# Coherent scalar-field oscillations in an expanding universe

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Motivated by the cosmological importance of coherent (classical), scalar-field oscillations in the context of the invisible axion and the new inflationary-universe scenario, we analyze, in general, the classical evolution of a scalar field in an isotropic and homogeneous cosmology. For a scalar potential of the form $V(\phi)=a\phi^n$, the energy density of the scalar-field oscillations decreases as $R^{-6n/(n+2)}$ when the oscillations are rapid compared to the expansion rate ($R$ = cosmic scale factor). We also investigate the effect of higher-order terms in the potential perturbatively, and analyze the decay of the coherent field oscillations due to quantum particle creation.

## I. INTRODUCTION

Recently there has been great interest in the evolution of a classical scalar field in a cosmological setting. Both in the context of the invisible axion[1–6] and in the context of the new inflationary-universe scenario,[7–13] the energy associated with coherent field oscillations can contribute significantly to, or even dominate, the energy density of the Universe. Here, we consider, in general, the evolution of the energy density associated with the coherent (classical) oscillations of a scalar field. However, we restrict our analysis to isotropic and homogeneous cosmologies, and only treat the case where the oscillation frequency of the scalar field, $\omega$, is always much greater than the expansion rate of the Universe, $H=\dot{R}/R$ ($R$ = the Friedmann-Robertson-Walker scale factor). In this limit the energy density of the scalar field $\phi$ red-shifts away $\propto R^{-6n/(n+2)}$, where the leading term in the potential $V(\phi)$ is $\phi^n$. For $n=2$ the energy density of the scalar-field oscillations behaves like nonrelativistic matter; for $n=4$ it behaves like relativistic matter. We also consider the effects of higher-order terms in the potential. Finally, we analyze the decay of the scalar-field oscillations due to the radiation of quanta (particles) of the fields which couple to $\phi$.

Consider the scalar field whose Lagrangian density is given by

$$\mathscr{L}=-\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\ , \tag{1}$$

where our metric signature is $(-1,1,1,1)$ and $\hbar=c=k_B=1$. The stress-energy tensor for this scalar field is

$$T^{\mu\nu}=\partial^\mu\phi\partial^\nu\phi+\mathscr{L}g^{\mu\nu}\ . \tag{2}$$

The equations of motion for $\phi$ can be obtained either by varying its action

$$\left[=\int d^4x\sqrt{-g}\,\mathscr{L}\right]$$

or, equivalently, from the conservation of its stress-energy, $T^{\mu\nu}{}_{;\mu}=0$.

We shall restrict ourselves to homogeneous and isotropic cosmologies, and use the Robertson-Walker line element

$$ds^2=-dt^2+R(t)^2[dr^2/(1-kr^2)+r^2d\theta^2+r^2\sin^2\theta\,d\phi^2]\ . \tag{3}$$

In such models the scalar field is necessarily homogeneous, and its equation of motion is just

$$\ddot{\phi}+3H\dot{\phi}+V'(\phi)=0\ , \tag{4}$$

where an overdot denotes $d/dt$, a prime denotes $d/d\phi$, and $H\equiv\dot{R}/R$ is the expansion rate. Equation (4) can also be written in the more suggestive forms

$$\frac{d}{dt}(\tfrac{1}{2}\dot{\phi}^2+V)=-3H\dot{\phi}^2\ , \tag{5a}$$

$$d\rho/dt=-3H(\rho+p)\ , \tag{5b}$$

$$d(\rho R^3)=-p\,d(R^3)\ , \tag{5c}$$

where $\rho=\frac{1}{2}\dot{\phi}^2+V(\phi)$ $(=-T^0{}_0)$ and $p=\frac{1}{2}\dot{\phi}^2-V(\phi)$ $(=T^i{}_i$, no sum). The underlying physics in Eq. (5) is manifest; the energy density of the scalar field $\phi$ decreases, because of the "red-shifting away" of the kinetic part $(\frac{1}{2}\dot{\phi}^2)$. These equations must, of course, be supplemented by the Friedmann equation for $H$,

$$H^2\equiv(\dot{R}/R)^2=8\pi G\rho_T/3-k/R^2\ , \tag{6}$$

where $\rho_T$ is the total energy density, and $k$ is the curvature signature.

