac{m^2}{M_1\hat{E}}\right]$$

$$+\mathrm{Re}F_1'G_2^*\left[-1+\frac{M_2}{M_1}+\frac{\hat{E}}{M_1}-\beta\frac{\hat{E}_\nu}{M_1}\hat{x}+\frac{m^2}{M_1\hat{E}}\left[1-\frac{M_1}{M_2}\right]+\frac{m^2}{M_1M_2}\right]$$

$$+\mathrm{Re}G_1'F_2^*\left[1+\frac{M_2}{M_1}+\frac{\hat{E}}{M_1}-\beta\frac{\hat{E}_\nu}{M_1}\hat{x}+\frac{m^2}{M_1\hat{E}}\left[1+\frac{M_1}{M_2}\right]+\frac{m^2}{M_1M_2}\right]$$

$$+\mathrm{Re}F_2G_2^*\left[\frac{M_2}{M_1}\right]\left[-\frac{\hat{E}_\nu}{M_1}-\beta\frac{\hat{E}_\nu}{M_1}\hat{x}-\frac{m^2\hat{E}_\nu}{M_2^2M_1}+\beta\frac{\hat{E}_\nu}{M_1}\frac{m^2}{M_2^2}\hat{x}-\frac{2\hat{E}_\nu m^2}{\hat{E}M_1M_2}\right]+\mathrm{Re}F_1'G_3^*\left[\frac{m^2}{M_1\hat{E}}\right]\left[1-\frac{M_1}{M_2}+\frac{\hat{E}}{M_2}\right]$$

$$+\mathrm{Re}G_1'F_3^*\left[\frac{m^2}{M_1\hat{E}}\right]\left[1+\frac{M_1}{M_2}+\frac{\hat{E}}{M_2}\right]+(\mathrm{Re}F_2G_3^*+\mathrm{Re}F_3G_2^*)\left[\frac{m^2}{M_1\hat{E}}\right]\left[-\frac{\hat{E}_\nu}{M_1}-\frac{\hat{E}\hat{E}_\nu}{M_1M_2}+\beta\frac{\hat{E}\hat{E}_\nu}{M_1M_2}\hat{x}\right]$$

$$+\mathrm{Re}F_3G_3^*\left[\frac{m^2}{M_1^2}\right]\left[-\frac{\hat{E}_\nu}{M_2}+\beta\frac{\hat{E}_\nu}{M_2}\hat{x}\right], \tag{10}$$

where

$$F_1'=f_1'+\left[1+\frac{M_2}{M_1}\right]f_2,\quad G_1'=g_1'-\left[1-\frac{M_2}{M_1}\right]g_2,\quad F_2=-2f_2,\quad G_2=-2g_2\,.$$

The model-independent parts of the radiative corrections are

$$\hat{\phi}_1+\hat{\theta}_1=2\left[\frac{1}{\beta}\tanh^{-1}\beta-1\right]\left[\frac{\hat{E}_m-\hat{E}}{3\hat{E}}-\frac{3}{2}+\ln\frac{2(\hat{E}_m-\hat{E})}{m}\right]+\frac{2}{\beta}L\left[\frac{2\beta}{1+\beta}\right]$$

$$+\frac{1}{2\beta}\tanh^{-1}\beta\left[2(1+\beta^2)+\frac{(\hat{E}_m-\hat{E})^2}{6\hat{E}^2}-4\tanh^{-1}\beta\right]-\tfrac{3}{8}+\begin{cases}\tfrac{3}{2}\ln\left[\dfrac{M_1}{m}\right] & \text{(NEH)},\\[2ex] \dfrac{\pi^2}{\beta}+\tfrac{3}{2}\ln\left[\dfrac{M_2}{m}\right] & \text{(CEH)}\end{cases} \tag{11}$$

and

$$\hat{\phi}_2+\hat{\theta}_2=\left[\frac{1}{\beta}\tanh^{-1}\beta-1\right]\left[\frac{(\hat{E}_m-\hat{E})^2}{12\beta^2\hat{E}^2}+\frac{2(\hat{E}_m-\hat{E})}{3\beta^2\hat{E}}+2\ln\frac{2(\hat{E}_m-\hat{E})}{m}-3\right]$$

