CP-violating effects in leptonic systems

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The electric dipole moment of leptons and *T*-odd correlations in leptonic decays are calculated in a wide class of gauge theories. These theories phenomenologically realize milliweak, microweak, and superweak models of *CP* violation. The *T*-odd transverse polarization in μ (or τ) leptonic decay is found to be superweak in magnitude and hence is intrinsically difficult to detect. Consideration is also devoted to the coherent admixture of opposite *CP* eigenstates in positronium. The upper limit of the mixing amplitude of the transition between states with the same *C*, but opposite *P* and *CP*, is of order 10^{-18}.

I. INTRODUCTION

Almost 20 years after the discovery of *CP* violation,[1] there has been much progress in the study of the phenomenology of *CP* nonconservation. However, the problem of why the weak interaction approximately conserves *CP* symmetry in the low-energy kaon system still remains an enigma. Indeed it is easy to accommodate *CP* violation in gauge models, but it appears rather difficult to fully understand the smallness of *CP* nonconservation.

Several models which have been put forward in trying to explain the small *CP* violation in the neutral-kaon system are based on the idea of spontaneous breakdown of *CP* symmetry. As we will discuss in Sec. II, these models are gauge theories which are realized phenomenologically as the milliweak, microweak, and superweak models, respectively, of *CP* violation. But unlike models of hard *CP* nonconservation, the approximate *CP* symmetry in the K^0-$\bar{K}^0$ system is understandable if the Kobayashi-Maskawa matrix[2] can be made real at the tree level. As a consequence, the small *CP*-violating phase in the gauge interaction arises from higher-order radiative corrections. Thus the smallness of *CP* breakdown is due either to the fact that quarks (of the first two generations) are lighter than the Higgs bosons, or to the fact that the horizontal gauge boson is very heavy, or to the very small mixing of the left- and right-handed gauge bosons.

The concept of spontaneous *CP* violation is appealing and attractive and provides a way of explaining the almost conserved *CP* symmetry in the weak interaction of six quark flavors. Nevertheless it also encounters some difficulties. These are as follows. (a) There is the domain-wall problem.[3] (b) There is a need in general for a rather complicated scalar-field structure. (c) Discrete symmetries, if needed, have to be imposed by hand. (d) Global symmetry gradually seems to be the automatic consequence of gauge symmetries, renormalizability, and the representation of the Higgs bosons. Hence, imposing *CP* symmetry on the initial Lagrangian appears to be quite *ad hoc.* Thus, as mentioned earlier, the smallness of *CP* nonconservation and its origin in the neutral-kaon system still remains a tantalizing problem.

In this paper we focus on *CP* violation in the leptonic sector. Possible *CP*-violating effects associated with the leptons include (1) the electric dipole moment of the lepton, (2) the *T*-odd transverse polarization of the lepton in purely leptonic decays, (3) neutrino oscillation if the neutrino is massive, and (4) the admixture of opposite *CP* eigenstates in positronium. For massless neutrinos none of the effects mentioned above can happen in the Kobayashi-Maskawa (KM) model. In the extension of the KM model for *CP* violation one has an analogous KM matrix for leptons. However, the *CP*-odd phases responsible for *CP* violation in the leptonic sector are not determined or constrained by any known experiment.[4] In analogy to the quark case, *CP* nonconservation in the leptonic sector with right-handed neutrinos may be introduced by adding more Higgs bosons[5] or by enlarging the gauge group. Nevertheless in these models the *CP*-violating phases for leptons are in general not related to those in the quark sector and hence are undetermined.[6] On the other hand, even if the neutrino is massless, *CP* nonconservation in leptons can still be achieved through the *CP*-violating neutral-current interaction, as discussed in Sec. II. Thus, except in the left-right-symmetric models, we will assume in this paper that neutrinos have only left-handed components and are massless. As a consequence the neutrino-mass problem which is itself at the center of another controversy, is also avoided.

In Sec. II we briefly review some milliweak, microweak, and superweak gauge models of *CP* violation. Then the electric dipole moment of the lepton is calculated in Sec. III. The coherent admixture of states with opposite eigenvalues of *CP* is discussed in Sec. IV. Section V is devoted to the computation of the *T*-odd transverse polarization of the lepton in $K_{\mu 3}^-$ and μ^- decays. Finally, Sec. VI presents our conclusions.

II. MODELS

To fix our notation, and for later purposes of calculation, we briefly review in this section some gauge models of *CP* nonconservation. We consider only models of spontaneous *CP* violation. Models of hard or intrinsic *CP* nonconservation, in which the breakdown of *CP* invariance arises from the complex Yukawa couplings or from the complex quartic terms in the Higgs potential, are not discussed here.[7] As mentioned in the Introduction, we will assume that neutrinos have only left-handed components and are massless in all models except the left-right-symmetry model.

Gauge models of *CP* violation may be classified into the following three categories according to the effective coupling strength F of their low-energy four-fermion interaction: milliweak, microweak, and superweak. Milliweak models have a $\Delta S=1$ *CP*-odd Hamiltonian H_-. The coupling strength of H_- is $10^{-3}G_F$ and it gives rise to $\mathrm{Im}M_{12}\sim 10^{-3}\mathrm{Re}M_{12}$, where M_{12} is the off-diagonal mass matrix element in the K^0-$\bar{K}^0$ system. As a result, in the milliweak model one has a direct *CP*-violating $K_2^0\to 2\pi$ decay. (Of course, this does not necessarily mean that $K_L\to 2\pi$ is totally explained by the $K_2^0\to 2\pi$ decay.) Superweak models have a flavor-changing neutral current which violates *CP* symmetry.[8] However, unlike the phenomenological superweak model,[9] $K_2^0\to 2\pi$ is not strictly forbidden in the gauge superweak model although it is very much suppressed. For a $\Delta S=1$ H_-, $F\simeq 10^{-3}G_F$ is a necessary but not a sufficient condition for the *CP* impurity ϵ in the neutral-kaon system to be of order 10^{-3}. Thus it is possible to have a microweak model which has a $\Delta S=1$ H_- with $F\sim 10^{-6}G_F$ but still gives the right amount of ϵ in the K^0-$\bar{K}^0$ system. An example of this is the left-right-symmetric model. It should be emphasized at the outset that the above classification of *CP* violating models is usually appropriate only for light quarks, (i.e., first two generations of quarks). Indeed superweak models may no longer be superweak for third-generation quarks.

