Quantum chromodynamics and the statistical hydrodynamical model of hadron production

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We analyze the Fermi-Landau statistical hydrodynamical model of hadron-hadron multiplicities in the framework of QCD, using the Pokorski—Van Hove model wherein the collision of preexisting glue dominates the multiplicity. It is noted that previous dismissal of the possibility of thermalization in the basis of nuclear "transparency" is circumvented in this picture because the valence quarks pass through, whereas the gluon clouds interact strongly. Assuming that the gluons equilibrate to a thermalized plasmoid within the Fermi-Landau (FL) Lorentz-contracted initial volume, we derive a simple formula for the multiplicity with the form $N_{\text{ch}} \approx 2.5 f^{1/4} W_{\text{had}}{}^{1/2}$ (three flavors excited), where $1-f$ is the fraction of energy carried away by the leading particles and $W_{\text{had}} = fW$ is the energy left behind. If f were fixed at a constant value of $\frac{1}{2}$, the formula would agree extremely well with data up to and including $\bar{p}p$ collider energies. (The widely held belief that collider multiplicities rule out the Fermi power law was based on the use of W rather than W_{had}.) However, using the data of Basile *et al.*, in which multiplicities are broken down as a function of W_{had} for different W values, we find that the $f^{1/4}$ dependence is ruled out. We conclude that thermalization of the colliding gluon clouds in the FL volume is also ruled out, although thermalization in the gluon fragmentation and central regions remains a possibility.

I. INTRODUCTION

The quantitative application of QCD to the description of hadron-hadron collisions has been limited principally to the calculation of "hard," typically large-$P_\perp$ process up to now, since the use of perturbation theory sanctioned by asymptotic freedom is plausible only in this domain. Moreover, the gluon content of the proton has only slowly become clear (though indirectly) through the analysis of lepton-proton deep-inelastic reactions. Although the shape of the gluon distribution function is still uncertain, the energy-momentum sum rule indicates that the gluons carry half the momentum of the proton. It is gradually becoming realized that this "preexisting" glue, no doubt connected with the confinement mechanism, plays an important role in jet production and evolution.[1]

Since only a small subset of hadronic events can be analyzed completely by perturbation theory, different techniques need to be used for the bulk of the production events. Even though cascade calculations of the evolution of hadron jets produced in e^+e^- annihilation give reasonable answers, no persuasive analog for hadron-hadron collisions has been proposed. The most popular models of hadronic production mechanisms involve color separation[2,3] (induced by single-gluon exchange) along with gluon bremsstrahlung[4,5] from colliding quarks. Neither of these mechanisms is compulsory provided the principal mechanism for converting kinetic energy to internal energy involves collision of the gluon clouds, which evolve finally to ordinary hadrons.

The intent of this work is to apply the latter physical picture, which was already proposed in 1974 by Pokorski and Van Hove,[6,7] to the description of hadron-hadron multiparticle production. In particular, we explore the possibility that a portion of the evolution involves sufficient equilibration to allow a modernized interpretation of the Fermi-Landau[8,9] statistical hydrodynamical model (SHM). (This would be useful since the numerical predictions of the SHM have been surprisingly good[10–12] for the description of multiplicities, rapidity distributions, scaling violation, and other gross features of inclusive data.) Our aim is to separate the overall collision process into space-time regions where, in turn, perturbative QCD is correct and others where some sort of statistical description is suitable. Furthermore, it is possible that various phases of QCD matter, particularly the quark-gluon plasma, might be involved at some stage of the collision.

Besides assessing the relative significance of the several mechanisms mentioned above, a full description will require knowledge of behavior of QCD matter in regimes not yet understood, especially highly nonequilibrium transport mechanisms. Nonetheless, we believe the separation into various space-time and kinematical regions to be essential for the unraveling of the complex dynamics involved. A similar point of view has been expressed by Bjorken.[13]

Over the years various models of multiparticle production have come and gone, finally falling into oblivion because of their lack of predictive power. We may mention the multiperipheral picture, the j-plane extension thereof, and the parton model, none of which bear up under critical analysis. QCD now provides flesh to the general scheme of the parton model and gives hope that honest labor will be rewarded. In the background there have been predictions based on statistical hydrodynamical reasoning.

