Gauge theories with composite bosons

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We develop an approach for introducing bosons into gauge theories as composite states of fermions. It is scale and gauge invariant from the outset and utilizes invariant regularization procedures. This results in the acquisition of gauge-covariant kinetic energy by the bosons and the growth of coupling constants and masses. Relationships among the coupling constants and masses are derived.

I. INTRODUCTION

In recent years there has been a considerable amount of activity directed toward introducing the bosons (both spin zero and one) as composite states of fermions. Many of these models utilize a four-fermion coupling following the work of Nambu and Jona-Lasinio.[1,2] Their equivalence, in certain cases, to gauge theories both for Abelian[3] and non-Abelian[4] examples has also been shown. However, the couplings are non-scale-invariant and/or non-gauge-invariant initially with a compensating non-gauge-invariant renormalization procedure used to arrive at a gauge-invariant formulation.

Another, very attractive approach starts with just fermions using scale-invariant and gauge-invariant couplings involving derivatives and inverses of the fermion fields.[5] However, the implementation of this model appears to be extremely difficult and it does not appear to be renormalizable.

Here, we offer another approach which is scale invariant and gauge invariant from the outset and employs a gauge-invariant renormalization procedure.

In Sec. II, we exhibit the procedure by working with a prototype based on invariance under SU(3)×SU(2)×U(1). In Sec. III we determine the relationships among the coupling constants and masses which emerge from the calculation and in Sec. IV we present some concluding remarks.

II. SU(3)×SU(2)×U(1)

We choose this group as our prototype. It is a very easy one with which to work and will exhibit the method we have in mind very nicely. It also contains most of the features we believe exist in the weak, electromagnetic, and strong interactions. It does not explain why we have more than one family of fermions and if for no other reason is therefore seriously lacking.

However, we do introduce three families of Dirac fermions where Ψ_k refers to leptons and Q_k to the quarks. The index k refers to the families. We let the left-handed fermions transform as isotopic doublets. Thus,

$$\Psi_{kL}=\begin{bmatrix}\nu_k\\ E_k\end{bmatrix}_L \tag{2.1}$$

and

$$Q_{kL}=\begin{bmatrix}U_k\\ D_k\end{bmatrix}_L , \tag{2.2}$$

while the right-handed fermions (excluding neutrinos) transform as SU(2) singlets. We denote them by E_{kR}, U_{kR}, and D_{kR}.

We require that the model be locally gauge invariant and to this end introduce the gauge fields T^a_μ for SU(3), $\vec{V}_\mu$ for SU(2), and S_μ for U(1). However, we are not required to intoduce kinetic energies for the gauge fields and hence we shall not.

Our Lagrangian may thus be written as

$$\begin{aligned}\mathscr{L}_1=\sum_{k=1}^{3}\Bigg[&i\bar{\Psi}_{kL}\left(\partial\!\!\!/-i\frac{\vec{\tau}}{2}\cdot\vec{V\!\!\!/}-i\frac{Y_{lL}}{2}S\!\!\!/\right)\Psi_{kL}+i\bar{E}_{kR}\left(\partial\!\!\!/-i\frac{Y_{lR}}{2}S\!\!\!/\right)E_{kR}\\&+i\bar{Q}_{kL}\left(\partial\!\!\!/-i\frac{\vec{\tau}}{2}\cdot\vec{V\!\!\!/}-i\frac{Y_{qL}}{2}S\!\!\!/-i\frac{\lambda_a}{2}T\!\!\!/^a\right)Q_{kL}+i\bar{U}_{kR}\left(\partial\!\!\!/-i\frac{Y_{uR}}{2}S\!\!\!/-i\frac{\lambda_a}{2}T\!\!\!/^a\right)U_{kR}\\&+i\bar{D}_{kR}\left(\partial\!\!\!/-i\frac{Y_{dR}}{2}S\!\!\!/-i\frac{\lambda_a}{2}T\!\!\!/^a\right)D_{kR}\Bigg]+\left\{\left[\sum_{k=1}^{3}(f_{kl}\bar{\Psi}_{kL}E_{kR}-f_{ku}\bar{Q}^G_{kR}U^c_{kL}+f_{kd}\bar{Q}_{kL}D_{kR})\right]\times(\text{H.c.})\right\}^{2/3}.\end{aligned} \tag{2.3}$$

Q^G_k is the G-conjugate field of Q_k, while U^c_k is the charge-conjugate field of U_k. There is of course a summation on color indices where quarks are involved. The above choice for the fermion interactions leads to the most economical symmetry-breaking scheme that is scale invariant. Obvious generalizations are possible.

