Production of vector-meson pairs in hadronic collisions

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(Received 14 April 1983; revised manuscript received 7 July 1983)

Production in hadronic collisions of 2^{++} $Q^2\overline{Q}^2$ states which decay predominantly to vector-meson pairs are discussed based on a Drell-Yan-type mechanism with gluon fusion via the color vector-dominance model. The 2^{++} $\phi\phi$ resonance at 2.16 GeV produced in pp and πp collisions is interpreted as a 2^{++} $ss\bar{s}\bar{s}$ four-quark state. The calculated mass and width of this four-quark state, its characteristic S-wave decay, and the theoretical production cross sections in pp reaction at $p_L=400$ GeV/c and πp reaction at $p_L=100$ GeV/c are consistent with experimental data. Estimates of the hadronic production cross sections and their longitudinal- and transverse-momentum distributions for various channels of vector-meson pairs are also presented.

I. INTRODUCTION

Do multiquark hadrons, especially $Q^2\overline{Q}^2$ states, exist in nature? This is an intriguing question both experimentally and dynamically. The spectroscopy of the S-wave $Q^2\overline{Q}^2$ mesons of light quarks ($Q=u,d,s$) has been studied both in the MIT bag model[1,2] and in the potential model.[3] The salient features of these states are the following: (1) The wave functions of the $Q^2\overline{Q}^2$ states consist of two parts—in one part, the $Q\overline{Q}$ pairs are in the color singlet representations and in another part, the $Q\overline{Q}$ pairs are in the color-octet representations. (2) Their decays obey the Okubo-Zweig-Iizuka (OZI) rule—most of the $Q^2\overline{Q}^2$ states can "fall apart" into two constituent color-singlet $Q\overline{Q}$ mesons, making them too broad to be observed. However, we have shown[4] that there are three types of $Q^2\overline{Q}^2$ states whose widths are not too broad to allow them to be detected as resonances. These are the states whose decays are dominated by a pair of vector mesons. For example, the recoupling coefficients of the S-wave 2^{++} $Q^2\overline{Q}^2$ states show that they decay mainly through vector-meson pairs. In Refs. 4 and 5, we showed that the $\gamma\gamma\rightarrow VV$ (V stands for vector meson) reaction is a preferred process to search for these four-quark states. The $\rho^0\rho^0$ enhancement in this reaction near the threshold[6–8] has been interpreted[4,5,9] as due to three 2^{++} $Q^2\overline{Q}^2$ states at 1.65 GeV. It was also noted[4,5] that in the $Q^2\overline{Q}^2$ wave functions there are color-octet-vector—color-octet-vector ($\underline{V}\cdot\underline{V}$) parts which can couple to two gluons directly. Therefore, these 2^{++} $Q^2\overline{Q}^2$ states are expected to be produced via two gluons. And indeed, $\rho\rho$ enhancement at 1.65 GeV (Ref. 10) is observed in the radiative decay of charmonium which is produced via two gluons. Of course, some 0^{++} $Q^2\overline{Q}^2$ states with large recoupling coefficients for the $\underline{V}\cdot\underline{V}$ (Refs. 2 and 11) parts can also be produced via two gluons, but their widths are usually so large that they may easily escape detection. In this paper, we shall concentrate on the hadronic production of 2^{++} $Q^2\overline{Q}^2$ states which decay almost exclusively via two vector mesons.

There are several experimental data on the production of vector-meson pairs in hadronic collisions:

(1) $\phi\phi$ production in p-nucleon and π-p collisions.[12,13] These processes are supersuppressed by the OZI rule. However, the experimental data show that the $\phi\phi$ production cross section is quite large. The invariant mass of $\phi\phi$ peaks at 2.16 GeV with $J^{PC}=2^{++}$ and $\Gamma=315\pm60$ MeV. According to the bag-model calculation of Jaffe,[2] there is a 2^{++} $ss\bar{s}\bar{s}$ four-quark state at 2.25 GeV which decays predominantly to $\phi\phi$ and its width has been calculated to be 360 MeV.[4] Its characteristic S-wave decay is also consistent with experiment. All these seem to favor the interpretation of a 2^{++} $ss\bar{s}\bar{s}$ state.

(2) JJ production in π-p scattering process.[14] This is also supersuppressed by the OZI rule. The invariant mass of JJ is found[14] to spread from 6.5 to 8.5 GeV where we expect a 2^{++} $cc\bar{c}\bar{c}$ state to contribute. A detailed discussion about this will be presented elsewhere.

(3) $\rho\rho$ enhancement at 1.7 GeV in $\bar{p}p\rightarrow3\pi^+3\pi^-\pi^0$ at 5.7 GeV.[15] This could be the same structure observed in $\gamma\gamma\rightarrow\rho^0\rho^0$ (Refs. 6 and 7) and recently in $J/\psi\rightarrow\gamma\rho^0\rho^0$ (Refs. 10 and 16), which are interpreted as due to the 2^{++} $q^2\bar{q}^2$ ($q=u,d$) states.[4,5,9]

The fact that these structures observed in various hadronic collisions share the common features that their main decay channels are two vector mesons, that their masses are just above the thresholds of the two vector-meson masses, and that some of them are 2^{++} states and observed in other processes (i.e., $\gamma\gamma$ reactions and radiative decay of charmonium) seem to indicate that some of them, if not all, may have the common origin—namely, 2^{++} $Q^2\overline{Q}^2$ states which honor precisely these features. As a further check, we need to calculate the production cross sections in order to compare with experiments. This is done in Sec. II. In Sec. III, we discuss the differential cross sections for $pp\rightarrow\phi\phi+\cdots$ and $\pi p\rightarrow\phi\phi+\cdots$. Remarks on the interpretation of the two $\phi\phi$ states are given in Sec. IV. Section V contains a summary and the concluding remarks.

