# Self-dual gauge field, its quantum fluctuations, and interacting fermions

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The quantum fluctuations about a self-dual background field in SU(2) are computed. The background field consists of parallel and equal uniform chromomagnetic and chromoelectric fields. Determination of the gluon fluctuations about this field yields zero modes, which are naturally regularized by the introduction of massless fermions. This regularization makes the integrals over all fluctuations convergent, and allows a simple computation of the vacuum energy which is shown to be lower than the energy of the configuration of zero field strength. The regularization of the zero modes also facilitates the introduction of heavy test charges which can interact with the classical background field and also exchange virtual quanta. The formalism for introducing these heavy test charges could be a good starting point for investigating the relevant physics of the self-dual background field beyond the classical level.

## I. INTRODUCTION

Non-Abelian gauge field theories are known to admit nontrivial solutions to the classical equations of motion. These field configurations are potentially of great interest in determining the vacuum structure of the underlying field theory. To be of physical relevance, these solutions should have lower energy density than the trivial perturbative ground state of vanishing field strength, and they should also be stable against quantum fluctuations corresponding to local deformations of the vacuum field. Indeed, many authors have considered field configurations of lower energy than the naive perturbative ground state for one such theory, quantum chromodynamics (QCD).[1] These configurations then serve as a starting point for models of the QCD vacuum.

One of the simplest examples of this type of field configuration which has lower vacuum energy is a pure uniform chromomagnetic field. The drawback to this solution is that it is unstable against quantum fluctuations.[2] However, it seems possible to obtain a stabilized ground state by introducing a complicated domain structure of randomly oriented chromomagnetic fields, which eliminates the long-wavelength destabilizing eigenmode. This forms the basis of what is commonly called the "Copenhagen vacuum."[3]

Another example of a field configuration with lowered vacuum energy has been considered by Leutwyler for an SU(2) gauge theory.[4] It consists of a constant (anti) self-dual Abelian vacuum field given by the vector potential

$$A_\mu^a(x) = -\tfrac{1}{2}F_{\mu\nu}x_\nu\delta^{a3}\,, \tag{1.1}$$

$$F_{\mu\nu} = \pm\tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}\,, \tag{1.2}$$

with $F_{\mu\nu}$ a constant matrix. In distinction to the uniform constant chromomagnetic field of the Copenhagen solution, the field strength of Eq. (1.1) corresponds to uniform constant parallel chromoelectric and chromomagnetic fields, due to the requirement of self-duality. This requirement is sufficient to ensure stability against localized deformations of the given field configuration, and this is explicitly shown in Leutwyler's one-loop calculation.[4] The major difficulty in this beautiful calculation is the existence of zero-energy modes which greatly complicates the analysis. In this work, we will introduce massless fermions to the former analysis and show that the fermions succeed in damping the zero modes by giving them an effective mass, and simplifying certain aspects of the calculation. The result is that once the zero modes have been lifted, all quantum fluctuations about the field of Eq. (1.1) become easily integrable to one loop. This also allows simplified expressions for quantum field propagators, and may lend itself more easily to further investigation of the physical implications of this self-dual vacuum field.

In Sec. II of this paper we will establish our notation and begin the computation of the effective Lagrangian generated by the gluon fluctuations about the self-dual solution of Eq. (1.1). We will proceed up to the point where the fermions are needed to damp the zero modes. In Sec. III, it is explicitly shown how the fermions damp the zero modes, and the magnitude of the effective mass generated for the zero modes is computed to one loop in the fermion fields. Section IV contains the completion of the computation of the effective Lagrangian generated by the gluon fluctuations begun in Sec. II, utilizing the stabilization of the zero modes. In Sec. V it is shown how very heavy quarks would be included in the Lagrangian, and effective interactions induced as the light degrees of freedom are integrated out. This gives a formalism of "test charges" in the theory that will be useful in determining the physical implications of this self-dual vacuum field configuration. Finally, in Sec. VI we summarize and make some concluding remarks.

## II. GLUON FLUCTUATIONS ABOUT THE CLASSICAL FIELD

For simplicity we will restrict ourselves to the gauge theory of SU(2). The analysis of the vacuum fluctuations will be carried out in Euclidean space, recalling that the

 

Euclidean functional integral is a legitimate representation of physical amplitudes defined in Minkowski space.[5] The schematic correspondence is

$$\langle A' | e^{-HT} | A \rangle = N \int [\mathscr{D}A] e^{S_E} , \tag{2.1}$$

where all quantities on the left side are defined in physical space, with $|A\rangle$ a gauge field configuration at $t=0$ in the Schrödinger representation, and $H$ the Hamiltonian. The right-hand side involves an integral over unphysical Euclidean field configurations with the proper boundary conditions $A(t=0)=A$ and $A(t=T)=A'$. The Euclidean action is $S_E$ and $N$ is a normalization constant. Our concern will be the use of the Euclidean functional integral,

$$Z_E = N \int [\mathscr{D}A] \exp\left[\int d^4x\, \mathscr{L}_E\right] \equiv N' \exp\left[\int d^4x\, \mathscr{L}_E^{\text{eff}}\right] , \tag{2.2}$$

to compute the effective Lagrangian $\mathscr{L}_E^{\text{eff}}$ generated by vacuum fluctuations about a classical field configuration. The Lagrangian for the pure SU(2) theory is given by

$$\mathscr{L}_E = -\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \tag{2.3a}$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\epsilon^{abc} A^b_\mu A^c_\nu . \tag{2.3b}$$

The classical equations of motion generated by this Lagrangian are

$$D^{ab}_\mu F^b_{\mu\nu} = 0 \tag{2.4}$$

with

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - g\epsilon^{abc} A^c_\mu . \tag{2.5}$$

As stated in the Introduction, the field configuration of interest that satisfies Eq. (2.4) is explicitly given by

$$\bar{A}^a_\mu = -\tfrac{1}{2}\bar{F}_{\mu\nu} x_\nu \delta^{a3} \tag{2.6a}$$

with the imposed self-duality condition

$$\bar{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\alpha\beta}\bar{F}_{\alpha\beta} . \tag{2.6b}$$

This corresponds to space-time constant parallel chromomagnetic and chromoelectric fields. A space-time coordinate rotation aligns the fields in the $z$ direction, corresponding to the specific form

$$\bar{F}_{03} = \bar{F}_{12} = B, \quad \text{all other } \bar{F}_{\mu\nu} = 0 \tag{2.7}$$

with $B$ the constant field strength of, as yet, arbitrary magnitude.

