SU(4) symmetry and meson-baryon processes in a quark model

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We evaluate the $PQ \to PQ$ and $PQ \to VQ$ (P=pseudoscalar meson, V=vector meson, Q=quark) amplitudes within the $QQ\bar{Q}$ model proposed by Mitra assuming SU(4) as the relevant internal symmetry. Compared with SU(3), a different combination of the residues of the spectator functions is found to appear in the meson-quark amplitudes. We also show the structures of the amplitudes and their connections with relevant cross sections for various meson-baryon processes assuming SU(4) symmetry. Predictions for charmed-meson production are made. Possible symmetry-breaking effects are also discussed.

Although the original quark model (with u,d,s quarks) in its "constituent" or "current" version[1] is the most economical way of describing hadron physics, the unified theory of weak and electromagnetic interactions suggested the existence of a fourth quark (c=charm quark) to suppress the strangeness-changing neutral currents.[2] Subsequent discovery of $\psi(3100)$, $\psi'(3700)$, and other new particles[3] has indeed indicated the existence of the c quark and hence an SU(4) type of internal symmetry of hadrons rather than SU(3).

Some time back Mitra[4] (hereafter referred to as I) developed a three-body quark model for the meson-baryon processes. The motivation to the model was to take into account the nonadditivity idea within the framework of the quark model on the basis of the hierarchy argument. In that model, the additivity assumption was relaxed by taking into account basic $QQ\bar{Q}$ amplitudes (Fig. 1) rather than QQ amplitudes. The model was developed as a natural improvement of the "elementary" meson model.[5] We recollect that in the "elementary" meson model, meson-baryon scattering and production processes proceed via the basic mechanism

$$P+Q^{(i)} \to P+Q^{(i)}$$

(P=incident pseudoscalar meson, $Q^{(i)}$=ith quark inside the baryon), which takes place at each individual $Q^{(i)}$ of the baryon. This model has found interesting applications in many production processes.[6–9]

The elementarity assumption for mesons in this later kind of model can be somewhat relaxed through the following device. If we assume a certain hierarchy of compositeness for various hadrons, the assumption of elementarity of mesons should first be modified by the statement that mesons are tighter structures than baryons. However, quarks, being supposed to be the ultimate constituents of hadrons, must be considered to be more elementary than mesons. With this consideration, meson-baryon scattering may be supposed to proceed through a two-step process: the first step is

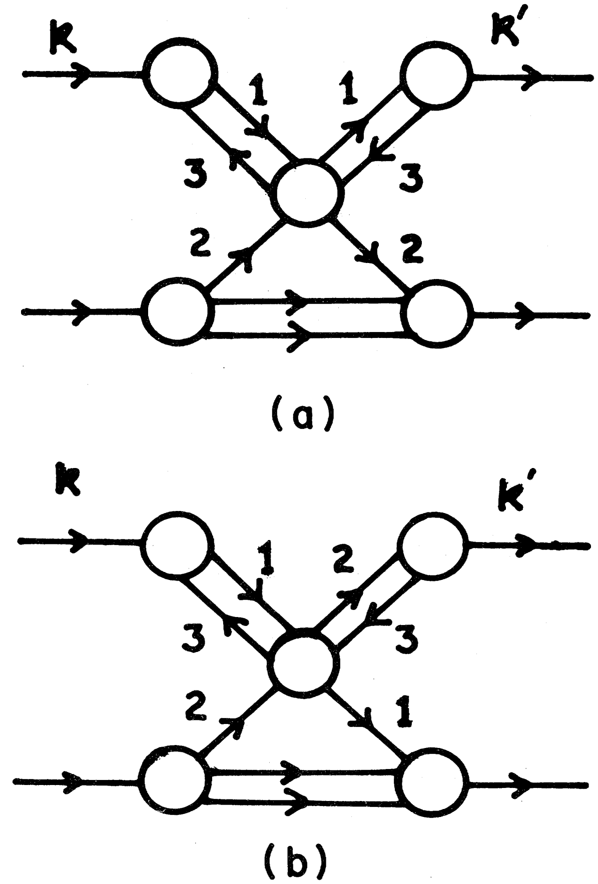

FIG. 1. (a) Meson-baryon scattering as viewed in a $QQ\bar{Q}$ model with direct term. (b) Meson-baryon scattering as viewed in a $QQ\bar{Q}$ model with exchange term.

represented by $P+Q^{(i)}\to P+Q^{(i)}$ and the second step either by the impulse approximation or by multiple scattering. However, within a quark-meson system, it is reasonable to assume that the quark can "see through" the structure of the meson as a $Q\bar{Q}$ composite (but not *vice versa*) so that the meson-quark scattering can be viewed as a three-particle ($QQ\bar{Q}$) process. This relaxation of the original elementarity of the meson will enable us to take into account all multiple-scattering corrections, at least within the tightly bound meson-quark subsystem of the full meson-baryon state. Thus such a model seems to emerge naturally as a first step to take into account the nonadditivity effects in the meson-baryon processes on the basis of hierarchy arguments.

Subsequent investigations[10–12] have shown that the model compares fairly well with experiments in various hadron-hadron and photon-hadron reactions. Prediction of the model was also shown to be in complete agreement[12] with the corresponding prediction[13,14] using the Regge-pole model.[15]

Since quark physics has gained a new dimension by the discovery of charmed particles during the last few years as mentioned above, it is therefore tempting to restudy the $QQ\bar{Q}$ model embedding an SU(4) type of internal symmetry rather than an SU(3) type. This is the aim of the present paper.

