Quantum chromodynamics and the rise of hadronic total cross sections

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We have computed the sum of all multiparton-production cross sections induced by one hard-gluon exchange in the leading-logarithm approximation of QCD for high-energy hadronic collisions. The rise of hadronic total cross sections is explained by the excess of the rise of parton multiplicities with respect to the decrease of each individual cross section. We have shown that this rise is very sensitive to only one free parameter, namely the mass of the constituent quark, but not to other nonperturbative effects. With a reasonable value of 350 MeV, we have obtained a rise of 15 mb between CERN ISR and collider energies.

I. INTRODUCTION

It is now a rather well established fact that the observables of hadronic multiple production exhibit similar behavior both in purely hadronic reactions and in hard lepton-hadron reactions, at least at moderate values of Q^2. For instance, the increase of multiplicities and average transverse momentum, and long-range correlations have been observed in high-energy hadronic reactions as well as in e^+e^- annihilation.[1] Since QCD is the theoretical framework used to describe these behaviors in hard lepton-hadron reactions, the observed similarities suggest that perturbative QCD may be a good tool to explain some features of purely hadronic reactions.

In this paper we answer the question of the relevance of QCD to describing hadronic total cross sections. Indeed we show by means of a quantitative estimate that the gluon radiation induced in hard parton collisions may be responsible for the rise of hadronic total cross sections. We discuss the theoretical significance of this result.

We exclude from the domain of application of perturbative QCD the constant component of total cross sections, which is obviously related to nonperturbative effects. We concentrate on the rise of the total cross sections, and the problem is to explain a rise of order of about 15 mb from CERN ISR to $p\bar{p}$ collider energies, by means of small cross sections typical of hard processes.

We obtain this effect thanks to the increase of the number of gluons radiated by bremsstrahlung.[2,3] Qualitatively, the higher the momentum transfer, the lower the individual parton cross section but the higher the multiplicity of partons. Indeed, it has been observed that the rise of hadronic total cross sections is connected with an increase of multiplicities faster than logarithmic. This increase is related to the increase of the height of the central plateau, where gluons and sea quarks are supposed to be dominant. On the other hand, large-P_T inclusive cross sections increase very rapidly with energy, for instance, at $P_T = 10$ GeV, $E\,d\sigma/d^3p$ increases by orders of magnitude from ISR to collider[4] energies. It has also been observed, in calorimeter experiments,[5] that the cross section to deposit a large transverse energy is large and is built up by events involving large multiplicities. On a more theoretical ground Gribov, Levin, and Ryskin[6] have shown that the cross section for the production of high-P_T hadrons in the pionization region may be much larger than expected in commonly used parametrizations.

II. TOTAL CROSS SECTION OF VALONS[7] IN THE LEADING-LOGARITHM APPROXIMATION

A. Valons and the additive quark model

In order to study hadronic total cross sections we have to proceed in two steps. In a first step one extracts from the incoming hadrons the so-called constituent quarks or valons by means of a nonperturbative Q^2-independent distribution. One then applies QCD at the leading-logarithm approximation to describe the interaction of these valons. This separation is necessary since QCD applies only for inclusive distribution of partons inside a parton and not inside a hadron. On the other hand this two-step procedure guarantees that one obtains the results of the so-called additive quark model which are known to agree with data.

Let A and B be the incoming hadrons and $\dot{q}_A$ and $\dot{q}_B$ their valons. By constituent quarks or valons we mean valence quarks (or antiquarks in the case of an antiproton) accompanied by sea quarks and gluons, carrying the whole momentum of the incoming hadron and interacting independently.

Valons are characterized by their rest mass m_q (expected to be of the order of one third of the nucleon mass) and by a Q^2-independent probability distribution

$$\psi_{A\to\dot{q}_A}(z)$$

which is constrained by

$$\int_0^1 \psi_{A\to\dot{q}_A}(z)dz=3 \text{ for a proton or an antiproton} \tag{1}$$

(conservation of the number of valence quarks; note that we do not distinguish u and d quarks) and by

$$\int_0^1 z\psi_{A\to\dot{q}_A}(z)dz=1 \tag{2}$$

(energy-momentum conservation).