The nonpolynomial interaction in Eq. (2.3) can be converted into the standard polynomial form upon the introduction of a scalar, SU(2)-doublet, constraint field χ. We then have

$$\mathscr{L}_1=\sum_{k=1}^{3}\Bigg[i\bar{\Psi}_{kL}\left[\partial-i\frac{\vec{\tau}}{2}\cdot\vec{\mathscr{V}}-i\frac{Y_{lL}}{2}\mathscr{S}\right]\Psi_{kL}+i\bar{E}_{kR}\left[\partial-i\frac{Y_{lR}}{2}\mathscr{S}\right]E_{kR}$$
$$+i\bar{Q}_{kL}\left[\partial-i\frac{\vec{\tau}}{2}\cdot\vec{\mathscr{V}}-i\frac{Y_{qL}}{2}\mathscr{S}-i\frac{\lambda_a}{2}T^a\right]Q_{kL}+i\bar{U}_{kR}\left[\partial-i\frac{Y_{uR}}{2}\mathscr{S}-i\frac{\lambda_a}{2}T^a\right]U_{kR}$$
$$+i\bar{D}_{kR}\left[\partial-i\frac{Y_{dR}}{2}\mathscr{S}-i\frac{\lambda_a}{2}T^a\right]D_{kR}$$
$$-g_{kl}(\bar{\Psi}_{kL}E_{kR}\chi+\chi^\dagger\bar{E}_{kR}\Psi_{kL})-g_{ku}(\bar{Q}_{kL}U_{kR}\chi^G+\chi^{G\dagger}\bar{U}_{kR}Q_{kL})-g_{kd}(\bar{Q}_{kL}D_{kR}\chi+\chi^\dagger\bar{D}_{kR}Q_{kL})\Bigg]-\lambda_0(\chi^\dagger\chi)^2\ . \quad (2.4)$$

Here $\chi^G=i\tau_2\chi^*$ is the G-conjugate field of χ. We note that in Eqs. (2.3) and (2.4) no coupling constants have been introduced along with the gauge fields because they can be absorbed into these fields. This in turn stems from the fact that no kinetic energy has been introduced for the gauge fields.

The equations of motion obtained by varying χ and $\chi^\dagger$ yield, using Eqs. (2.3) and (2.4),

$$f_i=\left[\frac{27}{16}\frac{g_i{}^4}{\lambda_0{}^3}\right]^{1/2}, \quad (2.5)$$

where the index i takes on the values k_l, k_μ, or k_d. The coupling constant λ_0 is superfluous and could be absorbed into a redefined χ with a corresponding redefinition of the g_i. However, its appearance will help us make a connection with the standard theory.

It will be convenient to introduce the two-component Weyl spinors for doing integrations over the fermion fields. However, we shall not use the full machinery of the dotted and undotted notation. Thus if Ψ is a Dirac field with

$$\Psi_L=\frac{(1-\gamma_5)}{2}\Psi\ ,$$

and ψ_L is the corresponding two-component spinor with equivalent relationships for Ψ_R and ψ_R, we have

$$\bar{\Psi}_L\gamma^\mu\Psi_L=\psi_L^\dagger\bar{\sigma}^\mu\psi_L\ ,$$
$$\bar{\Psi}_R\gamma^\mu\Psi_R=\psi_R^\dagger\sigma^\mu\psi_R\ , \quad (2.6)$$
$$\bar{\Psi}_L\Psi_R\chi=i\psi_L^\dagger\psi_R\chi\ ,$$

and

$$\chi^\dagger\bar{\Psi}_R\Psi_L=-i\chi^\dagger\psi_R^\dagger\psi_L\ .$$

In Eq. (2.6), σ^μ and $\bar{\sigma}^\mu$ are 2×2 matrices given by

$$\sigma^\mu=(1,\vec{\sigma})$$

and (2.7)

$$\bar{\sigma}^\mu=(1,-\vec{\sigma})\ .$$

We use the notation ψ_L, e_R, q_L, u_R, and d_R for the Weyl fields. We then substitute the relations Eq. (2.6) into the Lagrangian Eq. (2.4). Following the authors of Ref. 2, we next integrate over the Fermi fields in the generating functional to obtain an effective Lagrangian. This makes use of the quantum fluctuations inherent in quantum field theories to develop dynamics for the boson fields. Thus, we define $\mathscr{L}_{\rm eff}$ by

$$e^{i\int d^4x\mathscr{L}_{\rm eff}}=\int\prod_{k\neq1}^{3}\mathscr{D}\psi_{kL}^\dagger\mathscr{D}\psi_{kL}\mathscr{D}e_{kR}^\dagger\mathscr{D}e_{kR}\mathscr{D}q_{kL}^\dagger\mathscr{D}q_{kL}\mathscr{D}u_{kL}^\dagger\mathscr{D}u_{kL}\mathscr{D}d_{kR}^\dagger\mathscr{D}d_{kR}e^{i\int d^4x\mathscr{L}_1}\ . \quad (2.8)$$