$$+\frac{2}{\beta}L\left[\frac{2\beta}{1+\beta}\right]-\frac{2}{\beta}\tanh^{-1}\beta(\tanh^{-1}\beta-1)-\tfrac{3}{8}+\begin{cases}\tfrac{3}{2}\ln\left[\dfrac{M_1}{m}\right] & \text{(NEH)}\,,\\[2mm] \dfrac{\pi^2}{\beta}+\tfrac{3}{2}\ln\left[\dfrac{M_2}{m}\right] & \text{(CEH)}\,.\end{cases} \quad (12)$$

NED and CEH mean "neutral emitted hyperon" and "charged emitted hyperon," respectively. $\hat{\phi}_1$ and $\hat{\phi}_2$ are certain linear combinations of $\hat{P}_1$ and $\hat{P}_2$, Eqs. (4) and (5); $\hat{\theta}_1$ and $\hat{\theta}_2$ come from the inner-bremsstrahlung contribution. Comparing with Ref. 1, one can see that they are formally identical with their counterparts there. As a matter of fact, they agree numerically when terms of order $(\alpha/\pi)q$ are neglected, as is the case here.

Equation (8) for $dw(A\to Bl\nu)$ is useful for a model-independent experimental analysis. It clearly shows that only the effective form factors f_1' and g' can be experimentally determined. Equation (8) can be used even in muon-mode decays since we have not neglected the charged-lepton mass. Also, no approximation about the smallness of q has been made in $\hat{D}'_5$ and $\hat{D}'_6$.

IV. RADIATIVE CORRECTIONS TO ASYMMETRY COEFFICIENTS

When low-statistics experiments are performed it is customary to measure integrated observables such as the total decay rate, angular correlation coefficients, and asymmetry coefficients, instead of the detailed Dalitz plots. Four asymmetry coefficients have been proposed[2] when the emitted hyperon is polarized; namely, the electron and neutrino asymmetries, $\hat{\alpha}_l$ and $\hat{\alpha}_\nu$, and two more asymmetries $\hat{\alpha}_\alpha$ and $\hat{\alpha}_\beta$, which correspond to replacing $\hat{l}^*$ and $\hat{p}^*_\nu$ by a new orthonormal basis. The advantage of the latter two over the former two is that they are governed directly by a theorem due to Weinberg.[11] Specifically, when the charged-lepton mass can be neglected, $\hat{\alpha}_\alpha$ depends only on cross-product terms of vector and axial-vector form factors, while $\hat{\alpha}_\beta$ does not contain any such interference terms. The first measurements of $\hat{\alpha}_\alpha$ and $\hat{\alpha}_\beta$ in $\Sigma^-\to\Lambda e\nu$ have been reported in Ref. 3.

The radiative corrections to $\hat{\alpha}_l$, $\hat{\alpha}_\nu$, and the decay rate R are obtained directly by straightforward integration of Eq. (8),

$$\hat{\alpha}_l=\hat{\alpha}_l^0\left[1+\frac{\alpha}{\pi}\left[\frac{\hat{\Phi}_2}{\hat{a}}-\frac{\hat{\Phi}_1}{\hat{b}}\right]\right], \quad (13)$$

$$\hat{\alpha}_\nu=\hat{\alpha}_\nu^0, \quad (14)$$

and

$$R=R^0\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_1}{\hat{b}}\right], \quad (15)$$

where

$$\hat{a}=\int_m^{\hat{E}_m}d\hat{E}\,\hat{l}^2(\hat{E}_m-\hat{E})^2\,, \quad (16)$$

$$\hat{b}=\int_m^{\hat{E}_m}d\hat{E}\,\hat{E}\,\hat{l}(\hat{E}_m-\hat{E})^2\,, \quad (17)$$

$$\hat{\Phi}_1=\int_m^{\hat{E}_m}d\hat{E}\,\hat{E}\,\hat{l}(\hat{E}_m-\hat{E})^2(\hat{\phi}_1+\hat{\theta}_1)\,, \quad (18)$$

$$\hat{\Phi}_2=\int_m^{\hat{E}_m}d\hat{E}\,\hat{l}^2(\hat{E}_m-\hat{E})^2(\hat{\phi}_2+\hat{\theta}_2)\,. \quad (19)$$

The uncorrected asymmetries and rate are $\hat{\alpha}_l^0$, $\hat{\alpha}_\nu^0$, and R^0, respectively; they contain the contributions proportional to q to all orders.