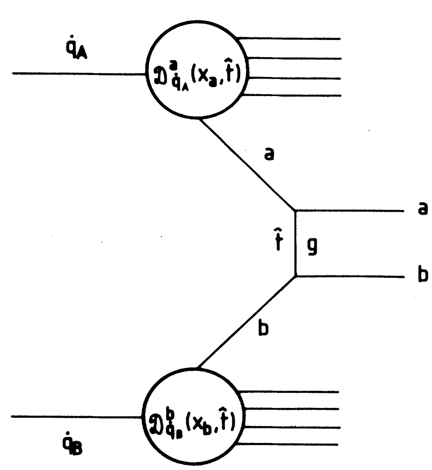

FIG. 1. Valon-valon total cross section; $\hat{t}$ is the virtual mass squared of the gluon exchanged between partons a and b.

With the help of the ψ distributions we write the hadron total cross section in terms of the valon total cross section as

$$\sigma_{AB}^{\text{tot}}(s)=\int_{2m_q/\sqrt{s}}^1 dz_a \int_{2m_q/\sqrt{s}}^1 dz_b \psi_{A\to\dot{q}_A}(z_a)\psi_{B\to\dot{q}_B}(z_b)\sigma_{\dot{q}_A\dot{q}_B}^{\text{tot}}(z_a z_b s) \tag{3}$$

which simply expresses that valons interact independently.

In a first step we shall study the valon total cross section by means of QCD, afterward we shall show that the hadronic total cross section is not very sensitive to the peculiar form of the ψ distributions.

B. One-gluon-exchange contribution to the valon cross section

High-$P_\perp$ events in hadron-hadron collisions are reasonably well described by perturbative QCD.[8] At the parton level they correspond to the exchange of a gluon with large invariant squared mass or virtualness (of modulus $\hat{t}$) between quarks or gluons carrying a large fraction of hadron longitudinal momenta. These events are characterized by a jetlike structure.

But, let us consider the same one-gluon-exchange contribution without a large-$P_\perp$ trigger. Now, gluon radiation due to the bremsstrahlung of the valons may be important. From the infrared properties of perturbative QCD, one expects a rather large accumulation of events corresponding to small relative longitudinal momenta and intermediate $\hat{t}$ values (say $\hat{t}$ of order of a few GeV^2) which contributes to the central plateau of hadronic multiplicity distributions. Experimental data on large transverse energy inclusive cross sections confirm this expectation.[5]

The contribution to the valon total cross section from one-gluon exchange between partons (see Fig. 1) is obtained by summation over all final-state configurations. One obtains

$$\tilde{\sigma}\equiv\sigma_{\dot{q}_A\dot{q}_B}^{\text{tot}}(\tilde{s})=\sum_{a,b}\int\int\int_\Delta d\hat{t}\,dx_a dx_b \frac{d\sigma}{d\hat{t}}(ab\to ab)\mathscr{D}^a_{\dot{q}_A}(x_a,\hat{t})\mathscr{D}^b_{\dot{q}_B}(x_b,\hat{t}) \,. \tag{4}$$

The integration domain Δ will be discussed later on. $\tilde{s}=z_a z_b s$ [see Eq. (3)] is the c.m. squared energy of the valon reaction, $\mathscr{D}^a_{\dot{q}_A}(x_a,\hat{t})$ [$\mathscr{D}^b_{\dot{q}_B}(x_b,\hat{t})$] is the probability of finding parton a in valon $\dot{q}_A$ [b in $\dot{q}_B$] with the fraction x_a [x_b] of the incident valon momentum, with a resolution scale $\hat{t}$, and $d\sigma/d\hat{t}(ab\to ab)$ (Ref. 9) is the differential cross section in the one-gluon-exchange approximation

$$\frac{d\sigma}{d\hat{t}}(ab \to ab) = \frac{\pi}{2}\alpha_s{}^2(\hat{t})\frac{1}{\hat{t}^2}\left[(C_2\delta_{aq}+N_c\delta_{ag})(C_2\delta_{bq}+N_c\delta_{bg})+O\left[\frac{\hat{t}}{\hat{s}}\right]\right]. \tag{5}$$

The Kronecker δ symbols select the vertices with quarks or antiquarks $(a,b=q)$ and gluons $(a,b=g)$. $\hat{s}=x_a x_b \tilde{s}$ is the c.m. squared energy in the parton collision. As usual we define

$$\alpha_s(\hat{t}) = \frac{1}{b \ln \hat{t}/\Lambda^2} \text{ with } b = \frac{11N_c - 2N_f}{12\pi}$$

$C_2=(N_c{}^2-1)/2N_c$, where $N_c=3$ is the number of colors and $N_f=4$ the number of flavors. Λ is the usual QCD scale, which is set at 0.1 GeV according to the theoretical expectations of Ref. 10 and in agreement with the current world average.[11] Note that only elastic (but for color) differential cross sections appear in Eq. (4) because we have neglected parton reactions involving quark exchange. The summation over colors in the final state has been performed in Eq. (5). In terms of hadrons, the processes which we study are obviously inelastic since they involve one-gluon exchange. Our calculation thus applies to the total inelastic cross section.