The functional integral will be analyzed in the region of the field configuration $\bar{A}^a_\mu$. The fields will be parametrized as

$$A^a_\mu(x) = \bar{A}^a_\mu(x) + b^a_\mu(x) , \tag{2.8}$$

and the Lagrangian can be expanded in powers of the small fluctuation $b^a_\mu$. With this parametrization for the fields, and introducing a background gauge-fixing term with the associated Faddeev-Popov determinant $\Delta_{\text{FP}}$, the Euclidean functional integral becomes

$$Z_E = N \int [\mathscr{D}b] \Delta_{\text{FP}} \exp\left[\int d^4x \{ -\tfrac{1}{4}\bar{F}^a_{\mu\nu}\bar{F}^a_{\mu\nu} + \tfrac{1}{2} b^a_\mu [\delta_{\mu\nu}(\bar{D}_\sigma\bar{D}_\sigma)_{ac} - (\bar{D}_\mu\bar{D}_\nu)_{ac} - 2g\epsilon^{adc}\bar{F}^d_{\mu\nu}] b^c_\nu + \tfrac{1}{2}\alpha b^a_\mu (\bar{D}_\mu\bar{D}_\nu)^{ac} b^c_\nu + O(b^3) \} \right] , \tag{2.9}$$

where "barred" terms depend only upon the background field. Choosing the gauge parameter to be $\alpha=1$, and rewriting the appropriate Faddeev-Popov term yields

$$Z_E = N \int [\mathscr{D}b] \exp\left\{ \int d^4x [ -\tfrac{1}{4}\bar{F}^a_{\mu\nu}\bar{F}^a_{\mu\nu} + \tfrac{1}{2} b^a_\mu \bar{\Theta}^{ac}_{\mu\nu} b^c_\nu + \ln \text{Det}(-\bar{D}_\sigma\bar{D}_\sigma) + O(b^3)] \right\} \tag{2.10a}$$

with

$$\bar{\Theta}^{ac}_{\mu\nu} = \delta_{\mu\nu}(\bar{D}_\sigma\bar{D}_\sigma)^{ac} - 2g\epsilon^{adc}\bar{F}^d_{\mu\nu} . \tag{2.10b}$$

The one-loop approximation will be used in computing the effective Lagrangian from Eq. (2.10). This corresponds to retaining only the terms quadratic in $b_\mu$ in the exponent. In order for the one-loop computation to make sense,

$$\int d^4x\, b^a_\mu(x) \bar{\Theta}^{ac}_{\mu\nu} b^c_\nu(x) < 0 . \tag{2.11}$$

If this is not the case, the background field $\bar{A}^a_\mu$ is unstable against quantum fluctuations in the one-loop approximation.

Formally, the integration over the $b^a_\mu$ fields can be done using

$$\int [\mathscr{D}b] \exp\left[ -\int d^4x\, b^a_u M^{ac}_{\mu\nu} b^c_\nu \right] = \text{Det}^{-1/2} M^{ac}_{\mu\nu} = \exp(-\tfrac{1}{2}\,\text{Tr}\ln M^{ac}_{\mu\nu}) . \tag{2.12}$$

Using Eq. (2.2) yields the effective Lagrangian

$$\mathscr{L}_E^{\text{eff}} = -\tfrac{1}{4}\bar{F}_{\mu\nu}\bar{F}_{\mu\nu} - \tfrac{1}{2}\,\text{Tr}\ln(-\bar{\Theta}^{ab}_{\mu\nu}) + \text{Tr}\ln(-\bar{D}_\sigma\bar{D}_\sigma)^{ab} . \tag{2.13}$$

The traces can be most easily evaluated by determining the eigenvalues of the operators $-\bar{\Theta}^{ab}_{\mu\nu}$ and $-\bar{D}_\sigma\bar{D}_\sigma$ and summing. The eigenvalue equation to solve is

$$\bar{\Theta}^{ac}_{\mu\nu} b^c_\nu = \lambda b^a_\mu . \tag{2.14}$$

From the explicit form of $\bar{A}^a_\mu$ from Eqs. (2.6) and (2.7), it follows that the eigenvalue equation for $b^3_\mu$ does not contain the background field, and becomes

$$\nabla^2 b^3_\nu = \lambda b^3_\nu \ , \tag{2.15a}$$

with solution

$$b^3_\nu = \epsilon_\nu e^{ipx}, \ \ \lambda = -p^2 \le 0 \ . \tag{2.15b}$$

The eigenvalue equations for the eigenmodes in the color directions orthogonal to the 3-direction are

$$\left[\delta_{\mu\nu}\left[\nabla^2 - \frac{g^2 B^2 x^2}{4} \mp i g x_\alpha \bar{F}_{\alpha\sigma}\partial_\sigma\right] \mp 2ig\bar{F}_{\mu\nu}\right] b^\pm_\nu = \lambda b^\pm_\mu \ , \tag{2.16}$$

where $b^\pm_\mu = b^1_\mu \pm i b^2_\mu$. The equation is further diagonalized by considering the following linear combinations of Lorentz indices, $b^-_{0\pm i3} = b^-_0 \pm i b^-_3$, giving

$$\left[\nabla^2 - \frac{g^2 B^2 x^2}{4} + i g x_\alpha \bar{F}_{\alpha\sigma}\partial_\sigma \mp 2gB\right] b^-_{0\pm i3} = \lambda b^-_{0\pm i3} \ , \tag{2.17a}$$

$$\left[\nabla^2 - \frac{g^2 B^2 x^2}{4} + i g x_\alpha \bar{F}_{\alpha\sigma}\partial_\sigma \mp 2gB\right] b^-_{1\pm i2} = \lambda b^-_{1\pm i2} \ . \tag{2.17b}$$