For the purpose of illustration we will consider only one milliweak, one microweak, and two superweak models of *CP* violation. Other relevant models[10] are not examined here.

(a) Milliweak Higgs model. This is the "spontaneous" version of the Weinberg model of *CP* noninvariance.[11,12] Natural flavor conservation (NFC) is imposed, and at least three Higgs doublets are needed to implement *CP* violation. The breakdown of *CP* symmetry arises from the exchange of a light Higgs boson whose mass is of the order of 10 GeV. The charged- and neutral-Higgs-boson Yukawa interactions are given by

$$\mathscr{L}_Y^{\pm}=(2\sqrt{2}G_F)^{1/2}\sum_{i=1}^{2}(\alpha_i\bar{U}_LKM_DD_R+\beta_i\bar{U}_RM_UKD_L+\gamma_i\bar{N}_LM_EE_R)H_i^{+}+\mathrm{H.c.} \tag{2.1}$$

and

$$\mathscr{L}_Y^{0}=(\sqrt{2}G_F)^{1/2}\sum_{i=1}^{5}(\zeta_{1i}\bar{D}M_DD+i\zeta_{2i}\bar{D}M_D\gamma_5D+\zeta_{3i}\bar{U}M_UU+i\zeta_{4i}\bar{U}M_U\gamma_5U+\zeta_{5i}\bar{E}M_EE+i\zeta_{6i}\bar{E}M_E\gamma_5E)H_i\ , \tag{2.2}$$

respectively. For notation see Ref. 13. The coupling constants ζ_{ij} in Eq. (2.2) are real and of order unity. In purely leptonic interactions, the coefficient γ in (2.1) can always be made real by redefining the lepton states, so there is no *CP*-odd charged current. Two difficulties often encountered in this model are (1) the ratio ϵ'/ϵ in the neutral-kaon system is unacceptably large[14] and (2) the electric dipole moment of the neutron d_N is of order -10^{-26} (with negative sign),[15] which may not be consistent with the recent measured upper bound , 6×10^{-25} e cm.[16]

(b) Superweak Higgs model. This is the original model of Lee.[17] Two Higgs doublets are required but NFC is not imposed on the theory so that flavor can be changed by the neutral Higgs boson. This model is superweak in nature if the charged Higgs boson is as heavy as the neutral Higgs boson. The Higgs-boson—fermion couplings are not fixed by the theory, but there are two cases of interest[18]:

(i) The couplings are roughly of the order of the mass of the fermion

$$\mathscr{L}\sim(G_F)^{1/2}(m_i+m_j)\bar{\psi}_i(x)H(x)\psi_j(x)\ . \tag{2.3}$$

In this case, the mass of the Higgs boson lies in the range $1\mathrm{TeV}>m_H>500$ GeV.[19]

(ii) The couplings are roughly of the order of the mass of some heavy fermion M_F:

$$\mathscr{L}\sim(G_F)^{1/2}M_F\bar{\psi}_i(x)H(x)\psi_j(x)\ . \tag{2.4}$$

Shanker[20] has estimated $m_H\gtrsim 150$ TeV from the K_L-K_S mass difference. Because the Higgs particle in this case is so heavy that the quartic Higgs-boson couplings become very large,[21] perturbation theory may break down in this case.

In either case, the Yukawa interactions for charged and flavor-changing neutral currents can be parametrized as

$$\mathscr{L}_Y^{\pm}=(2\sqrt{2}G_F)^{1/2}\bar{\psi}_i[\alpha_{ij}^{L}(1-\gamma_5)/2+\alpha_{ij}^{R}(1+\gamma_5)/2]\psi_jH^{+}+\mathrm{H.c.} \tag{2.5}$$

and

$$\mathscr{L}_Y^{0}=\left[\frac{G_F}{\sqrt{2}}\right]^{1/2}H^0\{\bar{\psi}_i[\alpha_{ij}^{L}(1-\gamma_5)/2+\alpha_{ij}^{R}(1+\gamma_5)/2]\psi_j+\mathrm{H.c.}\}\ , \tag{2.6}$$

where $\alpha_{ij}^{L,R}=(m_i+m_j)C_{ij}^{L,R}$ for case (i) and $M_FC_{ij}^{L,R}$ for case (ii). The dimensionless coefficients $C_{ij}^{L,R}$ are in general complex and of order unity.

(c) Horizontal superweak model. In this class of models the vertical gauge interaction is *CP*-conserving at the tree level. *CP* violation arises from the flavor-changing horizontal gauge interaction, and is thus superweak in character. The first such model was discussed

in Ref. 21a. The horizontal gauge group G_H is often chosen as $U_H(1)$,[22] $SU_H(2)$,[23] $SU_H(3)$,[24] $O_{RH}(3)$,[25] or $SU_{LH}(N)\times SU_{RH}(N)$ (Ref. 26) (N is the number of generations). In any case the horizontal interaction has the form

$$\mathscr{L}=\frac{g_H}{2\sqrt{2}}\sum_a Y_a^\mu[(g_{La}^{ij}\bar{\psi}_L^i\gamma_\mu\psi_L^j+g_{Ra}^{ij}\bar{\psi}_R^i\gamma_\mu\psi_R^j)+\text{H.c.}]\ , \tag{2.7}$$

where g_H is the gauge coupling constant of G_H, g_L and g_R are mixing angles between different flavors with the same charge, and a is the species index of the horizontal gauge boson Y. The mass of Y^a and the mixing angles g_L as well as g_R depend very much on the details of the model (or the horizontal gauge group). In general m_Y is in the range 10^5-10^6 GeV.