C. The domain of integration Δ

We rely on the leading-logarithm approximation (LLA) of QCD which has been extensively studied by Dokshitzer, Dyakonov, and Troyan[2] (hereafter denoted as DDT) to determine the domain of integration Δ in Eq. (4) and the parton structure functions $\mathscr{D}^a_{\dot{q}_A}$ and $\mathscr{D}^b_{\dot{q}_B}$. DDT have shown that, in a planar gauge, at the LLA the contributions which dominate the process of Fig. 1 are the ones corresponding to tree diagrams which, in the modulus squared of the amplitude give rise to generalized ladder diagrams (namely, ladder diagrams with vertex and propagator renormalizations) (see Fig. 2).

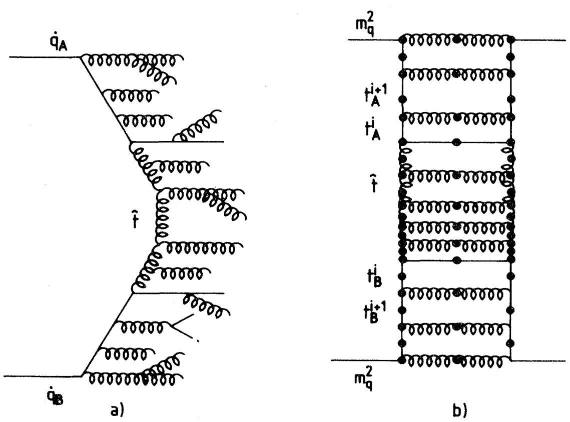

FIG. 2. Dominant contributions of valon-valon cross section in the planar gauge at the LLA. (a) Typical tree diagram in the amplitude. (b) Typical generalized ladder diagram in the cross section.

The kinematics for which the process depicted in Fig. 2 is dominant is characterized by the strong ordering of the moduli of the transfers of momentum (as in heavy-lepton-pair production)[2]

$$\begin{aligned} m_q{}^2 &\ll \cdots t_A^{i+1} \ll t_A^i \ll \cdots \hat{t}\,, \\ m_q{}^2 &\ll \cdots t_B^{i+1} \ll t_B^i \ll \cdots \hat{t}\,. \end{aligned} \tag{6}$$

On the other hand the subsequent relative longitudinal momenta are also ordered. To establish this property which is of crucial importance for our purpose we have first to consider the cascading of a *timelike* parton of virtual mass squared $Q^2>0$ down to a valon (see Fig. 3).

At step i we have

$$P_\perp{}^2 = t_i z_i(1-z_i) - t_{i+1}(1-z_i) - m^2 z_i\,, \tag{7}$$

where z_i is the relative energy of parton $i+1$ with respect to parton i. Because of the strong ordering of the virtual mass squared [see Eq. (6)], the condition that $P_\perp{}^2$ is positive, implies that

$$z_i > \frac{t_{i+1}}{t_i}\,. \tag{8}$$

Repeating this reasoning step by step we obtain that z, the relative energy of the final valon is bounded by

$$z > \frac{m_q{}^2}{Q^2}\,. \tag{9}$$

The process depicted in Fig. 2 involves spacelike partons, but the Gribov-Lipatov reciprocity relation[12] holds in the LLA and condition (9) provides us with a first constraint for the domain Δ:

$$x_a > \frac{m_q{}^2}{\hat{t}}\,,\quad x_b > \frac{m_q{}^2}{\hat{t}}\,; \tag{10}$$

the second one is given by

$$\hat{t} < \hat{s} = x_a x_b \tilde{s} \tag{11}$$

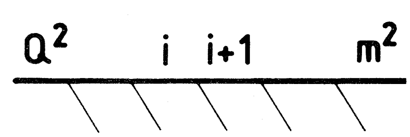

FIG. 3. Cascading of a timelike virtual parton.