In the notation of I, the residues of the spectator functions for the even- and odd-parity cases have the expressions

$$R^{(+)}=D^{(+)}\chi_s\phi_s+\bar{D}^{(+)}\chi_a\phi_a+F^{(+)}\chi_s\phi_a+\bar{F}^{(+)}\chi_a\phi_s+k'_\mu k'_\nu\chi^{\mu\nu}(f^{(+)}\phi_a+d^{(+)}\phi_s) \tag{1}$$

and

$$R^{(-)}=D^{(-)}\vec{k}'\cdot\vec{\chi}_s\phi_s+\bar{D}^{(-)}\vec{k}'\cdot\vec{\chi}_a\phi_a+F^{(-)}\vec{k}'\cdot\vec{\chi}_s\phi_a+\bar{F}^{(-)}\vec{k}'\cdot\vec{\chi}_a\phi_s+f^{(-)}\vec{k}'\cdot\vec{\chi}^s\phi_a+d^{(-)}\vec{k}'\cdot\vec{\chi}^s\phi_s\ . \tag{2}$$

Here $\phi_{s,a}$ are the SU(4)-symmetric and antisymmetric functions defined as

$$\phi_{s,a}=\frac{1}{\sqrt{2}}[1\pm(12)_u^{\mathrm{SU}(4)}]\phi\ , \tag{3}$$

where $(12)_u^{\mathrm{SU}(4)}$, the (12) permutation operator in SU(4) space, is given by

$$(12)_u^{\mathrm{SU}(4)}=\tfrac{1}{4}+\tfrac{1}{2}\lambda^{(1)}\cdot\lambda^{(2)} \tag{4}$$

to be compared with its SU(3) counterpart

$$(12)_u^{\mathrm{SU}(3)}=\tfrac{1}{3}+\tfrac{1}{2}\lambda^{(1)}\cdot\lambda^{(2)}\ . \tag{5}$$

The basic meson-quark amplitudes are obtained by taking the overlap of Eqs. (1) and (2) with the basic functions ϕ and χ corresponding to the $\underline{20}_m$ target baryons.

In order to evaluate spin overlaps, we define, as in I,

$$\begin{aligned}
&\chi^\dagger\chi_a=\frac{1}{2\sqrt{2}}(1-\sqrt{3}V_0),\quad \chi^\dagger\chi_s=\frac{1}{2\sqrt{2}}(3+\sqrt{3}V_0),\quad V_0=\sigma^{(2)}\cdot\hat{V},\quad \sigma^{(1)}=\sqrt{3}\hat{V}\ ,\\
&\chi^\dagger\chi_{\mu\nu}k'_\mu k'_\nu=(\tfrac{3}{2})^{1/2}V_3=(\tfrac{3}{2})^{1/2}k'_\mu k'_\nu\hat{V}_\lambda(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-\tfrac{2}{3}\sigma_\lambda^{(2)}\delta_{\mu\nu})\ ,\\
&(\vec{k}\cdot\vec{\chi})^\dagger(\vec{k}'\cdot\vec{\chi}_a)=\frac{1}{2\sqrt{2}}(P-\sqrt{3}V_1),\quad (\vec{k}\cdot\vec{\chi})^\dagger(\vec{k}'\vec{\chi}_s)=\frac{1}{2\sqrt{2}}(3P+\sqrt{3}V_1)\ ,\\
&(\vec{k}\cdot\vec{\chi})^\dagger(\vec{k}'\cdot\chi^s)=(\tfrac{3}{2})^{1/2}V_2,\quad P=k_\mu k'_\nu(\delta_{\mu\nu}+i\epsilon_{\mu\nu\lambda}\sigma_\lambda^{(2)})\ ,\\
&V_1=\hat{V}_\lambda k_\mu k'_\nu(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-i\epsilon_{\mu\nu\lambda}-\delta_{\mu\nu}\sigma_\lambda^{(2)})\ ,\\
&V_2=\hat{V}_\lambda k_\mu k'_\nu(\sigma_\mu^{(2)}\delta_{\nu\lambda}-\tfrac{1}{2}\sigma_\nu^{(2)}\delta_{\mu\lambda}+\tfrac{1}{2}i\epsilon_{\mu\nu\lambda}+\tfrac{1}{2}\delta_{\mu\nu}\sigma_\lambda^{(2)})\ .
\end{aligned} \tag{6}$$

Here $\vec{k}$ and $\vec{k}'$ represent the momenta of initial and final mesons.

In order to evaluate the SU(4) overlaps, we note Eq. (4) and the SU(4) relations

$$\lambda_i^{(1)}\lambda_j^{(1)}=\tfrac{1}{2}\delta_{ij}+if_{ijk}\lambda_k^{(1)}+d_{ijk}\lambda_k^{(1)}\ , \tag{7}$$

and

$$\lambda_\beta^{(1)}\lambda_\gamma^{(1)}\lambda_\gamma^{(2)}\lambda_\alpha^{(1)}=\tfrac{1}{2}(-if_{\beta\alpha\gamma}+d_{\beta\alpha\gamma})\lambda_\gamma^{(2)}+\tfrac{1}{2}\lambda_\beta^{(2)}\lambda_\alpha^{(1)}+\lambda_\epsilon^{(1)}\lambda_\gamma^{(2)}(if_{\beta\gamma\delta}f_{\delta\alpha\epsilon}+if_{\beta\gamma\delta}d_{\delta\alpha\epsilon})+\lambda_\epsilon^{(1)}\lambda_\gamma^{(2)}(id_{\beta\gamma\delta}f_{\delta\alpha\epsilon}+d_{\beta\gamma\delta}d_{\delta\alpha\epsilon})\ . \tag{8}$$

If we assume that higher-dimensional SU(4) representations than 15-plet for mesonic states are highly suppressed one can neglect the last two terms of Eq. (8).