(d) Left-right-symmetric model. In the left-right-symmetric (LRS) model based on the gauge group $SU_L(2)\times SU_R(2)\times U_{B-L}(1)$,[27] *CP* violation in the four-quark case lies in the right-handed gauge sector, since the mass matrix is real for left-handed quarks but complex for right-handed quarks. However, because *CP* symmetry is assumed to be spontaneously broken, if there were no W_L-W_R mixing quark mass matrices could always be made real and hence there would be no *CP* violation at all. Thus the degree of *CP* symmetry breakdown is a function of the W_L-W_R mixing parameter. It is known that apart from the sign, the W_L-W_L and W_L-W_R box diagrams have the same contributions to $\text{Re}M_{12}$ in the neutral-kaon systems if $M_R\sim 1.6$ TeV.[28] With the assumption that *CP* violation in K^0-$\bar{K}^0$ is dominated by the W_L-W_R box graph[29] the ratio of *CP*-odd to *CP*-even amplitude due to the exchange of a W_R boson must be of order 10^{-3}. This means that the effective *CP*-violating coupling strength is of order

$$10^{-3}\left[\frac{M_L}{M_R}\right]^2 G_F\sim 10^{-6}G_F\ . \tag{2.8}$$

Therefore the LRS model of spontaneous *CP* violation is microweak in nature.[30]

Following the conventions and notation of Ref. 31, the vacuum expectation value of the Higgs multiplet ϕ in representation $(\frac{1}{2},\frac{1}{2},0)$ reads

$$\langle\phi\rangle=\begin{bmatrix}\kappa & 0\\ 0 & \kappa' e^{i\alpha}\end{bmatrix}\ . \tag{2.9}$$

The left- and right-handed gauge bosons $W_{L,R}$ are related to their mass eigenstates $W_{1,2}$ by

$$\begin{aligned}W_1&=W_L\cos\xi-W_R e^{-i\alpha}\sin\xi\ ,\\ W_2&=W_L e^{i\alpha}\sin\xi+W_R\cos\xi\ .\end{aligned} \tag{2.10}$$

The W_L-W_R *CP*-violating mixing is independent of the quark and leptonic sectors. The gauge interaction of four quarks in terms of mass eigenstates is

$$\begin{aligned}\mathscr{L}=&\frac{g}{\sqrt{2}}W_1^{+\mu}(\cos\xi\,\bar{U}_L\gamma_\mu K_L D_L-e^{i\alpha}\sin\xi\,\bar{U}_R\gamma_\mu K_R D_R)\\ &+\frac{g}{\sqrt{2}}W_2^{+\mu}(e^{-i\alpha}\sin\xi\,\bar{U}_L\gamma_\mu K_L D_L\\ &+\cos\xi\,\bar{U}_R\gamma_\mu K_R D_R)+\text{H.c.}\end{aligned} \tag{2.11}$$

with

$$\begin{aligned}K_L&=\begin{bmatrix}\cos\theta_C & \sin\theta_C\\ -\sin\theta_C & \cos\theta_C\end{bmatrix}\ ,\\ K_R&=\begin{bmatrix}e^{-i\delta_2}\cos\theta_C & e^{-i\delta_1}\sin\theta_C\\ -e^{i\delta_1}\sin\theta_C & e^{i\delta_2}\cos\theta_C\end{bmatrix}\ ,\end{aligned} \tag{2.12}$$

and[31]

$$\begin{aligned}\delta_1&\cong-\frac{3}{2}\frac{m_c}{m_s}r\sin\alpha\ ,\\ \delta_2&\cong-\frac{1}{2}\frac{m_c}{m_s}r\sin\alpha\ ,\end{aligned} \tag{2.13}$$

where $r=\kappa'/\kappa$, and θ_C is the Cabibbo angle. The W_L-W_R mixing angle ξ

$$\xi\cong\tfrac{1}{2}\tan 2\xi=\frac{\kappa'}{\kappa}\left[\frac{M_L}{M_R}\right]^2=r\beta\ , \tag{2.14}$$

is determined by the *CP*-violating parameter ϵ in the K^0-$\bar{K}^0$ system[32]

$$|\epsilon|\cong\frac{430}{2\sqrt{2}}r\beta\frac{m_c}{m_s}\sin\alpha\ . \tag{2.15}$$

With $\sin\alpha\sim 1$, one has

$$\xi\cong 1.5\times 10^{-6}\ . \tag{2.16}$$

It should be noted that no assumption about M_R has been made in obtaining (2.16). Again one may check the microweak feature of this model from Eq. (2.11). For a generalization to the six-quark case, see also Ref. 31.

Together with Higgs multiplets $\Delta_L\equiv(1,0,2)$ and $\Delta_R\equiv(0,1,2)$, this model leads to a light Majorana neutrino ν_e and a heavy Majorana lepton N_e.[33] In terms of mass eigenstates e, ν, and N, the gauge interaction in the leptonic sector reads

$$\begin{aligned}\mathscr{L}=&\frac{g}{2\sqrt{2}}W_1^{-\mu}[\bar{e}\gamma_\mu(1-\gamma_5)(\nu-\xi N)-e^{i\alpha}\xi\bar{e}\gamma_\mu(1+\gamma_5)e^{i(\theta_1-\theta_2)}(N+\xi\nu)]\\ &+\frac{g}{2\sqrt{2}}W_2^{-\mu}[\bar{e}\gamma_\mu(1+\gamma_5)e^{i(\theta_1-\theta_2)}(N+\xi\nu)+e^{-i\alpha}\xi\bar{e}\gamma_\mu(1-\gamma_5)(\nu-\xi N)]+\text{H.c.}\ ,\end{aligned} \tag{2.17}$$

where ζ is the ν_e-N_e mixing angle and is of order m_e/m_N,[33]

$$\theta_1-\theta_2=r\sin\alpha(h_1/h_2-h_2/h_1)$$

with h_1 and h_2 the Yukawa coupling constants in

$$\mathscr{L}_Y=h_1\bar{\psi}_L\phi\psi_R+h_2\bar{\psi}_L\tilde{\phi}\psi_R+\text{H.c.}+\cdots, \quad (2.18)$$

$$\psi_L=\begin{bmatrix}\nu_e\\ e\end{bmatrix}_L,\quad \psi_R=\begin{bmatrix}\nu_e\\ e\end{bmatrix}_R.$$

Although (2.17) is written for the first generation of leptons, it can be generalized to the three-generation case with more complicated mixings among the leptons.

III. THE ELECTRIC DIPOLE MOMENT OF THE LEPTON

The electromagnetic current of a fermion has the general form

$$\bar{\psi}(p')[F_1(q^2)\gamma_\mu-F_2(q^2)i\sigma_{\mu\nu}q^\nu+F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5]\psi(p). \quad (3.1)$$

T invariance requires that F_3 be imaginary, and hermiticity implies that F_3 is real. Thus the measurement of the electric dipole moment (EDM) of the lepton $d_l=F_3(0)$ is a sensitive test of time-reversal invariance in the leptonic sector.