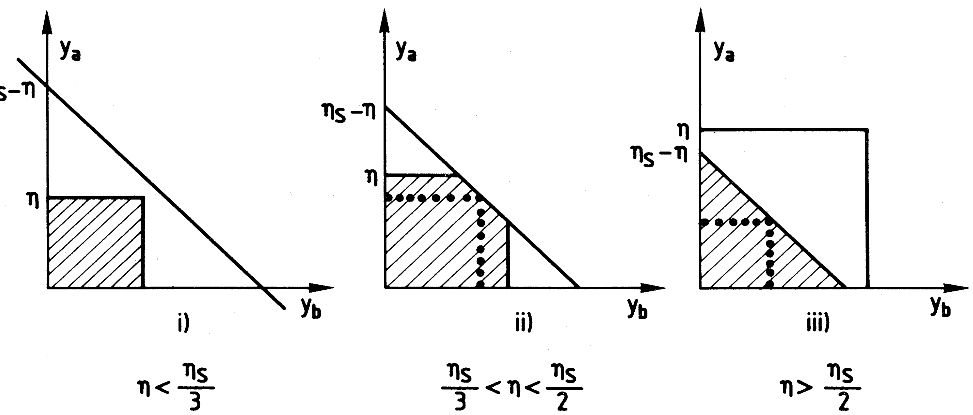

FIG. 4. The integration domain Δ_y (shaded area). Dotted lines are the limit of the domain leading to the factorizable approximation.

which reflects the kinematics of two-body reactions at the parton level. The integration domain Δ is simpler in terms of rapiditylike variables:

$$y_{a,b}=\ln\frac{1}{x_{a,b}}\ ,$$
$$\eta=\ln\frac{\hat{t}}{m_q{}^2}\ , \tag{12}$$
$$\eta_s=\ln\frac{\tilde{s}}{m_q{}^2}\ .$$

$0\leq\eta\leq\eta_s$; y_a, $y_b\in\Delta_y(\eta)$ where $\Delta_y(\eta)$ is defined as follows [it turns out that in order to make Eqs. (10) and (11) compatible one has three cases as shown in Fig. 4]:

(i) $0\leq\eta\leq\frac{\eta_s}{3}$ $(m_q{}^2\leq\hat{t}\leq m_q{}^{4/3}\tilde{s}^{1/3})$

$$\Delta_y=\{0\leq y_{a,b}\leq\eta\}\ ,$$

(ii) $\frac{\eta_s}{3}\leq\eta\leq\frac{\eta_s}{2}$ $(m_q{}^{4/3}\tilde{s}^{1/3}\leq\hat{t}\leq m_q\tilde{s}^{1/2})$

$$\Delta_y=\{0\leq y_{a,b}\leq\eta\}\cap\{y_a+y_b\leq\eta_s-\eta\}\ , \tag{13}$$

(iii) $\frac{\eta_s}{2}\leq\eta\leq\eta_s$ $(m_q\tilde{s}^{1/2}\leq\hat{t}\leq\tilde{s})$

$$\Delta_y=\{y_a+y_b<\eta_s-\eta\}\ .$$

D. Valon structure functions

It is well known that sea-quark and gluon structure functions are more singular than x^{-1} near $x=0$, which leads to multiplicities which increase more than logarithmically with energy. This is due to an essential singularity in the moment space near $j=1$. DDT have shown that this effect reflects in the following small-x behavior of gluon and sea-quark structure functions:

$$\begin{aligned} &x\mathscr{D}_q^g(x,\hat{t})\propto I_1(v)/v\ ,\\ &x\mathscr{D}_q^{q_S}(x,\hat{t})\propto I_2(v)/v^2\ , \end{aligned} \tag{14}$$

where $v=(16N_c\xi\ln 1/x)^{1/2}$, I_1 and I_2 are the modified Bessel functions which behave for large v as $\exp(v)/\sqrt{2\pi v}$, and

$$\xi=\int_{\mu^2}^{\hat{t}}\frac{dQ^2}{Q^2}\frac{\alpha_s(Q^2)}{4\pi}=\frac{1}{4\pi b}\ln\frac{\alpha_s(\mu^2)}{\alpha_s(\hat{t})}$$
$$=\frac{1}{4\pi b}\ln\frac{\ln\frac{\hat{t}}{\Lambda^2}}{\ln\frac{\mu^2}{\Lambda^2}}\ . \tag{15}$$