Thus far we have ignored both the coupling of the scalar field $\phi$ to other fields and quantum-mechanical effects. As $\phi$ oscillates these effects result in particle creation and the conversion of scalar-field energy to other particles.[10–12] We shall take this into account in Sec. IV; for now we are assuming that the damping time due to particle creation is very long ($\gg H^{-1}$).

## II. EVOLUTION OF $\rho=\frac{1}{2}\dot{\phi}^2+V(\phi)$

We are interested in the oscillations of $\phi$ about some local (or global) minimum of $V(\phi)$. If $V$ has a minimum (as it must if it is bounded), then at some point in its evolution $\phi$ will begin to oscillate around it. By shifting the field we can arrange for the minimum (of interest) to be at $\phi=0$. We shall further assume that the frequency of these oscillations, $\omega\simeq\dot{\phi}/\phi$, is always much greater than the expansion rate $H$. For example, if $V(\phi)=\frac{1}{2}m^2\phi^2$, then $\omega=m$, and our assumption is that $m \gg H$.

As $\phi$ oscillates, $\rho$ will be a slowly varying function of time, decreasing on a time scale characterized by $H^{-1}$. However $(\rho+p)=\dot{\phi}^2$ varies rapidly, changing on a time scale characterized by $\omega^{-1} \ll H^{-1}$. Therefore write

$$\dot{\phi}^2=\rho+p=(\gamma+\gamma_p)\rho \ , \tag{7}$$

where $\gamma$ represents the average of $(\rho+p)$ over an oscillation, and $\gamma_p$ represents the periodic part of $\dot{\phi}^2$. Depending upon the form of the potential, $\gamma$ might also vary slowly with time, as the amplitude of the oscillations decreases. From the periodicity of $\gamma_p$ it follows that

$$\int_0^t \gamma_p dt \lesssim O(\omega^{-1}) \ .$$

Using expression (7), Eq. (5) can be formally integrated:

$$\ln(\rho/\rho_0)=-3\int_{t_0}^{t}\gamma\, d\ln R-3\int_{t_0}^{t}H\gamma_p dt \ , \tag{8}$$

where $\rho_0=\rho(t_0)$. Integrating the second term on the right-hand side by parts, it follows that it is of the order of $H(t_0)/\omega$, which by assumption is much less than 1, and thus it can be neglected relative to the first term. That is, in the limit that the oscillations are much more rapid than the expansion, $\dot{\phi}^2$ can be replaced by its value averaged over an oscillation period. If $\gamma$ is constant, then Eq. (8) can be explicitly integrated to give

$$\rho=\rho_0(R/R_0)^{-3\gamma} \ , \tag{9}$$

where $R_0=R(t_0)$. Moreover, if $\rho$ dominates the total energy density $\rho_T$, then in the limit that the curvature can be ignored,

$$R(t)\propto t^{2/3\gamma} \ . \tag{10}$$

The quantity $\gamma$ is obtained by averaging $\dot{\phi}^2/\rho$ over one cycle. Writing $\rho$, which on time scales $\ll H^{-1}$ is constant, as $\rho=V(\phi_{\max})\equiv V_{\max}$, it follows that

$$\gamma=2\frac{\int_0^{\phi_{\max}}(1-V/V_{\max})^{1/2}d\phi}{\int_0^{\phi_{\max}}(1-V/V_{\max})^{-1/2}d\phi} \ , \tag{11}$$

where by only integrating over a half cycle we have assumed that $V(\phi)=V(-\phi)$. Physically, $\phi_{\max}$ is the amplitude of the oscillations, the point where $\dot{\phi}=0$. The quantity $(\gamma-1)\rho$ is the pressure averaged over one cycle, and in the limit $\omega \gg H$, plays the role of a fluid pressure.

