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Direct electromagnetic excitation $N \to \Delta(1232)$, final-state interactions, and the multipoles $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photopion production off nucleons in the range $160 \lesssim E_\gamma \lesssim 800$ MeV

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In this paper one of the classics of particle theory in the resonance region is reexamined, namely the prediction of the amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photopion production on nucleons around the resonance $\Delta(1232)$. The tool employed for this purpose is the final-state-interaction theory, and the principal aim of the study is to advocate for the appearance of two distinct dynamical mechanisms of photoexcitation in such an approach. Special emphasis is put then on the naturally emerging links between the present treatment and the isobaric approach and calculations based on the static quark model. It is argued that the dynamical meaning of the troublesome arbitrary constants (known to appear in most studies of the type presented in this paper) can be very convincingly explained when the resonances formed in the reaction [$\Delta(1232)$ in this specific case] are treated as "elementary" objects represented in the T-matrix elements by singularities of the Castillejo-Dalitz-Dyson kind. In the case discussed here, "direct" magnetic-dipole excitation $N \to \Delta(1232)$ seems to emerge very clearly as an independent dynamical mechanism of photoproduction in the p_{33} state, parallel to the usual electromagnetic excitation of nonresonant πN systems, both followed by resonant pion-nucleon rescattering. It turns out that the magnitude of such direct $\gamma N\Delta$ coupling is completely determined by the related strong interactions. The necessity of allowing for both specified mechanisms in this and other similar calculations is emphasized. The no-fit absolute predictions regarding the two multipole amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ agree then with the data over the whole interval of photon laboratory energy $E_\gamma \lesssim 800$ MeV currently covered by multipole analyses of experimental measurements. In the interpretation of the results, it is concluded that photoproduction of pions on nucleons in the p_{33} state by magnetic-dipole excitation proceeds mainly through "direct" resonance formation, which explains the well-known strongly resonant character of $M_{1+}^{(3/2)}$ *a priori* assumed in isobaric calculations and qualitatively emerging from static-quark-model analyses. On the contrary, photoproduction by electric-quadrupole excitation is a nonresonant process (despite resonant pion-nucleon interactions in the final p_{33} state) which explains the smallness of $E_{1+}^{(3/2)}$, also qualitatively predicted within the static quark model.

I. INTRODUCTION

The problem of predicting (or postdicting) reliably the amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ of photoproduction of pions on nucleons in the $\Delta(1232)$ region belongs to the classics of particle theory in the resonance energy range. As a great deal of excellent papers[1] on this subject have appeared since 1955, a natural question arises, whether there is still anything new to be told in this connection. In this paper I try to give an affirmative answer to this question by arguing that there exists an implicit common dynamical background of most of the past studies on photoproduction despite their apparent diversity and by showing that these common ideas, when made explicit and applied systematically, lead to surprisingly good predictions (or postdictions) regarding all measured quantities related to the electromagnetic excitation $N \to \Delta(1232)$. I find this point quite important, since in spite of a general success reported by most of the earlier studies regarding the amplitude $M_{1+}^{(3/2)}$, the effective determination of this multipole had in many cases no clear dynamical motivation. In what concerns the amplitude $E_{1+}^{(3/2)}$ much effort spent in calculating it in the past did not yield, unfortunately, any conclusive answer to the question of whether $E_{1+}^{(3/2)}$ had to be resonant or not. It is only through phenomenological fits that the nonresonant character of the latter multipole was firmly established.[4–7]

In the following I summarize and extend some of my recently published fragmentary studies[8–10] regarding the determination of the amplitudes of electromagnetic excitation $N\rightarrow\Delta(1232)$. I believe such extension necessary because the calculations, primarily intended to be applicable in the range of photon laboratory energy $E_\gamma \lesssim 450$ MeV, ultimately turned out to give very correct *no-fit* predictions over a much broader energy interval extending up to $E_\gamma \lesssim 800$ MeV, i.e., comprising the whole range covered by phenomenological multipole analysis.[6,11,12] There is hence good reason to advocate for a dynamically rooted interpretation of these results. It is then interesting to note how naturally the links between the approach in question, the isobar model, and the quark model show up in the course of the present calculation.

The study presented here is based on the equations of relativistic, unitary, final-state-interaction theory derived from fixed-angle dispersion relations. The latter have been discussed in detail some time ago[13] and later on applied to a simple calculation of photoproduction amplitudes.[14] The study which follows differs from that of Refs. 13 and 14 by some formal improvements in the integral equation, such as the use of subtracted dispersion relations and introduction of more suitable threshold factors. As the present calculation has been confined to the determination of the multipoles $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ a truncated system of integral equations, comprising these two amplitudes only, has been actually solved. The dynamical input to these equations contains two nontrivial assumptions in addition to the routine use of elastic πN phase shifts as required by the Watson theorem.[15] The first assumption is a "pseudoelastic" extrapolation of the multipole phases beyond the range of negligible inelasticity. The details of the extrapolation have already been presented earlier[10] and will also be recalled in Sec. III. The second, far more important assumption consists in introducing to the multipole amplitudes a Castillejo-Dalitz-Dyson (CDD) zero[16] located on the energy axis at the value where the multipole phase equals π. The necessity of introducing such a zero is evident from the present and earlier calculations, though the meaning of this device has not been made sufficiently clear before. The interpretation advocated in the present study tells that the CDD zero reflects the existence of the so-called "direct" photoexcitation[17–20] of $\Delta(1232)$ through $\gamma N\Delta$ coupling which is a mechanism of photoproduction alternative to photoexcitation of nonresonant states followed by resonant πN rescattering in the p_{33} state. This brings the present study close to the isobar model, the important difference between the two approaches lying in the way of setting the strength of the $\gamma N\Delta$ coupling. In the isobar model it has always been a new parameter of the calculation, while in the present case it is fixed by strong interactions alone. This can be easily seen here because the location of the CDD zero is ultimately connected with the use of the perfectly elastic unitarity condition in the whole range of integration over the energy variable, as mentioned in the foregoing. The latter assumption is very fair indeed, owing to the remarkably elastic character of πN scattering in the p_{33} state. In Sec. III, I elaborate on the last points thoroughly.

The results, as presented in Sec. IV and discussed in Sec. V, are in very good agreement with the data, which is considered to be an important check for a calculation without arbitrary parameters. The formalism is able to reproduce correctly the multipole $M_{1+}^{(3/2)}$. The multipole $E_{1+}^{(3/2)}$ turns out to be nonresonant and its values, though small, match the available data very reliably with the possible exception of those in the threshold region. It turns out, however, that the traditional approach, which treats the subsystem of mutually coupled integral equations for $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ as a closed one, might be too restrictive. The coupling of $E_{1+}^{(3/2)}$ to the s-wave multipoles could be large enough to affect final results close to the threshold.

The presence of the "direct" electromagnetic excitation $\gamma N\rightarrow\Delta(1232)$ and, owing to this, the treatment of $\Delta(1232)$ as an "elementary" (within the framework of the present formalism) object, combined with its known behavior under electromagnetic excitations as a three-quark state, allows for a very convincing explanation of differences in the energy dependence of the amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ around $\Delta(1232)$. Photoproduction of pions on nucleons in the p_{33} state by magnetic-dipole excitation proceeds overwhelmingly through "direct" $\gamma N\Delta$ excitation, which explains its known resonant character. On the contrary, photoproduction in the p_{33} state by electric-quadrupole excitation is an effect of formation of nonresonant pion-nucleon states, which notwithstanding the resonant πN rescattering in the p_{33} state leads in general to a nonresonant total amplitude as will be argued in Sec. V. This explains the presumed nonresonant character of the transition and the relative smallness of its amplitude $E_{1+}^{(3/2)}$.

Owing to such a sharp distinction between the characteristics of the two foregoing multipole transitions, the process of photo- and electroexcitation[20] $N\rightarrow\Delta(1232)$ turns out then to be a highly rewarding testing ground for a study of interplay between interactions in the final state and "direct" excitations in resonant scattering. The results of the present investigation could therefore pertain not only to the specific domain of photo- and electroproduction of pions in the energy region of $\Delta(1232)$, but to photo- and electroexcitation of hadronic resonances in general. Clearly, not in all such cases will the effective calculations be as easy and meaningful as they turned out to be here.

