Direct lepton production in high-energy collisions of nuclei

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Direct lepton-pair production is a potentially efficient probe of quark matter formed in high-energy collisions of nuclei. Pair-production rates in the central rapidity region are estimated by means of a hydrodynamical model of the expansion of the hadron plasma. The rates are expected to be independent of rapidity in the central region and exhibit a characteristic enhancement if phase transition from hadron to quark matter takes place.

I. INTRODUCTION

One of the interesting predictions of quantum chromodynamics (QCD) is the existence of a phase transition in bulk hadronic matter at sufficiently high net baryon densities and/or temperatures.[1] An experimental verification of this prediction is somewhat difficult due to several circumstances.

(i) Lack of a satisfactory theoretical understanding of the nature of the phase transition and its parameters, in particular at nonvanishing net baryon densities. Earlier, more-or-less *ad hoc* calculations of the critical curve contained substantial theoretical uncertainties. Recent progress made in numerical solutions of non-Abelian gauge theories at finite temperatures on a lattice[2] removes some of those uncertainties. In particular, the inclusion of fermion pairs in the SU(3) calculation by Kuti and Polonyi[3] removes uncertainties associated with the use of a quenched approximation to QCD. These results tend to lower the critical temperature T_c at a vanishing net baryon density; one is not unlikely to find a value as low as $T_c \simeq 1.5\Lambda$, where Λ is the characteristic scale of QCD.[4] With recent values of Λ suggested both by lattice calculations[3] and by scale-breaking effects in high-energy reactions, T_c may be as low as 120 to 170 MeV. (At the time of this writing, uncertainties associated with the effects of a nonvanishing chemical potential still remain.)

(ii) Difficulties in creating thermalized bulk hadronic matter. In a terrestrial environment, probably the best one can do is to study hadronic matter in the collisions of heavy nuclei. There exist rather convincing semiquantitative arguments that hadronic matter does indeed (approximately) thermalize in a large fraction of the time.[5] However, typical relaxation times, expected (on dimensional grounds) to be of the order of Λ^{-1}, are comparable to the lifetime of the typical "fireballs" created in such reactions; therefore one should be prepared to face rather large fluctuations of the various physical quantities around their equilibrium values.

(iii) It is problematic to find unambiguous experimental signatures of the existence of the phase transition. A number of possible such signatures was suggested (for a review, cf. Ref. 6). However, due to the fact that several of the proposed signatures involve details of the final hadronic spectrum, they are subject to theoretical uncertainties, due to our lack of understanding of the dynamics of hot hadronic matter. From a qualitative point of view, one can argue that, as a consequence of the strong interactions of the constituents $[\alpha_s(T/\Lambda) \gtrsim 1$ in the hadron-formation epoch of the fireball], any memory of the initial stages of the evolution tends to be washed out.

In view of this fact it was proposed that leptonic channels, in particular, direct pair production, carry less ambiguous signatures of the hadron-quark phase transition.[7,8] Admittedly, lepton-pair production is a rare process compared to hadron production (suppressed by factors of the order of α^2). However, once formed, virtual photons are less likely to thermalize than hadrons, due to their weaker interaction with the environment; as a consequence, their spectrum is more likely to carry information about the primary (hot) stages of the fireball.

At present, it seems to be unlikely that the phase boundary between the hadron and quark phases is crossed unless the colliding nuclei carry a substantial kinetic energy (perhaps of the order of 10 to 20 GeV/nucleon) in the center-of-mass systems (c.m.s.) for hadron matter may not be heated sufficiently at lower energies and/or the energy of excitation may be rapidly distributed between the degrees of freedom available in a "slow" collision. Therefore, in order to test the ideas outlined above, it is imperative to study pair production in *high-energy* nuclear interactions.

In this paper I consider a simple model in order to describe the pair-production process, in the same spirit as it was discussed in our previous work.[8] The main advantage of the model is its simplicity; in view of the fact that models of this type give results in qualitative agreement with the data for hadron production at high energies, one is encouraged to believe that the predictions are at least qualitatively (within a factor of 3 or so) correct. Our present level of understanding of the dynamics of hot hadronic matter does not justify the construction of very detailed models: any improvement on the accuracy of the predictions would be purely illusory in view of the drastic simplifying assumptions made at the outset. (In the same spirit, all relevant physical quantities are approximated by simple analytic expressions, thereby reducing the task of making numerical estimates to a "pocket-calculator level.")