Experimentally, the EDM of the electron d_e can either be measured directly or extracted from the experimentally determined EDM of an atom, such as Xe, Cs, and Tl. The atomic-beam experiments are indirect, but are more sensitive to d_e. On the other hand, d_μ has been measured at the CERN storage ring by assuming that the discrepancy between the measured $g-2$ value of the muon and the theoretical prediction is totally due to the EDM of the muon.[34] The experimental upper limits for d_e and d_μ are 10^{-24} e cm,[35] and 10^{-18} e cm,[34] respectively. On theoretical grounds, the calculation of d_l is less ambiguous and uncertain than that of d_N, the EDM of the neutron. There are no QCD corrections, no contributions from strong CP violation, and no similar ambiguity in choosing the quark mass, current, or constituent. The early calculations of d_l in the context of different gauge models of CP violation can be found in Refs. 36 and 37. It has also been discussed recently by Gavela and Georgi.[38]

Naively d_l is expected to be of order

$$d_l=e\frac{G_F}{4\pi}m_lf=2\times10^{-20}f(m_l/1\text{GeV})\,e\text{ cm}, \quad (3.2)$$

where the parameter f characterizes the degree of T violation. However we note in passing that the classification of CP-violating models into milliweak, microweak, and superweak categories is suitable only for the first-two-generation quarks.

Milliweak Higgs model

The dominant contribution to d_l comes from the loop diagram with the neutral-Higgs-boson exchange. From Eq. (2.2), we find

$$d_l=\frac{eG_Fm_l^3}{4\pi^2}\sum_{i=1}^{5}\frac{g_i^{(1)}g_i^{(2)}}{m_{H_i}^2}\frac{1}{(1-x_i^2)^2}\left[-\frac{3}{2}+\frac{x_i^2}{2}-\frac{\ln x_i^2}{1-x_i^2}\right], \quad (3.3)$$

where $x_i=m_l/m_{H_i}$. In deriving (3.3) the external lepton mass has been neglected in the denominator of the amplitude. Since $g_i^{(1)}\equiv\zeta_{5i}$ and $g_i^{(2)}\equiv\zeta_{6i}$ are of order unity, it is reasonable to assume that $g_i^{(1)}g_i^{(2)}\sim0.1$. Taking $m_H=10$ GeV, the EDM of the lepton is

$$d_e=1.4\times10^{-32}\,e\text{ cm},\quad d_\mu=5.1\times10^{-26}\,e\text{ cm},\quad d_\tau=6.6\times10^{-23}\,e\text{ cm}. \quad (3.4)$$

Because d_l is proportional to m_l^3, the EDM of the electron is far below the present experimental upper bound.

Superweak Higgs model

From Eq. (2.6) it is easy to show that

$$d_l=\frac{eG_F}{16\sqrt{2}\pi^2}\sum_{l'}\text{Im}(\alpha_{ll'}^L\alpha_{ll'}^{R*})\frac{m_{l'}}{m_H^2}\frac{1}{(1-x^2)^2}\left[-\frac{3}{2}+\frac{x^2}{2}-\frac{\ln x^2}{1-x^2}\right], \quad (3.5)$$

where $x=m_{l'}/m_H$ and l' is the intermediate leptonic state. Since the neutral Higgs boson has a flavor-changing coupling, the main contribution to d_l is from the τ intermediate state, and thus $d_{e,\mu}$ are enhanced relative to d_τ. As discussed in Sec. II, there are in general two different types of Higgs-boson–fermion couplings. For the present problem, both cases have the same couplings:

$$\alpha_{l\tau}^{L,R}\simeq m_\tau C_{l\tau}^{L,R}, \quad (3.6)$$

where the dimensionless parameter $C_{l\tau}$ is complex in general. Therefore

$$d_l\cong\frac{eG_F}{16\sqrt{2}\pi^2}\frac{m_\tau^3}{m_H^2}\text{Im}(C_{l\tau}^LC_{l\tau}^{R*})\left[\ln\frac{m_H^2}{m_\tau^2}-\frac{3}{2}\right]. \quad (3.7)$$

Taking into account the mixing angles between different leptons and the CP-odd phase, it is reasonable to take the following values for purposes of the present calculation:

$$\text{Im}(C_{\tau\tau}^LC_{\tau\tau}^{R*})\simeq10^{-1},$$

$$\text{Im}(C_{e\tau}^LC_{e\tau}^{R*})\simeq\text{Im}(C_{\mu\tau}^LC_{\mu\tau}^{R*})\simeq10^{-2}. \quad (3.8)$$

For case (i), 1 TeV $> m_H >$ 500 GeV, we find

$$d_{e,\mu}\simeq 10^{-27}\ e\ \text{cm}\ ,\quad d_\tau\simeq 10^{-26}\ e\ \text{cm}\ . \tag{3.9}$$

For the second case, $M_H \gtrsim 150$ TeV, the EDM is much suppressed

$$d_{e,\mu}\simeq 10^{-31}\ e\ \text{cm}\ ,\quad d_\tau\simeq 10^{-30}\ e\ \text{cm}\ . \tag{3.10}$$

Although for both cases d_τ is not as big as in the milliweak Higgs model, the EDM of the electron in the first case is significant because of the flavor-changing interaction.

Horizontal superweak model

From Eq. (2.7) the loop diagram involving the exchange of the horizontal gauge boson Y yields the gauge-invariant result[39]

$$d_l=\frac{eg_H{}^2}{128\pi^2}\sum_{a,l'}\text{Im}(g_{La}^{ll'}g_{Ra}^{ll'*})\frac{m_{l'}}{M_a{}^2}\left[\frac{1}{2}+\frac{3}{(1-x^2)^3}\left[\frac{1}{2}-\frac{x^4}{2}+x^2\ln x^2\right]\right]\ , \tag{3.11}$$

where $x=m_{l'}/M_a$ and M_a is the mass of Y_a. It should be noted that in Eq. (3.11) $l'\neq l$ because the flavor-conserving amplitude is real. In obtaining d_l we have neglected contributions such as $\text{Im}(g_{La}^{ll'}g_{Lb}^{ll'*})$ which come from the mixing of Y_a and Y_b. Such mixing is suppressed by a factor of order $M_W{}^2/M_a{}^2$.[25] Since $M_a \gg m_{l'}$, (3.11) simplifies to

$$d_l=\frac{eg_H{}^2}{64\pi^2}\sum_{a,l'}\text{Im}(g_Lg_R^*)\frac{m_l'}{M_a{}^2}\ . \tag{3.12}$$

For $d_{e,\mu}$ the main contribution is from $l'=\tau$. For d_τ the μ intermediate state gives the dominant contribution. The prediction of d_l is quite model dependent: It depends on the details of the mixing angles of leptons and masses of various horizontal gauge bosons. To have a conservative estimate we assume $g_H{}^2\text{Im}(g_Lg_R^*)\sim 10^{-2}$ as in Eq. (3.8) and take the heaviest mass of Y_a, namely, 3×10^6 GeV.[25] Then

$$d_e\simeq 10^{-32}\ e\ \text{cm}\ . \tag{3.13}$$

This result (3.13) may be regarded as a lower-bound prediction of d_e in the horizontal superweak model because for a light horizontal boson and large mixing angle the EDM of the electron can easily become as large as 10^{-27} e cm.