The parameter μ controls the way how ξ and thus the gluon and sea-quark multiplicities vary with $\hat{t}$. DDT have determined μ by demanding a realistic value for the fraction of momentum carried by gluons in a nucleon at $Q^2\simeq 2.5$ GeV^2. They have obtained this way $\mu=0.15$ GeV for $\Lambda=0.1$ GeV. Now according to Eq. (15) ξ vanishes at $\sqrt{\hat{t}}=\mu$, which is too small to be interpreted as the mass of the valon. Nevertheless for our quantitative calculations we have used the DDT parametrization with $\mu=0.15$ GeV but we have allowed the mass of the valon (that is, the scale at which ξ should vanish) to be different from μ by means of a slight modification of ξ, namely,

$$\xi=\frac{1}{4\pi b}\ln\frac{\ln\frac{\hat{t}-m_q{}^2+\mu^2}{\Lambda^2}}{\ln\frac{\mu^2}{\Lambda^2}}\ . \tag{16}$$

This ansatz does not affect at all the structure functions at the Q^2 values which are relevant for the rise of total cross sections ($\hat{t} \gg m_q{}^2-\mu^2$).

The LLA applies for small values of x (Ref. 13) provided that

$$1 \ll \ln\frac{1}{x} \lesssim \ln\frac{\hat{t}}{\Lambda^2} . \tag{17}$$

In our calculations $x_{a,b} > m_q{}^2/\hat{t}$ [region (i), in Eq. (13)], and provided that $(1/\pi)\alpha_s(m_q{}^2)$ is small, i.e., $m_q{}^2/\Lambda^2 \gg 1$, Eq. (17) is satisfied and the LLA applies. As will be shown below $m_q{}^2/\Lambda^2$ turns out to be of order 10 which yields $(1/\pi)\alpha_s(m_q{}^2) \sim 0.2$.

E. Factorizable approximation

From Eqs. (4), (5), and (14) we can discuss qualitatively the contributions of sea quarks and gluons in the valon total cross section. In Eq. (4) the parton differential cross section favors small values of $\hat{t}$ [see Eq. (5)]. But, because of the domain of integration defined in Eq. (13) and because of the behavior of structure functions near $x=0$ [see Eq. (14)] the parton multiplicities favor large values of $\hat{t}$. We thus expect that the maximum valon total cross section is obtained for intermediate $\hat{t}$.

From Eq. (14) we see that the integrand in Eq. (4) is maximum when both y_a and y_b are large. In region (i) the integrals in y_a and y_b are factorized which simplifies the calculations. By replacing regions (ii) and (iii) by the domain Δ'_y (see Fig. 4)

$$\Delta'_y = \left\{ y_{a,b} \le \frac{\eta_s-\eta}{2} \right\} , \tag{18}$$

we obtain the best factorizable approximation to the contribution of the integral in regions (ii) and (iii). In any case this factorizable approximation provides us with a lower bound for the effect we are studying.

F. Truncated multiplicities

Let us consider truncated parton multiplicities in valons, which are nothing but the parton multiplicities in valons obtained in the LLA, namely,

$$n^p(\eta) = \begin{cases} \int_0^{\eta} \mathscr{D}_q^p(y,\eta)dy & \text{for } \eta \le \frac{\eta_s}{3} \\ \int_0^{\eta_s-\eta/2} \mathscr{D}_q^p(y,\eta)dy & \text{for } \eta > \frac{\eta_s}{3} , \end{cases} \tag{19}$$

where $p=v$ for valence quark, S for sea quarks, and g for gluons. These multiplicities are truncated because the y integrals are performed in the domain Δ'_y.

In terms of these truncated multiplicities, Eq. (4) becomes

$$\tilde{\sigma} = \frac{\pi}{2m_q{}^2} \int_0^{\eta_s} d\eta \{ C_2[n^v(\eta)+n^S(\eta)] + N_c n^g(\eta) \}^2 e^{-\eta} \frac{1}{b^2(\eta+\ln m_q{}^2/\Lambda^2)^2} . \tag{20}$$