Collecting all the results of spin overlaps and SU(4) overlaps, the SU(4) elastic terms of the meson-quark amplitude are contained in the expression

$$\pi_\beta^\dagger \pi_\alpha [\tilde{A}_{\mathrm{SU}(4)}(\tfrac{1}{2}\delta_{\beta\alpha} + U_{\beta\alpha}^{(+)}) + \tilde{B}_{\mathrm{SU}(4)} U_{\beta\alpha}^{(-)}] , \tag{9}$$

where

$$U_{\beta\alpha}^{(+)} = (if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda_\gamma^{(1)}, \quad U_{\beta\alpha}^{(-)} = (-if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda_\gamma^{(2)} , \tag{10}$$

$$\begin{aligned}\tilde{A}_{\mathrm{SU}(4)} = \tfrac{3}{16}[&(3+\sqrt{3}V_0)(5D^{(+)}+3F^{(+)}) + (1-\sqrt{3}V_0)(3\bar{D}^{(+)}+5\bar{F}^{(+)}) \\ &+2\sqrt{3}V_3(3f^{(+)}+5d^{(+)}) + (3P+\sqrt{3}V_1)(5D^{(-)}+3F^{(-)}) \\ &+(P-\sqrt{3}V_1)(3\bar{D}^{(-)}+5\bar{F}^{(-)}) + 2\sqrt{3}V_2(3f^{(-)}+5d^{(-)})] ,\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{B}_{\mathrm{SU}(4)} = \tfrac{3}{16}[&(3+\sqrt{3}V_0)(D^{(+)}-F^{(+)}) + (1-\sqrt{3}V_0)(\bar{F}^{(+)}-\bar{D}^{(+)}) \\ &+2\sqrt{3}V_3(d^{(+)}-f^{(+)}) + (3P+\sqrt{3}V_1)(D^{(-)}-F^{(-)}) \\ &+(P+\sqrt{3}V_1)(\bar{F}^{(-)}-\bar{D}^{(-)}) + 2\sqrt{3}V_2(d^{(-)}-f^{(-)})] .\end{aligned} \tag{12}$$

This is to be compared with corresponding SU(3) quantities[4]

$$\begin{aligned}\tilde{A}_{\mathrm{SU}(3)} = &\tfrac{1}{2}A^{(+)}(3+\sqrt{3}V_0) + \tfrac{1}{2}\bar{A}^{(+)}(1-\sqrt{3}V_0) + \sqrt{3}a^{(+)}V_3 \\ &+\tfrac{1}{2}A^{(-)}(3P+\sqrt{3}V_1) + \tfrac{1}{2}\bar{A}^{(-)}(P-\sqrt{3}V_1) + \sqrt{3}a^{(-)}V_2 ,\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{B}_{\mathrm{SU}(3)} = &\tfrac{1}{4}B^{(+)}(3+\sqrt{3}V_0) - \tfrac{1}{4}\bar{B}^{(+)}(1-\sqrt{3}V_0) + \frac{\sqrt{3}}{2}b^{(+)}V_3 \\ &+\tfrac{1}{4}B^{(-)}(3P+\sqrt{3}V_1) - \tfrac{1}{4}\bar{B}^{(-)}(P-\sqrt{3}V_1) + \frac{\sqrt{3}}{2}b^{(-)}V_2 ,\end{aligned} \tag{14}$$

where

$$A^{(\pm)} = 2D^{(\pm)} + F^{(\pm)}, \quad \bar{A}^{(\pm)} = \bar{D}^{(\pm)} + 2\bar{F}^{(\pm)}, \tag{15}$$

$$B^{(\pm)} = D^{(\pm)} - F^{(\pm)}, \quad \bar{B}^{(\pm)} = \bar{D}^{(\pm)} - \bar{F}^{(\pm)}, \tag{16}$$

$$a^{(\pm)} = 2d^{(\pm)} + f^{(\pm)}, \quad b^{(\pm)} = d^{(\pm)} - f^{(\pm)} . \tag{17}$$

Here terms associated with $\tilde{A}_{\mathrm{SU}(4)}$ are called direct terms while those associated with $\tilde{B}_{\mathrm{SU}(4)}$ are called exchange terms as explained in I. Similarly the SU(4)-singlet production mechanism in the present model yields the structure

$$\sqrt{2}\lambda_\alpha^{(2)}\pi_\alpha\pi_0^\dagger\tilde{B}_{\mathrm{SU}(4)} \tag{18}$$

to be compared with the SU(3) counterpart[4]

$$(\tfrac{2}{3})^{1/2}\lambda_\alpha^{(2)}\pi_\alpha\pi_0^\dagger\tilde{B}_{\mathrm{SU}(3)} . \tag{19}$$

As in I, it is now a straightforward calculation to obtain the various scattering and production amplitudes for meson-baryon processes by folding expressions like (9) and (18) into the initial and final baryons as $3Q$ composites. For that purpose one has to use the spin—unitary-spin—orbital wave functions defined for $\underline{20}_m$ and $\underline{20}_s$ representations of SU(4) as

$$\Psi_{20_s} = \chi^s\phi^s\psi^s , \tag{20}$$

$$\Psi_{20_m} = \psi^s(\chi'\phi' + \chi''\phi'')/\sqrt{2} . \tag{21}$$

In Table I, parts (a) and (b), we list the amplitudes for transitions into various meson-baryon SU(3) states for $\underline{8}\to\underline{8}$ and $\underline{8}\to\underline{10}$ transitions, respectively, using the following notations:

$$A' = \langle\chi'|\tilde{A}_{\mathrm{SU}(3)}|\chi'\rangle, \quad A'' = \langle\chi''|\tilde{A}_{\mathrm{SU}(3)}|\chi''\rangle , \tag{22}$$

$$B' = \langle\chi'|\tilde{B}_{\mathrm{SU}(3)}|\chi'\rangle, \quad B'' = \langle\chi''|\tilde{B}_{\mathrm{SU}(3)}|\chi''\rangle , \tag{23}$$

$$A^s = \langle\chi^s|\tilde{A}_{\mathrm{SU}(3)}|\chi''\rangle, \quad B^s = \langle\chi^s|\tilde{B}_{\mathrm{SU}(3)}|\chi''\rangle . \tag{24}$$