Consider a potential of the form

$$V(\phi)=a\phi^n \ , \tag{12}$$

where $a$ must have dimensions of $(\text{mass})^{4-n}$. It is straightforward to integrate (11) to obtain $\gamma=2n/(n+2)$, so that

$$\rho/\rho_0=(R/R_0)^{-6n/(n+2)} \ , \tag{13a}$$

$$R(t)\propto t^{(n+2)/3n} \ , \tag{13b}$$

where (13b) is only applicable if the coherent field oscillations dominate the energy density ($\rho_T\simeq\rho$). Since $\rho\propto V_{\max}$, it follows that $\phi_{\max}\propto R^{-6/(n+2)}$ ($\propto t^{-2/n}$ if $\rho_T\simeq\rho$) (see Fig. 1).

The case of $n=2$ is of particular interest since it corresponds to a massive scalar field with $m^2=V''(\phi=0)$, and since formally it is the leading term in the expansion of any potential around a local minimum. For $n=2$ the coherent field energy behaves just like nonrelativistic matter: $\gamma=1$, $\langle p\rangle=(\gamma-1)\rho=0$, $\rho\propto R^{-3}$, and $R(t)\propto t^{2/3}$; here brackets indicate the average over one oscillation period.

For $n=4$ the coherent field energy behaves like relativistic matter: $\gamma=\frac{4}{3}$, $\langle p\rangle=\rho/3$, $\rho\propto R^{-4}$, and $R(t)\propto t^{1/2}$. For a renormalizable theory $n$ must be at most 4. However, if the Lagrangian (1) is just an effective, low-energy Lagrangian, then $n$ could be larger than 4, with the higher-order terms being suppressed by powers of some large energy scale.

If the potential $V(\phi)$ is a polynomial in $\phi$, it is clear that eventually, as the amplitude of the oscillations decrease, the lowest power of $\phi$ in $V$ will come to dominate the potential. Suppose the lowest power is $\phi^n$; consider the perturbative effect of a higher-order term by writing

$$V(\phi)=a\phi^n(1+\epsilon\phi^l) \ . \tag{14}$$

The ratio of the higher-order term to the leading $\phi^n$ term is $\epsilon\phi^l\lesssim\epsilon\phi_{\max}^l$, which is less than unity so long as $\epsilon\lesssim\phi_{\max}^{-l}$. It is straightforward (but tedious) to compute $d\gamma/d\epsilon$, and we find that

$$d\gamma/d\epsilon=\phi_{\max}^l\frac{4l(l+1)}{(2l+n+2)(n+2)} \times\frac{\Gamma((n+2)/2n)\Gamma((l+1)/n)}{\Gamma(1/n)\Gamma((2l+n+2)/2n)} \ . \tag{15}$$

Using expression (15), we can expand $\gamma$ in a Taylor series in $\epsilon\phi_{\max}^l$,

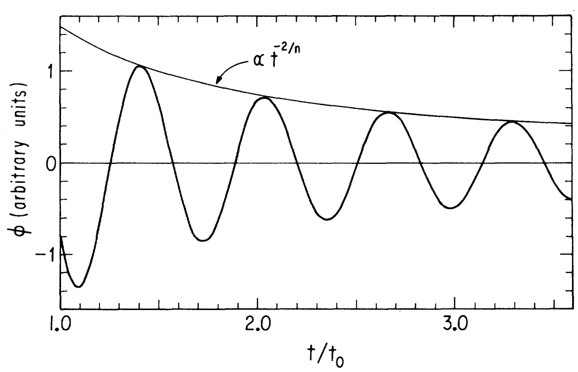


FIG. 1. The time evolution of $\phi$ in a model universe dominated by coherent scalar-field oscillations. The amplitude of the oscillations $\phi_{\max}$ decreases as $t^{-2/n}$, where $V(\phi)=a\phi^n$. Here $t=$ the age of the Universe, $R/R_0=(t/t_0)^{2/3\gamma}$, $H=(2/3\gamma)t^{-1}$, $\gamma=2n/(n+2)$, and for the model shown $\omega=10t_0^{-1}$ and $n=2$.