The radiative corrections to $\hat{\alpha}_\alpha$ and $\hat{\alpha}_\beta$ require a little more effort. The orthonormal basis introduced to define $\hat{\alpha}_\alpha$ and $\hat{\alpha}_\beta$ is $\hat{\alpha}=(\hat{l}^*+\hat{p}^*_\nu)/a'$, $\hat{\beta}=(\hat{l}^*-\hat{p}^*_\nu)/b'$, and $\hat{n}=\hat{\beta}\times\hat{\alpha}$; a' and b' are normalization coefficients. In order to use Eq. (8) to get the radiative corrections to $\hat{\alpha}_\alpha$, we change the angular variables of the electron to those of the vector $\hat{\alpha}$ using

$$\hat{l}^*=-\hat{p}^*_\nu+(2\hat{p}^*_\nu\cdot\hat{\alpha})\hat{\alpha}\,.$$

The solid angle $d\hat{\Omega}_l$ is replaced by $(4\hat{p}^*_\nu\cdot\hat{\alpha})d\Omega_\alpha$. The part containing $\hat{S}_2$ of Eq. (8) then becomes

$$dw(A\to Bl\nu)_{\hat{S}_2}=\frac{1}{2}\frac{G_V^2}{(2\pi)^5}\frac{(\hat{E}^*_m-\hat{E})^2\hat{l}\hat{E}\,d\hat{E}\,d\Omega_\alpha d\hat{\Omega}_\nu}{\left[1+\dfrac{\hat{E}}{M_2}(1-\beta\hat{x})\right]^3}(4\hat{p}^*_\nu\cdot\hat{\alpha})[(2\hat{p}^*_\nu\cdot\hat{\alpha})\hat{S}_2\cdot\hat{\alpha}\hat{D}'_5+\hat{S}_2\cdot\hat{p}^*_\nu(-\hat{D}'_5+\hat{D}'_6)]\,. \quad (20)$$

Integrating Eq. (20) to get $\hat{\alpha}_\alpha$, we get[12]

$$R\hat{\alpha}_\alpha=\frac{G_V^2}{2\pi^3}\hat{a}\frac{2}{3}\left[\bar{\hat{D}}'_5\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_2}{\hat{a}}\right]+\frac{\hat{b}}{\hat{a}}\bar{\hat{D}}'_6\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_1}{\hat{b}}\right]\right], \quad (21)$$

where $\bar{\hat{D}}'_5$ and $\bar{\hat{D}}'_6$ are the integrated coefficients $\hat{D}'_5$ and $\hat{D}'_6$.

The radiative corrections to $\hat{\alpha}_\beta$ are obtained using $\hat{l}^*=\hat{p}^*_\nu-(2\hat{p}^*_\nu\cdot\hat{\beta})\hat{\beta}$ to replace $\hat{l}^*$ and $d\hat{\Omega}_l$ in Eq. (8), so that $\hat{\beta}$ and $d\Omega_\beta$ appear instead. The result is

$$R\hat{\alpha}_\beta=\frac{G_V^2}{2\pi^3}\hat{\alpha}\frac{2}{3}\left[\bar{\hat{D}}'_5\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_2}{\hat{a}}\right]-\frac{\hat{b}}{\hat{a}}\bar{\hat{D}}'_6\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_1}{\hat{b}}\right]\right]. \tag{22}$$