II. NOTATION AND KINEMATICS

I mention here only a few basic points necessary to provide a link with earlier papers,[13,14] to which the reader should refer for more details.

Invariant functions appearing in the spin and isospin decomposition of the S-matrix elements of photoproduction are denoted $\mathscr{H}_i^{(\alpha)}(s,t,u)$ where the superscript and subscript refer to isospin and spin indices, respectively. In the c.m. system of the direct-photoproduction channel (s channel) s is the total energy squared, t is the squared four-momentum transfer between bosons, and $s+t+u=2m^2+\mu^2$ where m and μ denote the mass of the nucleon and the pion, respectively.

In the process of deducing the fixed-angle dispersion relations we introduce the new variables

$$\nu=\frac{1}{4s}[s^2-2s(m^2+\mu^2)+(m^2-\mu^2)^2] \; , \qquad (2.1)$$

$$c=\frac{s^2+s(2t-2m^2-\mu^2)+m^2(m^2-\mu^2)}{(s-m^2)[s^2-2s(m^2+\mu^2)+(m^2-\mu^2)^2]^{1/2}} \; , \qquad (2.2)$$

which mean, in the c.m. system of the s channel, the final three-momentum squared and the cosine of the production angle, respectively. The invariant amplitudes $\mathscr{H}_i^{(\alpha)}(s,t,u)$ expressed in terms of the new variables ν, c will be denoted $H_{ij}^{(\alpha)}(\nu,c)$ where the second subscript $j=0,1,2,3$ labels the four Riemann sheets of the ν,c domain resulting from the transformation from (s,t,u) to (ν,c). The ordering of the sheets has been explained earlier.[13]

The symmetry properties

$$H_{i0}^{(\alpha)}(\nu,c)=H_{i3}^{(\alpha)}(\nu,-c) \; , \qquad (2.3)$$

$$H_{i1}^{(\alpha)}(\nu,c)=H_{i2}^{(\alpha)}(\nu,-c) \qquad (2.4)$$

combined with the property of independence of $H_{ij}^{(\alpha)}(0,c)$ on c enable us to write, at $c=$const, the integral representations for $H_{ij}^{(\alpha)}(\nu,c)$ with one subtraction at $\nu=0$ which does not introduce arbitrary constants. These expressions are more convenient than those used previously[13,14] because, firstly, the integrals converge even more rapidly than before, and secondly, owing to the explicit use of properties (2.3) and (2.4), we avoid some spurious singularities inherent in earlier calculations and requiring particular treatment especially in numerical work.

Calculations and approximations perfectly analogous to those described in Ref. 13 lead to the following integral relations:

$$H_{i0}^{(\alpha)}(\nu,c)=b_{i0}^{(\alpha)}(\nu,c)+\frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu'-\nu}\sum_{j=0}^{3}\overline{\Omega}_j(\nu,\nu')\mathrm{Im}H_{ij}^{(\alpha)}(\nu',c), \quad i=1,2,3,4 \qquad (2.5)$$

where $b_{i0}^{(\alpha)}(\nu,c)$ denotes the pole terms which comprise the minimal gauge-invariant set corresponding to N and π exchange, and

$$\overline{\Omega}_j(\nu,\nu')=\frac{1}{4}\left[1\pm\frac{\nu\kappa(\nu)}{\nu'\kappa(\nu')}\pm\frac{\nu\lambda(\nu)}{\nu'\lambda(\nu')}+\frac{\nu\chi(\nu)}{\nu'\chi(\nu')}\right], \quad j=\begin{cases}0\\1\end{cases} \qquad (2.6a)$$

$$\overline{\Omega}_j(\nu,\nu')=\frac{1}{4}\left[1\pm\frac{\nu\kappa(\nu)}{\nu'\kappa(\nu')}\mp\frac{\nu\lambda(\nu)}{\nu'\lambda(\nu')}-\frac{\nu\chi(\nu)}{\nu'\chi(\nu')}\right], \quad j=\begin{cases}2\\3\end{cases} \qquad (2.6b)$$

with $\kappa(\nu)=[(\nu+m^2)/\nu]^{1/2}$, $\chi(\nu)=[(\nu+\mu^2)/\nu]^{1/2}$, $\lambda(\nu)=\kappa(\nu)\chi(\nu)$. The approximations mentioned above consist in neglecting all contributions to the dispersion integral from the regions other than the positive ν semiaxis on the four sheets. This formal procedure stems from a dynamical assumption that in the integrands we may confine ourselves to contributions of the direct- and crossed-photoproduction channels (s and u channels, respectively) only. A more extensive discussion of this point can again be found in earlier papers.[13,14]

A remark regarding the crossing properties of $H_{ij}^{(\alpha)}(\nu,c)$ is in order. Since the integral representation (2.5) holds for $c=$const (instead of $t=$const), an additional minus sign in the usual crossing condition[13,21] is necessary in the range where $(\partial s/\partial u)_{c=\text{const}}>0$ for $j=1,2$.

By performing known multipole projections,[13,22] we finally obtain the following system of equations:

$$\mathcal{M}_i^{(\alpha)}(\nu)=B_i^{(\alpha)}(\nu)+\frac{1}{\pi}\int_0^\infty \frac{d\nu'}{\nu'-\nu}\,\mathrm{Im}\mathcal{M}_i^{(\alpha)}(\nu')+\sum_{\beta,j}\int_0^\infty d\nu'\mathcal{K}_{ij}^{(\alpha\beta)}(\nu',\nu)\,\mathrm{Im}\mathcal{M}_j^{(\beta)}(\nu') \quad , \tag{2.7}$$

where, according to the convention adopted in the earlier papers, $\mathcal{M}_i^{(\alpha)}(\nu)$ denotes an arbitary multipole amplitude multiplied by a suitable threshold factor to be specified later. Functions $B_i^{(\alpha)}(\nu)$ denote the pole contributions and $\mathcal{K}_{ij}^{(\alpha\beta)}(\nu',\nu)$ are nonsingular functions which couple the multipole amplitude $\mathcal{M}_i^{(\alpha)}(\nu)$ to all the others. In what follows the phase of $\mathcal{M}_i^{(\alpha)}(\nu)$ will be denoted $\varphi_i^{(\alpha)}(\nu)$. One should remember an otherwise known fact,[23] namely that the dependence of the diagonal coupling functions $\mathcal{K}_{ii}^{(\alpha\alpha)}(\nu',\nu)$ on threshold factors is not trivially multiplicative, if the form of the singular integral in Eq. (2.7) is to be kept invariant under a change of threshold factors. This property should be remembered especially when the strength of the coupling of a multipole $\mathcal{M}_i^{(\alpha)}(\nu)$ to itself is concerned.

III. THE INTEGRAL EQUATION AND DYNAMICAL ASSUMPTIONS

In order to transform dispersion relations of the form (2.7) into a system of manageable integral equations, to be ultimately solved for multipole amplitudes, some essential assumptions of dynamical character are unavoidable. The most crucial among them is probably that which stems from the necessity of extrapolating the multipole phase somehow from the region where the elastic unitarity condition $\varphi_i^{(\alpha)}(\nu)=\delta(\nu)$ holds [where $\delta(\nu)$ denotes the proper πN phase shift[15]] to the region beyond the inelastic thresholds. This allows for transforming[24,25] dispersion relations (2.7) into a system of Fredholm-type equations.[26] Such a way of handling the dispersion relations for photoproduction amplitudes has been discussed by many authors[3,21,22,27–31] and has often been used in practical applications despite strong criticism. The reason is that alternative proposals, trying to overcome the traps inherent in the above-mentioned treatment, are usually based on assumptions which can hardly be judged more sound. Fortunately enough, the particular case of the multipole transitions leading to the p_{33} pion-nucleon final state is far easier to handle than any other owing to the remarkably elastic character of the δ_{33} phase shift over that part of the integration interval giving the most important contributions. The identification of the multipole phase with δ_{33} in this region is therefore well motivated. On the other hand, however, the ascent of δ_{33} with energy to the value of π and the presumed identical behavior of the multipole phase gives rise to a known polynomial ambiguity[18,27,29] in the final solution of the respective integral equations. The ambiguity which needs special attention is believed here to be connected with the "elementary" or "CDD-type"[32] character of the $\Delta(1232)$ hadronic state. This will be discussed later. Being concerned with the truncated subsystem of equations for the mutually coupled multipole amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ we leave apart any discussion regarding all the other multipoles.