$$\gamma=\frac{2n}{n+2}+\epsilon\phi_{\max}^{l}\frac{4l(l+1)}{(2l+n+2)(n+2)}$$
$$\times\frac{\Gamma((n+2)/2n)\Gamma((l+1)/n)}{\Gamma(1/n)\Gamma((2l+n+2)/2n)}+O((\epsilon\phi_{\max}^{l})^2)\ . \quad (16)$$

For a harmonic potential ($n=2$), consider the effect of an anharmonic ($l=2$) perturbation:

$$\gamma=1+\tfrac{3}{8}\epsilon{\phi_{\max}}^2+\cdots$$
$$\simeq 1+\tfrac{3}{8}\epsilon(\rho/a)+\cdots\ , \quad (17)$$

the second equality following from the fact that $\rho=V_{\max}\simeq a{\phi_{\max}}^2$. Using expansion (17) for $\gamma$ in Eq. (8), and integrating, we find that

$$(\rho/\rho_0)\exp[-\tfrac{3}{8}\epsilon(\rho-\rho_0)/a]=(R/R_0)^{-3}\ , \quad (18)$$

or for late times ($R\gg R_0$, $\rho\ll\rho_0$)

$$\rho=\exp\left[-\frac{3}{8}\frac{\epsilon\rho_0}{a}\right](R/R_0)^{-3}\ . \quad (19)$$

Recall that $\epsilon\rho/a=\epsilon{\phi_{\max}}^2$ is the ratio between the anharmonic and harmonic terms at the maximum amplitude of the oscillation, which must be less than unity for this analysis to be valid. Thus, for late times, the anharmonic term changes the energy density by a factor $\exp(-\tfrac{3}{8}r)$, where $r$ is the ratio of the anharmonic term to the harmonic term (at $\phi=\phi_{\max}$) when $\rho=\rho_0$. Since this factor is at most of order unity, anharmonic effects do not substantially modify the conclusion of Refs. 1—3, namely that unless the scale of Peccei-Quinn symmetry breaking is less than about $10^{13}$ GeV, the energy density contributed by coherent axion field oscillations will presently exceed the observed mass density of the Universe.

Finally, consider a periodic potential of the form

$$V=V_0\sin^2(\phi/f)\ , \quad (20)$$

where $f$ is some energy scale. If $\rho>V_0$, then the oscillations of $\phi$ are not bounded. To find $\gamma$ though, we need only average over the interval $\phi=0\rightarrow(\pi/2)f$. In this case we find that

$$\gamma/2=\frac{\int_0^{\pi/2}(1-\beta^2\sin^2\theta)^{1/2}d\theta}{\int_0^{\pi/2}(1-\beta^2\sin^2\theta)^{-1/2}d\theta}=\frac{E(\beta,\pi/2)}{F(\beta,\pi/2)}\ , \quad (21a)$$
$$\simeq 1-\tfrac{1}{2}\beta^2+O(\beta^4)\ , \quad (21b)$$

where $\beta^2=V_0/\rho$, $F$ is an elliptic integral of the first kind, and $E$ an elliptic integral of the second kind. For small $\beta^2$, $\gamma$ is about 2, so that $\langle p\rangle\simeq\rho$, $\rho\propto R^{-6}$, and if $\rho_T\simeq\rho$, $R(t)\propto t^{1/3}$. That is, in the limit that $\dot\phi^2\gg V(\phi)$, the coherent field oscillations behave like a fluid with $p=\rho$.