Models of this type, originated by Fermi[8] and Landau,[9] have been mysteriously successful in predicting phenomena beyond the reach of the more fashionable models. The relative lack of interest in such models in the high-energy community is partly a result of prejudice (historically rooted in a perturbation-theory mentality) and partly in genuine obscurities in formulation of the model. We believe it is now possible to settle these questions in an objective way provided QCD is the correct theory of strong interactions. Not surprisingly, no single model is expected to describe the complex history of a typical multiparticle event.

Curiously, the revival of interest in the collective and statistical aspects of QCD has come from speculations on relativistic heavy-ion collisions and lattice-gauge-theory calculations. The realization that new phases of matter might be created in the collision of ultrarelativistic heavy ions has led to many recent theoretical speculations.[14–16] The most likely new phase is the quark-antiquark-gluon "plasma," which we shall call the QCD plasma. It has also been noted that the behavior of such a system is likely to be described to a first approximation by statistical hydrodynamical models[17–19] (SHM) of the sort originating thirty years ago in the works of Fermi and Landau, in their studies[8,9] of the mechanism for multiparticle production in proton-proton collisions.

Despite the phenomenological successes[10–12] of the "modernized" SHM in predicting (for CERN ISR energies of 30–60 GeV in the c.m.) the gross features of hadron-hadron collisions such as the energy dependence of charged multiplicities, rapidity and transverse-momentum dependence of secondaries, scaling violations, and the existence of and energy dependence of the height of the "central" region, few have taken the SHM seriously as a physical model of particle-production dynamics. The reason for this lack of acceptance is partly rooted in the "leading-particle effect" which seems to contradict the physical picture assumed in the original work of Fermi and Landau. The latter imagined the interactions to be so strong that the protons stuck together completely on collision, so that all of the incident kinetic energy was dumped into a c.m. Lorentz-contracted volume of size $4\pi/(3m_\pi^3\gamma)$ (where γ is E/M_p, E and M_p being the c.m. proton energy and mass). Since the leading particles retain about one-half the incident total energy W on the average, it would seem that hadronic material is too transparent to justify the geometrical initial conditions of the SHM. A second serious objection was raised by Moravscik and Teper,[20] who observed that the rapid deceleration required in the original Fermi-Landau picture would lead to massive radiation removal of the internal energy (even by electromagnetism).

Recently the increasing evidence for the universality of hadronic multiplicities, i.e., dependence only on available energy (rather than total energy) and independence of the initial state (e^+,e^-), (p,p), etc., led us to reexamine the whole issue. The analyses of Basile *et al.*,[21] Brick *et al.*,[22] and Breakstone *et al.*[23] are particularly interesting in supporting the idea that the *pp* multiplicity depends to first approximation only on the residual energy left (or the invariant mass) after identifying and removing the energy of leading particles. After this adjustment, the *pp* charged multiplicity plotted as a function of the available energy $W_{\rm had}$ also agrees closely with e^+e^- hadronic multiplicities with the identification $W_{\rm had}=(Q^2)^{1/2}$ over the joint c.m. energy available at PETRA/PEP and the ISR. Even diffractive multiplicities follow this law.[24]

Secondly, we wish to stress that as a function of *available energy* $W_{\rm had}$ the hadronic multiplicity varies as $N \approx 2.2\ W_{\rm had}^{1/2}$ over a vast range of initial energies.[25] Claims to the contrary result universally from the use of the total kinetic energy, rather than $W_{\rm had}$. Moreover, there is a discouraging lack of uniformity in data presentation and analysis, which causes continuing confusion. We urge that when charged multiplicities are analyzed, both the energies and charges of the leading particles be removed insofar as possible. Although the gross charged multiplicity seems indeed universal for hadron-hadron, lepton-hadron, and lepton-lepton induced reactions, inspection of more detailed features, e.g., $P_\perp$ behavior and composition of secondaries, reveals interesting and significant dependence on the initial state. In this paper, we are not going to attempt an analysis of these differences. We shall be completely concerned with the dynamics of hadron-hadron collisions.