II. CROSS SECTIONS

In this section we shall calculate the cross section of the processes $h_1+h_2\rightarrow V_1+V_2+\cdots$ via $Q^2\overline{Q}^2$ states. For OZI-supersuppressed processes (e.g., $\phi\phi$ production in pp

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and πp reactions), a gluon fusion picture is needed. As mentioned earlier, there are color-octet-vector—color-octet-vector ($\underline{V}\cdot\underline{V}$) parts in the wave functions of the four-quark states. The quantum numbers of a flavorless color-octet-vector $q\bar{q}$ pair are identical to those of a gluon. Therefore a gluon is expected to couple directly to $\underline{V}$. This is similar to the vector-dominance model (VDM). In this paper, we will use this picture to describe the VV productions. This is done with the help of a Drell-Yan-type mechanism depicted in Fig. 1. The glue partons originating from the colliding hadrons couple to the $\underline{V}\cdot\underline{V}$ parts of the $Q^2\bar{Q}^2$ states which in turn decay via the VV channels.

In the spirit of the Drell-Yan mechanism, the cross section of the process can be written as

$$\sigma(h_1+h_2\to V_1+V_2\cdots)=\int_{x_{1\min}}^{1}dx_1\int_{x_{2\min}}^{1}dx_2[G_{g_1}^{h_1}(x_1)G_{g_2}^{h_2}(x_2)+G_{g_2}^{h_1}(x_1)G_{g_1}^{h_2}(x_2)]\sigma(g_1+g_2\to Q^2\bar{Q}^2\to V_1V_2)\ , \tag{2.1}$$

where

$$x_{2\min}=\frac{(m_1+m_2)^2}{x_1 s},\quad x_{1\min}=\frac{(m_1+m_2)^2}{s}\ ,$$

where $\sqrt{s}$ is the total energy in the center-of-mass frame and $G_g^h(x)$ is the gluon distribution function inside the hadron h. The gluon distribution function of the π meson is taken to be

$$G_g^{\pi}(x)=2(1-x)^3/x\ , \tag{2.2}$$

and we take gluon distribution function of the proton to be[17]

$$G_g^{p}(x)=2.62(1+3.5x)(1-x)^{5.9}/x\ . \tag{2.3}$$

Since

$$G_g^{\pi^+}(x)=G_g^{\pi^-}(x)\ , \tag{2.4}$$

we obtain the following relation in this picture:

$$\sigma(\pi^+ +N\to V_1+V_2+\cdots)$$
$$=\sigma(\pi^- +N\to V_1+V_2+\cdots)\ , \tag{2.5}$$

where N is the nucleon. Similarly we have the relation

$$\sigma(K^+ +N\to V_1+V_2+\cdots)$$
$$=\sigma(K^- +N\to V_1+V_2+\cdots)\ . \tag{2.6}$$

In general, we expect

$$G_g^{\pi}(x)\cong G_g^{K}(x)\ . \tag{2.7}$$

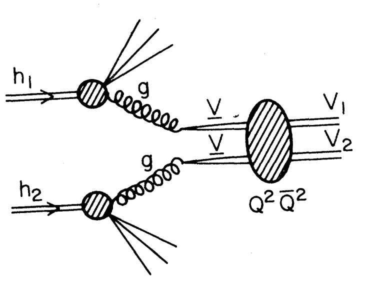

FIG. 1. Drell-Yan-type mechanism for the production of two vector mesons. $\underline{V}$ denotes a color-octet vector state, V_1 and V_2 denote the two vector mesons. The intermediate state is a four-quark state.

We then have

$$\sigma(\pi+N\to V_1+V_2+\cdots)$$
$$\cong\sigma(K+N\to V_1+V_2+\cdots)\ . \tag{2.8}$$

Similarly, since

$$G_g^{p}(x)=G_g^{n}(x)=G_g^{\bar{p}}(x)=G_g^{\bar{n}}(x)\ , \tag{2.9}$$

we obtain further relations:

$$\sigma(p+N\to V_1+V_2+\cdots)=\sigma(n+N\to V_1+V_2+\cdots)$$
$$=\sigma(\bar{p}+N\to V_1+V_2+\cdots)\ . \tag{2.10}$$

Strictly speaking, these relations [Eqs. (2.5), (2.6), (2.8), and (2.10)] hold for OZI-rule supersuppressed processes which proceed via gluons. For OZI-rule-allowed reactions, these relations either do not hold or need modifications. For OZI-rule-suppressed reactions, these relationships can be used to test the production mechanism (production via two gluons) of two vector mesons.

There are some data[13] on $\phi\phi$ production in low-energy K^-p and π^-p reactions. The data shows $\sigma(K^-p\to\phi\phi\Lambda/\Sigma^0)/\sigma(\pi^-p\to\phi\phi n)\sim 6$. It is seemingly in contradiction with Eq. (2.8). But one needs to exercise caution in this case. These reactions are OZI-rule-suppressed processes alright, but they involve recombinations. From a naive gluon-counting rule, the diagrams for these two processes are shown in Figs. 2(a) and 2(b). In the $K^-p\to\phi\phi\Lambda/\Sigma^0$ reaction, the $\phi\phi$ can be constructed by several different combinations of the strange and anti-strange quarks; yet in the reaction $\pi^-p\to\phi\phi n$, the $\phi\phi$ is formed only by one combination. For this reason the cross section of the process $K^-p\to\phi\phi\Lambda/\Sigma^0$ is several times larger than that of the process $\pi p\to\phi\phi n$. The gluon counting is further supported by the data[13] $\sigma(K^-p\to\phi\Lambda)/\sigma(\pi^-p\to\phi n)\sim 60$ which simply reflects the fact that the process $K^-p\to\phi\Lambda$ involves one gluon, whereas at least two gluons are needed in the process $\pi^-p\to\phi n$. For the same reason, we expect that $\sigma(\pi^-p\to\phi n)$ to be in the same magnitude with, but several times smaller than, the cross section of the process $K^-p\to\phi\phi\Lambda/\Sigma^0$, at the same beam energy since both involve two gluons and the latter has more final state recombinations. If we take into account the fact these cross sections decrease by several times for each increase