For valence quark we have approximated the truncated multiplicity by the full multiplicity which is finite: $n^v(\eta)=1$. For gluon and sea quarks the truncated multiplicities are obtained from Eq. (14) and the normalization is fixed by means of the second moments. This yields

$$n^g(\eta) = \lambda_g [I_0((16N_c\xi\bar{\eta})^{1/2}) - 1] , \tag{21}$$

$$n^S(\eta) = \lambda_S \left[\frac{I_1((16N_c\xi\bar{\eta})^{1/2})}{(16N_c\xi\bar{\eta})^{1/2}} - \frac{1}{2} \right] , \tag{22}$$

where

$$\bar{\eta} = \begin{cases} \eta & \eta \le \frac{\eta_s}{3} \\ \frac{\eta_s-\eta}{2} & \eta > \frac{\eta_s}{3} , \end{cases} \tag{23}$$

$$\lambda_g = \frac{4C_2\exp(-4N_c\xi)}{4C_2+N_f} \frac{1-\exp[-\frac{2}{3}(4C_2+N_f)\xi]}{1-\exp(-4N_c\xi)} , \tag{24}$$

$$\lambda_S = \frac{8N_c\xi \left\{ \frac{N_f}{4C_2+N_f} - \exp(-\frac{8}{3}C_2\xi) + \frac{4C_2}{4C_2+N_f}\exp[-\frac{2}{3}\xi(4C_2+N_f)] \right\}}{\exp(4N_c\xi)-1-4N_c\xi} . \tag{25}$$

It is possible now to understand qualitatively the origin of the rise of total cross sections. From the asymptotic behavior of modified Bessel function one can evaluate the behavior of the integrand in Eq. (20) as a function of η. Neglecting the slow variation of ξ (which behaves like $\ln\eta$) we obtain for the gluon-gluon contribution to the integrand

$$I_{gg} \sim \exp(-\eta + 2\sqrt{16N_c\xi\eta}) \quad \text{in region (i), i.e., } \eta \leq \frac{\eta_s}{3} \qquad (26)$$

and

$$I_{gg} \sim \exp\left\{-\eta + 2\left[16N_c\xi\left[\frac{\eta_s-\eta}{2}\right]\right]^{1/2}\right\} \quad \text{in regions (ii) and (iii), i.e., } \eta > \frac{\eta_s}{3}\,. \qquad (27)$$

It appears that mostly region (i) contributes to the rise of total cross section because there exists a region in which the integrand is increasing, whereas the behavior of the integrand in Eq. (27) is continuously decreasing. Indeed we rewrite Eq. (26) as

$$I_{gg} \sim \exp[-(\sqrt{\eta} - \sqrt{16N_c\xi})^2]$$

which is a Gaussian centered at

$$\eta = 16N_c\xi \qquad (28)$$

which corresponds to a very large value of η. With ξ, as defined in Eq. (16), one can solve Eq. (28). For $m_q{}^2 \sim 0.125$ GeV^2 we find that the logarithmic derivative of the cross section is positive [increasing integrand in region (i)], for $\eta_s \lesssim 56$, i.e., $\tilde{s} \lesssim 10^{23}$ GeV^2. We thus see that for the rise of the cross section the most important region is the upper end of region (i), i.e.,

$$\eta \simeq \frac{\eta_s}{3} \quad (\hat{t} \sim m_q{}^{4/3}\tilde{s}^{1/3})\,.$$

The approximations to calculate the contributions of regions (ii) and (iii) are furthermore justified by the fact that the integrand is decreasing there.

III. THE RISE OF THE *NN* TOTAL INELASTIC CROSS SECTION, A QUANTITATIVE ESTIMATE

To evaluate hadron cross sections for the knowledge of valon total cross sections one needs to know the nonperturbative distributions $\psi_{A\to\dot{q}_A}(z_a)$ and $\psi_{B\to\dot{q}_B}(z_b)$ which appear in Eq. (3). However the valon total cross section depends on z_a and z_b only through the upper bond in Eq. (4) $\tilde{s} = z_a z_b s$, and

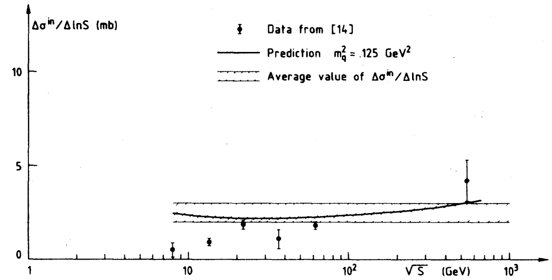

FIG. 5. $\Delta\sigma/\Delta\ln s$. The solid curve is our prediction for $m_q = 350$ MeV. Data are from UA4 (Ref. 14).

it turns out that the valon total cross section is a slowly rising function of the energy. As a consequence the hadron total cross section does not depend very much on the specific form of the ψ distributions. Provided that these distributions obey constraints (1) and (2), a good approximation of the nucleon-nucleon (or nucleon antinucleon) cross section is given by

$$\sigma_{NN}(s) \simeq 9\sigma_{\dot{q}\dot{q}}^{\text{tot}}(s/9) \qquad (29)$$

which is nothing but the result obtained from the additive quark model using the quark kinematics.