The assumptions relative to the multipole phases as adopted in the present approach can be summarized as follows. The elastic-unitarity condition $\varphi_i^{(3)}(\nu)=\delta_{33}(\nu)$ $(i=2,3)$ has been applied wherever possible, i.e., up to $\sqrt{s}\cong 1500$ MeV.[33] Above this limit, i.e., where inelasticity becomes non-negligible, $\varphi_2^{(3)}(\nu)=\varphi_3^{(3)}(\nu)$ is smoothly extrapolated in such a way that $\exp[i\varphi_2^{(3)}(\nu)]\sin\varphi_2^{(3)}(\nu)$ would represent the p_{33} amplitude of πN scattering if inelasticity were ignored. Consequently, $\varphi_2^{(3)}(\nu)=\varphi_3^{(3)}(\nu)=\pi$ at a finite $\nu=\nu_c$, corresponding to $\sqrt{s}\cong 1780$ MeV (Ref. 10) counter to many earlier extrapolations which used to assume an asymptotic rise of the multipole phase to π in the limit $s\rightarrow\infty$. A more detailed discussion of the foregoing extrapolation and some related technical particulars can be found in earlier papers.[8,10]

Owing to the kinematic suppression of the kernels $\mathcal{K}_{ij}^{(33)}(\nu',\nu)$ $(i,j=2,3)$ and of the threshold factors at large ν and ν', we can safely neglect all contributions to the integrands in Eq. (2.7) from the range of "large ν" which extends, in the following, from $\nu=\nu_c$ on. This fair assumption, being essentially technical, has also the advantage of dispensing us from approximating the multipole phase in the region where any extrapolation would be questionable. The present calculation does not differ in this respect from many others carried out in the past. The relation $\mathrm{Im}\mathcal{M}_i^{(3)}(\nu)/\mathrm{Re}\mathcal{M}_i^{(3)}(\nu)=\tan\varphi_i^{(3)}(\nu)$ $(i=2,3)$ expressing the unitarity of the calculated multipole amplitude is therefore satisfied throughout the integration range.

We are then led to the system of integral equations which can be cast in the following form:

$$\frac{\mathrm{Im}\mathscr{M}_k^{(\alpha)}(\nu)}{\sin\varphi_k^{(\alpha)}(\nu)}=b_k^{(\alpha)}(\nu)+\frac{1}{\pi}\sum_{\beta,q}\int_0^{\nu_c}\mathscr{R}_{kq}^{(\alpha\beta)}(\nu',\nu)\frac{\mathrm{Im}\mathscr{M}_q^{(\beta)}(\nu')}{\sin\varphi_q^{(\beta)}(\nu')}d\nu'+c_k^{(\alpha)}\exp[\rho_k^{(\alpha)}(\nu)],\quad \alpha=\beta=3,\quad k,q=2,3\ , \tag{3.1}$$

where

$$b_k^{(\alpha)}(\nu)=B_k^{(\alpha)}(\nu)\cos\varphi_k^{(\alpha)}(\nu)+\frac{\exp[\rho_k^{(\alpha)}(\nu)]}{\pi}\mathrm{P}\int_0^{\nu_c}\frac{d\nu'}{\nu'-\nu}B_k^{(\alpha)}(\nu')\exp[-\rho_k^{(\alpha)}(\nu')]\sin\varphi_k^{(\alpha)}(\nu')\ , \tag{3.2}$$

$$\mathscr{R}_{kq}^{(\alpha\beta)}(\nu',\nu)=\mathscr{K}_{kq}^{(\alpha\beta)}(\nu',\nu)\sin\varphi_q^{(\beta)}(\nu')\cos\varphi_k^{(\alpha)}(\nu)+\frac{\exp[\rho_k^{(\alpha)}(\nu)]}{\pi}\mathrm{P}\int_0^{\nu_c}\frac{d\nu''}{\nu''-\nu}\mathscr{K}_{kq}^{(\alpha\beta)}(\nu',\nu'')\exp[-\rho_k^{(\alpha)}(\nu'')]\sin\varphi_k^{(\alpha)}(\nu'')\sin\varphi_q^{(\beta)}(\nu')\ , \tag{3.3}$$

$$\rho_k^{(\alpha)}(\nu)=\frac{1}{\pi}\mathrm{P}\int_0^{\nu_c}\frac{\varphi_k^{(\alpha)}(\nu')}{\nu'-\nu}d\nu'-\ln(\nu_c-\nu)\ . \tag{3.4}$$

Multipole amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ are connected with $\mathscr{M}_i^{(3)}(\nu)$ $(i=2,3)$ in the following way:

$$\begin{bmatrix} M_{1+}^{(3/2)}(\nu) \\ E_{1+}^{(3/2)}(\nu)\end{bmatrix}=\frac{\nu^{1/2}kE(\nu)(s-m^2)[E(\nu)+m]^{1/2}}{8\pi W(2W)^{1/2}}\begin{bmatrix}\mathscr{M}_2^{(3)}(\nu)\\ \mathscr{M}_3^{(3)}(\nu)\end{bmatrix}\ , \tag{3.5}$$

where $k=(s-m^2)/(2W)$, $E(\nu)=(s+m^2-\mu^2)/(2W)$, $W=\sqrt{s}$. The kinematic threshold factor which appears in (3.5) differs from those used in similar calculations.[29,31,34] Its form has been chosen so as to ensure a better asymptotic decrease of the kernels $\mathscr{R}_{kq}^{(33)}(\nu',\nu)$ $(k,q=2,3)$ as $\nu,\nu'\to\infty$. This is by no means surprising since the kernels in question result from fixed-angle dispersion relations and therefore may differ from those resulting from the fixed-t dispersion approach more commonly used in the past. For $\nu\to 0$, the kinematic factor appearing in Eq. (3.5) has of course the usual threshold behavior. From the discussion of the multipole phases earlier in this section it follows that $\varphi_2^{(3)}(\nu)=\varphi_3^{(3)}(\nu)$ and, consequently $\rho_2^{(3)}(\nu)=\rho_3^{(3)}(\nu)$. The arbitrary constants denoted $c_j^{(3)}$ $(j=2,3)$ represent the polynomial ambiguities (of order zero) mentioned earlier. Their appearance in Eq. (3.1) is a direct consequence of the fact that the variation of the p_{33} scattering phase [identical with the two multipole phases $\varphi_i^{(3)}(\nu)$] over the integration interval is[25] $\delta_{33}(\nu_c)-\delta_{33}(0)=\pi$. In the present and the related calculations[9,10,20] the values of $c_j^{(3)}$ $(j=2,3)$ have been fixed by enforcing a zero (actually a CDD zero[16]) at $\nu=\nu_c$ in both calculated amplitudes $\mathscr{M}_j^{(3)}(\nu)$ $(j=2,3)$. This leads to

$$c_j^{(3)}=-\left[B_j^{(3)}(\nu_c)\exp[-\rho_j^{(3)}(\nu_c)]\cos\varphi_j^{(3)}(\nu_c)+\frac{1}{\pi}\int_0^{\nu_c}\frac{d\nu'}{\nu'-\nu_c}B_j^{(3)}(\nu')\exp[-\rho_j^{(3)}(\nu')]\sin\varphi_j^{(3)}(\nu')\right.$$
$$+\frac{1}{\pi}\sum_{k=2,3}\int_0^{\nu_c}d\nu'\left\{\mathscr{K}_{jk}^{(33)}(\nu',\nu_c)\exp[-\rho_j^{(3)}(\nu_c)]\cos\varphi_j^{(3)}(\nu_c)+\frac{1}{\pi}\int_0^{\nu_c}\frac{d\nu''}{\nu''-\nu}\mathscr{K}_{jk}^{(33)}(\nu',\nu'')\right.$$
$$\left.\left.\times\exp[-\rho_j^{(3)}(\nu'')]\sin\varphi_j^{(3)}(\nu'')\right\}\mathrm{Im}\mathscr{M}_k^{(3)}(\nu')\right]\ . \tag{3.6}$$