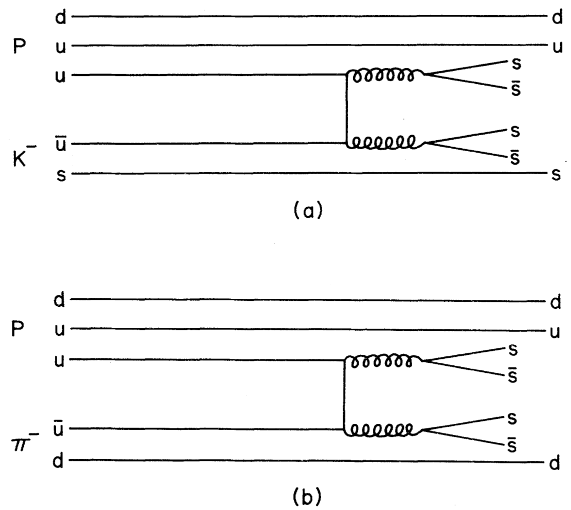

FIG. 2. Diagrams for the reactions (a) $K^-p \to \phi\phi\Lambda/\Sigma^0$ and (b) $\pi^-p \to \phi\phi n$.

in the step of 1 GeV/c of the beam momentum,[13] the available data[13] which give $\sigma(\pi^-p \to \phi n) = 0.47 \pm 0.14$ μb at $p_\pi = 6$ GeV/c and $\sigma(K^-p \to \phi\phi\Lambda) = 0.7 \pm 0.2$ μb at $p_{K^-} = 8.25$ GeV/c would support our thesis that $\sigma(K^-p \to \phi\phi\Lambda/\Sigma^0)$ is several times larger than $\sigma(\pi^-p \to \phi n)$ at the same beam energy, similar to the comparison between $\sigma(K^-p \to \phi\phi\Lambda/\Sigma^0)$ and $\sigma(\pi^-p \to \phi\phi n)$.

As shown in Fig. 2(a), the $\phi\phi$ production in the process $K^-p \to \phi\phi\Lambda/\Sigma^0$ involve several possible combinations of the strange quarks and strange antiquarks. Only one combination comes from $C^{ss}(36)$ which is produced by two gluons, the rest will involve the s quark in the K^- meson. Therefore, if we collect such events in which Λ/Σ^0 possesses large forward momentum, i.e., the strange quark in Λ/Σ^0 comes from the incoming K^- meson directly, a $\phi\phi$ resonance which is seen in π^-p reaction could also be seen in K^-p reaction. It is also in this sense that Eq. (2.8) is expected to hold.

In Refs. 4 and 5, the VDM has been used to obtain the cross section of the process

$$\gamma\gamma \to Q^2\bar{Q}^2 \to V_1 + V_2 \ . \tag{2.11}$$

The physical picture is depicted in Fig. 3. In the VDM, the photon is coupled to a neutral vector meson. We assume that a mechanism analogous to the VDM can be employed to evaluate

$$g + g \to Q^2\bar{Q}^2 \to V_1 + V_2 \ , \tag{2.12}$$

where the gluons couple to the $q\bar{q}$ pairs in the color-octet vector representation which in turn couple to the $\underline{V}\cdot\underline{V}$ part of the $Q^2\bar{Q}^2$ state. This is shown in Fig. 4. In order to calculate the cross section, we need to determine the coupling constant between a gluon and the color-octet vector $q\bar{q}$. Thus we invoke a "color VDM" to do the job. Recall that in the VDM the coupling constant of the photon and the neutral vector meson can be determined from the electromagnetic annihilation of a neutral vector meson

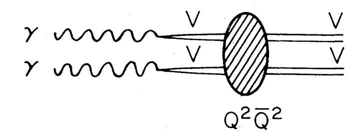

FIG. 3. Diagram for the reaction $\gamma\gamma \to VV$. The symbol V stands for the vector meson. The intermediate state is a four-quark state.

$$\frac{4\pi}{f_V^{\,2}} = 48\pi\, |\psi_V(0)|^2 \frac{1}{m_V^{\,3}} (\mathrm{Tr}Q\phi_V)^2 \ , \tag{2.13}$$

where $\psi_V(0)$ is the wave function of the neutral vector meson at the origin and m_V is the mass of the vector meson. Q is the charge operator and ϕ_V is the flavor wave function of the vector meson. Similarly, we can determine the color-VDM constant $4\pi/f_{\underline{V}}^{\,2}$ from the annihilation of a color-octet vector $q\bar{q}$ to a gluon. According to the MIT bag model,[2] the masses of the 2^{++} $Q^2\bar{Q}^2$ states are close to the masses of two constitutent vector mesons. Therefore the effective mass $m_{\underline{V}}$ of the color-octet vector $q\bar{q}$ is almost the same as the mass m_V of the corresponding vector meson (the flavor contents of $\underline{V}$ and V are the same), i.e.,

$$m_{\underline{V}} \cong \tfrac{1}{2} M_{2^{++}} \cong m_V \ . \tag{2.14}$$

Similar to gluons and quarks, a color-octet vector $q\bar{q}$ pair does not exist in free space, but can exist under certain boundary conditions, such as in a bag together with another color octet object to form a total color singlet. With the MIT bag boundary condition, the bag radius R grows as $M^{1/3}$. Given Eq. (2.14) and the fact that $|\psi(0)|^2 \propto 1/R^3$, we also expect the $\underline{V}$ wave function at the origin to be very close to that of V, i.e.,