The paper is organized as follows. In Sec. II, a simple model of high-energy nuclear collisions is reviewed; this is mostly based on a recent work of Bjorken,[9] who revived the idea of a similarity flow of hadron matter at early stages of a fireball expansion.[10] In Sec. III, simple analytic approximations are derived for lepton-pair-production rates, both from the quark and hadron phases. Observable pair-production rates are estimated in Sec. IV; the results are discussed in Sec. V. Geometrical formulas, useful in a semiclassical treatment of nuclear collisions, are collected in the Appendix. Throughout this paper, natural units are used ($\hbar=c=1$); temperatures are measured in energy units (Boltzmann's constant, $k=1$).

II. MODEL OF HIGH-ENERGY NUCLEAR COLLISIONS

The qualitative picture of high-energy ($E_{\text{c.m.}} \gtrsim 20$ GeV/nucleon nuclear collisions appears to differ significantly from the one valid at lower energies ($E_{\text{c.m.}} \simeq 1-3$ GeV/nucleon); this change is probably due to the increase of nuclear transparency as one goes to higher energies. An extrapolation from nucleon-nucleon and hadron-nucleus data suggests the existence of well-distinguishable fragmentation and central regions in the longitudinal-rapidity plot of secondaries,[11] in contradistinction with low-energy collisions. (The latter are *qualitatively* consistent with a one-fireball picture, without very distinct fragmentation and central regions; for a recent review, cf. Hallman.[12]) I made the standard simplifying assumptions, viz., (i) hadron matter is in thermal equilibrium, and (ii) the initial temperatures are sufficiently high so that the QCD scale Λ plays no significant role in the early dynamics of the hadron matter.

Bjorken[9] emphasized that the central region carries negligible net baryon number; consequently, the corresponding chemical potential is zero outside the fragmentation regions.

This has two important consequences.

(a) The initial flow in the central region is scale free; hence it is a similarity flow.[13]

(b) The central region is hotter than the fragmentation regions; therefore it is more likely to be in the quark phase. Indeed, under our assumptions, the density of the grand potential is of the form

$$\omega = -T^4 f\left[\frac{\mu^2}{T^2}\right], \tag{2.1}$$

where $f(x)>0$, $f'(x)>0$. (The fact that ω is an even function of the chemical potential μ is a consequence of C invariance.) The energy density is given by the well-known thermodynamical relation

$$\epsilon = \omega - \left[T\frac{\partial}{\partial T} + \mu\frac{\partial}{\partial \mu}\right]\omega , \tag{2.2}$$

which with (2.1) becomes $\epsilon=-3\omega$. Thus, given the initial energy density deposited in the collision, the temperature is a monotonically *decreasing* function of the chemical potential or, equivalently, of the net baryon density B by using the relation

$$B = -\frac{\partial \omega}{\partial \mu} . \tag{2.3}$$

These relationships also tell us that hadron matter behaves as an ideal gas, since the pressure is given by $p=-\omega$; thus the speed of sound is $u=1/\sqrt{3}$.

The *longitudinal* expansion of the hadron matter in the central region is governed by the equations of hydrodynamics of an ideal fluid, viz,

$$\nabla_\nu T^{\mu\nu}=0 , \quad T^{\mu\nu}=-pg^{\mu\nu}+wu^\mu u^\nu , \\ g_{\mu\nu}u^\mu u^\nu = 1 , \tag{2.4}$$

where $\mu,\nu=0,1$ and $w=\epsilon+p$ is the density of the heat function. In order to generate a similarity flow, it is convenient to introduce curvilinear coordinates with the definition

$$\tau=(t^2-x^2)^{1/2} , \quad y=\tfrac{1}{2}\ln\frac{t+x}{t-x} , \\ t\equiv x^0 , \quad x\equiv x^1 . \tag{2.5}$$

In these coordinates, the nonvanishing components of the metric tensor are $g_{\tau\tau}=1$, $g_{yy}=-\tau^2$, whereas, on setting $u^0=\cosh\eta$, $u^1=\sinh\eta$, the components of the four-velocity of the flow become

$$u^\tau=\cosh(\eta-y) , \quad u^y=\frac{1}{\tau}\sinh(\eta-y) \tag{2.6}$$

and Euler's equations, $\nabla_\nu T^{\mu\nu}=0$, read

$$-\partial_\tau p+\frac{1}{\tau}\partial_\tau(\tau w u^{\tau 2})+\partial_y(wu^\tau u^y)+\tau u^{y2}=0 , \\ \frac{1}{\tau^2}\partial_y p+\frac{1}{\tau}\partial_\tau(\tau w u^\tau u^y)+\partial_y(wu^{y2})+\frac{2}{\tau}u^\tau u^y=0 . \tag{2.7}$$