Left-right-symmetric model

The evaluation of d_l in the LRS model is similar to that of d_N, which has been calculated by Beall and Soni,[40] thus we will take over their result directly. Equation (2.17) can be recast in the form

$$\mathcal{L}=\bar{e}\gamma_\mu(a+b\gamma_5)NW_1^{-\mu}+\cdots\ , \tag{3.14}$$

with

$$a=-\frac{g}{2\sqrt{2}}(\zeta+e^{i\alpha}\xi)\ ,\qquad b=\frac{g}{2\sqrt{2}}(\zeta-e^{i\alpha}\xi)\ . \tag{3.15}$$

The phase $(\theta_1-\theta_2)$ is neglected in (3.14). As shown in Fig. 1 the loop graph with the intermediate state N gives the dominant contribution to d_e, which is given by

$$d_e=-\frac{em_N}{8\pi^2M_1{}^2}\text{Im}(ab^*)\left[\frac{1}{2}+\frac{3}{(1-x^2)^3}\left(\tfrac{1}{2}-2x^2+\tfrac{3}{2}x^4-x^4\ln x^2\right)\right]\ , \tag{3.16}$$

where $x=m_N/M_1$, and $M_1(\cong M_W)$ is the mass of mass eigenstate W_1. In (3.16) m_N is an unknown parameter. Since m_N is proportional to v_R, which is the vacuum expectation value of Δ_R, it is natural to assume that $m_N\simeq M_R$ (Ref. 33) (recall that $M_R\simeq gv_R$). Because the ν-N mixing angle $\zeta\simeq m_e/m_N$, d_e is insensitive to the magnitude of M_R. Taking the conservative lower-bound $M_R/M_L\gtrsim 3$,[41] (3.16) reduces to

$$d_e\cong\frac{eG_F}{4\sqrt{2}\pi^2}m_e\xi\sin\alpha\ . \tag{3.17}$$

For $\sin\alpha\sim 1$, it follows that

$$d_e\cong 10^{-30}\ e\ \text{cm}\ . \tag{3.18}$$

Similarly d_μ, and d_τ are expected to be 10^{-27} e cm and 10^{-26} e cm, respectively, which agree with the naive esti-

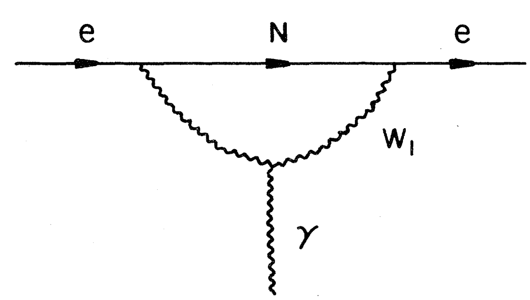

FIG. 1. Lowest-order contribution to the electric dipole moment of the electron in the left-right-symmetric model. N is a heavy Majorana lepton and W_1 is one of the mass eigenstates of gauge bosons.

mate (3.2).

To conclude this section we notice that the Majorana lepton cannot have an electric dipole or magnetic moment if CPT invariance holds, as pointed out by many authors.[42]

IV. CP VIOLATION IN POSITRONIUM

In this section we study CP nonconservation in the positronium atom. The eigenvalues of charge conjugation and parity of positronium are given by

$$C=(-1)^{L+S}\ , \quad P=(-1)^{L+1}\ , \quad CP=(-1)^{S+1}\ , \tag{4.1}$$

where L and S denote the orbital and spin angular momenta of positronium. Thus, it follows from (4.1) that orthopositronium has $CP=+1$ and parapositronium has $CP=-1$. Consequently a CP-violating interaction can admix states with the same total orbital angular moment but opposite eigenvalues of CP.

The first investigation of a possible coherent admixture of opposite CP states in positronium was due to Fischbach, Freeman, and Cheng.[43] They studied the fermion-antifermion system in the presence of an external gravitational field, and computed the gravity-induced effective potential which leads to transitions between states with the same C but opposite P, and hence opposite CP. Since the gravity-induced potential is T invariant, the changes in CP induced in positronium are compensated for by corresponding changes in the external gravitational field.

In this section we wish to examine such an admixture in the context of gauge models of CP violation. The main contribution which gives rise to mixing of opposite CP eigenstates comes from the one-photon-exchange Feynman diagram with vertices $e\gamma_\mu$ and $d_e\sigma_{\mu\nu}q^\nu\gamma_5$, respectively, plus the annihilation diagram. The amplitude is given by

$$M=\frac{ed_e}{(p'-p)^2}[\bar{u}(p')\gamma_\mu u(p)\bar{v}(q)\sigma^{\mu\nu}(q'-q)_\nu\gamma_5(q')v(q')+\bar{u}(p')\sigma^{\mu\nu}(p'-p)_\nu\gamma_5 u(p)\bar{v}(q)\gamma_\mu v(q')]$$
$$+\ \text{amplitude of annihilation diagram}\ , \tag{4.2}$$

where d_e is the EDM of the electron at $q^2=0$. The Fourier transform of (4.2) in the nonrelativistic limit yields an effective long-range potential

$$\delta V=\frac{ed_e}{4\pi}(\vec{\sigma}_- - \vec{\sigma}_+)\cdot\vec{r}/r^3\ , \tag{4.3}$$

which is spin-dependent. In (4.3) $\vec{\sigma}_-$ and $\vec{\sigma}_+$ are spin operators acting on the electron and positron spinors, respectively. We note that in the nonrelativistic limit the annihilation diagram has no contribution to δV. In addition to the long-range potential there is another contribution in the Higgs model of CP violation, namely, the tree diagram with neutral-Higgs-boson exchange. This gives the following spin-dependent short-range potential:

$$\delta V_H=\frac{G_F m_e}{4\sqrt{2}\pi}g^{(1)}g^{(2)}e^{-m_H r}(\vec{\sigma}_- - \vec{\sigma}_+)\cdot\vec{r}\left[\frac{m_H}{r^2}+\frac{1}{r^3}\right]\ , \tag{4.4}$$

where $g^{(1)}$ and $g^{(2)}$ are real and of order unity. Both δV and δV_H can induce transitions between states with the same eigenvalue of C, but opposite eigenvalues of P and CP.