Now it is a matter of simple algebra[27] to show that the foregoing choice of $c_j^{(3)}$ $(j=2,3)$ is *formally* equivalent to skipping over the terms containing $c_j^{(3)}$ $(j=2,3)$ everywhere in Eqs. (3.1)–(3.3) with the simultaneous substitution

$$\exp[\rho_j^{(3)}(\nu)]\to(\nu_c-\nu)\exp[\rho_j^{(3)}(\nu)]\ . \tag{3.7}$$

This means of course that we introduce the enhancement factors due to rescattering (or Jost functions) $D_j^{(3)}(\nu)=(\nu_c-\nu)^{-1}\exp[-\rho_j^{(3)}(\nu)-i\varphi_j^{(3)}(\nu)]$ $(j=2,3)$ with CDD poles at ν_c explicitly built in. Although the mere appearance of these poles formally follows from the behavior of the multipole phase, their location is an arbitrary choice so far, as emphasized repeatedly in earlier discussions.[9,10,20] Yet, the location of the poles adopted in the present approach seems to acquire a very plausible motivation if only perfect elasticity of the p_{33} pion-nucleon scattering amplitude (responsible for final-state interactions) and, consequently, elastic unitarity of the multipoles $\mathscr{M}_j^{(3)}(\nu)$ $(j=2,3)$ are enforced throughout the integration range, just as it is done in the present calculation. As the enhancement factors, by definition, are supposed to carry all information on rescattering in the final state of a given process, it seems natural in the present case to identify $D_j^{(3)}(\nu)$ $(j=2,3)$ with the analogous enhancement factor appearing in the N/D representation of the πN scattering amplitude in the p_{33} state. But $N(\nu)/D(\nu)=\exp[i\delta_{33}(\nu)] \times \sin\delta_{33}(\nu)/\nu^{1/2}$ and, unless the pole at $\nu=\nu_c$ coincides with that of $N(\nu)$ (which would be a pathology, however), the condition $\sin\delta_{33}(\nu_c)=0$ or $\delta_{33}(\nu_c)=n\pi$ for the location of this pole follows immediately. This observation justifies the *ad hoc* condition enforced on ν_c in the foregoing provided the assumptions regarding the multipole phases $\varphi_j^{(3)}(\nu)$ $(j=2,3)$ are kept in mind. Note that $\nu_c \rightarrow \infty$ will be a particular case of this condition if the generalized Levinson theorem[32] is taken for granted, whereas another possibility, namely ν_c being an unphysical point, does not need discussion here. The possibility of fixing the constants in the amplitudes of photoproduction evolves then into the possibility of identifying the enhancement factors in these amplitudes with the respective factors in the amplitudes of πN scattering, if only the influence of inelasticity can be neglected. Such identification is hardly anything new in calculations regarding photoproduction of pions on nucleons in the p_{33} state. Its implicit use should be credited for the success of the widely known formula proposed by Chew *et al.*[21] For if in the simplest approximation accommodating rescattering the magnetic-dipole amplitude can be expressed as $\mathscr{M}_2^{(3)}(\nu)=B_2^{(3)}(\nu)/D_2^{(3)}(\nu)$ and likewise for the p_{33} pion-nucleon scattering amplitude, then by identifying the two enhancement factors one is immediately led to the conclusion that the ratio of the two amplitudes in question is equal to the ratio of their one-particle terms, just as conjectured by Chew *et al.*[21] Then $\mathscr{M}_2^{(3)}(\nu)$ can be calculated without knowing $D_2^{(3)}(\nu)$ explicitly. In particular, no assumption regarding the zeros of the multipole amplitude needs to be anticipated as all such information should be contained in the p_{33} scattering amplitude used in the calculation. Another example of an approach of this sort has been due to Finkler[30] who has devised a fitting procedure to construct explicitly a "correct" D function of pion-nucleon scattering in the p_{33} state in order to use it for the purpose of calculating $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$. Since no attempt has been made in the past to interpret dynamically Finkler's device, it has remained just another way of grappling with the problem of resolving the polynomial ambiguity.[27,29] A possible dynamical picture which I advocate in this connection will be elaborated thoroughly in Sec. V. For the moment it is worth emphasizing that the idea of identifying the enhancement factors of photoproduction of pions on nucleons and of pion-nucleon scattering in the p_{33} state, their CDD singularities comprised, receives phenomenological backing going well beyond the static formula of Chew *et al.*[21] Such a conclusion comes first from the conviction that inelastic channels in the two processes are irrelevant over the studied energy range, and second from the striking similarity of the Argand diagrams of the multipole amplitude $M_{1+}^{(3/2)}$ (Refs. 4 and 6) and the p_{33} amplitude of πN scattering.[33] The situation in the case of $E_{1+}^{(3/2)}$ is less clear and will be discussed in Sec. V.

I have alluded earlier in this section to a possible connection between the appearance of the zero now in question and the existence of an unstable, "elementary" (for all the purposes of this discussion) state $\Delta(1232)$. The position of the zero $(\nu=\nu_c)$ on the energy axis will then be related to that of the unstable state $(\nu=\nu_r)$ in such a way that the first will be determined by the condition $[D_j^{(3)}(\nu_c)]^{-1}=0$ and the second by $\mathrm{Re}D_j^{(3)}(\nu_r)=0$. The dynamical link between ν_r and ν_c will therefore be implicit in the functional dependence of the phase shift (or, equivalently, of the πN scattering amplitude) on energy which is given *a priori* in our photoproduction calculation, but should in principle emerge somehow from a complete study of πN scattering. Note also that the just-described way of determining ν_r and ν_c is more general[35] than that which starts from singularities on an unphysical sheet of the complex energy plane, and that ν_r is not necessarily close to ν_c, but their relative distance depends on the details of the dynamics. For these reasons there is no mention of $\Delta(1232)$ as a "second-sheet pole" here, and no surprise that ν_r and ν_c are quite distant from each other.

IV. PROPERTIES OF THE SOLUTIONS OF THE INTEGRAL EQUATIONS

The purpose of the calculations, as reported here, has been to obtain possibly adequate *no-fit* predictions regarding the values of the two multipole amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ in the energy region of $\Delta(1232)$. Actually, a more ambitious goal has been scored, namely a very good agreement of the calculated values with the data over the whole energy interval covered by the presently available multipole analysis[4,5,11,12] ranging from threshold to ~ 800 MeV/c photon laboratory momentum. This makes a considerable extension of the results published previously[8-10] which covered only the interval below ~ 450 MeV/c photon momentum in the laboratory.

To begin with, a few remarks regarding the procedure of solving the system (3.1) numerically are in order. The use of Eq. (3.6) to calculate the constants $c_j^{(3)}$ $(j=2,3)$ would be very impractical in computational work. Instead, to ensure the vanishing of the solution of (3.1) at ν_c the following device was applied. The system (3.1) was solved on an interval slightly truncated at its upper end. Then $\varphi_j^{(3)}(\nu) < \pi$ ($j=2,3$) throughout the interval and no ambiguity can appear in Eq. (3.1). Owing to the presence of a branching point which emerges now at the upper integration limit, the calculated $\mathscr{M}_j^{(3)}(\nu)$ ($j=2,3$) fall to zero there,[25] and automatically satisfy the enforced property. By extrapolating slightly the result (with respect to the upper integration limit), I recover the desired amplitudes. The actual values of $c_j^{(3)}$ ($j=2,3$) remain therefore unknown, but they are of no relevance to the present discussion. If needed, they could be computed from Eq. (3.6). The details of the just-mentioned extrapolation and other connected technical problems have been described in earlier papers.[8,10]

Some differences in the procedures of solving the system (3.1) for $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ motivate the following separate discussions of the two cases.

A. The multipole $M_{1+}^{(3/2)}$

The calculation has been carried out in three steps. As they have been already discussed in detail before,[8,10] only the essentials need outlining here.