On the other hand, for $V_0>\rho$, $\phi_{\max}\simeq(\rho/V_0)^{1/2}f$ and

$$V\simeq V_0(\phi/f)^2[1-(\phi/f)^2/3+\cdots]\ .$$

In this limit $\gamma\simeq 1-\tfrac{1}{8}\rho/V_0+O((\rho/V_0)^2)$.

## III. TIME-DEPENDENT POTENTIALS

If $V$ is explicitly time dependent, then the equation of motion (4) is still valid; however the first integral obtained in Eqs. (5a)—(5c) is not. Since $dV/dt=\partial V/\partial t+\dot\phi V'$, an additional term arises in constructing the analog of Eq. (5):

$$\frac{d}{dt}(\tfrac{1}{2}\dot\phi^2+V)=-3H\dot\phi^2+\partial V/\partial t\ . \quad (22)$$

Using the fact that $\langle\dot\phi^2\rangle=\gamma\rho$ and $\langle V\rangle=(1-\gamma/2)\rho$, and assuming that the explicit time variation of $V$ is not rapid ($\partial\ln V/\partial t\ll\omega$), Eq. (22) can be formally integrated to give

$$\rho/\rho_0=(R/R_0)^{-3\gamma}\exp\left[(1-\gamma/2)\int_{t_0}^{t}\frac{\partial\ln V}{\partial t}dt\right]\ . \quad (23)$$

If $V$ factors, $V(\phi,t)=f(t)v(\phi)$, then Eq. (23) can be integrated directly,

$$\rho/\rho_0=(R/R_0)^{-3\gamma}(f/f_0)^{1-\gamma/2}\ . \quad (24)$$

The invisible axion provides an example of a time-dependent potential which factors: $V=\tfrac{1}{2}{m_a}^2(t)\phi^2$, so that $\gamma=1$ and $\rho/\rho_0=(R/R_0)^{-3}m_a(t)/m_a(t_0)$. This result was derived and used by the authors of Refs. 1—3.

## IV. COHERENT FIELD OSCILLATIONS ARE NOT FOREVER

In Sec. I, we neglected the effects of quantum particle creation by the oscillating scalar field on the evolution of $\rho$. The damping of the oscillations due to the radiation of light particles can be taken into account by adding a term $-\dot\phi^2\Gamma$ to the right-hand side of Eq. (5) (Ref. 10), where $\Gamma$ is the total decay width of the scalar $\phi$ particle (Ref. 12). For simplicity assume that the particles created are photons (i.e., very relativistic particles). Then the equations for the evolution of $\rho$ and $\rho_\gamma$ are

$$\dot\rho=-(3H+\Gamma)\gamma\rho\ , \quad (25a)$$
$$\dot\rho_\gamma=-4H\rho_\gamma+\gamma\Gamma\rho\ . \quad (25b)$$

It is straightforward to integrate these equations to obtain

$$\rho=\rho_0[R(t)/R_0]^{-3\gamma}\exp[-\gamma\Gamma(t-t_0)]\ , \quad (26a)$$
$$\rho_\gamma=\rho_{\gamma 0}[R(t)/R_0]^{-4}+\rho_0[R(t)/R_0]^{-4}\times\int_{\gamma\Gamma t_0}^{\gamma\Gamma t}[R(t')/R_0]^{4-3\gamma}e^{u_0-u}du\ , \quad (26b)$$

where the subscript zero denotes the value of that quantity at $t=t_0$, and $u=\gamma\Gamma t'$. [If the particles created are nonrelativistic, all the 4's in Eqs. (26a) and (26b) become 3's.]