In order to have coherent admixture of opposite CP states we will focus on states with the same quantum numbers J and m_J. Before proceeding to the calculation of matrix elements of δV and δV_H, we note that the radial matrix elements of the CP-nonconserving potential δV vanish when taken between nonrelativistic Coulomb wave functions with the same principal quantum number n. For the ground state and first excited state the matrix elements of δV and δV_H are given by

$$\left\langle 1\,^3S_1(S_z=\pm 1)\,|\,\delta V\,|\,2\,^1P_1(L_z=\pm 1)\right\rangle=\pm\frac{ed_e}{27\sqrt{2}\pi a_0^2}\ ,$$

$$\left\langle 1\,^1S_0\,|\,\delta V\,|\,2\,^3P_0\right\rangle=-\frac{4ed_e}{27\sqrt{6}\pi a_0^2}\ ,$$

$$\left\langle 1\,^3S_1(S_z=\pm 1)\,|\,\delta V_H\,|\,2\,^1P_1(L_z=\pm 1)\right\rangle=\pm\frac{G_F m_e}{8\pi a_0^4 m_H^2}\ ,$$

$$\left\langle 2\,^3S_1(S_z=\pm 1)\,|\,\delta V_H\,|\,2\,^1P_1(L_z=\pm 1)\right\rangle=\pm\frac{G_F m_e}{16\sqrt{2}\pi a_0^4 m_H^2}\ ,$$

$$\left\langle 2\,^1S_0\,|\,\delta V_H\,|\,2\,^3P_0\right\rangle=-\frac{G_F m_e}{4\sqrt{6}\pi a_0^4 m_H^2}\ , \tag{4.5}$$

where $a_0=2\hbar^2/m_e e^2$ is the Bohr radius for positronium and the spatial wave functions are assumed to be those obtained using only the Coulomb interaction. The amplitude of admixture of opposite-CP components induced by δV is given by

$$a=\frac{\langle\beta\,|\,\delta V\,|\,\alpha\rangle}{E_\alpha-E_\beta+i\Gamma/2}\ . \tag{4.6}$$

Similarly for a_H, which is induced by δV_H. The relevant fine-structure energy splittings in the $n=2$ energy level are

$$E(2\,^3P_0)-E(2\,^1S_0)=2.9\times 10^{-5}\ \text{eV}\ ,$$
$$E(2\,^3S_1)-E(2\,^1P_1)=4.6\times 10^{-5}\ \text{eV}\ . \tag{4.7}$$

Together with

$$E(n_2)-E(n_1)=5.1\ \text{eV}\ ,$$

we find

$$a(1\,^3S_1,2\,^1P_1)=2.1\times10^{-18}(d_e/10^{-24}\ e\,\mathrm{cm})\ ,$$
$$a(1\,^1S_0,2\,^3P_0)=4.9\times10^{-18}(d_e/10^{-24}\ e\,\mathrm{cm})\ ,\qquad(4.8)$$
$$a_H(2\,^3S_1,2\,^1P_1)=1.9\times10^{-22}\ ,$$
$$a_H(2\,^1S_0,2\,^3P_0)=7.0\times10^{-22}\ ,$$

where the decay width difference Γ is negligible. To calculate the matrix elements of δV, and hence the amplitude a, between states with the same principal quantum number n, we need the next-order wave functions which take account of the fine-structure and radiative corrections. This is because, as noted earlier, the matrix elements of δV vanish when taken between pure Coulomb wave functions. However, since the lowest-order correction to the Coulomb wave function is of order α^2, this offsets the enhancement due to the small fine-structure energy splitting compared to the principal energy level difference in the denominator of the amplitude a. Thus we anticipate that $a(2\,^3S_1,2\,^1P_1)$ is of the same order as $a(1\,^3S_1,2\,^1P_1)$.

Since the computation of d_e is model-dependent, the mixing amplitude a thus depends on the models of CP violation. Because the experimental upper limit of d_e is 10^{-24} e cm, we then have an upper bound for a:

$$a\lesssim10^{-18}\ .\qquad(4.9)$$

From (4.8) it follows that if $d_e>10^{-28}$ e cm, the main contribution to the mixing amplitude comes from the spin-dependent long-range potential δV. It is interesting to notice that the amplitude of admixtures induced by gravity is of order 10^{-22}.[43] If the experimental sensitivity in searching for the admixture of states with opposite CP in positronium is only at the 1% level, then it would be impossible to observe such CP-violating effect.

Thus far we have not considered the transition between $2\,^3P_1$ and $2\,^1P_1$ states because both T-odd potentials δV and δV_H cannot lead to this admixture directly. However such a C-odd CP-odd transition can be induced by a combination of δV (or δV_H) and C-odd P-odd (hence CP-even) part of the weak interaction. Therefore the magnitude of the admixture of $2\,^3P_1$ and $2\,^1P_1$ states is even smaller compared to the transition, say, between $2\,^1P_1$ and $2\,^3S_1$ states. An experiment looking for admixtures of opposite-CP components in the positronium atom is now underway in Michigan.[44]