Complex-conjugate equations exist for $b^+_\mu$. These equations can be easily solved by the following procedure. Define the operators

$$a_\mu \equiv \partial_\mu + \frac{g}{2} B x_\mu, \ \ a^\dagger_\mu \equiv -\partial_\mu + \frac{g}{2} B x_\mu \tag{2.18a}$$

and form the linear combinations

$$\begin{aligned} C^\dagger &\equiv a^\dagger_0 + i a^\dagger_3, \ \ C \equiv a_0 - i a_3 \ , \\ D^\dagger &\equiv a^\dagger_1 + i a^\dagger_2, \ \ D \equiv a_1 - i a_2 \ , \end{aligned} \tag{2.18b}$$

which satisfy the commutation relations

$$\begin{aligned} &[C^\dagger, D^\dagger] = [C,D] = [C^\dagger, D] = 0 \ , \\ &[C, C^\dagger] = [D, D^\dagger] = 2gB \ . \end{aligned} \tag{2.18c}$$

The eigenvalue equation (2.17) can be rewritten as

$$[-(C^\dagger C + D^\dagger D) - 2gB \mp 2gB] b^-_{0\pm i3} = \lambda b^-_{0\pm i3} \ . \tag{2.19}$$

The commutation relations quickly yield the following eigenvalue spectrum:

$$\begin{aligned} b^-_{0+i3}&: \ \lambda = -2gB(n+m+2) \ , \\ b^-_{0-i3}&: \ \lambda = -2gB(n+m) \ , \\ b^-_{1+i2}&: \ \lambda = -2gB(n+m+2) \ , \\ b^-_{1-i2}&: \ \lambda = -2gB(n+m) \ , \end{aligned} \tag{2.20}$$

for $n,m = 0,1,2,\ldots,$ and similar expressions for $b^+_\mu$. Identical analysis goes through for the operator $-\bar{D}_\sigma \bar{D}_\sigma$, with the eigenvalue equation

$$-(\bar{D}_\sigma \bar{D}_\sigma)^{ac}\phi^c = \lambda \phi^a \ , \tag{2.21a}$$

yielding the eigenvalue spectrum

$$\phi^\pm: \ \lambda = 2gB(n+m+1) \ . \tag{2.21b}$$

Now, knowledge of the normal mode spectrum allows the evaluation of $\mathscr{L}^{\rm eff}_E$ from Eq. (2.13) using the identity

$$\ln a = -\int_0^\infty \frac{ds}{s} e^{-as} \ . \tag{2.22}$$

Ignoring constant terms that do not depend upon the background field,

$$\begin{aligned} \mathscr{L}^{\rm eff}_E = &-\tfrac{1}{4}\bar{F}^a_{\mu\nu}\bar{F}^a_{\mu\nu} \\ &+ 2c\int_0^\infty \frac{ds}{s} \sum_{m,n=0}^{\infty} (e^{-2gB(n+m)s} + e^{-2gB(n+m+2)s} \\ &\qquad - e^{-2gB(n+m+1)s}) \ , \end{aligned} \tag{2.23}$$

where $c$ is determined from the eigenmode normalization when taking the trace, and $c = g^2 B^2/4\pi^2$ as shown in the Appendix. This expression for $\mathscr{L}^{\rm eff}_E$ appears divergent for $s \to 0$, but this is the normal ultraviolet singularity removed by standard renormalization, as will be shown in Sec. IV. The divergence that does need further consideration is the infrared singularity as $s \to \infty$ when $n = m = 0$. The origin of this problem is the existence of zero modes of the operator $\bar{\Theta}^{ab}_{\mu\nu}$, and the lack of damping for the Gaussian integrations in these directions of field space. Our solution to this problem will be to show that the introduction of massless fermions gives these zero modes an effective mass term, making the integrations of Eq. (2.23) well behaved.

## III. MASSLESS QUARKS AND THE GLUON ZERO MODES

Massless quarks in the fundamental representation of SU(2) can be introduced into our previous analysis at a point just before the integrations over the small gluon fluctuations $b^a_\mu$ were begun. The integrand of the Euclidean functional integral of Eq. (2.10) changes by a multiplicative factor

$$Z^{\rm quarks}_E = \int [\mathscr{D}\psi][\mathscr{D}\bar{\psi}] \exp\left[\int d^4x \, \bar{\psi}(i\partial\!\!\!/ - g\bar{A\!\!\!/} - g b\!\!\!/)\psi\right] \tag{3.1}$$

with

$$b\!\!\!/ = b^a_\mu \gamma^E_\mu \frac{\sigma^a}{2} \ . \tag{3.2}$$

The $\sigma$ matrices are the usual Pauli matrices of SU(2) and the Euclidean $\gamma$ matrices have the convention

$$\{\gamma^E_\mu, \gamma^E_\nu\} = -2\delta_{\mu\nu} \ . \tag{3.3}$$

The integration over the quark fields of Eq. (3.1) can formally be done yielding

$$\begin{aligned} Z^{\rm quarks}_E &= {\rm Det}(i\partial\!\!\!/ - g\bar{A\!\!\!/} - g b\!\!\!/) \\ &= \exp\{{\rm Tr}[\ln(i\partial\!\!\!/ - g\bar{A\!\!\!/} - g b\!\!\!/)]\} \ . \end{aligned} \tag{3.4}$$

This constitutes a contribution to the effective Lagrangian of Eq. (2.13), which will be denoted as

$$\Delta\mathcal{L}_E^{\text{eff}}=\text{Tr}[\ln(i\partial\!\!\!/-g\bar{A\!\!\!/}-gb\!\!\!/)]\ . \tag{3.5}$$