After discarding matrices which only change $\mathscr{L}_{\rm eff}$ by a constant, we obtain

$$\int d^4x\mathscr{L}_{\rm eff}=\sum_{k=1}^{3}[-i\,{\rm tr}\ln(1+M_{k1})-i\,{\rm tr}\ln(1+M_{k2})]\ . \quad (2.9)$$

M_{k1} arises from integrations over the lepton fields, while M_{k2} arises from integrations over the quark fields.

For each family there will be an identical result except for the coupling constants g_{kl}, etc. Thus, suppressing the family labeling for the present we have

$$M_1=\begin{bmatrix}\dfrac{1}{i\bar{\sigma}^\nu\partial_\nu}\bar{\sigma}^\mu\left[\dfrac{\vec{\tau}}{2}\cdot\vec{\rm V}_\mu+\dfrac{Y_{lL}}{2}S_\mu\right] & \dfrac{1}{i\bar{\sigma}^\mu\partial_\mu}(-ig_l\chi)\\[2ex] \dfrac{1}{i\sigma^\mu\partial_\mu}(ig_l\chi^\dagger) & \dfrac{1}{i\sigma^\nu\partial_\nu}\sigma^\mu\dfrac{Y_{lR}}{2}S_\mu\end{bmatrix}, \quad (2.10a)$$

$$M_2=\begin{bmatrix} \frac{1}{i\bar{\sigma}^\nu\partial_\nu}\bar{\sigma}^\mu\left[\frac{\vec{\tau}}{2}\cdot\vec{V}_\mu+\frac{Y_{qL}}{2}S_\mu+\frac{\lambda_a}{2}T^a_\mu\right] & \frac{1}{i\bar{\sigma}^\nu\partial_\nu}(-ig_u\chi^G) & \frac{1}{i\bar{\sigma}^\nu\partial_\nu}(-ig_d\chi) \\ \frac{1}{i\sigma^\nu\partial_\nu}(ig_u\chi^{G\dagger}) & \frac{1}{i\sigma^\nu\partial_\nu}\sigma^\mu\left[\frac{Y_{uR}}{2}S_\mu+\frac{\lambda_a}{2}T^a_\mu\right] & 0 \\ \frac{1}{i\sigma^\nu\partial_\nu}(ig_d\chi^\dagger) & 0 & \frac{1}{i\sigma^\nu\partial_\nu}\sigma^\mu\left[\frac{Y_{dR}}{2}S_\mu+\frac{\lambda_a}{2}T^a_\mu\right] \end{bmatrix}, \quad (2.10b)$$

where χ is a column matrix and $\chi^\dagger$ a row matrix:

$$\chi=\begin{bmatrix}\chi^{(+)}\\ \chi^{(0)}\end{bmatrix},\quad \chi^\dagger=(\chi^{(+)\dagger},\chi^{(0)\dagger}) . \quad (2.11)$$

In terms of Feynman diagrams, the logarithms in Eq. (2.9) are the sum of all one-fermion-loop contributions with any number of external boson legs. We of course obtain only one-fermion loops because the fermions appear only quadratically in $\mathscr{L}_1$.

The ultraviolet divergences arise only from those diagrams having four or fewer external legs. These are the ones that generate the dynamics for the boson fields and hence are the ones of interest. We use dimensional regularization, thus preserving gauge and scale invariance. For these particular diagrams, the infrared divergences disappear as we let the complex dimension d approach the neighborhood of 4. Diagrams with more than four legs need not be considered as a result of the procedure we use in what follows.