The corresponding amplitudes for transitions into various meson-baryon SU(4) states for $\underline{20}_m\to\underline{20}_m$ and $\underline{20}_m\to\underline{20}_s$, are listed in Table II, parts (a) and (b), respectively. The notations used are as follows:

$$\begin{aligned}A'_{\mathrm{SU}(4)} &= \langle\chi'|\tilde{A}_{\mathrm{SU}(4)}|\chi'\rangle , \\ A''_{\mathrm{SU}(4)} &= \langle\chi''|\tilde{A}_{\mathrm{SU}(4)}|\chi''\rangle ,\end{aligned} \tag{25}$$

$$\begin{aligned}B'_{\mathrm{SU}(4)} &= \langle\chi'|\tilde{B}_{\mathrm{SU}(4)}|\chi'\rangle , \\ B''_{\mathrm{SU}(4)} &= \langle\chi''|\tilde{B}_{\mathrm{SU}(4)}|\chi''\rangle ,\end{aligned} \tag{26}$$

$$\begin{aligned}A^s_{\mathrm{SU}(4)} &= \langle\chi^s|\tilde{A}_{\mathrm{SU}(4)}|\chi''\rangle , \\ B^s_{\mathrm{SU}(4)} &= \langle\chi^s|\tilde{B}_{\mathrm{SU}(4)}|\chi''\rangle .\end{aligned} \tag{27}$$

Comparing Tables I and II, we observe that for the processes involving the direct term alone, the

TABLE I. The SU(3) matrix elements for the baryonic $\underline{8}\to\underline{8}$ and $\underline{8}\to\underline{10}$ transitions. The initial meson is a pion or a kaon. The direct term corresponds to one associated with A' and A'' of Eq. (22) while the exchange term corresponds to one associated with B' and B'' of Eq. (23).

Transition	Direct	Exchange
(a) $\underline{8}\to\underline{8}$		
$\langle \pi^+p \mid \pi^+p \rangle$	$\frac{2}{3}A''$	$\frac{1}{3}B'$
$\langle K^+\Sigma^+ \mid \pi^+p \rangle$	$\frac{2}{3}A''$	0
$\langle \pi^-p \mid \pi^-p \rangle$	$A'+\frac{1}{3}A''$	$-\frac{1}{6}B'+\frac{1}{6}B''$
$\langle \pi^0n \mid \pi^-p \rangle$	$\frac{1}{\sqrt{2}}A'-\frac{1}{3\sqrt{2}}A''$	$-\frac{1}{2\sqrt{2}}B'+\frac{1}{6\sqrt{2}}B''$
$\langle \eta n \mid \pi^-p \rangle$	$-\frac{1}{\sqrt{6}}A'+\frac{1}{3\sqrt{6}}A''$	$-\frac{1}{2\sqrt{6}}B'+\frac{1}{6\sqrt{6}}B''$
$\langle X^0n \mid \pi^-p \rangle$	0	$-\frac{1}{\sqrt{3}}B'+\frac{1}{3\sqrt{3}}B''$
$\langle K^0\Sigma^0 \mid \pi^-p \rangle$	$-\frac{\sqrt{2}}{3}A''$	0
$\langle K^0\Lambda^0 \mid \pi^-p \rangle$	$(\frac{2}{3})^{1/2}A'$	0
$\langle K^+p \mid K^+p \rangle$	0	$\frac{1}{3}B'$
$\langle K^-p \mid K^-p \rangle$	$A'+\frac{1}{3}A''$	$-\frac{1}{6}B'-\frac{1}{6}B''$
$\langle \bar{K}^0n \mid K^-p \rangle$	$-A'+\frac{1}{3}A''$	0
$\langle \pi^-\Sigma^+ \mid K^-p \rangle$	0	$\frac{1}{3}B''$
$\langle \pi^0\Sigma^0 \mid \pi^-p \rangle$	0	$-\frac{1}{6}B''$
$\langle \pi^0\Lambda^0 \mid K^-p \rangle$	0	$\frac{1}{2\sqrt{3}}B'$
$\langle \eta\Sigma^0 \mid K^-p \rangle$	$\frac{2}{3\sqrt{3}}A''$	$-\frac{1}{6\sqrt{3}}B''$
$\langle \eta\Lambda^0 \mid K^-p \rangle$	$-\frac{2}{3}A'$	$\frac{1}{6}B'$
$\langle X^0\Sigma^0 \mid K^-p \rangle$	0	$-\frac{1}{6}(\frac{2}{3})^{1/2}B''$
$\langle X^0\Lambda^0 \mid K^-p \rangle$	0	$\frac{\sqrt{2}}{3}B'$
(b) $\underline{8}\to\underline{10}$		
$\langle \pi^0\Delta^{++} \mid \pi^+p \rangle$	$-(\frac{2}{3})^{1/2}A^s$	$\frac{1}{2}(\frac{2}{3})^{1/2}B^s$
$\langle \pi^+\Delta^+ \mid \pi^+p \rangle$	$-\frac{2}{3}A^s$	$\frac{1}{3}B^s$
$\langle \eta\Delta^{++} \mid \pi^+p \rangle$	$-\frac{1}{3}\sqrt{2}A^s$	$-\frac{1}{6}\sqrt{2}B^s$
$\langle \Delta^{++}X^0 \mid \pi^+p \rangle$	0	$-\frac{2}{3}B^s$
$\langle K^+Y^+ \mid \pi^+p \rangle$	$-\frac{2}{3}A^s$	0
$\langle \pi^0\Delta^0 \mid \pi^-p \rangle$	$-\frac{1}{3}\sqrt{2}A^s$	$\frac{1}{6}\sqrt{2}B^s$
$\langle \pi^-\Delta^+ \mid \pi^-p \rangle$	$\frac{2}{3}A^s$	$-\frac{1}{3}B^s$
$\langle \eta\Delta^0 \mid \pi^-p \rangle$	$\frac{1}{3}(\frac{2}{3})^{1/2}A^s$	$\frac{1}{6}(\frac{2}{3})^{1/2}B^s$
$\langle \Delta^0X^0 \mid \pi^-p \rangle$	0	$\frac{2}{3}(\frac{1}{3})^{1/2}B^s$
$\langle K^0Y^0 \mid \pi^-p \rangle$	$\frac{1}{3}\sqrt{2}A^s$	0
$\langle K^+\Delta^+ \mid K^+p \rangle$	0	$\frac{1}{3}B^s$
$\langle K^0\Delta^{++} \mid K^+p \rangle$	0	$-(\frac{1}{3})^{1/2}B^s$
$\langle K^-\Delta^+ \mid K^-p \rangle$	$\frac{2}{3}A^s$	0
$\langle \bar{K}^0\Delta^0 \mid K^-p \rangle$	$\frac{2}{3}A^s$	0
$\langle \pi^-Y^+ \mid K^-p \rangle$	0	$-\frac{1}{3}B^s$
$\langle \pi^0Y^0 \mid K^-p \rangle$	0	$-\frac{1}{3}B^s$
$\langle \eta Y^0 \mid K^-p \rangle$	$-\frac{2}{3}(\frac{1}{3})^{1/2}A^s$	$\frac{1}{6}(\frac{1}{3})^{1/2}B^s$
$\langle X^0Y^0 \mid K^-p \rangle$	0	$\frac{1}{3}(\frac{2}{3})^{1/2}B^s$