The logarithm can be expanded in the small field $b_\mu^a$ to quadratic order, in keeping with the one-loop approximation of Sec. II. Using the notation $i\bar{D\!\!\!\!/}\equiv i\partial\!\!\!/-g\bar{A\!\!\!/}$,

$$\Delta\mathcal{L}_E^{\text{eff}}=\text{Tr}[\ln(i\bar{D\!\!\!\!/})]+g\,\text{Tr}\left[\frac{1}{-i\bar{D\!\!\!\!/}}b\!\!\!/\right] -\frac{g^2}{2}\text{Tr}\left[\frac{1}{-i\bar{D\!\!\!\!/}}b\!\!\!/\frac{1}{-i\bar{D\!\!\!\!/}}b\!\!\!/\right]+O(b^3)\ . \tag{3.6}$$

The first term is the usual fermionic one-loop contribution to the vacuum polarization which will not be included here. The second term can easily be shown to give a vanishing contribution by using the short-distance form of the fermion propagator, while the third term is the source of the gluon zero-mode mass term. Keeping only this term in $\Delta\mathcal{L}_E^{\text{eff}}$ and writing everything in coordinate space, the contribution to the effective action becomes

$$\Delta S_E^{\text{eff}}=g^2\int b_\mu^a(x)M_{\mu\nu}^{ac}(x,y)b_\nu^c(y)\,d^4x\,d^4y\ , \tag{3.7a}$$

where

$$M_{\mu\nu}^{ac}(x,y)=-\tfrac{1}{2}\,\text{Tr}\left[\gamma_\mu\frac{\sigma^a}{2}S(x,y)\gamma_\nu\frac{\sigma^c}{2}S(y,x)\right]\ , \tag{3.7b}$$

and

$$S(x,y)=\left\langle x\left|\frac{1}{-i\bar{D\!\!\!\!/}}\right|y\right\rangle \tag{3.7c}$$

is the fermion propagator in the background field $\bar{A}_\mu^a$. Equation (3.7b) can be quickly evaluated if the fermion propagators are known. There is a technology for determining fermion propagators in background self-dual fields that was developed by Brown *et al.*[6] originally for use in instanton calculations. Since $\bar{A}_\mu^a$ is also a self-dual field, the formalism can be carried over directly.

There is one complication to this procedure which is easily ameliorated. The fermion propagator in a self-dual field contains zero modes, making the naive expressions ill defined. However, we can temporarily introduce a small fermion mass term $m$ to regulate the zero modes, and show that in the end, due to the chirality structure of the propagator our result is finite and independent of $m$ in the $m\to 0$ limit.

Brown *et al.*[7] derive a Laurent series in $m$ for the fermion propagator of which the first few terms are

$$S(x,y)=\frac{1}{m}S_{-1}(x,y)+S_0(x,y)+mS_1(x,y)+O(m^2) \tag{3.8a}$$

with

$$S_{-1}(x,y)=[\delta^4(x-y)-\bar{D\!\!\!\!/}_x\Delta(x,y)\overleftarrow{\bar{D\!\!\!\!/}}_y]\left[\frac{1+\gamma_5}{2}\right]\ , \tag{3.8b}$$

$$S_0(x,y)=i\bar{D\!\!\!\!/}_x\Delta(x,y)\left[\frac{1-\gamma_5}{2}\right]+\Delta(x,y)i\overleftarrow{\bar{D\!\!\!\!/}}_y\left[\frac{1+\gamma_5}{2}\right]\ , \tag{3.8c}$$

$$S_1(x,y)=\Delta(x,y)\left[\frac{1-\gamma_5}{2}\right]\ . \tag{3.8d}$$

The function $\Delta(x,y)$ is defined as

$$\Delta(x,y)=\left\langle x\left|\frac{1}{-\bar{D}^2}\right|y\right\rangle\ , \tag{3.9a}$$

which has the simple representation for the field $\bar{A}_\mu^a$ of

$$\Delta(x,y)=\frac{e^{-gB(x-y)^2/8}}{4\pi^2(x-y)^2}\exp\left[\frac{i\sigma_3 g\bar{F}_{\alpha\beta}x_\alpha y_\beta}{4}\right]\ . \tag{3.9b}$$

Simple Dirac algebra involving the chiral projectors in the $m\to 0$ limit yields

$$\begin{aligned}M_{\mu\nu}^{ac}(x,y)=&-\tfrac{1}{2}\,\text{Tr}\left[\gamma_\mu\frac{\sigma^a}{2}S_0(x,y)\gamma_\nu\frac{\sigma^c}{2}S_0(y,x)\right]\\&-\tfrac{1}{2}\,\text{Tr}\left[\gamma_\mu\frac{\sigma^a}{2}S_{-1}(x,y)\gamma_\nu\frac{\sigma^c}{2}S_1(y,x)\right]\\&-\tfrac{1}{2}\,\text{Tr}\left[\gamma_\mu\frac{\sigma^a}{2}S_1(x,y)\gamma_\nu\frac{\sigma^c}{2}S_{-1}(y,x)\right]\ .\end{aligned} \tag{3.10}$$

Note that this expression is independent of $m$ as previously stated.