The similarity solution is obtained by setting $u^y=0$ (i.e., $\eta=y$); in that case one has from (2.7)

$$-\partial_\tau p+\frac{1}{\tau}w+\partial_\tau w=0 , \\ \partial_y p=0 . \tag{2.8}$$

On using the equation of state $\epsilon=3p$ one obtains the solution

$$\epsilon(\tau)=\epsilon_0\left[\frac{\tau}{\tau_0}\right]^{-4/3} \tag{2.9}$$

from which, using (2.1) with $\mu\simeq 0$ the scaling law for the temperature,

$$T=T_0\left[\frac{\tau}{\tau_0}\right]^{-1/3} \tag{2.10}$$

follows. In these equations, τ_0 is roughly the time needed in order to establish the initial thermal equilibrium in the collision; on dimensional grounds we expect $\tau_0\simeq\Lambda^{-1}$. From their phenomenological analysis, Mueller and Bjorken[11] expect an initial energy density $\epsilon_0\simeq 3$ GeV fm$^{-3}\simeq 30\Lambda^4$. This energy density is probably a slowly (perhaps logarithmically) growing function of the primary energy, cf. Van Hove.[6] As a tentative fit, I take

$$\epsilon_0\approx 30\Lambda^4\ln\frac{s^{1/2}}{7.5\text{ GeV}} , \tag{2.11}$$

where $s^{1/2}$ is the c.m.s. energy per nucleon.

Unfortunately, it is difficult to convert the initial energy density into temperatures, because it is not clear what equation of state should be used for the hot hadronic matter in the central region. However, if T_c is of the order of 1.5Λ, then immediately above the phase transition, one can approximate the equation of state by an ideal quark gas with just two quark flavors; this gives $\epsilon_0 \simeq 37\pi^2 T^4/30$ (see, e.g., Ref. 8). Using (2.11), one then finds that $T_c = 1.5\Lambda$ is reached at $s^{1/2} \simeq 58$ GeV/nucleon. By contrast, if one assumes that initially the color degrees of freedom are still frozen in and one approximates baryonless hadronic matter by an ideal pion gas, one has (neglecting the pion mass), $\epsilon_0 \simeq \pi^2 T^4/10$. From this one finds that T_c is reached at a much lower energy $s^{1/2} \simeq 9$ GeV/nucleon. Probably, these are extreme estimates and in reality the phase transition takes place somewhere in between these energies.

One has to have some information about the initial transverse size of the hot central region. Lacking any reliable theory, I estimate the initial transverse size, R_T from the geometrical overlap, i.e., I write at a given impact parameter b

$$\sigma_g(b) = \pi R_T^2(b) , \tag{2.12}$$

where σ_g is the geometrical cross section (cf. the Appendix).

In the transverse direction there is a characteristic size, and thus, on the average, the lateral expansion can be roughly described in terms of an inward moving rarefaction front, moving with the speed of sound. Given the crudeness of the model, I assume cylindrical symmetry and take the equation of the front to be $\rho = R_T(b) - 3^{-1/2}\tau$. Behind the rarefaction front, cooling is very rapid and one expects that matter goes into its hadronic phase immediately behind the front. As a consequence, matter can exist in its quark phase for any length of time if the rarefaction front does not reach $\rho = 0$ too soon. Somewhat arbitrarily, I set the minimal required lifetime to be $\tau_{\min} = 4\Lambda^{-1}$. In order to get a feeling for the maximal impact parameters allowed, one may consider identical colliding nuclei and use the standard formula for nuclear radii $R = r_0 A^{1/3}$, $r_0 = 1.12$ fm. The resulting maximal allowed impact parameters calculated with $\sigma_g(b)$ given by Eq. (A7) in the Appendix are displayed in Table I. It is clear from this table that in the case of light nuclei (for instance, He and C), the hot plasma does not live long enough for thermal equilibrium to get established. For collisions of unequal nuclei, A can be roughly put equal to the atomic number of the lighter colliding nucleus [cf. Eq. (A9)]. It is evident that even for intermediate nuclei, one has to require essentially central collisions in order to have reasonable central plasma lifetimes, whereas at the end of the periodic table (for instance, in a collision $U + U \to X$), the lifetime of the central plasma will be sufficiently long for almost all collisions.