The trace to be taken in Eq. (2.9) is with respect to space-time points, the Pauli matrices, and the internal-symmetry matrices. In evaluating the traces and matrix multiplication indicated in Eq. (2.9) we have made use of the following identities:

$$(\sigma^\mu k_\mu)(\bar{\sigma}^\nu k_\nu)=(k^0)^2-\vec{k}^2=k^2 ,$$
$$\mathrm{tr}_p(\sigma^\mu\bar{\sigma}^\nu)=2\eta^{\mu\nu} , \quad (2.12)$$
$$\mathrm{tr}_p(\sigma^\mu\bar{\sigma}^\nu\sigma^\lambda\bar{\sigma}^\sigma)=2(\eta^{\mu\nu}\eta^{\lambda\sigma}-\eta^{\mu\lambda}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\lambda})+2i\epsilon^{\mu\nu\lambda\sigma} .$$

In Eqs. (2.12), $\eta^{\mu\nu}$ is the Lorentz metric, $\epsilon^{\mu\nu\lambda\sigma}$ is the completely antisymmetric Levi-Civita tensor, while tr_p is over the two-dimensional Pauli space only.

The ultraviolet-divergent pieces of Eq. (2.9) will be exhibited as poles at $d=4$. Letting $d=4$ in the residue at these poles, we obtain

$$\mathscr{L}_{\mathrm{UV}}=\frac{3Y^2}{48\pi^2(4-d)}[-\tfrac{1}{4}S_{\mu\nu}(x)S^{\mu\nu}(x)]+\frac{3I^2}{48\pi^2(4-d)}[-\tfrac{1}{4}\vec{V}_{\mu\nu}(x)\cdot\vec{V}^{\mu\nu}(x)]+\frac{3L^2}{48\pi^2(4-d)}[-\tfrac{1}{4}T^a_{\mu\nu}(x)T^{a\mu\nu}(x)]$$
$$+\frac{G^2}{8\pi^2(4-d)}[(D^\mu\chi)^\dagger(D_\mu\chi)]-\left[\lambda_0+\left[\frac{1}{8\pi^2(4-d)}\right]\sum_{k=1}^{3}[g_{kl}{}^4+3(g_{ku}{}^4+g_{kd}{}^4)]\right](\chi^\dagger\chi)^2 . \quad (2.13)$$

In Eq. (2.13), although I^2 and L^2 have different origins they both have the value $I^2=L^2=8$, a result specific to the model we have chosen,

$$Y^2=2Y_{lL}{}^2+Y_{lR}{}^2+3(2Y_{qL}{}^2+Y_{uR}{}^2+Y_{dR}{}^2) \quad (2.14)$$

and

$$G^2=\sum_{k=1}^{3}[g_{kl}{}^2+3(g_{ku}{}^2+g_{kd}{}^2)] , \quad (2.15)$$

where the index k in Eq. (2.15) refers to the families. We assign the hypercharge the values of the Weinberg-Salam model. Thus

$$Y_{lL}=-1 ,\quad Y_{lR}=-2 ,\quad Y_{qL}=\tfrac{1}{3} ,$$
$$Y_{uR}=\tfrac{4}{3}\text{ and }Y_{dR}=-\tfrac{2}{3}\text{ whence }Y^2=\tfrac{40}{3} .$$

The field strengths and covariant derivative of χ are given by

$$S_{\mu\nu}=\partial_\mu S_\nu-\partial_\nu S_\mu ,$$
$$\vec{V}_{\mu\nu}=\partial_\mu\vec{V}_\nu-\partial_\nu\vec{V}_\mu+\vec{V}_\mu x\vec{V}_\nu ,$$
$$T^a_{\mu\nu}=\partial_\mu T^a_\nu-\partial_\nu T^a_\mu+f^{abc}T^b_\mu T^c_\nu , \quad (2.16)$$
$$D_\mu\chi=\left[\partial_\mu-\frac{i}{2}S_\mu-i\frac{\vec{\tau}}{2}\cdot\vec{V}_\mu\right]\chi .$$

For the purpose of brevity we had anticipated in writing the next to last term of Eq. (2.13) the subsequent assignment of the hypercharge.

We now follow the procedure of Terazawa *et al.*,[4] and note that we may add to $\mathscr{L}_1$ the term $\mathscr{L}_{\mathrm{UV}}$ to form

$$\mathscr{L}_2=\mathscr{L}_1+\mathscr{L}_{\mathrm{UV}} . \quad (2.17)$$

The Lagrangian $\mathscr{L}_1$ is then equal to $\mathscr{L}_2$ plus counterterms ($-\mathscr{L}_{\mathrm{UV}}$). In fact, if we consider the generating functional constructed using $\mathscr{L}_2$ and integrate over the Fermi fields, we obtain the same result as is obtained using $\mathscr{L}_1$ once the fields have been rescaled and coupling constants defined (see below and Sec. III). As an aside, we are reminded that we are to take the limit $d=4$ only after all renormalizations have been performed.