In Sec. II, we explain how the Pokorski–Van Hove model naturally accounts for several decisive features of high-energy collisions when clarified by QCD. Section III pursues the consequences of assuming that the colliding glue equilibrates in the Fermi-Landau initial volume. This assumption is proved wrong on the basis of the data of Basile *et al.* Section IV briefly summarizes our conclusions.

II. THE POKORSKI–VAN HOVE MODEL; THE ROLE OF PREEXISTING GLUE

Leptonic probes do not interact directly with the gluon content of hadrons. Nevertheless, from studies of νN and lN deep-inelastic scattering, it has been learned that gluons can make up fully one-half the momentum content of a proton. It is reasonable to assume that this glue *preexists* in the proton as well as other hadrons as a consequence of confinement. Thus, in addition to the bremsstrahlung of glue due to valence quark collisions, there will be a very significant gluon cloud collision occurring in hadron-hadron collisions, but not in lepton-induced hadron production. Indeed, simple perturbation-theory estimates[15,26] show that the glue-glue cross sections are roughly an order of magnitude bigger than quark-quark cross sections (mainly by virtue of the color factors in the final state). This situation supports a simple physical picture put forward some time ago by Pokorski and Van Hove,[6,7] recently developed in the same spirit as here by Shuryak.[15] In this model the valence quarks of one nucleon penetrate the other nucleon without much interaction, while the remaining energy momentum, in the gluon field, is arrested in some collision volume by virtue of the much stronger interaction of the glue fields. This picture not only explains the leading-particle effect but might give a proper basis to the role of the Fermi-Landau model.

In the foregoing, it is the glue that sticks as suggested by Fermi and Landau, while the valence quarks proceed onward, only slightly excited as they redress themselves with the appropriate glue fields. There is little electromagnetic radiation since the glue is electrically neutral and since the valence quarks are not decelerated. In addition, gluon radiation caused by the stopping of the glue cloud is restrained by color confinement, ultimately resulting in the production of ordinary hadrons. In this way the "paradox" of Moravscik and Teper[20] is overcome.

We wish to further develop and to test this attractive physical picture. Before so doing we list some qualitative features of the model.

(1) The leading-particle effect and diffractive excitation are naturally accounted for. In particular, the average fraction of momentum (x) carried by the leading hadron should be independent of the total c.m. hadronic energy W.

(2) Approximate universality of total multiplicity seems natural if the bulk of the produced particles comes from the collision of the color-neutral (and charge-neutral) gluon clouds. The reaction-specific valence quarks simply go about their fate of becoming leading particles.

(3) It is natural to identify the localized glue as the "prematter,"[12] the ideal relativistic fluid comprising the stuff of the Landau hydrodynamical model, and to try to associate the large "central" (in rapidity space) multiplicities with this object.

(4) Diffraction dissociation is naturally accounted for provided unitarity is saturated in glue-glue collisions.[27]

(5) For nucleus-nucleus collisions, two very interesting features emerge. Nuclear transparency, or rather, the absence of cascades follows immediately from the above picture, since the bare outgoing valence quarks carry no glue during the dressing time $\tau \sim 1/\Lambda$ and hence because of time dilation do not interact until outside the nucleus. Because of the weak interaction of all the valence quarks comprising the nucleons, they have similar rapidities and easily reassemble into nuclear fragments. A similar picture was proposed by Bertsch *et al.*,[28] who require further that the color-neutral valence quarks be close together. The "central" excited bags begin to overlap; because of the positive surface bag energy, coalescence of the bags into a giant bag, with attendant increase in the space-time volume available for equilibration and hydrodynamic evolution.