TABLE I. Maximal allowed impact parameters in collisions $A + A \to X$. Required central-plasma lifetime $\tau_{\min} = 4\Lambda^{-1}$; $\Lambda = 100$ MeV; $R = (1.12 \text{ fm})A^{1/3}$.

A	$b_{\max}/2R$
<16	No solution with $b_{\max} \geq 0$
16	0.18
20	0.34
24	0.38
30	0.58
40	0.66
56	0.76
84	0.82
108	0.85
130	0.88
195	0.90
238	0.92
∞	1.00

III. PAIR-PRODUCTION RATES

Theoretical expressions of lepton-pair-production rates are well known both in the quark (Q) and hadron (H) phases.[14] The theoretical expressions are subject to some uncertainty due to rescattering corrections which are not very accurately known. I want to argue, however, that lowest-order (in α_s) formulas should be reasonably accurate in the quark phase. Indeed, the real danger to any perturbative expression for a quark-pair annihilation into leptons arises from "soft" (low-momentum transfer) scattering of the initial quarks in the environment, for that is the strong-coupling regime of QCD. However, as long as the initial momentum of the quark scattered is not too high, the phase space for the scattered quark is reasonably well populated. Therefore, soft-rescattering effects tend to be suppressed by the exclusion principle. This argument is inapplicable for high-momentum quarks, which after a soft scattering would go into a sparsely populated region of phase space. However, the number of quarks with $|\vec{P}| \gg T$ is of the order of $\exp(-|\vec{P}|/T)$; therefore only the tail of the invariant-mass distribution of pairs is affected. The situation is less clear in the hadron phase, where the dominant source of pairs is the annihilation of charged pions.[8] *Some* rescattering effects are obviously taken into account by the pion form factor. In order to account for all rescattering effects, one obviously would have to study the detailed kinetics of an interacting hadron (quark-gluon) gas, respectively; I believe that at the present level of our understanding of the phenomena involved, this is not justified.

Without further apologies, I quote therefore the lowest-order expressions for the pair-production rates in both phases (Q,H):

$$\left[\frac{dN}{dM^2 d^4x}\right]_Q = \frac{\alpha^2}{\pi^3}\frac{5}{18}\left[1+\frac{2m_l^2}{M^2}\right]\left[1-\frac{4m_l^2}{M^2}\right]^{1/2} T^2 F_+\left[\frac{M}{T}\right] \tag{3.1}$$

(two flavors);

$$\left[\frac{dN}{dM^2 d^4x}\right]_H = \frac{\alpha^2}{\pi^3}\frac{1}{24}\left[1+\frac{2m_l^2}{M^2}\right]\left[1-\frac{4m_l^2}{M^2}\right]^{1/2} T^2 \left[1-\frac{4m_\pi^2}{M^2}\right]^{1/2} |F_\pi(M^2)|^2 F_-\left[\frac{M}{T}\right] . \tag{3.2}$$

In these equations, m_l and m_π stand for the lepton and pion masses, respectively; M is the invariant mass of the pair, and $F_\pi(q^2)$ is the pion form factor. The quantities $F_\pm(u)$ are integrals over Fermi and Bose distributions, respectively; their expressions were given in Ref. 8. These functions are approximated as follows. First, all masses are neglected where this generates no divergences in the integrals; this is a reasonably good approximation, for say, $T \gtrsim m_\pi$. Then both functions $F_\pm(u)$ are of the same form, viz.,

$$F_\pm(u) = \int_{\mu_\pm}^{\infty} dx\, dy \frac{\Theta(4xy - u^2)}{(e^x \pm 1)(e^y \pm 1)} \tag{3.3}$$

with $\mu_- = m_\pi / T$, $\mu_+ = 0$. On introducing new integration variables by the substitution $x = \lambda\xi$, $y = \lambda(1-\xi)$ $(0 < \xi < 1)$, one gets

$$F_\pm(u) = \int_{\mu_\pm}^{\infty} \lambda\, d\lambda \int_0^1 d\xi \frac{\Theta(\lambda^2 \xi(1-\xi) - u^2/4)}{(e^{\lambda\xi} \pm 1)(e^{\lambda(1-\xi)} \pm 1)} .$$