The first step consisted in solving the so-called characteristic equation for $M_{1+}^{(3/2)}$, i.e., the equation obtained from Eq. (3.1) by skipping over all terms containing $\mathscr{K}_{ij}^{(33)}(\nu',\nu)$ ($i,j=2,3$). The dynamical meaning of this simplification is of course that pole terms are taken as the only "left-hand" singularities. This contribution to the "driving force" is generally believed to dominate in the present case.[28,31,34] The agreement of the calculated $M_{1+}^{(3/2)}$ with the data[8] is interpreted as a hint that the dynamical input to Eq. (3.2) comprising the values of $\varphi_2^{(3)}(\nu)$ as described in Sec. III and the presumed presence of a CDD zero at $\nu=\nu_c$ is basically feasible. Further speculation regarding a deeper rooting of this input would be, however, rash at this stage of investigation.

In the second step only the self-coupling of $M_{1+}^{(3/2)}$ represented by $\mathscr{K}_{22}^{(33)}(\nu',\nu)$ has been included, leaving us with an isolated integral equation to be solved for $M_{1+}^{(3/2)}$. Past experience with analogous kernels derived from fixed-t dispersion relations has taught us that the bearing of self-coupling terms should be small in the case of $M_{1+}^{(3/2)}$. The same conclusion has been found true also with regard to kernels derived from fixed-angle dispersion relations used here. The effect of self-coupling can be summarized as leading to a general decrease of $|M_{1+}^{(3/2)}|$ by a few per cent. Although quantitatively less satisfactory than the preceding one, this result should not be discouraging. Suitable adjustments in the procedure of extrapolating $\varphi_2^{(3)}(\nu)$ on the high-energy tail

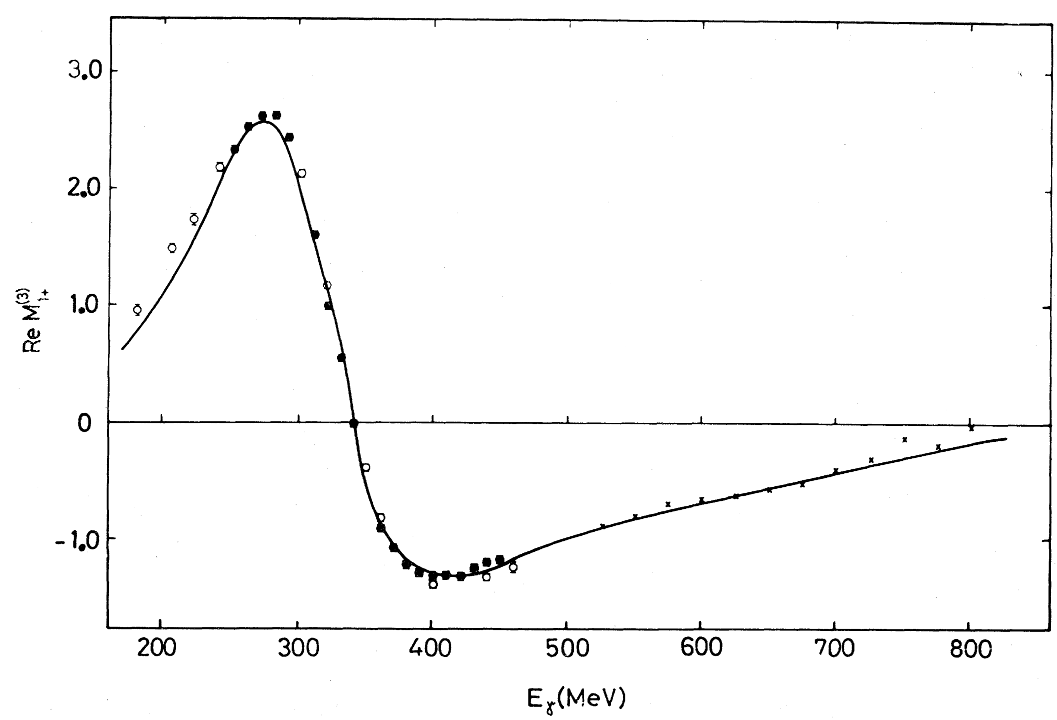

FIG. 1. Re$M_{1+}^{(3/2)}$ in units $10^{-2}\hbar/(\mu c)$ vs photon energy in the laboratory. The curve results from the present calculation, the points come from the multipole fits of Berends and Donnachie (Ref. 4) (●), Pfeil and Schwela (Ref. 12) (○), and Berends and Donnachie (Ref. 11) (×).

($\sqrt{s} \gtrsim 1500$ MeV), possibly including also a change in the value of ν_c, could certainly restore agreement with data as achieved in the first step, without altering the general spirit of the approach. Again I am reluctant to speculate at this moment on the interpretation of the dynamical input to Eq. (3.1).

The third step then consisted in solving the complete system (3.1), including all couplings represented by $\mathscr{K}_{ij}^{(33)}(\nu',\nu)$ $(i,j=2,3)$. As in most calculations carried out in the past,[28,31,34] the coupling of $M_{1+}^{(3/2)}$ to $E_{1+}^{(3/2)}$ described by $\mathscr{K}_{23}^{(33)}(\nu',\nu)$ was found negligible and the calculation has yielded the multipole $M_{1+}^{(3/2)}$ almost identical with that obtained in the preceding second step. The calculated values of $M_{1+}^{(3/2)}$ are shown in Fig. 1. This is a repetition of the results of Ref. 8 with supplements covering the range up to ~ 800 MeV/c photon laboratory momentum.

B. The multipole $E_{1+}^{(3/2)}$

In an earlier paper[9] only a brief account of the results regarding this multipole has been given, hence a more comprehensive presentation is due now. First of all, a calculation of the type described as the first step in Sec. IV A is expected to be misleading since the contribution from the crossed-cut integral is known to be large.[36] A trial solution of the characteristic equation in our case led indeed to completely wrong values of $E_{1+}^{(3/2)}$. Other conclusions which can be drawn from earlier papers flatly indicate that the coupling of $E_{1+}^{(3/2)}$ to $M_{1+}^{(3/2)}$ described by the kernel $\mathscr{K}_{32}^{(33)}(\nu',\nu)$ is important when equations analogous to (3.1) are solved for the multipole $E_{1+}^{(3/2)}$. There have been only differences in estimating the actual magnitude of the effect[28,34] which of course should give account of most of the $\Delta(1232)$ exchange in the crossed channel. In order to check this point here, the isolated integral equation analogous to that of the second step of Sec. IV A has been solved for $E_{1+}^{(3/2)}$. The outcome of the calculation is shown in Fig. 2. Although the agreement with data cannot be rated high, the order of magnitude and the sign of Re$E_{1+}^{(3/2)}$ below resonance are generally correct, and only the presence of a second zero at ~ 450 MeV contrasts with the desired double zero of Re$E_{1+}^{(3/2)}$ at the resonance $\Delta(1232)$. When the complete system (3.1) is now solved for $E_{1+}^{(3/2)}$ the role of the coupling kernel $\mathscr{K}_{32}^{(33)}(\nu',\nu)$ in the fine-tuning of the ultimate result becomes evident. Now the two previously mentioned zeros of Re$E_{1+}^{(3/2)}$ merge to one within the accuracy of the numerical work, whereas the values of the amplitude move almost perfectly into the experimental error band. This has been partly discussed in Sec. IV A as step three. Figure 3 displays the calculated values of the amplitude published earlier[9] together with these now added to cover a larger energy interval. Visible