$$|\psi_{\underline{V}}(0)|^2 \cong |\psi_V(0)|^2 \ . \tag{2.15}$$

Now, analogous to $4\pi/f_V^{\,2}$, the color-VDM assumption gives

$$\frac{4\pi}{f_{\underline{V}}^{\,2}} = 16\pi\, |\psi_{\underline{V}}(0)|^2 \frac{1}{m_{\underline{V}}^{\,3}} (\mathrm{Tr}\phi_V)^2 (\mathrm{Tr}\lambda^a \phi_{\underline{V}})^2 \ , \tag{2.16}$$

where $\phi_{\underline{V}}$ is the color wave function of the color-octet vector $q\bar{q}$ pair which is

$$\phi_{\underline{V}} = \sqrt{2}\lambda^a \ . \tag{2.17}$$

Using Eqs. (2.14), (2.15), and (2.17) we obtain

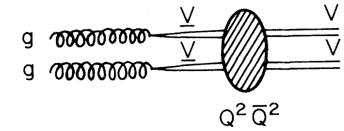

FIG. 4. The production of two vector mesons via two gluons. $\underline{V}$ denotes a color-octet vector state and V denotes a vector meson. The intermediate state is a four-quark state.

$$\frac{4\pi}{f_{\underline{V}}^2}=\frac{1}{6}\frac{(\mathrm{Tr}\phi_V)^2}{(\mathrm{Tr}Q\phi_V)^2}\frac{4\pi}{f_V^2}\ . \tag{2.18}$$

From the widths of the processes $\omega,\phi,J,\Upsilon\to e^+e^-$ we obtain

$$\frac{4\pi}{f_\omega^2}=0.046,\quad \frac{4\pi}{f_\phi^2}=0.058\ ,$$
$$\frac{4\pi}{f_J^2}=0.08,\quad \frac{4\pi}{f_\Upsilon^2}=0.0089\ . \tag{2.19}$$

Corresponding values of $4\pi/f_{\underline{V}}^2$ are thus determined from Eq. (2.18) to be

$$\frac{4\pi}{f_{\underline{\omega}}^2}=0.273,\quad \frac{4\pi}{f_{\underline{\phi}}^2}=0.087,$$
$$\frac{4\pi}{f_{\underline{J}}^2}=0.03,\quad \frac{4\pi}{f_{\underline{\Upsilon}}^2}=0.013\ . \tag{2.20}$$

We do not expect these values to be very accurate, after all even in VDM there are appreciable differences in the values of $4\pi/f_V^2$ determined from the photoproduction and the electromagnetic annihilations. We use these values to estimate the order of magnitude of the cross sections of the processes $h_1+h_2\to V_1+V_2+\cdots$ to test the picture of production of the four-quark states.

Analogous to the calculation of the cross section $\sigma(\gamma\gamma\to Q^2\bar{Q}^2\to V_1V_2)^5$, we obtain the cross section $\sigma(gg\to Q^2\bar{Q}^2\to V_1V_2)$ as follows:

$$\sigma(g_1+g_2\to Q^2\bar{Q}^2\to V_1+V_2)=\frac{1}{64}\frac{p}{128\pi W}\frac{7}{3}\left[1+\frac{p^2}{3}\left[\frac{1}{m_1^2}+\frac{1}{m_2^2}\right]+\frac{2}{15}\frac{p^4}{m_1^2m_2^2}\right]\sum_{\alpha\beta}\left|\sum_i\frac{a^i_{V_1V_2}b^i_{\alpha\beta}}{W-M_i+\frac{i}{2}\Gamma_i(W)}\right|^2, \tag{2.21}$$

where the factor $\frac{1}{64}$ stems from taking an average over the gluon's color index, and m_1 and m_2 are the masses of the vector mesons V_1 and V_2, respectively, and W is the c.m. energy of the two gluons. It is defined as

$$W^2=(k_1+k_2)^2\ , \tag{2.22}$$

where k_1 and k_2 are the momenta of gluons g_1 and g_2, respectively. Using the assumption of the parton model, we have

$$k_{1\mu}=x_1p_{1\mu},\quad k_{2\mu}=x_2p_{2\mu}\ , \tag{2.23}$$

where p_1 and p_2 are the momenta of the hadrons h_1 and h_2, respectively. Upon neglecting the hadron masses at high energies, we obtain

$$W^2=x_1x_2S\ , \tag{2.24}$$

where $S=(p_1+p_2)^2$ is the c.m. energy of the two hadrons. Further notation in Eq. (2.21) includes p which is the vector-meson momentum in the c.m. of the four-quark state

$$p^2=\frac{1}{4W^2}(W^2-m_1^2+m_2^2)^2-m_2^2\ , \tag{2.25}$$

and M_i which is the mass of the ith four-quark state. $\Gamma_i(W)$ is the total width of the ith four-quark state which is written as

$$\Gamma_i(W)=\frac{pC}{8\pi}\left[1+\frac{p^2}{3}\left[\frac{1}{m_1^2}+\frac{1}{m_2^2}\right]+\frac{2}{15}\frac{p^4}{m_1^2m_2^2}\right], \tag{2.26}$$

with $C=\frac{2}{3}$ for the flavor 9-plet states and $C=\frac{1}{3}$ for the flavor 36-plet states. $a^i_{V_1V_2}$ is the decay constant of the ith four-quark state decaying to two vector mesons and $b^i_{\alpha\beta}$ is the coupling constant between the ith four-quark state and the two gluons through two color-octet vector $q\bar{q}$ pairs. The subscripts α and β of $b^i_{\alpha\beta}$ are the color indices of the two gluons, respectively. The values of $a^i_{V_1V_2}$ and $b^i_{\alpha\beta}$ will be listed below for each channel.

Now we discuss the a and b constants for various channels of vector-meson pairs produced via $Q^2\bar{Q}^2$ states in hadronic collisions. Since the gluon's isospin is zero, only four-quark states with isospin zero can be produced by two gluons.