Equation (3.10) could be evaluated in a straightforward fashion using Eqs. (3.8) and (3.9). However, by making a brief digression into the form of the gluon zero modes which are contracted with $M_{\mu\nu}^{ac}(x,y)$, and then looking at the symmetries of the integrations over $x$ and $y$, the expressions to evaluate become much simpler. The equation for the gluon zero modes, generically denoted by $\phi(x)$ (representing $b_{0-i3}^-$, $b_{1-i2}^-$, $b_{0+i3}^+$, or $b_{1+i2}^+$), is gotten from Eq. (2.19),

$$(C^\dagger C+D^\dagger D)\phi(x)=0\ . \tag{3.11}$$

The solution is easily determined by demanding

$$C\phi(x)=0,\quad D\phi(x)=0\ , \tag{3.12}$$

which leads immediately to the solution

$$\phi(x)=Ne^{-gBx^2/4}\ , \tag{3.13}$$

where $N$ is a normalization constant. Using the fact that $\phi(x)$ is even in $x$, and that $M_{\mu\nu}^{ac}(x,y)$ will only be needed in the integrated form of Eq. (3.7a), allows one to average $M_{\mu\nu}^{ac}(x,y)$ over the coordinates $x$ and $y$ at any stage in the calculation. This greatly reduces the available tensor forms for $M_{\mu\nu}^{ac}(x,y)$ and we have the simple representation

$$M_{\mu\nu}^{ac}(x,y)=\frac{\delta_{\mu\nu}}{4}T_1(x,y)+\frac{\bar{F}_{\mu\nu}}{4B^2}T_2(x,y)\ , \tag{3.14}$$

where $T_1$ and $T_2$ can be calculated by doing the appropriate tensor projections of $M_{\mu\nu}^{ac}$ and doing the suitable coordinate averages. Straightforward calculation yields

$$M^{ac}_{\mu\nu}(x,y)=\frac{\delta_{\mu\nu}}{2}\Delta^2(\epsilon)\left[\frac{1}{\epsilon^2}+gB+g^2B^2\epsilon^2\right]\delta_{a3}\delta_{c3}+\frac{\delta_{\mu\nu}}{2}\Delta^2(\epsilon)\left[\frac{1}{\epsilon^2}+gB+\frac{3g^2B^2\epsilon^2}{2}\right]\cos(g\bar{F}_{\alpha\beta}\epsilon_\alpha R_\beta)(\delta_{a1}\delta_{c1}+\delta_{a2}\delta_{c2})$$
$$+\frac{\bar{F}_{\mu\nu}}{2B}\Delta^2(\epsilon)\left[gB+\frac{3g^2B^2\epsilon^2}{2}\right]\cos(g\bar{F}_{\alpha\beta}\epsilon_\alpha R_\beta)\epsilon_{3ac}\ , \qquad (3.15)$$

where

$$\Delta(\epsilon)=\frac{e^{-gB\epsilon^2/2}}{(4\pi\epsilon)^2} \qquad (3.16a)$$

and

$$\epsilon_\mu=\frac{(x-y)_\mu}{2},\quad R_\mu=\frac{(x+y)_\mu}{2}\ . \qquad (3.16b)$$

Equations (3.7a), (3.13), and (3.15) can now be used to compute the corrections to the gluon-zero-mode eigenvalue $\Delta\lambda$ due to the massless fermions. Denoting the gluon zero modes by $[b^a_\mu(x)]^{zm}$,

$$\Delta\lambda=\frac{g^2\int d^4x\,d^4y\,[b^a_\mu(x)]^{zm}M^{ac}_{\mu\nu}(x,y)[b^c_\nu(y)]^{zm}}{\int d^4x[b^a_\mu(x)]^{zm}[b^a_\mu(x)]^{zm}}\ , \qquad (3.17)$$

which can be reduced to

$$\Delta\lambda=\frac{\alpha_s}{8\pi^3}\int d^4\epsilon\frac{e^{-2gB\epsilon^2}}{\epsilon^4}\left[\frac{1}{\epsilon^2}+2gB+3g^2B^2\epsilon^2\right]. \qquad (3.18)$$

From this we must subtract the value of the eigenvalue one obtains for $B=0$ to get the contribution due to the fermions in the background field. This eliminates the $B$-independent singularity for $\epsilon\to 0$, and yields the finite result

$$\Delta\lambda=\Delta\lambda(B)-\Delta\lambda(0)=-\frac{\alpha_s gB}{16\pi}\ . \qquad (3.19)$$

As previously claimed, this is a nonzero stabilizing contribution to the gluon zero modes,[8] and must be added to the zero-mode eigenvalue of Eq. (2.20). All the integrations of Eq. (2.23) necessary to compute $\mathcal{L}^{\rm eff}_E$ become well defined due to the "lifting" of the zero modes, and these integrations will be done in the next section.

## IV. DETERMINATION OF $\mathcal{L}^{\rm eff}_E$

Including massless quarks in the preceding analysis has generated a contribution to the zero eigenvalue of the gluon zero modes of Sec. II. Specifically, for the eigenvalues of the $n=m=0$ modes for $b^-_{0-i3}$, $b^-_{1-i2}$, $b^+_{0+i3}$, and $b^+_{1+i2}$ of Eq. (2.20), the eigenvalue changes from zero to $(-\alpha_s gB/16\pi)$ due to the fermionic interactions. As a result, the expression for $\mathcal{L}^{\rm eff}_E$ in Eq. (2.23) must be altered by subtracting off the term corresponding to the ill-defined uncorrected zero mode, and adding the well-defined corrected term. Equation (2.23) becomes

$$\mathcal{L}^{\rm eff}_E=-B^2+\frac{g^2B^2}{2\pi^2}\int_0^\infty\frac{ds}{s}\sum_{n,m=0}^{\infty}(e^{-2gBs(n+m)}+e^{2gBs(n+m+2)}-e^{-2gBs(n+m+1)})$$
$$-\frac{g^2B^2}{2\pi^2}\int_0^\infty\frac{ds}{s}+\frac{g^2B^2}{2\pi^2}\int_0^\infty\frac{ds}{s}\exp\left[-s\left[\frac{\alpha_s gB}{16\pi}\right]\right]. \qquad (4.1)$$

Using the simple identity

$$\frac{1}{(1-x)}=\sum_{n=0}^{\infty}(x)^n\ \text{ for } x<1 \qquad (4.2)$$

gives the following expression for $\mathcal{L}^{\rm eff}_E$:

$$\mathcal{L}^{\rm eff}_E=-B^2+\frac{g^2B^2}{2\pi^2}\int_0^\infty\frac{ds}{s}\left\{\frac{1}{4\sinh^2(gBs)}+\exp\left[-s\left[\frac{\alpha_s gB}{16\pi}\right]\right]\right\}. \qquad (4.3)$$