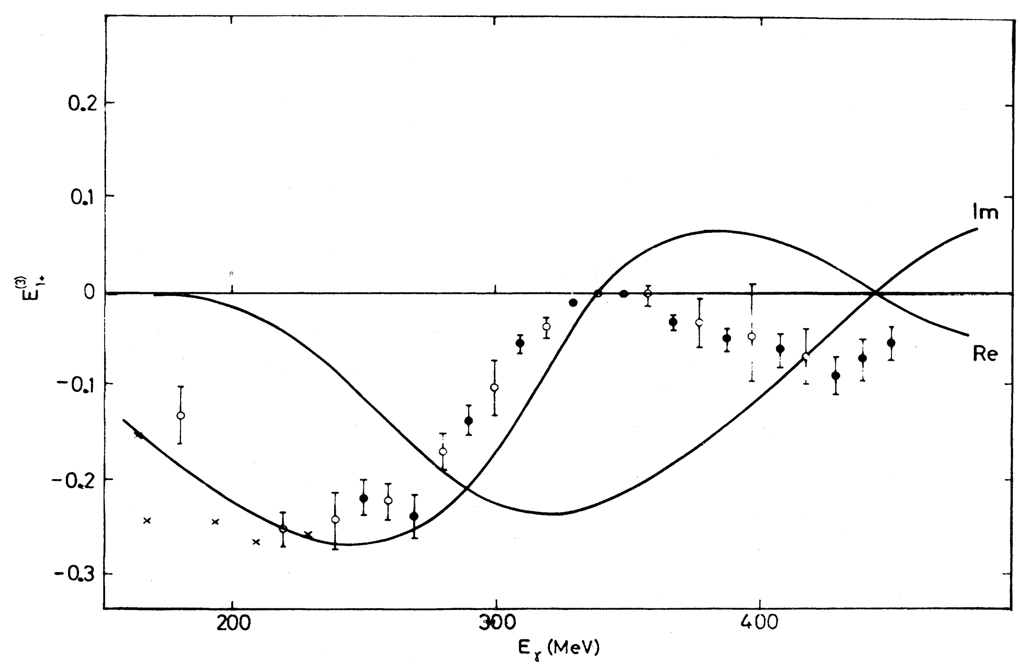

FIG. 2. The multipole amplitude $E_{1+}^{(3/2)}$ in units $10^{-2}\hbar/(\mu c)$ vs photon energy in the laboratory. The curves result from the "second-step" calculation, as described in the text. The points come from the multipole fits of Noelle (Ref. 37) (○), Berends and Weaver (Ref. 38) (×), and Berends and Donnachie (Ref. 4) (●).

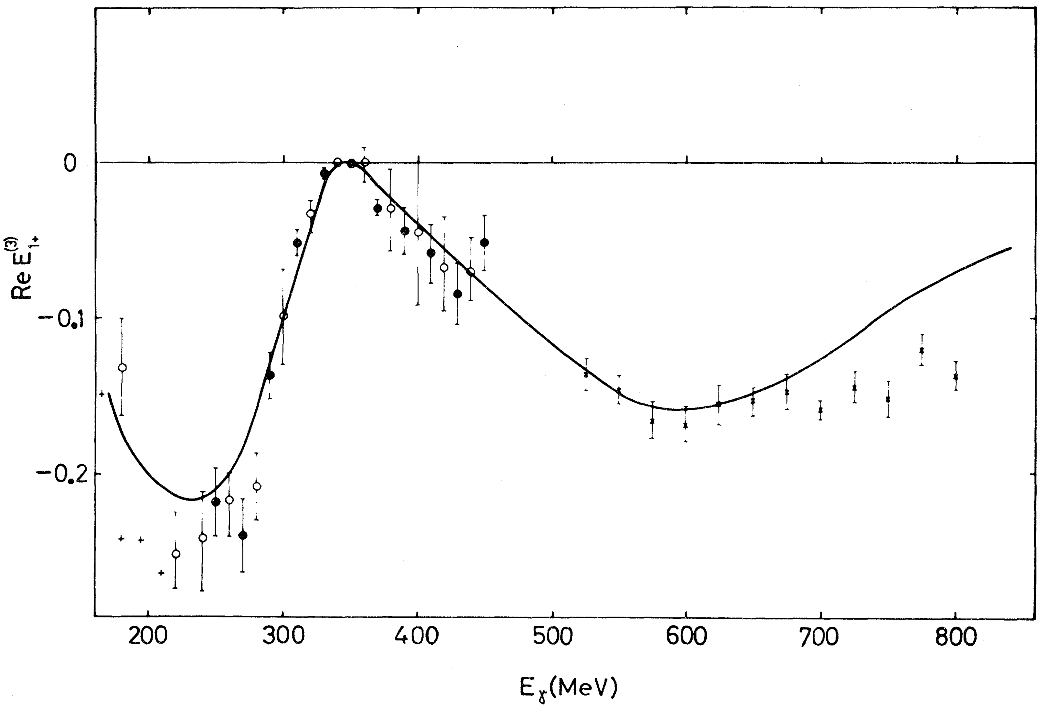

FIG. 3. $\mathrm{Re}E_{1+}^{(3/2)}$ in units $10^{-2}\hbar/(\mu c)$ vs photon energy in the laboratory. The curve results from the present calculation, the points come from the multipole fits of Berends and Weaver (Ref. 38) (+), Berends and Donnachie (Ref. 4) (●), Noelle (Ref. 37) (○), and Berends and Donnachie (Ref. 11) (×).

discrepancies with the data in the threshold region can be attributed to less precise measurements as well as to the omission of couplings with s-wave multipoles $E_{0+}^{(3/2)}$ and $E_{0+}^{(1/2)}$ in (3.1). Estimates of the relevance of these couplings made in the past vary considerably, and there has been no attempt to grapple with this problem here. Another discrepancy, growing with energy, shows up in the range $E_\gamma \gtrsim 700$ MeV. This might well be a consequence of condition (3.6) enforced on $E_{1+}^{(3/2)}(\nu_c)$ which, as we have seen, is correct only approximately.

It became customary to plot the ratio $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ against energy. This is shown in Fig. 4 together with a few curves of a much earlier date. The energy range in Fig. 4 has been confined to ~450 MeV to make the comparison more visible. There is little to comment on, as none of the older results is acceptable on the grounds of recent phenomenological fits. Regarding the differences between the fitted values of $E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and those of the present calculation around $E_\gamma = 450$ MeV, it should be emphasized that the calculated curve better represents the overall energy dependence in this energy region, as the results shown in Figs. 1 and 3 demonstrate.

The outcome of this part B of the calculation, as shown in Fig. 3, is by no means trivial if we reiterate that no freedom was left to make adjustments, once the "dynamical input" as defined earlier in this text was inserted into (3.1). Moreover, the just-described changes in the solution of the system (3.1) caused by switching the successive couplings on, indicate that casual coincidence between the calculated curve and the data is highly improbable. Incidentally, there seem to be no published calculations able to reproduce $E_{1+}^{(3/2)}$ with comparable accuracy, if the solution proposed somewhat *ad hoc* by Chew *et al.*[21] and those related to it are left apart.

The results discussed in Secs. IV A and IV B and the earlier successful prediction of the $N \to \Delta(1232)$ electromagnetic transition form factor,[20] make sufficient ground for drawing more general conclusions regarding the dynamical basis of the calculations of the electromagnetic excitation $N \to \Delta(1232)$ as reported here. This will be the subject of the next section.

V. DISCUSSION

As I mentioned repeatedly in the foregoing, all attempts to calculate the amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$