It is easily seen that the integrand is a sharply peaked function[15] around $\xi = \frac{1}{2}$; using this,

$$\int_0^1 d\xi \frac{\Theta(\lambda^2 \xi(1-\xi) - u^2/4)}{(e^{\lambda\xi} \pm 1)(e^{\lambda(1-\xi)} \pm 1)} \approx \frac{\Theta(\lambda - u)}{(e^{\lambda/2} \pm 1)^2} .$$

One can now safely set $\mu_\pm = 0$; this causes a spurious (logarithmic) increase of $F_-(u)$ for $u \ll 1$ (low invariant masses); that is adequately suppressed by the factor $(1 - 4m_\pi{}^2/M^2)^{1/2}$ in the production rate. The resulting one-dimensional integral is

$$F_\pm(u) \approx \int_u^{\infty} \frac{\lambda\, d\lambda}{(e^{\lambda/2} \pm 1)^2} .$$

In view of the elementary identity

$$\frac{1}{(e^t \pm 1)^2} = -e^{-t} \frac{d}{dt} \frac{1}{e^t \pm 1} ,$$

this integral can be brought to the form

$$F_\pm(u) \approx 4 \left[\frac{(u/2) e^{-u/2}}{e^{u/2} \pm 1} + \int_{u/2}^{\infty} \frac{dt\,(1-t)}{e^t \pm 1} \right] ,$$

or, on noting the rapid decrease of the integrand of the second term,

$$F_\pm(u) \approx 4 \left[\frac{(u/2) e^{-u/2}}{e^{u/2} \pm 1} + \left[1 - \frac{u}{2} \right] \int_{u/2}^{\infty} \frac{dt\, e^{-t}}{e^t \pm 1} \right] .$$

The remaining integral is elementary. The final result is

$$F_\pm(u) \approx 4 \left[\frac{(u/2) e^{-u/2}}{e^{u/2} \pm 1} + \left[1 - \frac{u}{2} \right] [\pm e^{-u/2} - \ln(1 \pm e^{-u/2})] \right] . \tag{3.4}$$

On substituting (3.4) into (3.1) and (3.2), one obtains approximate expressions for $dN(d^4x\, dM^2)^{-1}$ in the hadron and quark phases, respectively. The pion form factor can be reasonably well approximated by a Breit-Wigner formula centered at the mass of the ρ meson,

$$F_\pi(M^2) \approx \frac{m_\rho{}^2}{m_\rho{}^2 - M^2 - i m_\rho \Gamma} \tag{3.5}$$

with $m_\rho \simeq 780$ MeV, $\Gamma = 155$ MeV (cf. Ref. 8). The crucial point is that, in order to obtain the observable pair-production rates, one has to integrate (3.1) and (3.2) over the space-time history of the central region. In this respect, high-energy collisions differ significantly from ones, say, at typical Bevelac energies: this is the subject of Sec. IV.

IV. INTEGRATION OF PAIR-PRODUCTION RATES OVER THE HISTORY OF THE CENTRAL PLASMA

The observable pair-production rates are obtained from Eqs. (3.1) and (3.2) by integrating those equations over the space-time volume of the central plasma. Cooling is rapid behind the transverse rarefaction front; therefore one expects a negligible amount of pair production from that region. Consequently, it is sufficient to integrate over the space-time region of the plasma which has not been reached yet by the transverse front. The computation is further simplified by the observation that in the relevant region, the temperature decreases rather slowly, cf. Eq. (2.10). As a consequence,

$$\frac{dN}{dM^2} = \int_{(\text{plasma})} d^4x \frac{dN(T)}{dM^2 d^4x} \approx \Omega \frac{dN(\bar{T})}{dM^2 d^4x} , \tag{4.1}$$

where Ω is the volume of the relevant space-time region and $\bar{T}$ is the average temperature in that region.

One can now envisage two types of situations. Suppose that the initial temperature T_0 is sufficiently high, so that the central plasma is in the quark phase. As the plasma cools according to Eq. (2.10), it may reach T_c, the critical temperature before it is swept out by the transverse rarefaction front. In that case, one expects a mixed pair production, coming both from the quark and hadron phases. If, however, the cooling is sufficiently slow, so that the interior of the plasma remains in the quark phase until it is reached by the transverse rarefaction front, pair production from the quark phase is predominant, with a small amount of pairs produced from the hadronic phase behind the rarefaction front.