If the energy density of the Universe is dominated either by the coherent field oscillations ($\rho_T\simeq\rho$) or by the relativistic particles ($\rho_T\simeq\rho_\gamma$), then $R(t)/R_0=(t/t_0)^m$, where $m=\tfrac{1}{2}$ ($\rho_T\simeq\rho_\gamma$) or $m=2/3\gamma$ ($\rho_T\simeq\rho$), and $t$ is the age of the Universe. The quantity $(t_0/m)$ is just the expansion time scale ($=H^{-1}$) at $t=t_0$. If $\Gamma t_0$ is greater than about unity, then the coherent field energy $\rho$ will be rapidly (in an expansion time $\simeq t_0/m$, or less) converted into radiation, and become exponentially negligible. (In the inflationary-Universe scenario, this is referred to as "good reheating."[14])

Now consider the case where $\Gamma t_0$ is much less than unity, and assume (for convenience) that $\rho_{\gamma 0}$ is negligible. (In the inflationary-Universe scenario this is referred to as "poor reheating.") In addition to tracking $\rho$ and $\rho_\gamma$, it is also very useful to follow the evolution of the entropy in radiation per comoving volume which is produced by the decay of the coherent field oscillations,

$$R^3T^3 \propto S = [R(t)^4\rho_\gamma]^{3/4} .$$

The quantities $\rho$, $\rho_\gamma$, $\rho_\gamma/\rho$, and $S$ can be written as

$$\rho = \rho_0[R(t)/R_0]^{-3\gamma}\exp[-\gamma\Gamma(t-t_0)] , \quad (27a)$$

$$\rho_\gamma = \rho_0[R(t)/R_0]^{-4}I , \quad (27b)$$

$$\rho_\gamma/\rho = [R(t)/R_0]^{3\gamma-4}I\exp[\gamma\Gamma(t-t_0)] , \quad (27c)$$

$$S = (R_0{}^4\rho_0)^{3/4}I^{3/4} , \quad (27d)$$

where

$$I \equiv \int_{\gamma\Gamma t_0}^{\gamma\Gamma t}[R(t')/R_0]^{4-3\gamma}e^{u_0-u}du$$

$$\simeq \begin{cases} \gamma\Gamma t[R(t)/R_0]^{4-3\gamma}/[(4-3\gamma)m+1] , & \gamma\Gamma t \ll 1 \\ [(4-3\gamma)m]![R(t\simeq\gamma^{-1}\Gamma^{-1})/R_0]^{4-3\gamma} , & \gamma\Gamma t \gg 1 . \end{cases} \quad (28)$$

From Eqs. (27) and (28) it follows that $\rho_\gamma/\rho$ increases linearly with time until $t \simeq \gamma^{-1}\Gamma^{-1}$ (when the age of the Universe $\simeq$ decay time of a $\phi$), and thereafter, exponentially with time. That is, a coherent-field-energy-dominated Universe only remains so until $t \simeq \gamma^{-1}\Gamma^{-1}$. For the invisible axion this time is

$$t \simeq \gamma^{-1}\Gamma^{-1} \simeq \gamma^{-1}(\alpha^2 m_\pi{}^6/f_a{}^5)^{-1}$$
$$\simeq (f_a/10^{12}\ \text{GeV})^5\ 10^{40}\ \text{yr} ,$$

where $\Gamma$ is the width for the two-photon decay mode. Although an axion-dominated Universe (which corresponds to $f_a \simeq 10^{12}$ GeV) does not remain so forever, it is axion dominated for a long time.