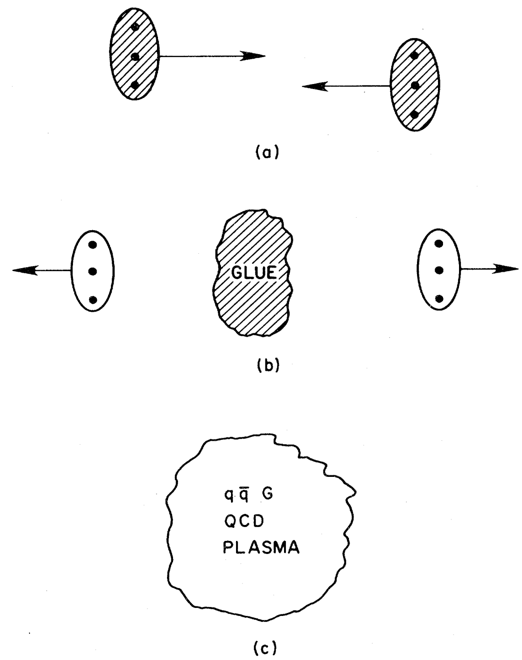

FIG. 1. In (a) two Lorentz-contracted protons are shown just before a nearly central collision in the c.m. frame. The shading represents the preexisting glue and the dots represent the valence quarks. (b) represents the situation just after collision: most of the glue has been stripped off the valence quarks, which have not yet had time to redress themselves. (c) suggests a possible evolution of the gluon into a QCD plasma of quarks, antiquarks, and glue.

III. TEST OF THE FERMI-LANDAU INITIAL VOLUME

Having reinterpreted the SHM by using the Pokorski–Van Hove model and QCD, we can test various geometrical assumptions which are a basic feature of the traditional model. Because of the assumptions of the model the question of how one-half the energy gets deposited is to first approximation not a dynamical issue.

We now focus on the glue (see Fig. 1). Just before collision, its appearance in the c.m. frame will be the usual pair of Lorentz-contracted pancakes. During the collision and after the separation of the leading particles, this localized ball of glue is presumably highly off shell, perhaps coherent and not yet a suitable object whose space-time evolution can be described by the SHM. We shall test the assumptions that the energy densities and the initial space-time scales are such that in a suitably short relaxation time τ the glue equilibrates to an expanding "plasmoid" of glue, quarks, and antiquarks in local thermodynamic equilibrium, having the geometrical shape of the traditional SHM. The plasmoid at time τ provides the proper initial condition for the Landau model, replacing the *ad hoc* boundary condition previously forced by the absence of a detailed microscopic picture of hadronic structure and interactions.

If the preceding assumptions are valid, the evolution now follows a trajectory on the multidimensional phase diagram of QCD matter. Equilibrium properties in QCD are a subject of high current interest[14,15] and not yet understood—even the equation of state remains to be discovered. The kinetics and transport properties required for a complete description are even further from our understandings for such calculations, although a covariant field-theoretic framework has been constructed.[29] For the moment we shall follow the tradition of ignoring irreversible processes except when inevitable, e.g., the latent heat discontinuities at phase boundaries for first-order phase transitions. Although many phases can be imagined for quark glue matter in various conditions, current wisdom in lattice calculations[30,31] indicates the existence of possibly three phases—ordinary (confined) hadronic matter, unconfined QCD plasma, and possibly an intermediate unconfined phase with breakdown of chiral symmetry.

A schematic trajectory[32] for the motion of a piece of hadronic matter as a function of proper time is shown in Fig. 2. As the matter cools and expands, it will undergo various phase transitions; in particular, for the situation shown in Fig. 2 we write for the final entropy

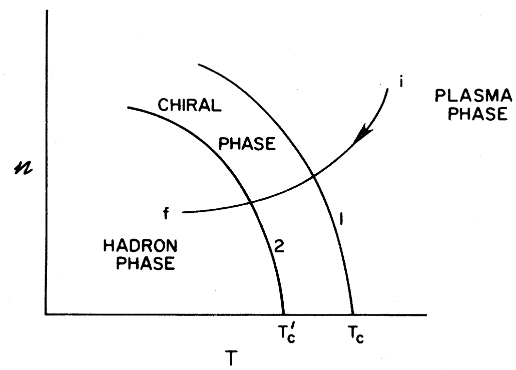

FIG. 2. The schematic evolution on a conjectured phase diagram is indicated for an initially hot dense system.

$$S_f = S_i + \sum_{j=1}^{2} \Delta S_j \ , \tag{1}$$

where ΔS_1 and ΔS_2 are possible entropy increments at the indicated phase boundaries. (It has to be realized that for the extremely small distances under consideration the phase boundary may be so diffuse that one might have to consider mixed phases.)

Note that, were it not for possible first-order phase transitions the hadronic multiplicity would follow from the initial entropy at $t=\tau$ according to the well-known argument of Landau, independently of details of the dynamics or the phase. For sufficiently high temperature, the multiplicity is completely given once the fraction of energy left in the initial state is known and the energy density at the beginning of the hydrodynamic phase is known.