TABLE II. The SU(4) matrix elements for the baryonic $\underline{20}_m \to \underline{20}_m$ and $\underline{20}_m \to \underline{20}_s$ transitions. The initial meson is a pion or kaon. The direct term corresponds to one associated with $\tilde{A}_{SU(4)}$ while the exchange term corresponds to one associated with $\tilde{B}_{SU(4)}$ of the text.

Transition	Direct	Exchange
(a) $\underline{20}_m \to \underline{20}_m$		
$\langle \pi^+ p \mid \pi^+ p \rangle$	$\frac{2}{3} A''_{SU(4)}$	$\frac{3}{4} B'_{SU(4)} + \frac{1}{12} B''_{SU(4)}$
$\langle K^+ \Sigma^+ \mid \pi^+ p \rangle$	$\frac{2}{3} A''_{SU(4)}$	0
$\langle \bar{D}^0 \Sigma_c^{++} \mid \pi^+ p \rangle$	$\frac{4}{3} A''_{SU(4)}$	0
$\langle \pi^- p \mid \pi^- p \rangle$	$A'_{SU(4)} + \frac{1}{3} A''_{SU(4)}$	$-\frac{1}{4} B'_{SU(4)} + \frac{5}{12} B''_{SU(4)}$
$\langle \pi^0 n \mid \pi^- p \rangle$	$\frac{1}{\sqrt{2}} A'_{SU(4)} - \frac{1}{3\sqrt{2}} A''_{SU(4)}$	$-\frac{1}{\sqrt{2}} B'_{SU(4)} + \frac{1}{3\sqrt{2}} B''_{SU(4)}$
$\langle \eta n \mid \pi^- p \rangle$	$-\frac{1}{\sqrt{6}} A'_{SU(4)} + \frac{1}{3\sqrt{6}} A''_{SU(4)}$	$-\frac{1}{\sqrt{6}} B'_{SU(4)} + \frac{1}{3\sqrt{6}} B''_{SU(4)}$
$\langle \chi n \mid \pi^- p \rangle$	0	$-B'_{SU(4)} + \frac{1}{3} B''_{SU(4)}$
$\langle K^0 \Sigma^0 \mid \pi^- p \rangle$	$-\frac{\sqrt{2}}{3} A''_{SU(4)}$	0
$\langle K^0 \Lambda^0 \mid \pi^- p \rangle$	$(\frac{2}{3})^{1/2} A'_{SU(4)}$	0
$\langle \eta' n \mid \pi^- p \rangle$	$-\frac{1}{2\sqrt{3}} A'_{SU(4)} + \frac{1}{6\sqrt{3}} A''_{SU(4)}$	$-\frac{1}{2\sqrt{3}} B'_{SU(4)} + \frac{1}{6\sqrt{3}} B''_{SU(4)}$
$\langle D^- \Sigma_c^+ \mid \pi^- p \rangle$	$-\frac{4}{3\sqrt{2}} A''_{SU(4)}$	0
$\langle K^+ p \mid K^+ p \rangle$	0	$\frac{3}{4} B'_{SU(4)} + \frac{1}{12} B''_{SU(4)}$
$\langle K^- p \mid K^- p \rangle$	$A'_{SU(4)} + \frac{1}{3} A''_{SU(4)}$	$-\frac{1}{4}(B'_{SU(4)} + B''_{SU(4)})$
$\langle \bar{K}^0 n \mid K^- p \rangle$	$-A'_{SU(4)} + \frac{1}{3} A''_{SU(4)}$	0
$\langle \pi^- \Sigma^+ \mid K^- p \rangle$	0	$\frac{2}{3} B''_{SU(4)}$
$\langle \pi^0 \Sigma^0 \mid K^- p \rangle$	0	$-\frac{1}{3} B''_{SU(4)}$
$\langle \pi^0 \Lambda^0 \mid K^- p \rangle$	0	$\frac{1}{\sqrt{3}} B'_{SU(4)}$
$\langle \eta \Sigma^0 \mid K^- p \rangle$	$\frac{2}{3\sqrt{3}} A''_{SU(4)}$	$-\frac{1}{3\sqrt{3}} B''_{SU(4)}$
$\langle \eta \Lambda^0 \mid K^- p \rangle$	$-\frac{2}{3} A'_{SU(4)}$	$\frac{1}{3} B'_{SU(4)}$
$\langle \chi \Sigma^0 \mid K^- p \rangle$	0	$-\frac{\sqrt{2}}{3} B''_{SU(4)}$
$\langle \chi \Lambda^0 \mid K^- p \rangle$	0	$\frac{2}{\sqrt{6}} B'_{SU(4)}$
$\langle F^- \Sigma_c^+ \mid K^- p \rangle$	$-\frac{4}{3\sqrt{2}} A''_{SU(4)}$	0
$\langle F^- \Lambda_c^+ \mid K^- p \rangle$	$\frac{2}{\sqrt{6}} A'_{SU(4)}$	0
$\langle \eta' \Sigma^0 \mid K^- p \rangle$	$-\frac{1}{3\sqrt{6}} A''_{SU(4)}$	$-\frac{1}{3\sqrt{6}} B''_{SU(4)}$
$\langle \eta' \Lambda^0 \mid K^- p \rangle$	$\frac{1}{3\sqrt{2}} A'_{SU(4)}$	$\frac{1}{3\sqrt{2}} B'_{SU(4)}$
(b) $\underline{20}_m \to \underline{20}_s$		
$\langle \pi^0 \Delta^{++} \mid \pi^+ p \rangle$	$-\frac{2}{\sqrt{6}} A^s_{SU(4)}$	$\frac{2}{\sqrt{6}} B^s_{SU(4)}$
$\langle \pi^+ \Delta^+ \mid \pi^+ p \rangle$	$-\frac{2}{3} A^s_{SU(4)}$	$\frac{2}{3} B^s_{SU(4)}$
$\langle K^+ \Sigma^{*+} \mid \pi^+ p \rangle$	$-\frac{2}{3} A^s_{SU(4)}$	0
$\langle \bar{D}^0 \Sigma_c^{*++} \mid \pi^+ p \rangle$	$-\frac{2}{3} A^s_{SU(4)}$	0
$\langle \eta \Delta^{++} \mid \pi^+ p \rangle$	$-\frac{\sqrt{2}}{3} A^s_{SU(4)}$	$-\frac{\sqrt{2}}{3} B^s_{SU(4)}$
$\langle \eta' \Delta^{++} \mid \pi^+ p \rangle$	$-\frac{1}{3} A^s_{SU(4)}$	$-\frac{1}{3} B^s_{SU(4)}$
$\langle \chi \Delta^{++} \mid \pi^+ p \rangle$	0	$-\frac{2}{\sqrt{3}} B^s_{SU(4)}$