We now rescale the Bose fields by letting

$$\vec{A}_\mu=\left[\frac{3I^2}{48\pi^2(4-d)}\right]^{1/2}\vec{V}_\mu\,,$$
$$B_\mu=\left[\frac{3Y^2}{48\pi^2(4-d)}\right]^{1/2}S_\mu\,,$$
$$F^a_\mu=\left[\frac{3L^2}{48\pi^2(4-d)}\right]^{1/2}T^a_\mu\,,\tag{2.18}$$
$$\phi=\left[\frac{G^2}{8\pi^2(4-d)}\right]^{1/2}\chi\,.$$

We also define coupling constants as follows:

$$g=\left[\frac{48\pi^2(4-d)}{3I^2}\right]^{1/2},$$
$$g'=\left[\frac{48\pi^2(4-d)}{3Y^2}\right]^{1/2},\tag{2.19}$$
$$h=\left[\frac{48\pi^2(4-d)}{3L^2}\right]^{1/2},$$

$$\lambda=\left[\frac{8\pi^2(4-d)}{G^2}\right]^2\left[\lambda_0+\frac{1}{8\pi^2(4-d)}\sum_{k=1}^{3}[g_{kl}{}^4+3(g_{ku}{}^4+g_{kd}{}^4)]\right],$$
$$G_{kl}=\left[\frac{8\pi^2(4-d)}{G^2}\right]^{1/2}g_{kl}\,,\ \text{etc.}$$

Then $\mathcal{L}_2$ is given by

$$\mathcal{L}_2=\sum_{k=1}^{3}\Bigg[i\bar{\Psi}_{kL}\left[\partial\!\!\!/-ig\frac{\vec{\tau}}{2}\cdot\vec{A\!\!\!/}-ig'\frac{Y_{lL}}{2}B\!\!\!/\right]\Psi_{kL}+i\bar{E}_{kR}\left[\partial\!\!\!/-ig'\frac{Y_{lR}}{2}B\!\!\!/\right]E_{kR}+i\bar{Q}_{kL}\left[\partial\!\!\!/-ig\frac{\vec{\tau}}{2}\cdot\vec{A\!\!\!/}-ig'\frac{Y_{qL}}{2}B\!\!\!/-ih\frac{\lambda_a}{2}F\!\!\!/^a\right]Q_{kL}$$
$$+i\bar{U}_{kR}\left[\partial\!\!\!/-ig'\frac{Y_{uR}}{2}B\!\!\!/-ih\frac{\lambda_a}{2}F\!\!\!/^a\right]U_{kR}+\bar{D}_{kR}\left[\partial\!\!\!/-ig'\frac{Y_{dR}}{2}B\!\!\!/-ih\frac{\lambda_a}{2}F\!\!\!/^a\right]D_{kR}-G_{kl}(\bar{\Psi}_{kL}E_{kR}\phi+\phi^\dagger\bar{E}_{kR}\Psi_{kL})$$
$$-G_{ku}(\bar{Q}_{kL}U_{kR}\phi^G+\phi^{G\dagger}\bar{U}_{kR}Q_{kL})-G_{kd}(\bar{Q}_{kL}D_{kR}\phi+\phi^\dagger\bar{D}_{kR}Q_{kL})\Bigg]$$
$$-\lambda(\phi^\dagger\phi)^2-\tfrac{1}{4}(B_{\mu\nu})^2-\tfrac{1}{4}(\vec{A}_{\mu\nu})^2-\tfrac{1}{4}(F^a_{\mu\nu})^2+|D_\mu\phi|^2\,,\tag{2.20}$$

where

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu\,,$$
$$\vec{A}_{\mu\nu}=\partial_\mu\vec{A}_\nu-\partial_\nu\vec{A}_\mu+g\vec{A}_\mu\times\vec{A}_\nu\,,$$
$$F^a_{\mu\nu}=\partial_\mu F^a_\nu-\partial_\nu F^a_\mu+hf^{abc}F^b_\mu F^c_\nu\,,\tag{2.21}$$
$$D_\mu\phi=\left[\partial_\mu-i\frac{g'}{2}B_\mu-i\frac{g}{2}\vec{\tau}\cdot\vec{A}_\mu\right]\phi\,.$$

$\mathcal{L}_2$ is the standard form that one would use if starting with a scale- and gauge-invariant theory of SU(3) $\times$SU(2)$\times$U(1). The difference of course is that there exists relationships among the coupling constants.