We now present the numerical predictions of the simplest possible version of our hybrid model in the case of proton-proton scattering. We assume that the fireball is described by (1) massless quarks and glue with the usual QCD degrees of freedom, (2) free-particle thermodynamic quantities, (3) naive geometrical initial volume determined by overlap of Lorentz-contracted spheres of radius $R=1/m_\pi$, and (4) adiabatic evolution of the system to break up at $T=m_\pi$.

Since the dynamics on the phase diagram remains to be studied, we assume an adiabatic trajectory, i.e., total entropy is conserved. Then the initial entropy of the plasmoid is also that of the final physical hadrons

$$S_i(q\bar{q}g) = S_\pi + S_{\bar{K}K} + S_\eta + S_{p\bar{p}} + \cdots \ . \tag{2}$$

According to the second law, a more general statement is that the final entropy will be greater than or equal to the initial, so any error is at least bounded.

To calculate thermodynamic quantities, we use the expressions for massless free quarks and glue. The initial temperature is calculated from the usual Landau argument according to

$$\epsilon_i = \frac{\pi^2}{30} N_d T_i^4 \ , \tag{3}$$

where ϵ_i is the initial energy density and N_d the effective number of degrees of freedom

$$N_d = (8\times 2) + (\tfrac{7}{8}\times N_f \times 2\times 2\times 3) \ , \tag{4}$$

where eight colored gluons and three colors of quarks are assumed. N_f is the number of quark flavors excited at temperature T_i and the factor $\frac{7}{8}$ derives from the different normalizations of Fermi-Dirac and Bose integrals. Our plasmoid is color neutral, has zero total chemical potential, and is electrically neutral or nearly neutral.

To compute ϵ_i we need to know the available fireball energy W_{had} and the initial volume V_i in the plasmoid. W_{had} is related to the total energy W by

$$W_{\text{had}} = fW \ . \tag{5}$$

Currently, the fraction f has to be taken from experiment. Averaged over events, a popular value is $f \approx \frac{1}{2}$. The energy density is then

$$\epsilon_i = \frac{W_{\text{had}}}{V_i} \ , \tag{6}$$

where the usual Fermi-Landau reference volume is

$$V_i^{\text{FL}} = \frac{V_0}{\gamma} = \frac{2M_p V_0}{W} \tag{7}$$

and V_0 is traditionally $4\pi/3{m_\pi}^3\gamma$. Note that this volume, determined by kinematics and geometry, is determined by W, not W_{had}. (The need for an equilibration time can make V_0 bigger, while nonzero impact parameters decrease V_0. Still this is the obvious reference volume in the problem. The energy dependence is, however, crucial.)

From (3) and (6) we find the initial temperature

$$T_i = \left[\frac{30 W_{\text{had}}}{\pi^2 N_d V_i} \right]^{1/4} \ . \tag{8}$$

Using the Fermi-Landau volume, we get, more explicitly,

$$\frac{T_i^{\text{FL}}}{m_\pi} = K(W_{\text{had}} W)^{1/4} \ , \tag{9}$$

where

$$K = \left[\frac{45}{4\pi^3 N_d M_p m_\pi} \right]^{1/4} = 1.29 {N_d}^{-1/4} \tag{10}$$

is measured in $\text{GeV}^{-1/2}$.

Although T_i is relatively insensitive to the number of degrees of freedom, there is a big difference in the pion gas ($N_d=3$) and a QCD plasma with $N_f=3$ ($N_d=47.5$): $K_\pi/K_{\text{QCD}}=1.85$. On the other hand, going from $N_f=2$ ($N_d=38$) to $N_f=3$ gives $\Delta K/K$ of 2%. Clearly the more degrees of freedom to share the energy, the lower the temperature. In principle, one could see thresholds, for example, when the energy increases beyond a threshold $T_f=m_f$, for example, the strange-quark threshold $T_s \approx 150$ MeV. The smallness of the effect (2%) and the many other factors to consider would make this effect hard to see by thermal considerations. Other phenomena may reveal the flavor content of the plasmoid, of course.