(a) $\rho\rho$ channel. According to Jaffe's classification, there are two 2^{++} $Q^2\bar{Q}^2$ states which decay to $\rho\rho$, their flavor contents are

$$C^0(9)=+\frac{\sqrt{3}}{2}(\rho\rho)^{I=0}+\tfrac{1}{2}\omega\omega\ ,$$
$$C^0(36)=-\tfrac{1}{2}(\rho\rho)^{I=0}+\frac{\sqrt{3}}{2}\omega\omega\ , \tag{2.27}$$

where 9 and 36 denote the SU(3)-flavor multiplets. The masses of these states are calculated to be 1.65 GeV. The decay constants for these four-quark states are

$$a_{\rho^0\rho^0}(9)=\sqrt{\tfrac{2}{3}}\times(-\tfrac{1}{2})a,$$
$$a_{\rho^0\rho^0}(36)=\frac{1}{\sqrt{3}}\times\frac{1}{2\sqrt{3}}a\ , \tag{2.28}$$

where a is a parameter which has been determined to be $\cong\sqrt{50}$ in Ref. 4 by fitting the data of the process $\gamma\gamma\to\rho^0\rho^0$. The first numbers on the right-hand side (RHS) the recoupling coefficients for VV in the color-spin representation of the four-quark states.[2,4,5,11] The second numbers are the Clebsch-Gordan (CG) coefficients for $\rho^0\rho^0$ in the flavor representation in Eq. (2.27). The b constants for $C^0(9)$ and $C^0(36)$ are related to a through the

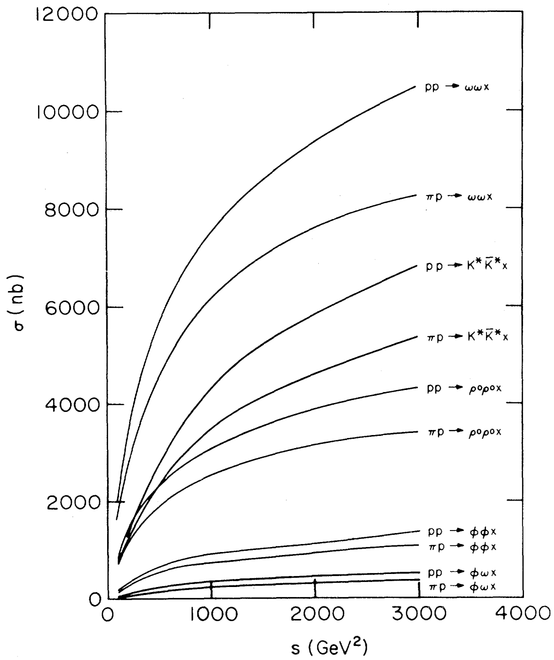

FIG. 5. The calculated cross sections of the processes $pp(\pi p)\to(\rho^0\rho^0$, $\omega\omega$, $\omega\phi$, $K^*\bar{K}^*$, and $\phi\phi)+\cdots$ as a function of s.

color VDM

$$b_{\alpha\beta}(9)=-\frac{1}{\sqrt{3}}\times\frac{1}{2}\times\frac{4\pi}{f_{\underline{\omega}}{}^2}\alpha_s a\frac{1}{\sqrt{8}}\delta_{\alpha\beta}\,,$$
$$b_{\alpha\beta}(36)=\sqrt{\tfrac{2}{3}}\times\frac{\sqrt{3}}{2}\times\frac{4\pi}{f_{\underline{\omega}}{}^2}\alpha_s a\frac{1}{\sqrt{8}}\delta_{\alpha\beta}\,. \tag{2.29}$$

The first numbers on the RHS are recoupling coefficients for $\underline{V}\cdot\underline{V}$ and the second numbers are the CG coefficients of $\underline{\omega}\cdot\underline{\omega}$ in its flavor representation. α_s is the running coupling constant which we take to be

$$\alpha_s(M_i)=\frac{4\pi}{\beta_0\ln(M_i{}^2/\Lambda^2)}\,, \tag{2.30}$$

where $\beta_0=11-\frac{2}{3}f$ with f being the flavor number we take to be 4. M_i is the mass of the ith four-quark state. Upon using $\alpha_s(m_J)=0.22$ to determine the parameter Λ to be 0.1 GeV, we get $\alpha_s(1.65)=0.27$. The cross sections of the processes $h_1+h_2\to\rho^0\rho^0+\cdots$ can therefore be calculated according to Eqs. (2.1) and (2.21) and the results for πp and for πp and pp collisions are shown in Fig. 5. For $\rho^+\rho^-$—channel we expect

$$\sigma(h_1+h_2\to\rho^+\rho^-+\cdots)$$
$$=2\sigma(h_1+h_2\to\rho^0\rho^0+\cdots)\,. \tag{2.31}$$

(b) $\omega\omega$ channel. The same states which contribute to the $\rho\rho$ channel also contribute to the $\omega\omega$ channel. The decay constants in this case are

$$a_{\omega\omega}(9)=\sqrt{\tfrac{2}{3}}\times\tfrac{1}{2}a,\;\; a_{\omega\omega}(36)=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}a\,. \tag{2.32}$$

The b constants are the same as in Eq. (2.29). As shown in Fig. 5, the cross sections of $pp\to\omega\omega+\ldots$ and $\pi p\to\omega\omega+\ldots$ are three times larger than the corresponding cross sections for $\rho^0\rho^0$ which is due to the dominance of $C^0(36)$ over $C^0(9)$.