This expression must be renormalized in the usual way, and we choose the renormalization conditions of Coleman and Weinberg.[9] The conditions on the renormalized Lagrangian $\bar{\mathcal{L}}$ are

$$\bar{\mathcal{L}}\,|_{B=0}=0\ , \qquad (4.4a)$$

$$\left.\frac{\partial\mathcal{L}}{\partial\mathcal{F}}\right|_{\mathcal{F}^{1/2}=\mu^2}=-1\ , \qquad (4.4b)$$

where $\mathcal{F}\equiv\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}=B^2$. The condition of Eq. (4.4a) merely corresponds to demanding that the energy density in the absence of background fields is zero. Condition (4.4b) is dependent upon the fact that we worked in background gauge.[10] In these gauges the gluon-wave-function and vertex-function renormalizations are equal and canceling, leading to a simple overall renormalization of the action. The counterterm has the universal form of $ZS^{\rm YM}_{\rm classical}$, with $Z$ being independent of the choice of gauge function. As a result, the usual renormalization conditions can be expressed by means of the function $\bar{\mathcal{L}}$ only, as in Eq. (4.4).

The renormalized Lagrangian $\overline{\mathscr{L}}_E^{\text{eff}}$ to be calculated is thus

$$\overline{\mathscr{L}}_E^{\text{eff}}=\mathscr{L}_E^{\text{eff}}-B^2\left.\frac{\partial\mathscr{L}}{\partial B^2}\right|_{B=\mu^2}-\mathscr{L}\,|_{B=0}-B^2\;. \tag{4.5}$$

A straightforward computation of this finite expression gives

$$\overline{\mathscr{L}}_E^{\text{eff}}=-B^2-\frac{11g^2B^2}{24\pi^2}[\ln(B/\mu^2)-\tfrac{1}{2}]\;. \tag{4.6}$$

Correspondence with Eq. (2.1) gives the vacuum energy density $\epsilon$:

$$\epsilon=B^2+\frac{11g^2B^2}{24\pi^2}[\ln(B/\mu^2)-\tfrac{1}{2}]\;, \tag{4.7}$$

which naively has an energy minimum away from the perturbative vacuum of $B=0$. This result agrees with the computation of Leutwyler,[4] and has the same caveats with regards to interpretation as the true vacuum energy. These caveats will be discussed in Sec. VI.

The simplification we have encountered in arriving at Eq. (4.6) is that the gluon zero modes have been effectively eliminated by introducing massless quarks. While this has made the computation of $\mathscr{L}_E^{\text{eff}}$ more straightforward, it also facilitates further analysis of the physical ramifications of the background self-dual field. The lifting of the zero modes has made the gluon propagator well defined in a simple way. Consequently, heavy test charges (quarks) can easily be introduced into the theory with well-defined interactions, and the physical effects of the background field can be determined beyond the classical dynamical level of heavy quarks interacting with the background field. We can now easily include how gauge quanta are exchanged between the test charges, which is presumably a crucial part of the dynamics in a confining field theory. This incorporation of heavy quarks is the subject of the next section.

## V. INTRODUCTION OF TEST CHARGES

In order to better investigate the physics dictated by the background field configuration of $\bar{A}_\mu^a$, we will introduce test charges in the form of massive quarks. They can be introduced as a multiplicative term in the integrand of the functional integral of Eq. (2.10). Let us further proceed to the point where the light fermions have been integrated out, regularizing the gluon zero modes. The form of the functional integral with the massive quarks included is

$$Z_E=N\int[\mathscr{D}b]\exp\left\{\int d^4x[-\tfrac{1}{4}\bar{F}_{\mu\nu}^a\bar{F}_{\mu\nu}^a+\tfrac{1}{2}b_\mu^a\bar{\Theta}_{\mu\nu}'^{ac}b_\nu^c+O(b^3)+\bar{\Psi}(i\partial\!\!\!/-g\bar{A\!\!\!/}-gb\!\!\!/-M_Q)\Psi+\operatorname{Tr}\ln(-\bar{D}_\sigma\bar{D}_\sigma)]\right\}, \tag{5.1}$$

where $\bar{\Theta}_{\mu\nu}'^{ac}$ is the operator of Eq. (2.10b) with the zero-mode eigenvalues corrected by the light-quark contribution. Since the zero modes have been eliminated, $\Theta_{\mu\nu}'^{ac}$ is an invertible operator. This allows the elimination of the term linear in $b_\mu^a$ by shifting the gluon field,

$$b_\mu^a\to b_\mu^a-g(\bar{\Theta}_{\mu\alpha}'^{ac})^{-1}\bar{\Psi}\gamma_\alpha\frac{\sigma_c}{2}\Psi\;. \tag{5.2}$$

The Jacobian of this transformation is unity, and the functional integral in terms of the shifted fields becomes

$$Z_E=N\int[\mathscr{D}b]\exp\left\{\int d^4x\left[-\tfrac{1}{4}\bar{F}_{\mu\nu}^a\bar{F}_{\mu\nu}^a+\tfrac{1}{2}b_\mu^a\bar{\Theta}_{\mu\nu}'^{ac}b_\nu^c+O(b^3)+\operatorname{Tr}\ln(-\bar{D}_\sigma\bar{D}_\sigma)+\bar{\Psi}(i\partial\!\!\!/-g\bar{A\!\!\!/}-M_Q)\Psi\right.\right.$$
$$\left.\left.-\frac{g^2}{2}\bar{\Psi}\gamma_\mu\frac{\sigma_a}{2}\Psi(\bar{\Theta}_{\mu\nu}'^{ac})^{-1}\bar{\Psi}\gamma_\nu\frac{\sigma_c}{2}\Psi\right]\right\}. \tag{5.3}$$