Table I lists some predicted initial temperatures for the case $N_f=3$, $f=\frac{1}{2}$. These temperatures are to be regarded as upper bounds (except insofar as possible compressional heating has been ignored), which to be interesting must exceed the transition temperature T_c (150—250 MeV) to allow the formation of the plasma. In addition, T must exceed $m_\pi=140$ MeV to allow the Landau model to be

TABLE I. Temperatures created in the Fermi-Landau volume in head-on proton-proton collisions are given (in pion mass units and MeV) for c.m. total energy W and equivalent laboratory energy E_{lab}. Equation (9) was used, with $N_f = 3$ and $W_{\text{had}} = \frac{1}{2} W$. Compression effects were ignored.

W (GeV)	4.52	9.76	13.7	19.4	30.6	540	1000	2×10^4
E_{lab} (GeV)	10	50	100	200	500			
T_i/m_π	0.87	1.28	1.52	1.80	2.27	9.52	12.96	58.0
T_i (MeV)	122	179	213	252	318	1.3×10^3	1.8×10^3	8.1×10^3

applicable even in the hadronic phase. Although V_0 may be larger than the canonical value (see below), T_i varies slowly with V_0 ($\sim V_0^{-1/4}$).

The entropy provides the key to final-state multiplicities. The connection is not trivial, however, due to conservation laws; for example, kaons and nucleons have to be produced in pairs.[9] As the system evolves, the number of a given species may change (for example, the freezing out of heavy quanta during cooling), while the entropy is conserved, or if not conserved, hopefully generated in a calculable way. For π's and η's, we can note that at the moment of separation [on the surface $T_0(\vec{x},t) = m_\pi$] the number N_k of particles of species k is related to the entropy by

$$N_k = \chi(T_k/m_\pi) S_k \,, \tag{11}$$

where $\chi \cong \frac{1}{4}$ for $m_\pi/T_k \lesssim 1$. The function χ is tabulated in the paper by Landau and Bilenkij.[9]

For the idealized case of $\pi^+\pi^-\pi^0$ final particles $N_f \approx \frac{1}{4} S_f = \frac{1}{4} S_i$ and the calculation is simple; further the charged fraction is $\frac{2}{3}$.

Before proceeding we compute the entropy of the plasmoid. The total initial entropy is

$$S_i = 2\pi^2 N_d T_i^3 V_i / 45 \,. \tag{12}$$

Using Eq. (8), we find

$$S_i = \frac{2\pi^2}{45} \left[\frac{30}{\pi^2} \right]^{3/4} (N_d V_i)^{1/4} W_{\text{had}}^{3/4} \,. \tag{13}$$

Further, using Eq. (7) leads to

$$S_i^{\text{FL}} = 7.4 (N_d f)^{1/4} W_{\text{had}}^{1/2} = 7.4 N_d^{1/4} W_{\text{had}}^{3/4} / W^{1/4} \,, \tag{14}$$

where $f = W_{\text{had}}/W$ and energies are measured in GeV.

For orientation, suppose that the final entropy $S_f = S_i$ is completely converted to pions. Since $N \approx S_f/4$, the charged multiplicity is

$$N_{\pi^\pm} \approx \tfrac{2}{3} \times \tfrac{1}{4} S_i \approx 1.23 (N_d f)^{1/4} W_{\text{had}}^{1/2} \,. \tag{15}$$

For two active flavors $N_d = 37$ this gives

$$N_{\pi^\pm} \approx 2.46 f^{1/4} W_{\text{had}}^{1/2} \approx 2.41 W_{\text{had}}^{1/2} \,, \tag{16}$$

choosing $f = 0.4$ as a typical fraction of deposited energy. [If $N_f = 3$, $N_d = 47.5$ the second coefficient in (16) increases to 2.56, not very different.]