(c) $\phi\phi$ channel. In the classification of the $ss\bar{s}\bar{s}$ states, there is only one S-wave 2^{++} state $C^{ss}(36)=\phi\phi$ which decays to $\phi\phi$. The mass of this state is calculated in the MIT bag model to be 2.25 GeV which is not far from the experimental value of $\cong 2.16$ GeV.[18] The coupling constants for this channel are

$$a_{\phi\phi}(36)=\frac{1}{\sqrt{3}}a,b_{\alpha\beta}(36)=\sqrt{\tfrac{2}{3}}\frac{4\pi}{f_{\underline{\Phi}}{}^2}\alpha_s a\frac{1}{\sqrt{8}}\delta_{\alpha\beta}\,,$$
$$\alpha_s=0.25\,. \tag{2.33}$$

We shall use the experimental mass ($M_{\phi\phi}=2.16$ GeV) to do the numerical calculation which is shown in Fig. 5. The comparison to experimental data will be carried out in Sec. V.

(d) $\omega\phi$ channel. There are two states contributing to this channel

$$C^s(9)=-\frac{1}{\sqrt{2}}K^*\bar{K}^*+\frac{1}{\sqrt{2}}\omega\phi\,,$$
and
$$C^s(36)=\frac{1}{\sqrt{2}}K^*\bar{K}^*+\frac{1}{\sqrt{2}}\omega\phi\,. \tag{2.34}$$

The masses of these two states are at 1.95 GeV. The coupling constants are

$$a_{\omega\phi}(9)=\sqrt{\tfrac{2}{3}}\times\frac{a}{2},\;\; a_{\omega\phi}(36)=\frac{1}{\sqrt{3}}\times\frac{a}{2}\,,$$
$$b_{\alpha\beta}(9)=-\frac{1}{\sqrt{3}}\times\frac{4\pi}{f_{\underline{\omega}}f_{\underline{\phi}}}\alpha_s a\frac{1}{\sqrt{8}}\delta_{\alpha\beta}\,,$$
$$b_{\alpha\beta}(36)=\sqrt{\tfrac{2}{3}}\times\frac{4\pi}{f_{\underline{\omega}}f_{\underline{\phi}}}\alpha_s a\frac{1}{\sqrt{8}}\delta_{\alpha\beta}\,,$$
$$\alpha_s=0.25\,. \tag{2.35}$$

The numerical results of the corresponding cross sections are shown in Fig. 5. Due to the cancellations between the 9-plet and the 36-plet, the cross section of the $\omega\phi$ channel are smaller than those of the $\phi\phi$ channel.

(e) $K^*\bar{K}^*$ channel. The same states in the $\omega\phi$ channel also contribute to this channel with the following constants:

$$a_{K^{*+}K^{*-}}(9)=\sqrt{\tfrac{2}{3}}\times(-\tfrac{1}{2})\frac{a}{\sqrt{2}},$$
$$a_{K^{*+}K^{*-}}(36)=\frac{1}{\sqrt{3}}\times\frac{1}{2}\frac{a}{\sqrt{2}}\,,$$
$$a_{K^{*0}\bar{K}^{*0}}(9)=\sqrt{\tfrac{2}{3}}\times(-\tfrac{1}{2})\frac{a}{\sqrt{2}}\,,$$
$$a_{K^{*0}\bar{K}^{*0}}(36)=\frac{1}{\sqrt{3}}\times\frac{1}{2}\frac{a}{\sqrt{2}}\,. \tag{2.36}$$

$b^i_{\alpha\beta}$ are the same as in the $\omega\phi$ channel. The numerical results of the corresponding cross sections are shown in Fig. 5.

III. DIFFERENTIAL CROSS SECTIONS

The differential cross section of the production of a 2^{++} four-quark state can be written as

$$\frac{d}{d\cos\theta}\sigma(h_1+h_2\to V_1+V_2+\cdots)=\int_{x_{1\,\min}}^{1}dx_1\int_{x_{2\,\min}}^{1}dx_2[G_{g_1}^{h_1}(x_1)G_{g_2}^{h_2}(x_2)+G_{g_1}^{h_1}(x_2)G_{g_2}^{h_2}(x_1)]\times\frac{d}{d\cos\theta}\sigma(g_1+g_2\to Q^2\bar{Q}^2\to V_1+V_2)\ . \tag{3.1}$$

In the c.m. frame of the two gluons,

$$\frac{d}{d\cos\theta}\sigma(g_1+g_2\to Q^2\bar{Q}^2\to V_1+V_2)=\frac{1}{64}\frac{p}{128\pi W}\left[\frac{7}{3}+p^2\left[\frac{1}{m_1^2}+\frac{1}{m_2^2}\right]\left[\tfrac{19}{18}-\tfrac{5}{6}\cos^2\theta\right]+\frac{p^4}{m_1^2m_2^2}\left[\tfrac{5}{9}-\tfrac{4}{3}\cos^2\theta+\cos^4\theta\right]\right]\times\sum_{\alpha\beta}\left|\sum_i\frac{a_{V_1V_2}^i b_{\alpha\beta}^i}{W-M_i+\frac{i}{2}\Gamma_i(W)}\right|^2 , \tag{3.2}$$

where θ is the angle between the vector meson and the gluon in the center-of-mass frame of two gluons. All other quantities in Eqs. (3.1) and (3.2) are defined in Sec. II. The numerical results for the differential cross sections as a function of the longitudinal momentum, $d\sigma/dk_{\parallel}$, and the transverse momentum, $d\sigma/dk_{\perp}^2$, for the processes $p(\pi)p\to\rho\rho$ and $\omega\omega$ at $p_L=400$ GeV/c are plotted in Figs. 6 and 7. They are calculated in the center-of-mass frame of the incoming hadrons.