TABLE II. *(Continued.)*

$\langle \pi^0\Delta^0 \mid \pi^- p \rangle$	$-\frac{\sqrt{2}}{3}A^s_{SU(4)}$	$\frac{\sqrt{2}}{3}B^s_{SU(4)}$
$\langle \pi^-\Delta^+ \mid \pi^- p \rangle$	$\frac{2}{3}A^s_{SU(4)}$	$-\frac{2}{3}B^s_{SU(4)}$
$\langle \eta\Delta^0 \mid \pi^- p \rangle$	$\frac{2}{3\sqrt{6}}A^s_{SU(4)}$	$\frac{2}{3\sqrt{6}}B^s_{SU(4)}$
$\langle \chi\Delta^0 \mid \pi^- p \rangle$	0	$\frac{2}{3}B^s_{SU(4)}$
$\langle K^0\Sigma^{*0} \mid \pi^- p \rangle$	$\frac{\sqrt{2}}{3}A^s_{SU(4)}$	0
$\langle \eta'\Delta^0 \mid \pi^- p \rangle$	$\frac{1}{3\sqrt{3}}A^s_{SU(4)}$	$\frac{1}{3\sqrt{3}}B^s_{SU(4)}$
$\langle D^-\Sigma_c^{*+} \mid \pi^- p \rangle$	$\frac{\sqrt{2}}{3}A^s_{SU(4)}$	0
$\langle K^+\Delta^+ \mid K^+ p \rangle$	0	$\frac{2}{3}B^s_{SU(4)}$
$\langle K^0\Delta^{++} \mid K^+ p \rangle$	0	$-\frac{2}{\sqrt{3}}B^s_{SU(4)}$
$\langle K^-\Delta^+ \mid K^- p \rangle$	$\frac{2}{3}A^s_{SU(4)}$	0
$\langle \bar{K}^0\Delta^0 \mid K^- p \rangle$	$\frac{2}{3}A^s_{SU(4)}$	0
$\langle \pi^-\Sigma^{*+} \mid K^- p \rangle$	0	$-\frac{2}{3}B^s_{SU(4)}$
$\langle \pi^0\Sigma^{*0} \mid K^- p \rangle$	0	$\frac{1}{3}B^s_{SU(4)}$
$\langle \eta\Sigma^{*0} \mid K^- p \rangle$	$-\frac{2}{3\sqrt{3}}A^s_{SU(4)}$	$\frac{1}{3\sqrt{3}}B^s_{SU(4)}$
$\langle \chi\Sigma^{*0} \mid K^- p \rangle$	0	$\frac{\sqrt{2}}{3}B^s_{SU(4)}$
$\langle F^-\Sigma_{c*}{}^+ \mid K^- p \rangle$	$\frac{\sqrt{2}}{3}A^s_{SU(4)}$	0
$\langle \eta'\Sigma^{*0} \mid K^- p \rangle$	$\frac{1}{3\sqrt{6}}A^s_{SU(4)}$	$\frac{1}{3\sqrt{6}}B^s_{SU(4)}$

amplitudes remain unaltered in going from SU(3) to SU(4) except the change of basis from $A'_{SU(3)}$ and $A''_{SU(3)}$ to $A'_{SU(4)}$ and $A''_{SU(4)}$. However we note that geometrical coefficients of the exchange terms (B',B'') are in general different for SU(3) and SU(4) symmetries. This is due to the fact that the structure of the permutation operator $(12)_u$ is different in the two symmetries.