Now, the integration over the gluon fluctuations can be done to one loop as before, giving the effective Lagrangian of Eq. (4.6) plus interaction terms for the massive quarks,

$$\mathscr{L}_Q=\bar{\Psi}(i\partial\!\!\!/-g\bar{A\!\!\!/}-M_Q)\Psi-\frac{g^2}{2}\bar{\Psi}\gamma_\mu\frac{\sigma_a}{2}\Psi(\bar{\Theta}_{\mu\nu}'^{ac})^{-1}\bar{\Psi}\gamma_\nu\frac{\sigma_c}{2}\Psi\;. \tag{5.4}$$

The computation of $(\bar{\Theta}_{\mu\nu}'^{ac})^{-1}$ is straightforward but tedious. It is defined by the integral

$$\langle x\,|\,(\bar{\Theta}_{\mu\nu}'^{ac})^{-1}\,|\,y\rangle=-\lim_{\epsilon\to0}\left[\int_0^\infty ds\,\exp\{s[\delta_{\mu\nu}(\bar{D}_\sigma\bar{D}_\sigma)^{ac}-2g\epsilon^{adc}\bar{F}_{\mu\nu}^d-\epsilon]\}\right.$$
$$\left.-\frac{[b_\mu^a(x)]_{zm}[b_\nu^c(y)]_{zm}^\dagger}{\epsilon}+\frac{[b_\mu^a(x)]_{zm}[b_\nu^c(y)]_{zm}^\dagger}{\lambda}\right], \tag{5.5}$$

where $\epsilon$ is used to regulate the original zero modes, which are then subtracted off and replaced by the proper expression for the modes regulated by the fermionic generated term, $\lambda=\alpha_s gB/16\pi$. Using the expression for the transverse $a,c=1,2$ components,

$$\langle x \mid \exp(s\bar{D}_\sigma\bar{D}_\sigma) \mid y \rangle = \left[ \frac{B}{4\pi \sinh(sB)} \right]^2 \exp\left[ -\frac{(x-y)^2 B}{4} \coth(sB) + i(F_3)\frac{\bar{F}_{\alpha\beta}x_\alpha y_\beta}{2} \right](F_3)^2 , \tag{5.6a}$$

which can be checked by verifying

$$\left[ \bar{D}_\sigma \bar{D}_\sigma - \frac{d}{ds} \right] \langle x \mid \exp(s\bar{D}_\sigma\bar{D}_\sigma) \mid y \rangle = 0 , \tag{5.6b}$$

and also using the explicit expressions for $[b_\mu^a(x)]_{zm}$ from Sec. III, we find Eq. (5.5) to be

$$\langle x \mid (\bar{\Theta}'^{ac}_{\mu\nu})^{-1} \mid y \rangle = -\frac{gB}{16\pi^2} \exp(-gB\epsilon^2) \exp[i(F_3) g\bar{F}_{\alpha\beta}\epsilon_\alpha R_\beta]$$

$$\times \left\{ \left[ \delta_{\mu\nu} + \frac{i(F_3)\bar{F}_{\mu\nu}}{B} \right] \left[ \frac{2gB}{\lambda} - \exp(2gB\epsilon^2) Ei(-2gB\epsilon^2) - C - \ln(2gB\epsilon^2) \right] \right.$$

$$\left. + \delta_{\mu\nu} \left[ \frac{1}{gB\epsilon^2} + 2\exp(2gB\epsilon^2) Ei(-2gB\epsilon^2) \right] \right\} (F_3)^2 - \frac{\delta_{a3}\delta_{c3}\delta_{\mu\nu}}{16\pi^2\epsilon^2} . \tag{5.7}$$

In these expressions, $(F_3)$ is the SU(2) adjoint generator in the three-direction, $Ei(x)$ is the exponential integral, $C$ is Euler's constant, and $\epsilon_\mu = (x-y)_\mu/2$, $R_\mu = (x+y)_\mu/2$ as before.

Given the closed form for $(\bar{\Theta}'^{ac}_{\mu\nu})^{-1}$ of Eq. (5.7) and $\mathscr{L}_Q^{\text{eff}}$ of Eq. (5.4), the dynamics of heavy quarks in the background field $\bar{A}_\mu^a$ can be investigated beyond the purely classical level. The first term in Eq. (5.4) corresponds to the classical background field interacting with the heavy quarks, and the second term allows the quarks to exchange virtual quanta. It must also be noted that the usual naive confinement criterion in terms of Wilson loops cannot be employed in this formalism due to the inclusion of dynamical fermions and their attendant screening effects. With this caveat, this would be the starting point for investigating the background field with heavy test particles.

## VI. SUMMARY AND CONCLUSIONS

The quantum fluctuations about a self-dual background field in SU(2) have been computed. The background field consists of parallel and equal uniform chromomagnetic and chromoelectric fields. Determination of the gluon flucutations about the background field yields zero modes, which are found to be naturally regularized by the introduction of massless fermions. This allows a simple computation of the vacuum energy by making the one-loop integrals over all normal modes Gaussian and damped. It also makes the gluon-fluctuation propagator well defined, and facilitates the introduction of heavy test charges which can interact with the background classical field and also exchange virtual quanta.

The one-loop computation of the vacuum energy yields the familiar expression

$$\epsilon = B^2 + \frac{11g^2B^2}{24\pi^2}[\ln(B/\mu^2) - \tfrac{1}{2}] , \tag{6.1}$$

which agrees with the formal (but unstable) case of the pure chromomagnetic field.[2] The vacuum energy has a minimum at nonzero $B = \mu^2 \exp(-24\pi^2/11g^2)$; however, this value of $B$ is too small for the one-loop approximation to be valid. It is well known from renormalization-group analysis that the loop expansion for the effective Lagrangian is only under control for strong fields, which corresponds to the short-range behavior of gauge theories.[11] However, the interesting existence of a minimum at nonzero $B$ can remain qualitatively valid beyond the one-loop approximation provided the $\beta$-function goes to infinity sufficiently fast for strong coupling.[12]

The physical significance of the field configuration is difficult to ascertain, even with the previously mentioned nice features. It is an extermely ordered state stable under local deformations, but it is not clear that this stability would not be overridden by phase space as large fluctuations are incorporated. A manifestation of this extreme ordering is the apparent breaking of Lorentz invariance due to singling out a direction for the field. (The problem of restoring this symmetry by averaging over field directions is under investigation, along with the attendant problem of violation of cluster decomposition for the unphysical gauge fields.)