IV. DISCUSSION ON TWO $\phi\phi$ STATES

In the recent $\pi^-p\to\phi\phi n$ experiment,[18] there were discovered two 2^{++} $\phi\phi$ states. Their masses, widths, and quantum numbers are reproduced in Table I. The first one at 2160 MeV seems to fit the description of the $C^{ss}(36)$ $ss\bar{s}\bar{s}$ state: the calculated mass at 2250 MeV is not far from the experimental value. The calculated width of 255 MeV (for $M=2160$ MeV) is within the experimental error. S-wave decay is also consistent with experiment. The calculated $\phi\phi$ production cross sections via $C^{ss}(36)$ in pp and πp reactions are listed in Table II together with the corresponding experimental cross sections.[12] The theoretical cross sections are slightly smaller than and within a factor of 2 of the experimental values. This is quite consistent with the fact that a K factor of $2-3$ is needed to explain the lepton pair productions in the Drell-Yan mechanism.[19] While the first $\phi\phi$ state at 2160 MeV favors an $ss\bar{s}\bar{s}$ four-quark state interpretation, it is not yet clear what the structure is for the second $\phi\phi$ state at 2310 MeV. It is conceivable that the latter could be an excited $ss\bar{s}\bar{s}$ four-quark state with an orbital angular moment $L=2$ which decays to $\phi\phi$ through a D wave making it narrower than the first state at a lower mass. However, a structure and reaction calculation has to be done in order to verify this speculation.

Under the interpretation of these $\phi\phi$ structures as $ss\bar{s}\bar{s}$ four-quark states, it is natural to understand their decay channels. If these states are glueballs as interpreted by others,[18] there should be other decay channels like $\rho\rho$, $\omega\omega$, $K^*\bar{K}^*$, $\pi\pi$, $\eta\eta$, $K\bar{K}$, etc., besides $\phi\phi$. Furthermore, one expects that the branching ratios for the other channels to be larger than that of $\phi\phi$ in this case.

V. SUMMARY AND DISCUSSION

(1) The first $\phi\phi$ enhancement (at 2160 MeV) discovered in pp and πp reactions favors a 2^{++} $ss\bar{s}\bar{s}$ four-quark state interpretation. The calculated mass, width, and cross sec-

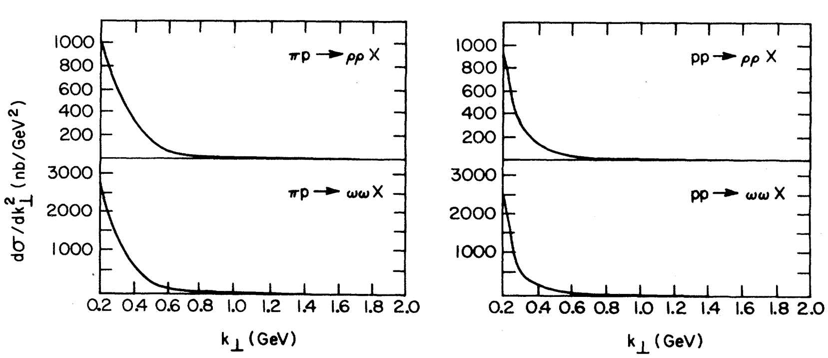

FIG. 6. Differential cross sections $d\sigma/dk_\perp^2$ for the processes $\pi p(pp)\to(\rho^0\rho^0$ and $\omega\omega)+\cdots$ at $p_L=400$ GeV/c, $k_\perp$ is the transverse momentum of the corresponding vector meson. They are calculated in the center-of-mass frame of the incoming hadrons.

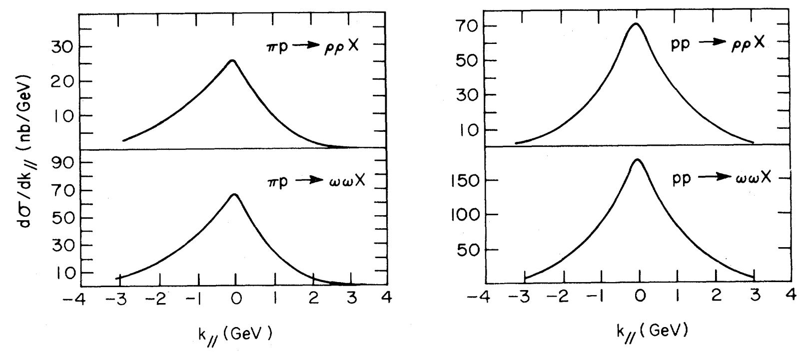

FIG. 7. Differential cross sections $d\sigma/dk_{\parallel}$ for processes $\pi p(pp)\rightarrow(\rho^0\rho^0$ and $\omega\omega)+\cdots$ at $p_L=400$ GeV/c. $k_{\parallel}$ is the longitudinal momentum of the corresponding vector meson in the center-of-mass frame of the incoming hadrons.

tions are consistent with the experimental findings. The production cross sections are calculated in a Drell-Yan-type mechanism where the $C^{ss}(36)$ four-quark state is produced via two gluons which couple to the $\underline{V}\cdot\underline{V}$ part of the four-quark state wave function with a color-VDM assumption. The results are within a factor of 2 of the experimental data. We have estimated the $C^{ss}(36)$ production where the s and $\bar{s}$ quarks originate from the sea quarks in the colliding hadrons π and p. It turns out that it is much smaller than the two-gluon contribution, due to the OZI-rule suppression. The second $\phi\phi$ enhancement at 2310 MeV could be due to an $ss\bar{s}\bar{s}$ four-quark state with an orbital angular momentum $L=2$. However, this needs to be verified.

(2) As seen in Fig. 5, the production cross sections of $\rho\rho(\rho^+\rho^-$ and $\rho^0\rho^0)$ and $\omega\omega$ are an order of magnitude larger than that of $\phi\phi$. The production of $\rho\rho$ and $\omega\omega$ peaks at 1.65 GeV. This may well be the origin of the $\rho\rho$ enhancement at 1.7 GeV observed in $\bar{p}p\rightarrow 3\pi+3\pi^-\pi^0$ experiment at 5.7 GeV/c.[15] In our calculation, the cross sections of the hadronic production of $\rho\rho$ and $\omega\omega$ are about the same, whereas in the case of $\gamma\gamma$ collision the predicted cross section of $\gamma\gamma\rightarrow\omega\omega$ is smaller than that of $\gamma\gamma\rightarrow\rho\rho$ by 2 orders of magnitude. This is due to the fact that while both the $I=0$ and $I=2$ $q^2\bar{q}^2$ states contribute to the case of $\gamma\gamma$ collision, only the isoscalar $q^2\bar{q}^2$ states contribute to the hadronic productions via gluons. A similar situation is expected in the radiative decay of J/ψ, i.e., $J/\psi\rightarrow\gamma\rho\rho$ and $J/\psi\rightarrow\gamma\omega\omega$.