We have performed a quantitative estimate of the QCD contributions to the nucleon-nucleon total inelastic cross section. We have used the approximations discussed both in Sec. II (one-gluon-exchange approximation, factorization of the truncated multiplicities obtained in the LLA) and in Eq. (29). The parameters are Λ, μ, and m_q. Λ and μ have been fixed at 100 MeV and 150 MeV, respectively, in agreement with the current world average for Λ[11] and the DDT estimate for μ.[2] The only free parameter for our estimate is m_q, the mass of the valon.

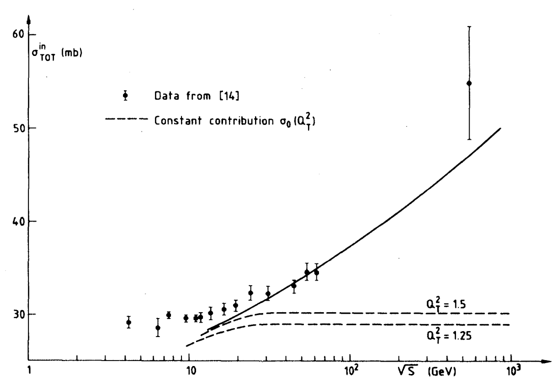

FIG. 6. The *N*-*N* total inelastic cross section as a function of energy. In dashed curves, the constant contributions corresponding to two different values of $Q_T{}^2$ are shown. Data are from UA4 (Ref. 14).

For the validity of the LLA one needs that $m_q{}^2/\Lambda^2 \gg 1$, and a reasonable expectation is that m_q is around one third of the nucleon mass. Our quantitative estimate is shown in Fig. 5. It is remarkable that with $m_q \sim 350$ MeV we obtain a rise of the cross section from ISR energy to the collider energy of about 15 mb, in agreement with data.[14] Surprisingly we even obtain a rough agreement (see Fig. 6) with the absolute normalization of the total inelastic cross section. To be more specific about the question of the *rise* of the cross section we have introduced a new scale $Q_T{}^2$ of the order of 1 GeV^2 which separates the soft region from the hard one in which perturbative QCD safely applies. In Eq. (4) we cut the $d\hat{t}$ integral into two pieces:

$$\int_{m_q{}^2}^{\tilde{s}} d\hat{t} = \int_{m_q{}^2}^{Q_T{}^2} d\hat{t} + \int_{Q_T{}^2}^{\tilde{s}} d\hat{t}\,. \qquad (30)$$

The first piece contributes a constant apart from a kinematical rise. Indeed as soon as $\eta_s/3 > \ln Q_T{}^2/m_q{}^2$, i.e., $\tilde{s} \simeq s/9 \gtrsim 20 Q_T{}^2$, the contribution of the first piece in Eq. (30) to the total cross section is energy independent. On the contrary the reasoning about the energy variation of the integrand displayed in Sec. II applies for the second piece in Eq. (30). So it is the hard region which is responsible for the rise of the cross section. In Fig. 6 we show the separation of σ_{NN}^{in} into two pieces according to Eq. (30) for a few values of $Q_T{}^2$.

We find that for $Q_T{}^2 = 1.50$ (GeV)^2 the constant contribution of the soft region is around 31 mb.

IV. DISCUSSION

We thus see that with the standard leading-logarithm approximation, with a natural value for the only free parameter one is able to obtain not only the rise of the total inelastic cross section but also its absolute normalization. To discuss this result we try in this concluding section to answer some questions about the theoretical meaning of our calculation.

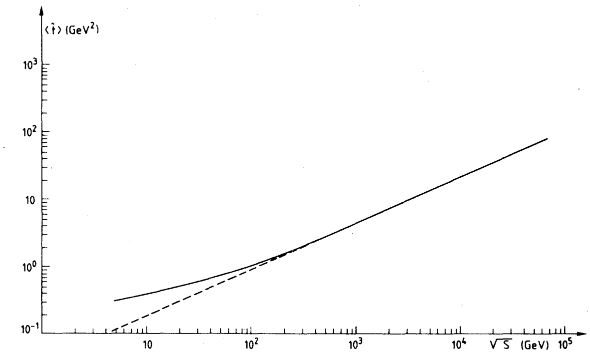

FIG. 8. Average value of the maximum $\hat{t}$ of the exchange gluon. The solid curve represents our calculation and the dashed curve the $s^{1/3}$ behavior predicted asymptotically by the model.