The proper time at which the plasma reaches the critical temperature is given by Eq. (2.10):

$$\tau_c = \tau_0 \left[\frac{T_0}{T_c} \right]^3 \approx 0.3 \Lambda^{-1} \left[\frac{T_0}{\Lambda} \right]^3 \tag{4.2}$$

with $T_c \simeq 1.5\Lambda$. This proper time is to be compared with the proper time at which the front reaches a given portion of the plasma; at a distance ρ from the axis, this is given by $\tau_F = \sqrt{3}(R_T - \rho)$ (cf. Sec. II). Clearly, the central region ($\rho \simeq 0$) is most likely to undergo a phase transition before it is reached by the front. Therefore, comparing (4.2) with τ_F at $\rho \simeq 0$, one finds that a phase transition takes place before the front reaches the central region if

$$\frac{T_0}{T_c} \lesssim \left[\frac{3}{\pi} \sigma_g \Lambda^2 \right]^{1/6} . \tag{4.3}$$

For near-central collisions this gives (cf. the Appendix)

$$\left[\frac{T_0}{T_c} \right] \lesssim 1.2 A_1{}^{1/9} , \tag{4.3'}$$

A_1 being the atomic number of the smaller nucleus. I conclude that the contamination coming from phase transition inside the plasma is negligibly small except perhaps for central collisions of very heavy nuclei, such as UU, UPb. In estimating the pair-production rates, one need not worry very much about interior phase transitions; interestingly, collisions such as OPb, FePb, appear to provide a cleaner laboratory for the study of the quark-hadron phase transition than UU, UPb, etc., collisions.

The calculation of the space-time volume is straightforward. It is convenient to compute Ω per unit rapidity. Using the scaling solution to the equations of hydrodynamics, we have in cylindrical coordinates

$$d\Omega = dv\, 2\pi \int dt\, \rho\, d\rho\, dx\, \Theta(\tau-\tau_0)\delta\left[\frac{x}{t}-v\right] \times \Theta(R_T-\sqrt{3}\tau-\rho)\,, \quad (4.4)$$

where $v=\tanh y$ is the flow velocity. On using $t=\tau(1-v^2)^{-1/2}$, this integral is easily evaluated; at a negligible loss of accuracy one can integrate over τ from $\tau=0$ instead of $\tau=\tau_0$. In this way one finds the simple expression

$$d\Omega \approx \sigma_g{}^2 \frac{dy}{4\pi}\,. \quad (4.5)$$

In order to find $\bar{T}$, one has to average both over the cross section of the plasma and over its history. One has

$$\bar{T} = \frac{2T_0}{R_T{}^2}\int_0^{R_T} \rho\, d\rho \frac{1}{\tau_F(\rho)-\tau_0}\int_{\tau_0}^{\tau_F(\rho)} d\tau \left[\frac{\tau}{\tau_0}\right]^{-1/3}, \quad (4.6)$$

with again, $\tau_F(\rho)=\sqrt{3}(R_T-\rho)$. In principle, this is a function of the impact parameter; however, we saw in Sec. II that (with the possible exception of the heaviest nuclei) pair production is expected to be small in peripheral collisions; therefore one can put $R_T \simeq r_0 A_1{}^{1/3}$. Moreover, one finds that $\bar{T}$ is not very sensitive to the precise choice of the value of R_T/τ_0; thus one approximates $R_T/\tau_0 \simeq r_0 \Lambda A^{1/4} \simeq A_1{}^{1/3}$. The integration over τ can be performed analytically; that over ρ was done numerically. The resulting average temperature is a slowly varying function over the periodic table of elements; one finds a variation of $\bar{T}/T_0$ between 0.99 (light nuclei) and 0.85 (heavy nuclei). As a reasonable estimate, one can take $\bar{T} \simeq 0.9T_0$ irrespective of the atomic numbers of the colliding nuclei.