(3) As shown in Fig. 5 the cross sections of $K^*\bar{K}^*$ production in pp and πp collisions are bigger than those of $\phi\phi$ production. At $p_L=400$ GeV/c, $\sigma(pp\rightarrow K^*\bar{K}^*+\cdots)=1.4\ \mu$b and $\sigma(\pi p\rightarrow K^*\bar{K}^*+-\cdots)=1.1\ \mu$b.

(4) Due to the cancellation mechanism mentioned in Sec. II, the cross sections of $\phi\omega$ production in pp and πp reactions are smaller than the corresponding cross sections of $\phi\phi$ production.

(5) The cross section of the production of a four-quark state in hadronic collisions increases as the total energy $\sqrt{s}$ of the two hadrons. This is due to the $1/x$ behavior of the gluon distribution function as x approaches zero. Also, the cross sections in pp collisions are larger than those in πp collisions at high energies. This is due to the fact that the gluon distribution function of the proton is proportional to $(1-x)^{5.9}$ as opposed to $(1-x)^3$ in the π meson.

(6) Recently, a $\pi^-N\rightarrow JJX$ experiment[14] shows two correlated JJ enhancements around 7 and 8 GeV. Explanations of these structures are given in terms of B-meson production[20] and $q\bar{q}$ annihilation $q\bar{q}\rightarrow\psi\psi$.[20] Interpreting the first structure around 7 GeV as due to the 2^{++} $cc\bar{c}\bar{c}$ state (with $a^2=30$ to fit its width of ~0.8 GeV), we calculated the cross sections for $\pi^-p\rightarrow JJX$ to be 43 pb at $p_L=280$ GeV/c and 8 pb at $p_L=150$ GeV/c. These are comparable to the experimental cross sections of 30 ± 10 and 16 ± 8 pb at these energies.

In our model, the JJ production is a resonance production with $J^{PC}=2^{++}$ and that JJ decays in an S wave. Due to the gluon fusion mechanism, the production cross section is predicted to increase fairly fast with energy and the relations in Eqs. (2.5), (2.6), (2.8), and (2.10) are expected to hold. All these predictions, which distinguish our production mechanism from others,[21,22] can be checked experimentally.

(7) The differential cross sections $d\sigma/dk_{\parallel}$ and $d\sigma/dk_{\perp}{}^2$ for the processes $p(\pi)p\rightarrow\rho\rho$ and $\omega\omega$ are plotted in Figs. 6

TABLE I. Masses, widths, and quantum numbers of the two $\phi\phi$ states discovered in the $\pi^-p\rightarrow\phi\phi n$ reaction (Ref. 18).

I^G	J^{PC}	M (MeV)	Full width (MeV)	Partial width
0^+	2^{++}	2160 ± 37	315 ± 62	$\Gamma_S/\Gamma_D\cong100$ (S wave)
0^+	2^{++}	2310 ± 72	192 ± 50	$\Gamma_D/\Gamma_S\cong37$ (D wave)

TABLE II. Theoretical and experimental $\phi\phi$ production cross sections for the 2^{++} state at 2160 MeV in pp and πp reactions.

Reaction	$p_{\rm lab}$ (GeV/c)	Theoretical cross section (μb)	Experimental cross section (μb)
$pp\rightarrow\phi\phi+\cdots$	400	0.57	0.7
$\pi p\rightarrow\phi\phi+\cdots$	100	0.17	0.34

and 7. $d\sigma/dk_\perp{}^2$ falls off fairly fast due to the fact that the 2^{++} $q^2\bar{q}^2$ resonances are only 100 MeV above the respective VV thresholds. $d\sigma/dk_{||}$ are symmetric with respect to $k_{||}$ for $pp\to\rho\rho$, and $\omega\omega$ since they are calculated in the center-of-mass frame of the incoming hadrons.

(8) We also expect the production of the 2^{++} four-quark states $C^0(9)$, $C^0(36)$, $C^{ss}(36)$, $C^s(9)$, and $C^s(36)$ in J/ψ radiative decays. The processes are

$$J/\psi\to\gamma+(\rho\rho,\omega\omega,\phi\phi,K^*\bar{K}^*,\phi\omega) \ . \tag{5.1}$$

The color-VDM assumption can also be tested in these processes. The recent Mark II experiments reports a signal in $J/\psi\to\gamma\rho^0\rho^0$.[16] The $\rho^0\rho^0$ spectrum can be interpreted as a combination of $\gamma\rho^0\rho^0$ phase space and a resonance described by a Breit-Wigner formula with a constant width. The fit gives

$$\begin{aligned} M &= 1650\pm 50 \text{ MeV} \ , \\ \Gamma &= 200\pm 100 \text{ MeV} \ . \end{aligned} \tag{5.2}$$

This mass is consistent with the 2^{++} four-quark states $C^0(9)$ and $C^0(36)$ observed in $\gamma\gamma\to\rho^0\rho^0$. The width is also consistent with the result presented in Refs. 4 and 5. It would be interesting to see whether the branching ratio can be reproduced with the four-quark state interpretation.

ACKNOWLEDGMENTS

This work was supported in part by U.S. Department of Energy Contract No. DE-AS05-82ER40074. One of the authors (K. F. Liu) acknowledges the hospitality of the University of Washingtion Summer Institute on Quantum Chromodynamics and Quarks (Seattle, Washington) where part of this work was done.

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