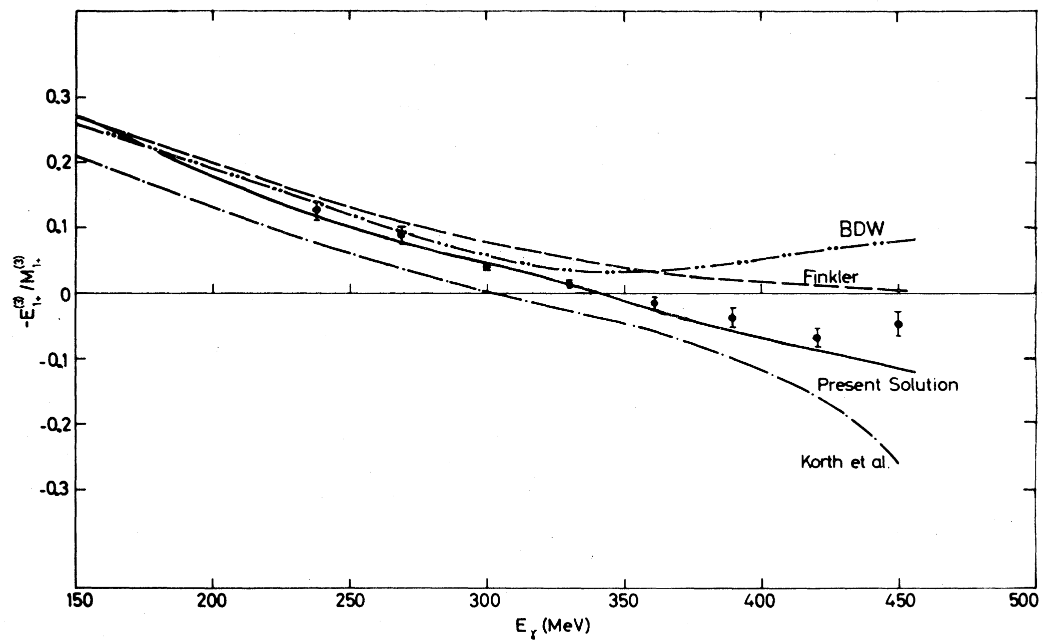

FIG. 4. The ratio $-E_{1+}^{(3/2)}/M_{+}^{(3/2)}$ vs photon energy in the laboratory. The curves result from the present calculation and those of Berends *et al.* (Ref. 34) (BDW), Finkler (Ref. 30), and Korth *et al.* (Ref. 40). The points come from the multipole fit of Berends and Donnachie. (Ref. 4).

with the aid of integral equations of the type (3.1) have been continuously plagued with a big problem, namely, how the arbitrary constants $c_j^{(3)}$, $(j=2,3)$ (or other equivalent ambiguities) should actually be fixed. A review of various devices proposed in this connection in the past can be found elsewhere,[3] while the way followed in the present study has been exposed in Sec. III. In an attempt to give a dynamical rationale to the assumptions and results discussed in the preceding sections, I shall cling to the idea that in analyzing the dynamics of $N \rightarrow \Delta(1232)$ electromagnetic excitation by means of final-state-interaction theory, it is essential to exploit the consequences of the view that $\Delta(1232)$ is, for all the purposes of the present calculation, an "elementary" or "CDD-type" object.[32] This standpoint strongly relies on the observation that the phase shift of πN scattering in the p_{33} state [by definition equal to the multipole phases $\varphi_j^{(3)}(\nu)$ $(j=2,3)$] rises with energy to $\pi/2$ and then steadily to π. Also, the notion of $\Delta(1232)$ as a bound state of quarks, whose photoexcitation is well described by the static properties of the system, offers very useful hints as to the possible magnitude of $N \rightarrow \Delta(1232)$ excitations of magnetic-dipole and electric-quadrupole type. This will be elaborated below. The foregoing ideas combined with the remarkable elasticity of πN scattering in the p_{33} state over a very wide energy interval are claimed now to form the basis for an appealing explanation of the results reported earlier in this study. Manifest difference between the interpretation as given below and those outlined in the earlier papers[9,10,20] should not surprise as it merely reflects an evolution in the views of the present author. The advocated interpretation is believed to be more coherent and sound than many others advanced in the past in this connection and is also convergent with proposals contained in some apparently unrelated studies.[17,19,39]

The first important consequence of the behavior of the multipole phase is that the desired resonant character of the amplitude $M_{1+}^{(3/2)}$ can not build up from final-state πN interactions alone in spite of the explicitly resonant character of the phase. Indeed, linear integral equations of the type (3.1) generally allow for solutions *falling to zero* near the resonance of the rescattering process provided $\varphi_i^{(\alpha)}(\nu-\nu_r)-\pi/2$ is an odd (or approximately odd) function of $\nu-\nu_r$ rising to $\pi/2$ and the inhomogeneous term slowly varies in the ν_r region, as pointed out, e.g., by Resnick.[18] These are the solutions with $c_i^{(\alpha)}$ set to zero or, equivalently, with $\nu_c=\nu_r$ (the so-called "fundamental" or "particular" solutions). Now, as the true $\varphi_2^{(3)}(\nu)$ and $B_2^{(3)}(\nu)$ (or the respective inhomogeneity with crossing terms included) do indeed vary approximately in the just-described fashion, it is no surprise then that also the "particular solution" to the equation for $M_{1+}^{(3/2)}$ vanishes near ν_r as observed, e.g., by Korth *et al.*[40] or by Schwela *et al.*[27] who did not refer apparently to the general case, however. The relevant point now is that by picking out such a "particular" solution as the ultimate one we enforce an arbitrary condition on ν_c (e.g., $\nu_c = \nu_r$ as in the foregoing example). Equation (3.6) for $j=2$ and with the left-hand side set to zero would be the form of such a condition imposed on ν_c and expressed in the language of the present study. This arbitrariness may bring a calculated production amplitude with a completely wrong shape, and the example[27] of $M_{1+}^{(3/2)}$ shows it explicitly. In order then to obtain the amplitudes $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ with correct shape and magnitude the values of $c_j^{(3)}$ $(j=2,3)$ have to be chosen suitably. In other words, we have to fix the position of the pole in the enhancement factor of the studied *production* amplitude [for example, a (3.3) amplitude of photoproduction] with regard to possible constraints on the admissible values of ν_c which may follow from the properties of the enhancement factor of the connected *scattering* amplitude (for example, the p_{33} amplitude of πN scattering). This has certainly been the main, though implicit, purpose of the previously mentioned procedure of Finkler[30] and a similar one due to Engels and Schmidt[31] who a long time ago successfully used the positions of the poles in their enhancement factors as free parameters to adjust the values of $M_{1+}^{(3/2)}$. The present claim is, however, that perfectly elastic unitarity holding, there is no freedom left at all in tuning $c_j^{(3)}$ $(j=2,3)$ as a suitable choice means here compliance with the postulated identity of the respective enhancement factors, one of which (namely, that corresponding to scattering) is, though implicitly, known *a priori*. Any difference between them may signify a change affecting an important dynamical ingredient of the calculation as pointed out in Sec. III. Exactly, this line has been strictly followed in the present study, which allows qualifying the latter as a *no-fit* approach.

As the first and the second terms on the left-hand side of (3.1) taken together come out as basically nonresonant around ν_r, it is the term $c_2^{(3)}\exp[\rho_2^{(3)}(\nu)+i\varphi_2^{(3)}(\nu)]$ whose contribution should be crucial[20,40] in predicting $M_{1+}^{(3/2)}$ with the desired magnitude and pronounced resonant shape. The question regarding a possible dynamical meaning of this term then arises naturally. In trying to answer this question let it be recalled first that the observed rise of the multipole phase $\varphi_2^{(3)}(\nu)$ through $\pi/2$ at ν_r to π at ν_c is generally believed to reflect an "elementary character" of the transition $N\to\Delta$, as opposed to the excitation of a "dynamical resonance" due to sufficiently strong forces between the interacting particles in which case the phase above ν_r would fall to zero again.[32] Room appears then naturally for a "direct" $\gamma N\to\Delta$ transition in our dynamical scheme, and this is what the polynomial ambiguity is supposed to represent. A study of Eq. (3.1) cannot prove such a statement but considerations based on nonrelativistic potential scattering[41] and a simple model calculation[19] convincingly support the belief that the term $c_2^{(3)}\exp[\rho_2^{(3)}(\nu)+i\varphi_2^{(3)}(\nu)]$ whose resonant behavior is evident[27,42] represents in fact the excitation $N\to\Delta(1232)$ through a so-called "contact term" $\gamma N\Delta$. For if $\Delta(1232)$ is a three-quark state and it is not generated dynamically from a proton and a pion, then even if one knew the phase shift of πN scattering perfectly and one knew the "driving forces" in Eq. (3.1) perfectly, then one would still not know the $\gamma N\to\Delta$ transition because this is inherently a quark-model calculation and it is inaccessible to a final-state-theory approach. A simple, based on the SU(6)-symmetry-group, nonrelativistic-quark-model calculation[43] known to predict a large magnetic-dipole excitation $N\to\Delta(1232)$ agrees then, not surprisingly, with the view that the contribution representing a "direct" $\gamma N\to\Delta$ transition is necessary if the system (3.1) has to yield a resonance solution for $M_{1+}^{(3/2)}$. This is believed to be the implicit, or even unconscious, content of various devices proposed in many earlier papers to determine the arbitrary constant $c_2^{(3)}$. The present, no-fit, result shown in Fig. 1 apart from extending the predictions regarding $M_{1+}^{(3/2)}$ to the energy range $450 \lesssim E_\gamma \lesssim 800$ MeV is actually typical to most calculations of the kind mentioned here. Incidentally, the curve representing the Argand diagram of $M_{1+}^{(3/2)}$ as available from the present fits, if extrapolated to higher energies, clearly points at a (CDD?) zero.[4,6] It would be desirable to have this tendency checked against the data from multipole analyses at larger E_γ.