V. T-ODD CORRELATION

A T-violating correlation, which is a scalar quantity and odd under T, is another place for testing T invariance. In four-body decays one may look at the T-odd angular correlation $\vec{P}_1\cdot\vec{P}_2\times\vec{P}_3$ of the decay products. In three-body decays the above angular correlation cannot occur, hence a T-odd correlation in these decays must involve at least one spin operator, such as $\vec{\sigma}_1\cdot\vec{P}_2\times\vec{P}_3$ or $\vec{\sigma}_1\cdot\vec{\sigma}_2\times\vec{P}_3$. The measurement of a T-odd correlation is a clean test of T invariance if the final-state interaction, which can masquerade as a T violation, can be neglected. However if the final-state correction is not negligible but comparable to the T-violating correlation induced by the violation of CP invariance, then CP-symmetry violation can still be observed. For instance in $K^0_{\mu3}$ semileptonic decays, the transverse polarization of the muon caused by the electromagnetic interaction is the same in $K^0_{\mu3}$ and $\bar{K}^0_{\mu3}$ decays, whereas the polarization due to the CP noninvariance has opposite signs in these cases. Hence the absolute magnitude of the muon's transverse polarization should be different in $K^0_{\mu3}$ and $\bar{K}^0_{\mu3}$ decays.[45]

Generally speaking it is not easy to have a T-violating correlation in most gauge models of CP violation. As emphasized by Donoghue,[37] to produce a T-odd correlation one usually needs an interference between two different processes (leading to the same final states) that have different phases and space-time structure. As we will see later on even so such an interference does not guarantee that a T-violating quantity is always observable.

In the purely leptonic sector a T-odd correlation can be studied in the four-body decay $\tau^\pm\to\pi^\pm\pi^-\pi^+\nu$ and in the three-body decay $\mu\to e\bar{\nu}\nu$ or $\tau\to\mu\bar{\nu}\nu$. The final-state interaction is electromagnetic and hence of order α. Unless it is suppressed by the phase-space factor, the electromagnetic correction is expected to be of order $10^{-2}\sim10^{-3}$. Thus if a milliweak model predicts a null T-odd correlation, then such correlation will be immensely difficult to detect. In this section we focus on μ (or τ) three-body decay. For purpose of comparison we also briefly discuss the muon's transverse polarization in $K_{\mu3}$ decays.

$K^-_{\mu3}$ decay

The degree of T-odd transverse polarization of the muon is usually expressed in terms of the parameter $\xi(t)\equiv f_-(t)/f_+(t)$, where f_+ and f_- are the form factors in the amplitude

$$M\propto\bar{\mu}\gamma_\mu(1-\gamma_5)\nu(P_K+P_\pi)_\mu f_+(t)$$
$$+m_\mu\bar{\mu}(1-\gamma_5)\nu f_-(t)\ .\qquad(5.1)$$

The imaginary part of ξ governs the size of the T-odd polarization. In the milliweak Higgs model it has been calculated by Zhitnitsky[46] and the result is given by[13]

$$-6.8\times10^{-4}v_2{}^2/v_3{}^2<\mathrm{Im}\xi<-2.0\times10^{-3}v_2{}^2/v_3{}^2\ ,\qquad(5.2)$$

where v_2 and v_3 are the vacuum expectation values of the Higgs fields ϕ_2 and ϕ_3, whose neutral components couple only to down-type quarks and leptons, respectively. The unknown factor v_2/v_3 in (5.2) is expected to be of order unity. The calculation in the superweak Higgs model is quite similar to that in the milliweak Higgs model and the result is

$$\mathrm{Im}\xi=\mathrm{Im}(\alpha^R_{su}\alpha^{R*}_{\nu\mu}+\alpha^L_{su}\alpha^{R*}_{\nu\mu})\frac{m_K{}^2}{m_s m_\mu m_H{}^2\sin\theta_C}\ .\qquad(5.3)$$

(For notation see Sec. II.) For the Higgs-boson—fermion couplings

$$\alpha_{ij}=(m_i+m_j)C_{ij}\qquad(5.4)$$

and (5.3) becomes

$$\mathrm{Im}\xi\simeq\mathrm{Im}C^{R*}_{\nu\mu}(C^R_{su}+C^L_{su})\frac{m_K{}^2}{m_H{}^2\sin\theta_C}\ .\qquad(5.5)$$

Assuming $\mathrm{Im}C^*C\simeq10^{-2}$ as in (3.8), and using $1\,\mathrm{TeV}>m_H>500$ GeV, it follows that

$$\text{Im}\xi \simeq 10^{-8} \ . \tag{5.6}$$

On the other hand for the couplings $\alpha_{ij} = M_F C_{ij}$, (5.3) becomes

$$\text{Im}\xi \simeq \text{Im} C_{\nu\mu}^{R*} (C_{su}^R + C_{su}^L) \frac{m_K{}^2 m_t m_\tau}{m_s m_\mu m_H{}^2 \sin\theta_C} \ . \tag{5.7}$$

For $m_t = 30$ GeV and $m_H = 150$ TeV,

$$\text{Im}\xi \simeq 10^{-9} \ . \tag{5.8}$$

It is easy to show that the LRS model, horizontal CP-violation model, and KM model all predict null T-odd transverse polarization of the muon to lowest order. The reason is that the T-violating polarization arises from the interference between vector and scalar parts of the amplitude (5.1). To lowest order the V and S amplitudes have the same phase in these three models, and thus there is no CP-violating effect.

For $K_{\mu 3}^{\pm}$ decays Zhitnitsky[46] has calculated the electromagnetic correction and found $P_T^{(\text{EM})} \leq 10^{-6}$. Accordingly, if the milliweak model is ruled out experimentally, then from the theoretical point of view it is impossible to detect any T-odd transverse polarization in $K_{\mu 3}^{\pm}$ decays.

μ decay

In $\mu \to e\bar{\nu}\nu$ decay, the quantity we are interested in is P_T which measures the transverse polarization of the electron perpendicular to the plane of $\vec{P}_e$ and $\vec{S}_\mu$, and has the form

$$P_T(x,\theta) = \frac{d^2\sigma(x,\theta,\phi=\pi/2,\psi=\pi/2) - d^2\sigma(x,\theta,\phi=-\pi/2,\psi=\pi/2)}{d^2\sigma(x,\theta,\phi=\pi/2,\psi=\pi/2) + d^2\sigma(x,\theta,\phi=-\pi/2,\psi=\pi/2)} \ , \tag{5.9}$$

where $x = 2E_e/m_\mu$ $(2m_e/m_\mu \leq x \leq 1)$, θ, ϕ, and ψ are defined in Fig. 2. For the effective Lagrangian in the charge-retention form

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_i [\bar{e}\Gamma_i \mu \bar{\nu}_\mu \Gamma_i (C_i + C_i' \gamma_5)\nu_e + \text{H.c.}] \ , \tag{5.10}$$

where Γ_i stands for S, V, T, A, and P operators, respectively, we have[47]

$$P_T \cong \frac{[3(1-x)\alpha' + 2\beta']\sin\theta}{8[3 - 2x + \xi(1-2x)\cos\theta]} \ , \tag{5.11}$$

where

$$\begin{aligned} \alpha' &= 2\,\text{Im}(C_S C_P'^* + C_S' C_P^*) \ , \\ \beta' &= 2\,\text{Im}(C_V C_A'^* + C_V' C_A^*) \ , \end{aligned} \tag{5.12}$$

and ξ is the standard asymmetry parameter in the μ decay.