The uncorrected asymmetries $\hat{\alpha}^0_\alpha$ and $\hat{\alpha}^0_\beta$ have been computed by Linke. They are given in Table 4(a) of Ref. 4. As we mentioned earlier, the numerical values of the model-independent part of the radiative corrections agree with the corresponding ones in Tables I and II of Ref. 1. For $\Sigma^-\to\Lambda e\nu$, they are $(\alpha/\pi)(\hat{\Phi}_1/\hat{b})=0.0012$ and $(\alpha/\pi)(\hat{\Phi}_2/\hat{a})=0.0001$. From the latter tables one can see that within our approximations, it turns out that $(\alpha/\pi)(\hat{\Phi}_1/\hat{b})\sim(\alpha/\pi)(\hat{\Phi}_2/\hat{a})$. Thus, in Eqs. (21) and (22) the model-independent radiative corrections factor out and we get

$$R\hat{\alpha}_\alpha=R^0\hat{\alpha}^0_\alpha\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_1}{\hat{b}}\right] \tag{23}$$

and

$$R\hat{\alpha}_\beta=R^0\hat{\alpha}^0_\beta\left[1+\frac{\alpha}{\pi}\frac{\hat{\Phi}_1}{\hat{b}}\right]. \tag{24}$$

Because of this factorization property, we can conclude that the above asymmetries, Eqs. (13), (14) and (23), (24), obey the same theorem that the angular coefficients of the initially-polarized-hyperon case obey.[13] Namely,

$$\hat{\alpha}_e=\hat{\alpha}^0_e, \tag{25}$$

$$\hat{\alpha}_\nu=\hat{\alpha}^0_\nu, \tag{26}$$

$$\hat{\alpha}_\alpha=\hat{\alpha}^0_\alpha, \tag{27}$$

$$\hat{\alpha}_\beta=\hat{\alpha}^0_\beta, \tag{28}$$

i.e., the form of these asymmetries is independent of radiative corrections.

For completeness, let us mention that for $q\simeq 0$, Eqs. (27) and (28) become

$$\hat{\alpha}_\alpha\simeq\tfrac{8}{3}\,\mathrm{Re}f'_1 g'^*_1/(f'^2_1+3g'^2_1)$$

and

$$\hat{\alpha}_\beta\simeq\tfrac{8}{3}\,|g'_1|^2/(f'^2_1+3g'^2_1),$$

in agreement with Refs. 2 and 4.

V. SUMMARY

We have calculated the radiative corrections to first order in α to semileptonic decays with a polarized emitted hyperon in close parallelism to the radiative corrections of semileptonic decays of an initially polarized hyperon. Our results are appropriate for high-statistics experiments that will be able to determine the form factors up to 1 or 2%. The effective form factors, Eqs. (6) and (7), determined from experiment are model independent and thus can be used for comparison with theoretical predictions without any bias in favor of or against any particular approach to estimate the model-dependent part of the radiative corrections.

The $V-A$ theory form of the asymmetry coefficients is not changed by radiative corrections. The only indication of the presence of radiative corrections is through the appearance of the primed form factors. The last paragraph of Sec. IV stresses this point.

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[3]M. Bourquin *et al.*, Z. Phys. C 12, 307 (1982).

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[5]This procedure was originally introduced for neutron β decay by A. Sirlin, Phys. Rev. 164, 1767 (1967).

[6]We follow the same conventions and notation of Ref. 1. Thus, M_1 is the decay hyperon mass, M_2 the emitted hyperon mass, m is the charged lepton mass, etc.

[7]We shall mark the kinematical variables in the recoil hyperon rest frame by a caret.

[8]β is the velocity of the charged lepton in the frame we are working in. We do not put a caret on it in order to avoid confusion with $\hat{\beta}$ that will be introduced in Sec. IV.

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[10]Equations (9) and (10) were essentially calculated before by Linke in Ref. 4. Our result agrees exactly with his.

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[12]$\hat{\alpha}_\alpha$ is defined as $2(N_\uparrow-N_\downarrow)/(N_\uparrow+N_\downarrow)$. The upwards arrow means all events with $\theta_\alpha<\pi/2$ and the downwards one means all events with $\theta_\alpha>\pi/2$. α_β is defined analogously.

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