(i) Does this model for the cross section obey the Froissart bound? We have already discussed in Sec. II the asymptotic behavior of the integrand in Eq. (20). From this discussion it turns out that at ultra asymptotic energies, the integrand [see Eq. (26)] begins to decrease, which means that the cross section finally flattens out. For instance we reach the value of 1 b at about 10^{30} GeV^2 (see Fig. 7). It thus appears that s-channel unitarity is not violated although it has not been taken into account explicitly.

(ii) How hard are the parton processes responsible for the rise of the total cross sections? To answer this question we have computed the average value of $\hat{t}$ (see Fig. 8). $\langle \hat{t} \rangle$ is the average squared transverse momentum of partons.

From Fig. 8 we notice at first that the average value of $\hat{t}$ is indeed rather moderate and slowly varying with respect to s. The asymptotic law of variation is, as expected proportional to $s^{1/3}$ corresponding to the maximum of the integrand [see Eq. (26)]. Note that this asymptotic form is valid only above $\sqrt{s} \sim 100$ GeV, $\hat{t} \sim 0.04\, s^{1/3}$.

At the collider energy we find for $\langle \hat{t} \rangle$ about 3 GeV^2. Assuming two or three hadrons per parton this value is not incompatible with the average transverse momentum of hadron which has been measured ($\langle P_\perp \rangle \sim 500$ MeV). This means that the main bulk of inelastic events can be described by means of our model.

(iii) How does the proposed model compare with theoretical expectations based on Pomeron calculus? To answer this question we rely on the scheme of correspondence between QCD and dual Pomeron calculus developed in Ref. 15. According to this approach there exists a transition scale at which the leading-logarithm approximation of QCD and the

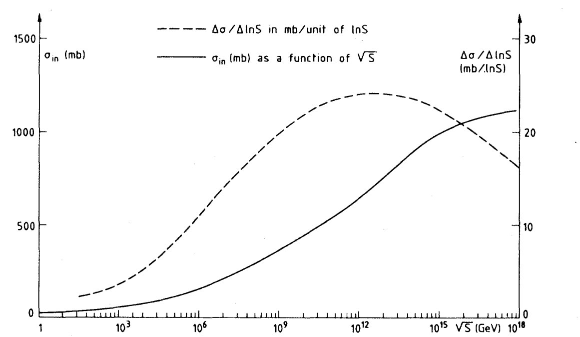

FIG. 7. Behavior of the total inelastic cross section at ultra asymptotic energies.

dual topological unitarization (DTU) scheme can be applied simultaneously. If we interpret Q_T^2 as this transition scale the soft component would be associated with the contribution of the "bare Pomeron" (lowest topology in DTU) and the hard component would be associated with the sum of all multi-Pomeron corrections to this bare Pomeron.

(iv) Why restrict the QCD calculations to the leading-logarithm approximation? For instance, can the effects of multigluon exchanges or those of parton configurations outside the Δ domain affect the obtained results? As far as perturbative effects are concerned, it has been shown in Ref. 13 that the LLA is sufficient in the region of interest for our calculations, namely, the small-$x_{a,b}$ region. On the other hand the above-mentioned equivalence between the LLA and the dual Pomeron calculus allows us to answer the question for nonperturbative contributions, namely, all these contributions are supposed to be taken into account either in the bare Pomeron or in the valon distributions. This conjecture is comparable with the "soft blanching" assumption proposed in Ref. 2 to account for the completeness of color states generated at the perturbative level.

In view of this discussion we conclude that perturbative QCD, at the leading-logarithm approximation may provide a quantitative theoretical description of the main bulk of inelastic cross sections at high energy. Indeed further tests are needed to draw a more affirmative conclusion.

A first test would be to describe, by means of our dynamical model, the inclusive cross section to produce one hadron at large $P_\perp$. It is well known that, for a not-too-large trigger transverse momentum, it is necessary to take into account the effect of gluon bremsstrahlung. In this respect, it is interesting to note that the recent calculations,[4] which take into account these bremsstrahlung effects, lead to a very decent agreement with ISR and collider data down to $P_\perp \sim 1$ GeV/c.

The best test would be to apply our model to the description of large-transverse-energy cross sections. We are currently investigating this extension of our model.

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