From Table II we can immediately write down the experimentally accessible cross section as

$$\left[\frac{d\sigma}{d\Omega}\right]=A_1^{P,V}(\theta)(u'+u'')^2+A_{12}^{P,V}(\theta)(u'+u'')(v'+v'')+A_2^{P,V}(\theta)(v'+v'')^2+B_1^{P,V}(\theta)(3u'-u'')^2$$
$$+B_{12}^{P,V}(\theta)(3u'-u'')(3v'-v'')+B_2^{P,V}(\theta)(3v'-v'')^2\;, \tag{28}$$

where superscripts P and V correspond to pseudoscalar and vector meson, respectively. $A_1(\theta)$, $A_{12}(\theta)$, $A_2(\theta)$, $B_1(\theta)$, $B_{12}(\theta)$, and $B_2(\theta)$ are the six independent $QQ\bar{Q}$ parameters for a particular scattering angle θ. u' and u'' are the SU(4) geometric coefficients of $A'_{SU(4)}$ and $A''_{SU(4)}$, respectively, of Table II, and v' and v'' are the corresponding geometric coefficients of $B'_{SU(4)}$ and $B''_{SU(4)}$, respectively, of the same table.

Using Eq. (28) and the Table II(a) we obtain the following cross-section sum rules for charmed-particle production:

$$4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to K^+\Sigma^+}=\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to\bar{D}^0\Sigma_c^{++}}\;, \tag{29}$$

$$4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Sigma^0}=\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Sigma_c^+}\;, \tag{30}$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\bar{K}^0n}+4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Sigma^0}=2\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Lambda_c^+}\;. \tag{31}$$

Equations (29)—(31) are valid above the relevant charm thresholds, viz., $P_{lab}\sim 9.38$, 10, and 9.38 GeV/c or $E_{c.m.}\sim 4.29$, 4.46, and 4.29 GeV, respectively. We note, however, that in deriving these naive relations, we have neglected SU(3)/SU(4) symmetry breaking due to mass differences among the observed hadrons. *A priori*, such symmetry-breaking effects are not expected to be negligible at an energy regime even much above the relevant charm thresholds.

Taking symmetry breaking into account due to initial and final hadron masses at the same $E_{\text{c.m.}}$, Eqs. (29)—(31) are modified to

$$4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to K^+\Sigma^+}=\left[\frac{p^{K^+\Sigma^+}}{p^{\bar{D}^0\Sigma_c^{++}}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to\bar{D}^0\Sigma_c^{++}}, \tag{29'}$$

$$4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Sigma^0}=\left[\frac{p^{K^0\Sigma^0}}{p^{F^-\Sigma_c^+}}\right]\left[\frac{p^{K^-p}}{p^{\pi^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Sigma_c^+}, \tag{30'}$$

$$\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\bar{K}^0n}+4\left[\frac{p^{\pi^-p}}{p^{K^0\Sigma^0}}\right]\left[\frac{p^{\bar{K}^0n}}{p^{K^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Sigma^0}=2\left[\frac{p^{K^-p}}{p^{F^-\Lambda_c^+}}\right]\left[\frac{p^{\bar{K}^0n}}{p^{K^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Lambda_c^+}. \tag{31'}$$

Here, the c.m. momentum p^{ed} and p^{ab} for a process $a+b\to e+d$ is defined to be

$$\begin{aligned}p^{ed}&=\frac{1}{2E_{\text{c.m.}}}\{[E_{\text{c.m.}}{}^2-(m_e+m_d)^2][E_{\text{c.m.}}{}^2-(m_e-m_d)^2]\}^{1/2},\\ p^{ab}&=\frac{1}{2E_{\text{c.m.}}}\{[E_{\text{c.m.}}{}^2-(m_a+m_b)^2][E_{\text{c.m.}}{}^2-(m_a-m_b)^2]\}^{1/2},\end{aligned} \tag{32}$$

where the total energy in the c.m. system is

$$E_{\text{c.m.}}=E_a+E_b=E_e+E_d$$

and m_a, m_b, m_d, and m_e are the masses of the particles a, b, d, and e, respectively.

We now make an estimation of $P_{\text{lab}}/E_{\text{c.m.}}$ above which the symmetry-breaking effects will be negligible and our naive relations Eqs. (29)—(31) would be true. For order-of-magnitude estimates, we observe that a typical momentum ratio occurring in Eqs. (29′)—(31′) can be approximated as

$$\frac{p^{ed}}{p^{ab}}\approx\left[\frac{1-\left[\dfrac{m_e+m_d}{E_c}\right]^2}{1-\left[\dfrac{m_a+m_b}{E_c}\right]^2}\right]^{1/2} \tag{33}$$

which becomes unity in the limit $E_c \gg m_a$, m_b, m_d, m_e. Using the magnitudes of relevant masses occurring in Eqs. (29′)—(31′) we find that the relevant momentum ratios reach approximate saturation values 1 ± 0.01 at

$$E_{\text{c.m.}}\sim 28.2,\ 26.8,\ \text{and}\ 28.8\ \text{GeV} \tag{34}$$

and

$$P_{\text{lab}}\sim 422,\ 381,\ \text{and}\ 440\ \text{GeV}/c\ ,$$

respectively.

In Table III, we evaluate the charmed production cross section $\sigma(\pi^+p\to\bar{D}^0\Sigma_c^{++})$, $\sigma(K^-p\to F^-\Sigma_c^+)$, and $\sigma(K^-p\to F^-\Lambda_c^+)$ using Eqs. (29′)—(31′), respectively, taking proper mass correction into account.

For completeness we also estimate the laboratory momenta and c.m. energies above which the purely SU(3) type of cross-section sum rules derived from our model will be valid. As an illustration, we note that our naive SU(3) predictions

TABLE III. Estimated cross section of charmed-particle production.