We note that the coupling constants go to zero as $d\to4$. However, this is exactly what one expects of the unrenormalized coupling constants in a standard field theory. One must first renormalize the theory before letting $d\to4$. The renormalization constants are of course infinite and hence the product is defined to be the renormalized physical constants which are nonzero and finite.

III. COUPLING-CONSTANT AND MASS RELATIONSHIPS

The Weinberg angle θ_W is related to the constants g and g' by

$$\sin^2\theta_W=\frac{g'^2}{g^2+g'^2}=\frac{3}{8}\,,\tag{3.1}$$

where we have used Eq. (2.19) for g and g' with $I^2=8$ and $Y^2=\frac{40}{3}$ as calculated in Sec. II. This result for the unrenormalized Weinberg angle is the same as that calculated by Georgi and Glashow in their SU(5) model.[6] This result was also obtained by Terazawa *et al.*[4] We also find that the bare strong-coupling constant $h=g$, since $L^2=8$. Hence there is only one independent gauge coupling constant.

There will also be a spontaneous symmetry breakdown due to radiative corrections as presented by Coleman and Weinberg[7] and in the related work of Weinberg.[8] The result for the mass of the Higgs boson is the same as that given in these papers (assuming the one-loop level). However, we do obtain a new relationship among the vector-boson and quark masses.

Thus if m_i is the fermion mass (where i takes on the values k_l, k_μ, and k_d), then

$$m_i = G_i \phi_v \ , \tag{3.2}$$

where the G_i are given in Eq. (2.19) and ϕ_v is the value of the classical scalar field at which the first derivative of the scalar potential vanishes. (We again restrict our considerations to the one-loop level.)

From Eq. (2.19), the definition of G^2 in Eq. (2.15), and setting $I^2=8$, we find that

$$g^2 = \tfrac{1}{4} \sum_{k=1}^{3} [G_{kl}{}^2 + 3(G_{ku}{}^2 + G_{kd}{}^2)] \ . \tag{3.3}$$

Now the mass of the W vector boson is given by

$$M_W{}^2 = \frac{g^2 \phi_v{}^2}{2} = \tfrac{1}{8} \sum_{k=1}^{3} [G_{kl}{}^2 + 3(G_{kl}{}^2 + G_{kd}{}^2)] \phi_v{}^2 \ . \tag{3.4}$$

Substituting Eq. (3.2) into Eq. (3.4) then gives M_W in terms of the fermion masses,

$$M_W{}^2 = \tfrac{1}{8} \sum_{k=1}^{3} [m_{kl}{}^2 + 3(m_{ku}{}^2 + m_{kd}{}^2)] \ . \tag{3.5}$$

Unfortunately, inserting the present known (or suspected) masses of the fermions into this relationship gives too small a value for $M_W{}^2$. There will of course be additional corrections coming from higher-order contributions. Furthermore, there is the possibility of more massive fermions appearing in additional families.

IV. SUMMARY AND CONCLUDING REMARKS

We have considered a model in which the bosons are composite states of the fermions. The starting Lagrangian is scale and gauge invariant and dimensional regularization has been used for calculating.

In this paper we have chosen SU(3)$\times$SU(2)$\times$U(1) as the invariance group of the model. However, we wish to emphasize that this is a prototype. Our primary purpose was to exhibit the procedure and show how the bosons acquire gauge-covariant kinetic energy and how coupling constants are grown. This in turn leads to the determination of the unrenormalized Weinberg angle as $\sin^2\theta_W = \frac{3}{8}$. We also obtained a relation for the mass of the W vector boson in terms of fermion masses.

Clearly it is of interest to look at other groups. For SU(5) and beyond an interesting new feature emerges: not all Higgs boson representations are allowed if only fermionic building blocks are available.

For example, in SU(5), $\underline{5}$ and $\underline{45}$ do couple to the fundamental fermions and so bound Higgs transforming as such will be grown along with their dynamics. On the other hand, the $\underline{24}$ and $\underline{75}$ (Ref. 9) do not couple directly to the fermions and if they are required such scalars would have to be introduced *ab initio* along with their dynamics. This would lead to a model in which the initial Lagrangian would contain only fermions and a set of scalar mesons (e.g., $\underline{24}$), and could still be locally gauge invariant and scale invariant with an appropriate choice of interactions.

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