Putting these results together, one gets the following estimates for the pair-production rates:

(a) Assuming phase transition into the quark phase,

$$\left[\frac{dN}{dM^2 dy}\right]_Q \approx 5.8\times 10^{-4}\alpha^2 T_0{}^2\sigma_g{}^2 \left[1+\frac{2m_l{}^2}{M^2}\right]\left[1-\frac{4m_l{}^2}{M^2}\right]^{1/2} F_+\left[\frac{M}{0.9T_0}\right]. \quad (4.7)$$

(b) Assuming no phase transition,

$$\left[\frac{dN}{dM^2 dy}\right]_H \approx 8.6\times 10^{-5}\alpha^2 T_0{}^2\sigma_g{}^2 \left[1+\frac{2m_l{}^2}{M^2}\right]\left[1-\frac{4m_l{}^2}{M^2}\right]^{1/2} |F_\pi(M^2)|^2 \left(1-\frac{4m_\pi{}^2}{M^2}\right]^{1/2} F_-\left[\frac{M}{0.9T_0}\right]. \quad (4.8)$$

In these equations, F_π and $F_\pm$ are given by Eqs. (3.5) and (3.4), respectively; y stands for the total rapidity of the pair (i.e., for the rapidity of the virtual photon). Not surprisingly, the distribution is independent of y: this is a consequence of the fact that the temperature, as measured in the comoving frame of a fluid element, is independent of y (similarity flow). Consequently, the total number of pairs produced is proportional to the length of the rapidity interval available. The averaging over impact parameters can be carried out numerically, using the geometrical formulas given in the Appendix. Only those impact parameters have to be taken into account for which pair production is significant (cf. Table I). Typically, one finds $\langle\sigma_g{}^2\rangle \simeq 7.8 r_0{}^4 A_1{}^{4/3}$, A_1 being, as usual, the atomic number of the smaller colliding nucleus.

V. DISCUSSION

High-energy nuclear collisions are well suited for a study of the phase transition to quark matter. This is mainly due to the fact that one can clearly distinguish a central region in the rapidity plot. In that region the physical properties of matter are relatively simple (small net baryon number, similarity flow); as a consequence, many complicating factors present at lower energies, disappear. If a phase transition indeed takes place, the change in the spectrum of pairs produced should be quite dramatic. The relevant quantity is the ratio of theoretical pair-production rates from the quark and hadron phases $P(M^2,T_0)=dN_Q/dN_H$. This quantity contains information about the invariant-mass region where signatures of a phase transition can be best observed. Table II gives $P(M^2,T_0)$ as a function of M^2 at an initial temperature

TABLE II. Sample values of $P(M^2,T_0)=(dN)_Q/(dN)_H$ as a function of the invariant mass M of lepton pairs. Initial temperature $T_0=200$ MeV; $\Lambda=100$ MeV. Pure quark phase is assumed at $T=T_0$.

M^2/Λ^2	$P(M^2,T_0)$
10	5.85
20	2.27
30	1.45
37	1.00
50	0.40
60	0.25

$T_0=200$ MeV, which is, one hopes, somewhat above the critical temperature (cf. Sec. I). Such temperatures of the central region should be within the reach of the next generation of colliding beam accelerators or perhaps, of suitable designed cosmic-ray experiments. It is worth emphasizing that cosmic-ray experiments may be quite competitive as a laboratory in which a phase transition between the hadron and quark phases of matter can be studied. This is due to two facts: (i) The pair-production rate is a slowly varying function of the primary energy [cf. Eqs. (2.11), (4.7), and (4.8)]; hence, a very accurate determination of the primary energy is not necessary. (ii) The critical temperature of the phase transition is expected to be quite low ($T_c<200$ MeV), thus one does not need extremely energetic nuclei; the luminosity of the primary cosmic radiation in the region of atomic numbers $A\simeq 56$ and $E\lesssim 2$ TeV/nucleon (corresponding to $s^{1/2}<60$ GeV/nucleon in the collision of an incident Fe with an average emulsion nucleus) is substantial.

One of the mildly surprising results of our analysis worth reiterating is that (contrary to widespread folklore) the collisions of very heavy nuclei may not provide the cleanest laboratory for the study of a hadron-quark phase transition: given T_0, $P(M^2,T_0)$ contains too much contamination from a late hadronic phase of the central plasma [cf. Eq. (4.3)]. Presumably, colliding beams of moderately heavy nuclei (O,Ar,Fe) or those incident on heavy targets will provide cleaner results.

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APPENDIX: GEOMETRICAL ASPECTS OF FAST NUCLEAR COLLISIONS

This is an old subject: High-energy collisions of nuclei can be treated classically with small quantum corrections. Authors of various "fireball," "firestreak," etc., models[16] give more-or-less explicit formulas for nuclear overlap volumes, geometrical cross sections, etc. The purpose of this appendix is, partly, to summarize known results, partly to make them as explicit as possible: As long as a classical approximation is valid, the computation of overlap volumes and geometrical cross sections is an elementary, albeit somewhat tedious, exercise in solid geometry.