Reaction	Equation	Estimated cross section σ (mb)	P_{lab} (GeV/c)	Reference
$\pi^+p\to\bar{D}^0\Sigma_c^{++}$	(29′)	$0.040\,24\pm0.004\,78$	16	16
$K^-p\to F^-\Sigma_c^+$	(30′)	$0.019\,04\pm0.002\,72$	15.2	17
$K^-p\to F^-\Lambda_c^+$	(31′)	$0.006\,99\pm0.001\,12$	$\sim$15	17 and 18

$$\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\bar{K}^0n}+2\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to K^+\Sigma^+}=2\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Lambda^0} \tag{35}$$

and

$$4\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\pi^0\Sigma^0}=\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\pi^-\Sigma^+} \tag{36}$$

at low energy become

$$\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\bar{K}^0n}+2\left[\frac{p^{\pi^+p}}{p^{K^+\Sigma^+}}\right]\left[\frac{p^{\bar{K}^0n}}{p^{K^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to K^+\Sigma^+}=2\left[\frac{p^{\pi^-p}}{p^{K^0\Lambda}}\right]\left[\frac{p^{\bar{K}^0n}}{p^{K^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Lambda} \tag{35'}$$

and

$$4\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\pi^0\Sigma^0}=\left[\frac{p^{K^-p}}{p^{\pi^-\Sigma^+}}\right]\left[\frac{p^{\pi^0\Sigma^0}}{p^{K^-p}}\right]\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\pi^-\Sigma^+}. \tag{36'}$$

Here we have used the notations as defined in Eq. (32). Using Eq. (33) we find that symmetry-breaking effects due to hadronic masses become negligible at $E_{\text{c.m.}}\sim 9.23$ and 1.5 GeV and $P_{\text{lab}}\sim 44$ and 0.7 GeV/c for Eqs. (35′) and (36′), respectively. Since these two equations, respectively, involve SU(3) and SU(2) symmetry-breaking effects to be compared with SU(4) symmetry-breaking effects of Eqs. (29′)—(31′), our estimates suggest that SU(2), SU(3), and SU(4) mass-breaking effects become negligible at the c.m. energies around 1.5, 10, and 30 GeV, respectively.

We note that as our meson-quark amplitude Eq. (9) can incorporate only $\Delta Y=0$ and $\Delta Y=1$ transitions and not $\Delta Y=2$ ones, our analysis cannot yield any information about processes like $K^-p\to K^+\Xi^-$, $K^0\Xi^0$.

We now discuss how our predictions (29)—(31) compare with those of Regge theory. Since the cross section in the Regge theory has a factor $\sim s^{2\mathscr{L}(0)}$, where $\mathscr{L}(0)$ is the Regge intercept, and since one expects that the trajectory of the charmed hadrons has a lower Regge intercept than the noncharmed one, the right-hand sides of Eqs. (29)—(31) will probably fall faster than the left-hand sides. This kind of behavior will be in conflict with our naive predictions which do not contain different energy dependences among the cross sections involved. Thus, in usual Regge theory such symmetry relations should perhaps be valid only among the relevant residues which have no strong energy dependences.

In order to incorporate such SU(4) symmetry-breaking effects within our formalism we have to assume that the initial meson-quark amplitude Eq. (9) satisfies only an SU(3) type of symmetry, rather than SU(4). This can be achieved if we assume that transitions between (u,d,s) quarks occur through a set of residue functions

$$D_L^{(\pm)},\bar{D}_L^{(\pm)},F_L^{(\pm)},\bar{F}_L^{(\pm)},f_L^{(\pm)},d_L^{(\pm)},$$

while those between c and (u,d,s) quarks occur through another set of residue functions

$$D_H^{(\pm)},\bar{D}_H^{(\pm)},F_H^{(\pm)},\bar{F}_H^{(\pm)},f_H^{(\pm)},d_H^{(\pm)}.$$

Then our relations Eqs. (29)—(31) get modified to be

$$\frac{\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to\bar{D}^0\Sigma_c^{++}}}{4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^+p\to K^+\Sigma^+}}=\mathscr{L}_{H/L}{}^2, \tag{37}$$

$$\frac{\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Sigma_c^+}}{4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to k^0\Sigma^0}}=\mathscr{L}_{H/L}{}^2, \tag{38}$$

and

$$\frac{2\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to F^-\Lambda_c^+}}{\left[\frac{d\sigma}{d\Omega}\right]_{K^-p\to\bar{K}^0n}+4\left[\frac{d\sigma}{d\Omega}\right]_{\pi^-p\to K^0\Sigma^0}}=\mathscr{L}_{H/L}{}^2, \tag{39}$$

where $\mathscr{L}_{H/L}$ is the asymptotic symmetry-breaking parameter of the model given by

$$\mathscr{L}_{H/L}=\frac{[3(5D_H^{(+)}+3F_H^{(+)})+(3\bar{D}_H^{(+)}+5\bar{F}_H^{(+)})]+(\vec{k}\cdot\vec{k}')[3(5D_H^{(-)}+3F_H^{(-)})+(3\bar{D}_H^{(-)}+5\bar{F}_H^{(-)})]}{[3(5D_L^{(+)}+3F_L^{(+)})+(3\bar{D}_L^{(+)}+5\bar{F}_L^{(+)})]+(\vec{k}\cdot\vec{k}')[3(5D_L^{(-)}+3F_L^{(-)})+(3\bar{D}_L^{(-)}+5\bar{F}_L^{(-)})]}. \tag{40}$$

Here $\vec{k}$ and $\vec{k}\,'$ are the momenta of the initial and final mesons.

Thus in our model, any probable violation of the relations Eqs. (29)—(31) above the limiting momenta and energies given by Eq. (34) must be interpreted as symmetry-breaking effects occurring through suitable parametrizations of $\mathscr{L}_{H/L}$. Since the present formalism cannot make any quantitative prediction about $\mathscr{L}_{H/L}$, our analysis falls short of any definite predictions about how production cross sections for charmed particles vary with increasing energies compared with noncharmed ones. For comparison, we note that in Regge theory, as noted earlier, cross sections for charmed-particle production fall at a faster rate than those for noncharmed-particle production. Such expected Regge behavior might unfortunately make the relevant cross sections for charmed-particle production quite small above the limiting energies of Eq. (34).

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