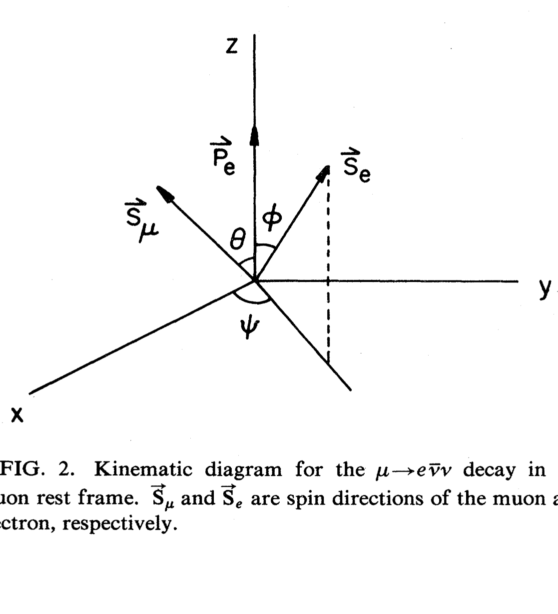

FIG. 2. Kinematic diagram for the $\mu \to e\bar{\nu}\nu$ decay in the muon rest frame. $\vec{S}_\mu$ and $\vec{S}_e$ are spin directions of the muon and electron, respectively.

We can now calculate P_T in some realistic models. Unfortunately α', β', and hence P_T vanish to lowest order in all of the models of CP violation that we considered in Sec. II. It is not difficult to see that in the milliweak, superweak, and horizontal superweak models (all of which assume no right-handed neutrinos) the CP-odd phase in the μ decay can be rotated away by redefining the μ and e states. In the LRS model we do have an interference between the left- and right-handed currents both having different phases, but it does not lead to a nonvanishing P_T.

However, a real T-odd correlation can be achieved if the effective Lagrangian responsible for the μ decay in the charge-retention order has the form

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{e}\gamma_\mu (g_V - g_A \gamma_5)\mu \bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\nu_e + \text{H.c.} \ , \tag{5.13}$$

with g_V and g_A close to unity but complex. Then

$$\alpha' = 0 \text{ and } \beta' = 4\,\text{Im}(g_V^* g_A) \tag{5.14}$$

and

$$P_T \simeq \frac{\text{Im}(g_V^* g_A)\sin\theta}{3 - 2x + \xi(1-2x)\cos\theta} \ . \tag{5.15}$$

Such a Lagrangian (5.13) can arise in the horizontal superweak model with massive Dirac neutrinos.[48] In this model we find

$$\beta' = -\frac{g_H{}^2}{2\sqrt{2}G_F} \sum_a \frac{\text{Im}(g_{Ra}^{e\mu} g_{La}^{\nu\nu*})}{M_a{}^2} \ . \tag{5.16}$$

Again the numerical result of β' and hence P_T depends on the details of the model, but in general it is quite small. The electromagnetic radiative correction to P_T has been discussed in Refs. 49 and 50. The radiative correction is a function of x and θ; for fixed θ, it is largest (of order 10^{-2}) at small x and then decreases with increasing x. Therefore unless the T-odd correlation is "milliweak" in magnitude (which is not the case in our discussion) it would be very difficult to observe a T-violating effect in the μ (or τ) leptonic decay.

To conclude this section we note that the prediction of T-odd correlation in the milliweak Higgs model of CP violation is quite different for the semileptonic $K_{\mu 3}$ and leptonic μ decays: The T-odd transverse polarization is appreciable in $K_{\mu 3}$ decay but vanishes in μ decay. It is not clear whether this is a general feature, i.e., that a T-violating correlation can be easily achieved in semileptonic decays but not in purely leptonic or hadronic decays.

VI. CONCLUSIONS

We have studied several CP-violating effects in leptons in the context of various gauge models of CP nonconservation. These models fall within the categories of milliweak, microweak, and superweak theories. In general even if neutrinos have no right-handed components and are massless, one can still have CP-odd neutral-current interaction in the purely leptonic sector.

The electric dipole moment of the lepton is examined in Sec. III. It turns out that the Weinberg model of CP nonconservation predicts $d_l = 10^{-32}$, 10^{-25}, and 10^{-22} e cm, respectively, for e, μ, and τ. In the superweak Higgs model d_l is in the range 10^{-26}–10^{-27} e cm or 10^{-30}–10^{-31} e cm depending on the mass of the Higgs boson. The prediction of d_l in the context of the horizontal CP violation model depends very much on the detailed structure of the model (i.e., the horizontal gauge group). A conservative estimate yields $d_e = 10^{-32}$ e cm. The left-right-symmetric model gives $d_l = 10^{-30}$, 10^{-27}, and 10^{-26} e cm for e, μ, and τ, respectively, which agree with the naive estimate of the EDM of the lepton.

CP violation may also be explored in positronium by measuring the amplitude of the coherent admixture of opposite CP eigenstates. We find a long-range and a short-range potential, both are CP-odd and spin dependent and can induce transitions between states with the same C, but opposite P and CP. The upper bound of the mixing amplitude is of order 10^{-18}, which is unfortunately quite small.

The transverse polarization of the electron $\vec{\sigma}_e \cdot \vec{\sigma}_\mu \times \vec{p}_e$ is a T-odd correlation in the $\mu \to e\bar{\nu}\nu$ decay. It turns out that all models given in Sec. II yield negative predictions about this T-violating polarization. Although a nonvanishing P_T can occur in the horizontal CP-violation model with massive Dirac neutrino, the T-odd correlation is superweak in magnitude. Therefore experimentally it will be extremely difficult to detect. For $K^{\pm}_{\mu 3}$ decays only the Higgs model of CP violation is able to produce a T-odd transverse polarization of the muon, which is not the case in the μ decay.

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