Even with these caveats, the study of this field configuration may yield insight into the vacuum structure of QCD. The formalism for introducing heavy test charges into the theory should be a good starting point for investigating the relevant physics.

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## APPENDIX

The normalization constant to be computed, $c$, that occurs in Eq. (2.23) is defined by the relation

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\langle x\mid \exp(s\Theta^{ab}_{\mu\nu})\mid y\rangle$$
$$\equiv c\sum_{m,n}\exp(-s\lambda_{mn})\ , \qquad \text{(A1)}$$

where $\lambda_{mn}$ are the eigenvalues of the operator $-\Theta^{ab}_{\mu\nu}$. The color and spin multiplicities of the eigenfunctions have already been incorporated in the main text, and here $c$ must be computed as the normalization of one eigenmode, with careful attention paid to the remaining degeneracies. Denoting the eigenfunctions generically as $\phi(x)$, Eq. (2.19) gives

$$\{-\Theta^{ac}_{\mu\nu}\}b^c_\nu\to\{C^\dagger C+D^\dagger D\}\phi(x)=\lambda\phi(x) \qquad \text{(A2)}$$

with

$$[C,C^\dagger]=[D,D^\dagger]=2gB\ . \qquad \text{(A3)}$$

Given the above commutation relations and the form of Eq. (A2), it is clear that the eigenfunctions can be cataloged by the quantum numbers of a two-dimensional harmonic oscillator $(n,m)$. Using this representation, Eq. (A1) can be simplified using the completeness of states:

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\sum_{nmn'm'}\langle x\mid n'm'\rangle\langle n'm'\mid \exp(s\Theta^{ab}_{\mu\nu})\mid nm\rangle\langle nm\mid y\rangle=\lim_{x\to y}\sum_{nm}\langle x\mid nm\rangle\langle nm\mid y\rangle\exp(-s\lambda_{nm})\ . \qquad \text{(A4)}$$

Furthermore, the excited states can be written as raising operators acting on the ground state $\mid 0\rangle$,

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\sum_{n,m}\frac{(C_x^\dagger)^n(D_x^\dagger)^m}{(2gB)^{n+m}n!m!}\langle x\mid 0\rangle\langle 0\mid y\rangle(\overleftarrow{D}_y)^m(\overleftarrow{C}_y)^n\exp(-s\lambda_{nm})\ . \qquad \text{(A5)}$$

What must now be calculated is $\langle x\mid 0\rangle\langle 0\mid y\rangle$ which is nontrivial due to the eigenfunction degeneracy, as will be shown below. The ground-state wave function is defined by

$$\phi_0(x)\equiv\langle x\mid 0\rangle,\ \ C\phi_0(x)=D\phi_0(x)=0\ . \qquad \text{(A6)}$$

Solving Eq. (A6) using the differential forms of $C$ and $D$ yields

$$\phi_0(x)\sim\phi_0(x;z)=\left[\frac{gB}{2\pi}\right]^2\exp\left[-\frac{gB(x-z)^2}{4}+\frac{ig\bar{F}_{\mu\nu}x_\mu z_\nu}{2}\right], \qquad \text{(A7)}$$

where $(gB/2\pi)^2$ is gotten from normalizing in $x$, and $z_\mu$ is an arbitrary parameter, revealing the previously mentioned eigenstate degeneracy. This degeneracy implies that a general solution can be formed from an arbitary linear combination of the solutions (A7),

$$\phi_0(x)=\int\phi_0(x;z)F(z)d^4z\ , \qquad \text{(A8)}$$

where $F(z)$ is any function. This implies that $\phi_0(x;z)$ can be interpreted as a projection operator onto the ground-state sector of function space, provided it also satisfies the relation

$$\phi_0(x;y)=\int\phi_0(x;z)\phi_0(z;y)d^4z\ . \qquad \text{(A9)}$$

This is easily verified using Eq. (A7). Thus we have shown

$$\phi_0(x;y)=\langle x\mid 0\rangle\langle 0\mid y\rangle\ , \qquad \text{(A10)}$$

which can be used in Eq. (A5). This yields

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\sum_{nm}\frac{(C_x^\dagger)^n(D_x^\dagger)^m}{(2gB)^{n+m}n!m!}\phi_0(x;y)(\overleftarrow{C}_y)^n(\overleftarrow{D}_y)^m\exp(-s\lambda_{nm})\ , \qquad \text{(A11a)}$$

which becomes, after using the differential forms of $C_x^\dagger$ and $D_x^\dagger$,

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\sum_{nm}\frac{[(x_0+ix_3)-(y_0+iy_3)]^n[(x_1+ix_2)-(y_1+iy_2)]^m}{2^{n+m}n!m!}\phi_0(x;y)(\overleftarrow{C}_y)^n(\overleftarrow{D}_y)^m\exp(-s\lambda_{mn})\ . \qquad \text{(A11b)}$$

The only terms in this sum that do not vanish in this limit $x\to y$ have the differential operators in $\overleftarrow{C}_y$ and $\overleftarrow{D}_y$ acting on the terms $(x-y)$, rather than $\phi_0(x;y)$. The simple derivatives give

$$\mathrm{Tr}[\exp(s\Theta^{ab}_{\mu\nu})]=\lim_{x\to y}\sum_{nm}\phi_0(x;y)\exp(-s\lambda_{nm})=\left[\frac{gB}{2\pi}\right]^2\sum_{nm}\exp(-s\lambda_{mn})\ ,$$

and thus $c=(gB/2\pi)^2$.

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