One year ago, the parameter-free result (16) would have seemed too good to be true, since our previous fit gave[25]

$$N_{\text{ch}} \cong 2.23 W_{\text{had}}^{0.46} \tag{17}$$

using data from $W_{\text{had}} = 9$ GeV up to ISR energies. Brick *et al.*[22] also found projectile independence as shown by

$$N_{\text{ch}}(pp) = (1.76 \pm 0.07) s_{\text{eff}}^{0.26 \pm 0.003} \,,$$
$$N_{\text{ch}}(\pi^+ p) = (1.76 \pm 0.07) s_{\text{eff}}^{0.26 \pm 0.003} \,, \tag{18}$$
$$N_{\text{ch}}(K^+ p) = (1.77 \pm 0.16) s_{\text{eff}}^{0.25 \pm 0.02} \,,$$

where their s_{eff}, found by identifying a leading particle in each hemisphere, is the same as our W_{had}^2.

However, the same data that support the dependence of the multiplicity on the available energy W_{had}, rather than W, allow us to test the geometrical factor $f^{1/4}$ in Eqs. (14)–(16) and hence to test the canonical initial condition of the SHM. For a given W, Eq. (14) predicts that the multiplicity varies as $W_{\text{had}}^{3/4}$. Alternatively, $N/f^{1/4}$ should depend on W_{had} alone, not on W. Figure 3 shows instead that $N/f^{1/4}$ does depend on W, and hence rules out the initial volume V_i of the Fermi-Landau model, which depends on the Lorentz-contraction factor of Eq. (7).

It is worth mentioning another experimental result bearing on the Fermi-Landau initial volume. Some time ago we derived the Q^2 dependence of hadronic multiplicities in lepton-proton collisions. Going to the Breit frame of the virtual photon and assuming the kinetic energy to be deposited in the Lorentz-contracted proton, the SHM predicts

$$N_{lp} = \frac{N_{pp}}{[1 + (M_p^2 + Q^2)/s]^{1/4}} \,. \tag{19}$$

The data of Allen *et al.*[33] demonstrate the Q^2 independence of the multiplicity over ranges of Q^2 and s which would easily cause a 20% variation. Nowadays the QCD reaction picture would be quite different, in that the virtual photon ejects a valence quark, creating a jet and a di-

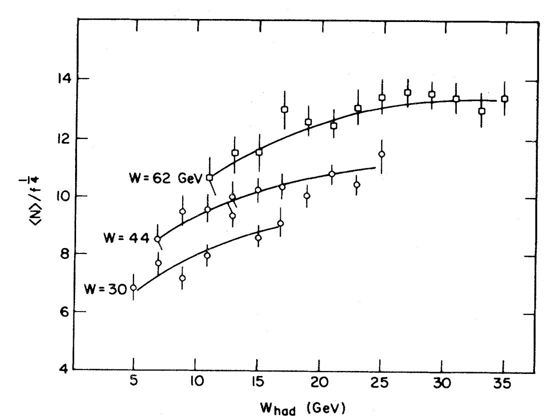

FIG. 3. The scaling law derived from Eq. (16) is tested using the data of Basile *et al.* (Ref. 21).

quark jet. The difficulty in the proton-proton case is more puzzling, however, especially since the coefficient and energy exponent of the SHM model are so close to experiment.

IV. CONCLUSIONS

The most straightforward conclusion of our analysis is that the Fermi-Landau initial geometry is wrong. We have not been able to find any physical argument to modify the Fermi-Landau geometrical determination of V_i. The tail of virtual wee partons attached[34] (by adjacent correlations in rapidity space) to the main contracted proton are most likely a small fringe of matter. We do not see how they could manage to dynamically change the longitudinal contraction from $1/W$ to $1/W_{\rm had}$. Refinements of the other assumptions following Eq. (1) do not seem to lead to any improvements. Nevertheless, it is extremely puzzling why the result (16) is so close to the experimental results of Eqs. (17) and (18). It is also puzzling that the QCD leading-logarithm multiplicity formula[35] for e^+e^- is numerically so close[25] to the Fermi power law for hadron-hadron multiplicities over a wide energy range. It is further puzzling that e^+e^-, lh, and hh multiplicities are so nearly identical (once the leading particles are removed) even though the microscopic picture is so different in these three cases. If we abandon a thermodynamic explanation for this similarity, how can we construct another argument for universality?

That the gluon clouds are not stopped in the Fermi-Landau volume does not mean that thermalization is completely ruled out. It is quite possible that the fragmentation region of the gluon cloud collision thermalizes along the lines of the analysis of Anishetty, Koehler, and McLerran.[16]

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