Consider two nuclei, of radii R_1 and R_2 (for the sake of definiteness, $R_1\leq R_2$) in their rest frames, colliding with a Lorentz factor γ in their overall center-of-mass frame, at impact parameter b. With an obvious choice of a coordinate frame, the nuclear surfaces are given by the equations

$$x^2+\left[y-\frac{b}{2}\right]^2+\gamma^2z^2=R_1^2\ , \tag{A1}$$

$$x^2+\left[y+\frac{b}{2}\right]^2+\gamma^2z^2=R_2^2\ . \tag{A2}$$

Clearly, the plane of smallest intersecting area is given by the equation

$$y_0=\frac{R_2^2-R_1^2}{2b}\ . \tag{A3}$$

Overlap volumes and geometrical cross sections are determined, in essence, by the quantity $\xi=b/y_0$: $\xi>1$ and $\xi<1$ lead to different-looking analytical expressions; however, the resulting formulas are continuous functions of the impact parameter. The method to be used in computing overlap volumes and cross sections is trivial; the main step is to introduce separate spherical polar coordinates for each colliding nucleus and compute the respective portions of overlap volumes and cross-sectional areas of interest. The result of such a calculation is summarized as follows. We define the variables

$$u=\frac{b^2+R_1^2-R_2^2}{2bR_1}\ ,\qquad v=\frac{b^2+R_2^2-R_1^2}{2bR_2}\ . \tag{A4}$$

(1) Geometrical cross section of overlap, σ_g is given by

$$\sigma_g=0 \text{ for } b>R_1+R_2\ ,$$

$$\sigma_g=R_1^2[1-\Theta(u)u^2]\cos^{-1}u+R_2^2(1-v^2)\cos^{-1}v \quad \text{for } R_2-R_1\leq b\leq R_2+R_1\ , \tag{A5}$$

$$\sigma_g=\pi R_1^2 \text{ for } 0\leq b\leq R_2-R_1\ .$$

(2) Overlap volume V is given by

$$V=0 \text{ for } b>R_1+R_2\ ,$$

$$V=\frac{2\pi}{3\gamma}\{R_1^3(1-u)[1-\Theta(u)u^3]+R_2^3(1-v)(1-v^3)\} \quad \text{for } R_2-R_1\leq b\leq R_2+R_1\ , \tag{A6}$$

$$V=\frac{4\pi}{3\gamma}R_1^3 \text{ for } 0\leq b\leq R_2-R_1\ .$$

(3) Collisions of identical nuclei $R_1=R_2=R$. Equations (A5) and (A6) are simplified. One gets the following expressions:

$$\sigma_g=2R^2\left[1-\frac{b^2}{4R^2}\right]\cos^{-1}\frac{b}{2R}\ , \tag{A7}$$

$$V=\frac{4\pi R^3}{3\gamma}\left[1-\frac{b^3}{8R^3}\right]\left[1-\frac{b}{2R}\right]\ . \tag{A8}$$

(4) Analytical interpolation formulas. The following formulas are obtained by comparing Eqs. (A5) and (A6) with (A7) and (A8), respectively. Useful interpolation formulas can be obtained for nuclei of comparable size $(R_2-R_1)\lesssim(R_1+R_2)/2$. We have

$$\sigma_g\approx 2R_1^2\left[1-\left[\frac{b}{R_1+R_2}\right]^2\right]\cos^{-1}\frac{b}{R_1+R_2}\ ,\qquad V\approx\frac{4\pi R_1^3}{3\gamma}\left[1-\left[\frac{b}{R_1+R_2}\right]^3\right]\left[1-\frac{b}{R_1+R_2}\right]\ . \tag{A9}$$

These formulas have the correct limits at $b\to 0$ and $b\to R_1+R_2$; Eqs. (A9) underestimate σ_g and V for central collisions $0<b<R_2-R_1$.

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[4]In what follows, we take $\Lambda \simeq \Lambda_{\overline{MS}}$ ($\overline{MS}$ refers to the modified minimal-subtraction scheme) and use a plausible numerical value, $\Lambda \simeq 100$ MeV.

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[15]In a comoving frame of a given fluid element this means that the probability for producing a virtual photon of energy E is maximal when each annihilating particle carries an energy $E/2$: This is intuitively obvious and it is borne out by Eq. (3.3).

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