The conclusion of this part of the discussion could then be that photoproduction of pions on nucleons in the p_{33} state by magnetic-dipole excitation seems due, in the first instance, to direct $\gamma N\to\Delta(1232)$ transition whereas the principal "driving forces" constructed of the minimal set of one-particle terms and of crossed terms due to self-coupling, play a secondary role in the framework of final-state interactions satisfying perfectly elastic unitarity.

On the other hand, a quark-model calculation parallel to that quoted before strongly suggests a nonresonant behavior of $E_{1+}^{(3/2)}$ in the $\Delta(1232)$ energy range. It is because the electric-quadrupole transition $N\to\Delta(1232)$ is forbidden in this model.[44] Possible corrections to this simple result should not alter the expectation that the direct coupling term in the system (3.1) for $j=3$ should play a minor role.

"Unitarity corrections" present in the "particular" solution would then make the dominant dynamical mechanism responsible for the actual magnitude of the amplitude $E_{1+}^{(3/2)}$. This happens indeed to be the case, as described in Sec. IV. Note in this connection that crossed terms, in particular those accounting for the coupling of $E_{1+}^{(3/2)}$ to $M_{1+}^{(3/2)}$, cannot be neglected in building up the respective "driving force." The "direct" magnetic-dipole excitation $\gamma N\rightarrow\Delta(1232)$ in the crossed channel contributes in this manner substantially to the ultimate result. Owing now to the smallness of $c_3^{(3)}$ and to the general shape of the "particular" solution discussed earlier in this section, the amplitude $E_{1+}^{(3/2)}$ falls to zero at ν_r as shown in Fig. 3. As the actual position of the zero results from destructive interference of a weak resonance with background of comparable magnitude, clear distinction between this zero and that of the inverse enhancement function (which is constantly kept at $\nu=\nu_c$) should be made. Examples of similar cancellations are known from studies regarding other processes.[19,39]

A conclusion of this part of the discussion would then be that photoproduction of pions on nucleons in the p_{33} state by electric-quadrupole excitation seems essentially a final-state-interaction effect with "driving forces" constructed of the minimal set of pole terms and of crossed terms mainly due to coupling with the multipole amplitude $M_{1+}^{(3/2)}$, i.e., corresponding to the exchange of $\Delta(1232)$ in the crossed channel. The role of the mechanism of "direct" interaction $\gamma N\rightarrow\Delta(1232)$ through electric-quadrupole excitation is in this case negligible.

It might be instructive now to get, by means of a rather pedagogical example, an idea about the interplay between the nonresonant and resonant parts of the calculated multipole amplitudes. In a narrow-resonance approximation the solutions of Eq. (3.1) have the form[27,30]

$$\mathscr{M}_j^{(3)}(\nu)=\frac{1}{D_j^{(3)}(\nu)}\frac{\nu_r-\nu}{\nu_c-\nu}F_j^{(3)}(\nu)+\frac{1}{D_j^{(3)}(\nu)}\frac{\nu_c-\nu_r}{\nu_c-\nu}F_j^{(3)}(\nu_c),\quad j=2,3\ ,\tag{5.1}$$

where $F_j^{(3)}(\nu)$ denote the inhomogeneous terms in the integral equation, i.e., one-particle terms and possible contributions from the crossed channel. The first term on the right-hand side of Eq. (5.1) represents the nonresonant contribution as it explicitly falls to zero at ν_r. The second term is resonant and corresponds obviously to the last term on the right-hand side of Eq. (3.1). According to the adopted line of reasoning the value ν_c is fixed *a priori.* When $F_j^{(3)}(\nu_c)=0$ there is no "direct" excitation of the resonance and the reaction proceeds in a nonresonant fashion despite a resonant behavior of the final-state interaction. Now, if $F_j^{(3)}(\nu_c)$ is large and $F_j^{(3)}(\nu_r)=0$, as discussed earlier in this section, the nonresonant background is minimal in the resonance region, and the excitation is purely "elementary." In general one would expect rather intermediate situations, but it happens that the two multipoles discussed here are close to the extremal cases: $E_{1+}^{(3/2)}$ to the former, $M_{1+}^{(3/2)}$ to the latter. This explains why they do represent such a rewarding testing ground for studies of the two components of this and other similar solutions.

The appearance of a "direct" $\gamma N\Delta$ term in the present calculation brings the latter very closely to those studies based on the concept of "isobar exchange." Yet the difference between the two approaches seems to be substantial. Firstly, because the "direct" term appears in the present calculation as a consequence of the adopted dynamics of pion-nucleon scattering and not as an *ad hoc* component conceived to fit the data, and secondly, because the strength of the $\gamma N\Delta$ coupling can neither be chosen here arbitrarily nor even fitted. The reason is that the value of ν_c has been determined by strong interactions alone. In other words, ν_c depends on the strength of the couplings $NN\pi$ and $N\Delta\pi$. Consequently, $c_j^{(3)}$ $(j=2,3)$ as given by Eq. (3.6) depends only on the foregoing coupling constants and, through $B_j^{(3)}(\nu_c)$, on the charges and magnetic moments of N and π. Therefore, contrary to what it might seem to be at first sight, the $\gamma N\Delta$ "direct coupling" constant, appearing implicitly in this calculation, is not a new independent quantity. Actually, this conclusion should come as no surprise if one realizes that the $\gamma N\Delta$ "direct coupling" constant measures the electromagnetic $N\rightarrow\Delta(1232)$ transition form factor at the squared four-momentum transfer $q^2=0$ which, like all electromagnetic form factors, is believed to originate from strong interactions. The just-presented way of thinking leads indeed to remarkably good effects when followed in the case of the $N\rightarrow\Delta(1232)$ electromagnetic form factor.[20]

It would be of course unwary to attribute very precise quantitative meaning to the foregoing statements regarding the two possible dynamical mechanisms of photoproduction in the $\Delta(1232)$ energy region. This is because the breakup between direct resonance production and the remainder is not unique. The terms proportional to $c_j^{(3)}$ $(j=2,3)$ in (3.1) could in fact implicitly contain a part of crossed terms, in addition to those shown explicitly. They could also give account for some necessary complements to the extrapolation procedure adopted for $\varphi_j^{(3)}(\nu)$ ($j=2,3$) or for some corrections due to inelastic effects certainly present in the uppermost part of the integration interval, but deliberately sacrificed here for the sake of perfectly elastic unitarity. Earlier experience teaches us, nevertheless, that crossed terms other than those explicitly shown in Eq. (3.1) cannot substantially alter the results concerning $M_{1+}^{(3/2)}$ and $E_{1+}^{(3/2)}$ in the resonance region.[28,34] As to other corrections, the changes they can introduce are certainly of a few per cent magnitude at most. Therefore, the first two terms on the right-hand side of Eq. (3.1) certainly represent the bulk of contributions to the "driving forces." For all these reasons the general idea about the term proportional to $c_j^{(3)}$ ($j=2,3$) as representing the direct $\gamma N \rightarrow \Delta(1232)$ transition has reasonable backing, though, e.g., attempts to extract the $\gamma N \Delta$ coupling constant from the present calculations would probably be shaky. Whatever the importance of the foregoing objections, the remarkable agreement of the results with the data as described here and elsewhere[8,9,20] strongly supports the belief that the presented calculation grasps correctly the essential dynamical content of the electromagnetic excitation $N \rightarrow \Delta(1232)$. If also the interpretation, as proposed in the foregoing, were substantially correct, it could serve as a guide in the studies regarding electromagnetic properties of other hadronic resonances.

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