The energy density of the radiation produced by the coherent field oscillations actually decreases with time, due to the red-shifting of $\rho$ and $\rho_\gamma$ by the expansion. The maximum temperature reached (using $\rho_\gamma \simeq T^4$) is

$$T_{\max} \simeq [\rho_\gamma(\text{few } t_0)]^{1/4} ,$$
$$\simeq (\gamma\Gamma t_0)^{1/4}\rho_0^{1/4} , \quad (29)$$

which occurs when $t \simeq$ few $t_0$. However, the entropy per comoving volume, $S$, increases until $t \simeq (\gamma\Gamma)^{-1}$, when it levels off at the value

$$S_{\max} \simeq (\rho_0 R_0{}^4)^{3/4}[R(t=\gamma^{-1}\Gamma^{-1})/R_0]^{3-9\gamma/4}$$
$$\simeq (\rho_0 R_0{}^4)^{3/4}(\gamma\Gamma t_0)^{-2/\gamma+3/2} .$$

The quantity $(\rho_0 R_0{}^4)^{3/4}$ is just the entropy per comoving volume that would have been produced had the oscillations decayed away rapidly ($\gamma\Gamma t_0 \gg 1$). Thus for $\gamma < \frac{4}{3}$, the final entropy released increases with decreasing $\gamma\Gamma$. The temperature reached at maximum entropy is

$$T(\gamma^{-1}\Gamma^{-1}) \simeq (\gamma\Gamma t_0)^{1/2}\rho_0{}^{1/4} .$$

The evolution of $\rho$, $\rho_\gamma$, and $S$ are shown in Fig. 2.

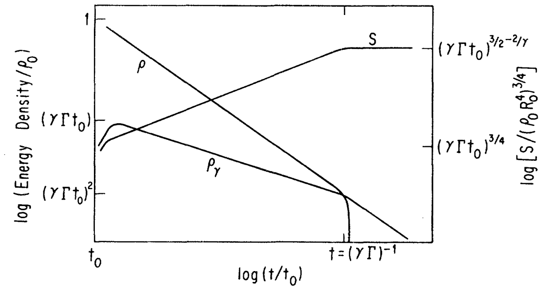


FIG. 2. The time evolution of $\rho$, $\rho_\gamma$, and $S$ for a model universe dominated by coherent scalar-field oscillations at $t=t_0$ and $\gamma\Gamma t_0 \ll 1$. From $t=t_0$ to $t \simeq (\gamma\Gamma)^{-1}$, $\rho$ decreases as $t^{-2}$; thereafter it decays exponentially. The energy density in radiation (assumed to be initially negligible) rises to a value of order $(\gamma\Gamma t_0)\rho_0$ by the time $t \simeq$ few $t_0$, and until $t \simeq (\gamma\Gamma)^{-1}$ decreases as $t^{-1}$. Its value at $t \simeq (\gamma\Gamma)^{-1}$ is about $\rho_0(\gamma\Gamma t_0)^2$, and thereafter $\rho_\gamma$ decreases as $t^{-2}$. The entropy per comoving volume $S$ increases as $t^{2/\gamma-3/4}$ until $t \simeq (\gamma\Gamma)^{-1}$ when it levels off at a value $S_{\max} \simeq (\rho_0 R_0{}^4)^{3/4}(\gamma\Gamma t_0)^{3/2-2/\gamma}$.

## V. SUMMARY

In the limit that $\omega$ is much greater than $H$ (oscillation period $\ll$ expansion time scale), coherent scalar field oscillations behave like a fluid with $p=(\gamma-1)\rho$, where $\gamma$ depends upon the form the scalar potential $V(\phi)$. As the amplitude of the oscillations decreases the lowest-order term in $V$ (say $\phi^n$) dominates $V$, and for $V = a\phi^n$, $\gamma = 2n/(n+2)$ and $\rho$ decreases as $R^{-6n/(n+2)}$. If the scalar-field energy density dominates the energy density of the Universe, then $R(t) \propto t^{(n+2)/3n}$. The perturbative effects of higher-order terms ($\phi^m, m > n$) modify $\rho \propto R^{-6n/(n+2)}$ by only a factor of $\exp[-O(1)]$. Because of particle creation due to the time variation of $\phi$, the energy density in scalar-field oscillations eventually decays away exponentially, in a time characterized by the lifetime of the scalar particle associated with $\phi$.


## ACKNOWLEDGMENT

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